APPLICATION OF THE METHOD OF FUNDAMENTAL SOLUTIONS FOR INVERSE PROBLEMS RELATED TO THE DETERMINATION OF ELASTO-PLASTIC PROPERTIES OF PRIZMATIC BAR

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Abstract. The problem of determining the elastoplastic properties of a prismatic bar from the given relation from experiment between torsional moment M_T and angle of twist per unit of rod's length θ is investigated as inverse problem. Proposed method of solution of inverse problem is based on solution of some sequences of direct problem with application of the Levenberg-Marquardt iteration method. In direct problem these properties are known and torsional moment as a function of angle of twist is calculated form solution of some non-linear boundary value problem. For solution of direct problem on each iteration step the method of fundamental solutions and method of particular solutions is used for prismatic cross section of rod. The non-linear torsion problem in plastic region is solved by means of the Picard iteration

1 INTRODUCTION

In the torsion problem of prismatic bars one can distinguish two cases of torsion: uniform and nonuniform torsion. When a bar is subjected to two concentrated torsional moments at its ends while warping of the cross section is not restrained, the angle of twist per unit length remains constant along its axis and the bar is under uniform torsion. The subject of this paper is the elastic-plastic uniform torsion.

The torsion analysis of bars has a long history, and can be traced back to Saint-Venant, who gave a final conclusion to the problem of elastic uniform torsion. The Saint-Venent semi-inverse method is used not only for the elastic torsion but very often for the elastic-plastic torsion analysis (see for example books: [1] chapter 3, [2] chapter 11, [3] chapter 4). The main interest from designer point of view is torsional rigidity. Its can be easily obtained from relations between torsional moment and angle of twist per unit length. If elastoplastic material properties of bar are known this relation is obtained by solution some non-linear boundary value problem. Such problem here is called as direct problem of elastoplastic torsion. Now there is many methods for solution direct problem.

If elastoplastic material properties are not known and are determined from experimentally given discrete values of torsional moment $M_{Ti} = M_T(\theta_i)$ and angle of twist per unit length θ_i we have inverse problem of elasto-plastic torsion. Such inverse problem has received relatively little attention in literature in comparison with direct problem. Mamedov [4-5] considered inverse problem for determining so-called plasticity function in Hencky correlation. Inverse problem was solved by solution of sequence direct problem using finite element method. In paper [6] also plasticity function is identified within the range of J_2-deformation theory. The method used by authors is based on the finite-difference discretization of the non-linear elastoplastic problem, and parametrization of the unknown plasticity curve. Similar considerations are given in paper [7] when author consider power-law material.

In mentioned above papers for solution inverse elastoplastic problem the mesh methods were used (FEM and FDM). In last decades the meshless methods have been become popular in computational mechanics. One of meshless method is method of fundamental solutions (MFS). This method was used with success for solve inverse heat conduction problems. As of right now, the MFS was applied in following inverse heat conduction problems involving the identification of heat sources, e.g. [8], boundary heat flux, e.g. [9], Cauchy problem, e.g. [10], backward heat conduction problem, e.g. [11], Stefan problem [12], or identification of geometry of boundary, e.g. [13]. Mentioned above application of MFS are related with 2-D problems.

The purpose of this paper is the application of the MFS method to the inverse elasticplastic torsion problem in prismatic rod case. There are many different models of isotropic elastoplasticity. Therefore, it is difficult sometimes to make a decision of what model to use for numerical implementation. In this paper we chose the Ramberg-Osgood stress-strain relation [14]. To the best knowledge of the authors, this paper is a first application of this method to the inverse elastoplastic problem.

2 FORMULATION OF THE PROBLEM

The governing equation of elastic torsion of a prismatic bar has form:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -2 \cdot \theta \cdot G \text{ for } (x, y) \in \Omega_E , \tag{1}$$

where $\psi(x, y)$ is a Prandtl stress function, θ is a angle of twist per unit length, G the shift modulus.
The torsion moment is express as:

$$M_T = 2\iint \psi \, dxdy \tag{2}$$

and magnitude of the resultant shear stress as:

$$\sqrt{\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2} = \left| grad\, \psi \right| \tag{3}$$

Since the lines of shear stress at each point of the boundary of the section must be directed along the tangent to the boundary, the lateral surface of the bar being stress free, the boundary curve Γ must be a line of constant stress function. For simply connected cross section, we may take

$$\psi = 0 \ \text{ for } (x, y) \in \Gamma . \tag{4}$$

According the Saint-Venant torsion theory only stress τ_{xz}, τ_{yz} are not equal zero:

$$\tau_{xz} = \frac{\partial \psi}{\partial y}, \ \tau_{yz} = -\frac{\partial \psi}{\partial x} . \tag{5}$$

In the elastic region the torsional rigidity is constant and there is linear relation between torsion moment (2) and angle of twist per unit length θ.

3.1 Plastic torsion

For elastoplastic torsion there is few different models of plastic behavior. Generalized form of the Ramberg-Osgood stress-strain law has been used in deformation theory of Nadai:

$$\varepsilon = \frac{\sigma}{E}\left[1 + \alpha\left(\frac{\sigma}{\sigma_y}\right)^{n-1}\right] \tag{6}$$

where σ_y is yield stress, $\alpha = 0.02$ (for example).
If all of the stress components are normalized by the yield stress σ_y and strains are normalized with respect to the corresponding tensile yield strain $\varepsilon_y = \frac{\sigma_y}{E}$ then

$$\bar{\varepsilon} = \bar{\sigma} + \alpha \bar{\sigma}^n \tag{7}$$

where $\bar{\varepsilon} = \varepsilon / \varepsilon_y$, $\bar{\sigma} = \sigma / \sigma_y$.
The invariant can be introduced in the form of the effective stress σ_e:

$$\bar{\sigma}_e^2 = \frac{3}{2} \bar{S}_{ij} \bar{S}_{ij} . \tag{8}$$

where $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$ is the second invariant of the stress deviator, $\frac{1}{3}\sigma_{kk}$ is hydrostatic component of stress.
In simple tension Hencky-Misses criterion reduces to $\bar{\sigma}_e = 1$

$$\varepsilon_{ij} = (1 + \nu)\bar{S}_{ij} + \frac{1 - 2\nu}{3}\bar{\sigma}_{kk}\delta_{ij} + \frac{3}{2}\bar{\sigma}_e^{n-1}\bar{S}_{ij} \tag{9}$$

Since only two components of stress, namely $\tau_{13} = \tau_{xz}$ and $\tau_{23} = \tau_{yz}$ have been assumed to be non-zero (5), that $\sigma_{kk} = 0$, $\varepsilon_{ij} = (1 + \nu)\bar{S}_{ij} + \frac{3}{2}\bar{\sigma}_e^{n-1}\bar{S}_{ij}$ and $\sigma_e = \sqrt{3\left(\tau_{xz}^2 + \tau_{yz}^2\right)}$.

Assuming that $\tau_{xz} = \frac{\partial \psi}{\partial y}$, $\tau_{yz} = -\frac{\partial \psi}{\partial y}$:

$$\bar{\sigma}_e = \frac{\sqrt{3}}{\sigma_y}\left(\frac{\partial \psi}{\partial x}^2 + \frac{\partial \psi}{\partial y}^2\right)^{1/2} \tag{10}$$

The generalized stress-strain relation can be written in form:

$$\varepsilon_{xz} = \left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial \psi}{\partial y}\bigg/\sigma y \tag{11}$$

$$\varepsilon_{yz} = -\left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial \psi}{\partial x}\bigg/\sigma y$$

By expressing the strain component in terms of the displacement and its derivatives the stress-strain relations have form:

$$\frac{1}{2}\left(\frac{\partial w}{\partial x} - \theta \cdot y\right)E = \left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial \psi}{\partial y} \tag{12}$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial y} - \theta \cdot x\right)E = -\left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial \psi}{\partial x}$$

Differentiating first equation (12) by *y* and second by *x:*

$$\frac{1}{2}\left(\frac{\partial^2 w}{\partial x \partial y} - \theta\right)E = \left\{\frac{3}{2}(n-1)\alpha(\bar{\sigma}_e)^{n-2}\frac{\partial \bar{\sigma}_e}{\partial y}\right\}\frac{\partial \psi}{\partial y} + \left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial^2 \psi}{\partial y^2} \tag{13}$$

$$\frac{1}{2}\left(\frac{\partial^2 w}{\partial x \partial y} - \theta\right)E = -\left\{\frac{3}{2}(n-1)\alpha(\bar{\sigma}_e)^{n-2}\frac{\partial \bar{\sigma}_e}{\partial x}\right\}\frac{\partial \psi}{\partial x} - \left\{(1+\nu) + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\right\}\frac{\partial^2 \psi}{\partial x^2}$$

and subtracting and after simplified we obtain the differential equation which is supposed governed torsion loading in the plastic region:

$$-\theta \cdot E = \left\{(1+\nu) + \frac{3}{2}(n-1)\alpha(\bar{\sigma}_e)^{n-2}\right\}\left(\frac{\partial}{\partial x}\left(\sigma_e \frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\sigma_e \frac{\partial \psi}{\partial y}\right)\right) + \left\{(1+\nu) + \frac{3}{2}(2-n)\alpha(\bar{\sigma}_e)^{n-1}\right\}\nabla^2 \psi \tag{14}$$

For the dimensionless variable: $\Psi = \frac{\psi}{L\sigma_y}$, $X = \frac{x}{L}$, $Y = \frac{y}{L}$, $\tilde{G} = \frac{G}{\sigma_y}$, $\tilde{\theta} = \theta \cdot L$ we can written the stress components as:

$$\tau_{xz} = \frac{\partial \Psi}{\partial Y}, \; \tau_{yz} = -\frac{\partial \Psi}{\partial X} \tag{15}$$

and therefore eq. (7) has form

$$\sigma_e = \sqrt{3}\left(\frac{\partial \Psi}{\partial X}^2 + \frac{\partial \Psi}{\partial Y}^2 \right)^{1/2} \tag{16}$$

and the plastic condition:

$$\frac{\partial \Psi}{\partial X}^2 + \frac{\partial \Psi}{\partial Y}^2 = \frac{1}{3} \tag{17}$$

For elastic region the governing equation (1) has form:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -2\tilde{\theta}\tilde{G} \text{ for } (X,Y) \in \Omega_E , \tag{18}$$

and for plastic region the governing equation (14) has form:

$$\left\{ 1 + 3^{\frac{n-1}{2}} \kappa \cdot \left[\left(\frac{\partial \Psi}{\partial X} \right)^2 + \left(\frac{\partial \Psi}{\partial Y} \right)^2 \right]^{\frac{n-1}{2}} \right\} \nabla^2 \Psi + \tag{19}$$

$$+ 3^{\frac{n+1}{2}} \kappa (n-1) \left[\left(\frac{\partial \Psi}{\partial X} \right)^2 + \left(\frac{\partial \Psi}{\partial Y} \right)^2 \right]^{\frac{n-3}{2}} \left[\left(\frac{\partial \Psi}{\partial X} \right)^2 \frac{\partial^2 \Psi}{\partial X^2} + 2 \frac{\partial^2 \Psi}{\partial X \partial Y} \frac{\partial \Psi}{\partial X} \frac{\partial \Psi}{\partial Y} + \left(\frac{\partial \Psi}{\partial Y} \right)^2 \frac{\partial^2 \Psi}{\partial Y^2} \right] = -2\tilde{\theta} \cdot \tilde{G}$$

$$\text{for } (X,Y) \in \Omega_P ,$$

where: $\kappa = \dfrac{\alpha}{2(1+\nu)}$.

The Torsion moment (2) for dimensionless variable is express by:

$$\tilde{M}_T = 2\iint \Psi \, dX dY \tag{20}$$

where $\tilde{M}_T = M_T \big/ \left(L^3 \sigma_y \right)$.

Direct elastoplastic problem depend on solution Eq. (19) with boundary condition (4) for prismatic cross section of rod. In such case the non-dimensional angle of twist $\tilde{\theta}$ and the non-dimensional material parameters κ and n are known. After determination stress function Ψ the torsional moment $\tilde{M}_T$ is calculated. As was mentioned above purpose of this paper is application of MFS for solution of inverse elastoplastic problem. Proposed method is based on Leveberg-Marquadt iteration what requires solution of direct problem at each iteration steps.

4 APPLICATION OF THE METHOD OF FUNDAMENTAL SOLUTIONS FOR SOLUTION OF DIRECT AND INVERSE PROBLEM

In the direct problem the non-dimensional angle of twist $\tilde{\theta}$ and the non-dimensional material parameters κ and n are known. Then problem is in solution non-linear differential equations (19) with boundary condition (4).

4.1 Algorithm 1 – direct problem for prismatic cross section of rod

Step 1 Choose initial values for the parameters $\kappa = 0$ and $n = 0$
Take $j = 0$ and solve simple problem by use the Method of Fundamental Solutions

$$\nabla^2\Psi_j = -2\widetilde{\theta}\cdot\widetilde{G}\,,\ (X,Y)\in\Omega$$

$$\Psi_j(X,Y)=0\,,\ (X,Y)\in\Gamma_1$$

$$\frac{\partial\Psi_j(X,Y)}{\partial n}=0\,,\ (X,Y)\in\Gamma_2$$

$$\Psi_j(X,Y)=\sum_{i=1}^{N}c_i\ln\left(\sqrt{(X-X_i)^2+(Y-Y_i)^2}\right)-\frac{1}{2}\left(X^2+Y^2\right) \tag{21}$$

Remark: In numerical experiment the cross section of bars can have axis of symmetry. In such case it is convenient consider some repeated element of cross section. On axis of symmetry Γ_2 in repeated element one have boundary condition with normal derivative and other part of boundary Γ_1 Dirichlet boundary conditions (Fig. 1 a).

Calculate: $W_j(X,Y)=\left(\frac{\partial\Psi_j}{\partial X}\right)^2+\left(\frac{\partial\Psi_j}{\partial Y}\right)^2$

Step 2 For known κ, n approximate the right hand side function

if $W_j(X,Y)\geq\frac{1}{3}$ then

$$f(X,Y)=\frac{-2\widetilde{\theta}\cdot\widetilde{G}-3^{\frac{n-1}{2}}\kappa\cdot(n-1)(W_j)^{\frac{n-3}{2}}\left[\left(\frac{\partial\Psi_j}{\partial X}\right)^2\frac{\partial^2\Psi_j}{\partial X^2}+2\frac{\partial^2\Psi_j}{\partial X\partial Y}\frac{\partial\Psi_j}{\partial X}\frac{\partial\Psi_j}{\partial Y}+\left(\frac{\partial\Psi_j}{\partial Y}\right)^2\frac{\partial^2\Psi_j}{\partial Y^2}\right]}{\left\{1+3^{\frac{n-1}{2}}\kappa\cdot(W_j)^{\frac{n-1}{2}}\right\}}$$

else

$f(X,Y)=-2\widetilde{\theta}\cdot\widetilde{G}$,

by Radial Basis Function and Mononomials:

$$f(X,Y)\cong\sum_{m=1}^{Ni}\alpha_m\varphi(\hat{R}_m)+\sum_{k=1}^{K}\beta_k\widetilde{\varphi}_k(X,Y) \tag{22}$$

where $\hat{R}_m=\sqrt{(X-X_m)^2+(Y-Y_m)^2}$

$$\sum_{m=1}^{M}\alpha_m\varphi(\hat{R}_{lm})+\sum_{k=1}^{K}\beta_k\widetilde{\varphi}_k(X_l,Y_l)=f(X_l,Y_l),\ I=1,..,M$$

$$\sum_{m=1}^{M}\alpha_m\widetilde{\varphi}_k(X_m,Y_m)=0\,,\ k=1,..,K$$

Step 2 Calculate the particular solution:

$$\Psi^{p}{}_{j+1}(X,Y)=\sum_{m=1}^{M}\alpha_m^{(j+1)}\psi(\hat{R}_m)+\sum_{k=1}^{K}\beta_k^{(j+1)}\widetilde{\psi}_k(X,Y) \tag{23}$$

Step 3 Solve homogenous problem

$$\nabla^2 \Psi_{j+1}^h(X,Y)=0\,,\ (X,Y)\in\Omega$$
$$\Psi_{j+1}^h(X,Y)=-\Psi_{j+1}^p(X,Y),\ (X,Y)\in\Gamma_1$$
$$\frac{\partial\Psi_{j+1}^h(X,Y)}{\partial n}=-\frac{\partial\Psi_{j+1}^p(X,Y)}{\partial n}\,,\ (X,Y)\in\Gamma_2$$

by use the Method of Fundamental Solutions.

Step 4 Calculate the solution as a sum of homogenous and particular solution:

$$\Psi_{j+1}(X,Y)=\sum_{i=1}^{N}c_i^{(j+1)}\ln\left(\sqrt{(X-X_i)^2+(Y-Y_i)^2}\right)+\sum_{m=1}^{M}\alpha_m^{(j+1)}\varphi(\hat{R}_m)+\sum_{k=1}^{K}\beta_k^{(j+1)}\varphi_k(X,Y) \tag{24}$$

Step 5 Evaluate $\varepsilon_\Psi=\left\|\Psi_{j+1}-\Psi_j\right\|_2$

If $\varepsilon_\Psi\le tol$ calculate $\widetilde{M}_T(\widetilde{\theta},\kappa,n)=2\iint_{\Omega_e}\Psi_{j+1}dXdY$ and STOP

Else $j=j+1$ and go back to Step 2

In the inverse problem the non-dimensional material parameters κ and n are unknown, but we known the non-dimensional torsional moment as a function of the non-dimensional angle of twist $\widetilde{M}_T=\widetilde{M}_T(\widetilde{\theta})$. For solving this problem for prismatic cross section of rod the Levenberg-Marquardt method [15] can be used according with following algorithm:

4.2 Algorithm 2 – inverse problem for prismatic and circular cross section

Step 1 Identification of the linear ranges of function $\widetilde{M}_T(\widetilde{\theta})$, $\tilde{\theta}_{\min}$

Step 2 For the nonlinear ranges ($\tilde{\theta}_{\min}$ - $\tilde{\theta}_{\max}$) choose initial guess for the fitted parameters $\kappa=\kappa_0$, $n=n_0$, and $\Delta\kappa$, Δn

Step 3 Compute $\varepsilon(\kappa,n)$ according to formula:

$$\frac{\partial\varepsilon(\kappa,n)}{\partial n}=2\sum_{i=1}^{Ne}\left[\widetilde{M}_T(\widetilde{\theta}_i,\kappa,n)-\widetilde{M}_{Ti}\right]\frac{\widetilde{M}_T(\widetilde{\theta}_i,\kappa,n+\Delta n)-\widetilde{M}_T(\widetilde{\theta}_i,\kappa,n-\Delta n)}{2\cdot\Delta n} \tag{25}$$

$$\varepsilon(\kappa,n)=\sum_{i=1}^{Ne}\left[\widetilde{M}_T(\widetilde{\theta}_i,\kappa,n)-\widetilde{M}_{Ti}\right]^2$$

$$\frac{\partial\varepsilon(\kappa,n)}{\partial\kappa}=2\sum_{i=1}^{Ne}\left[\widetilde{M}_T(\widetilde{\theta}_i,\kappa,n)-\widetilde{M}_{Ti}\right]\frac{\widetilde{M}_T(\widetilde{\theta}_i,\kappa+\Delta\kappa,n)-\widetilde{M}_T(\widetilde{\theta}_i,\kappa-\Delta\kappa,n)}{2\cdot\Delta\kappa},$$

Remark: This Step requires solution of **direct problem** (Algorithm 1) $3\cdot N_e$ times.

Step 4 Pick a modest value for λ, say λ=0.001

Step 5 Solve the linear system of equations:

$$\begin{bmatrix}A_{1,1} & A_{1,2}\\ A_{2,1} & A_{2,2}\end{bmatrix}\begin{bmatrix}\delta\kappa\\ \delta n\end{bmatrix}=\begin{bmatrix}b_1\\ b_2\end{bmatrix} \tag{26}$$

where: $b_1=-\frac{\partial\varepsilon}{\partial\kappa}$, $b_2=-\frac{\partial\varepsilon}{\partial n}$, $A_{1,1}=(1+\lambda)\left(\frac{\partial\varepsilon}{\partial\kappa}\right)^2$, $A_{1,2}=A_{2,1}=\frac{\partial\varepsilon}{\partial\kappa}\frac{\partial\varepsilon}{\partial n}$, $A_{2,2}=(1+\lambda)\left(\frac{\partial\varepsilon}{\partial n}\right)^2$

Step 6 Evaluate $\varepsilon(\kappa+\delta\kappa,n+\delta n)$ (solve direct problem - Algorithm 1 N_e times)

Step 7 If $\varepsilon(\kappa+\delta\kappa, n+\delta n) \geq \varepsilon(\kappa, n)$, $\lambda = 10 \cdot \lambda$ and go to Step 5

Step 8 If $\varepsilon(\kappa+\delta\kappa, n+\delta n) < \varepsilon(\kappa, n)$, $\lambda = \lambda/10$, update the trial solution $\kappa = \kappa + \delta\kappa$, $n = n + \delta n$

If $\|[\delta\kappa, \delta n]\|_2 \leq tol$ STOP

Else go back to Step 5

5 NUMERICAL EXAMPLE

The first and second numerical experiment are effect for a rod of square cross section for $\kappa = 0.023076923$ and $n = 2.0$ for first and $n = 3.0$ for second case.

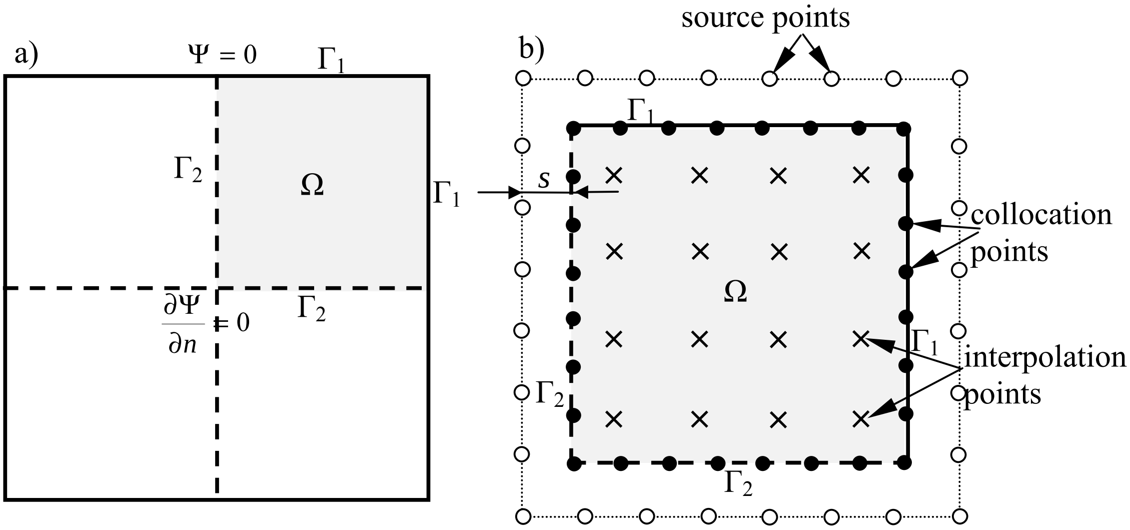

Figure 1: The consider repeated element Ω of cross section of bar a) and distributed collocation, source and interpolation points b).

All numerical experiments for prismatic cross section are carried out for Nc = 176 number of collocation points $(X_c, Y_c) \in (\Gamma_1 \cup \Gamma_2)$, and for Nz = 60 number of source points (Fig. 1 b). Source points are located on the fictitious contour similar to the boundary of the area at a distance s = 0.2 from it (Fig. 1 b). The figure 1b shows the distribution of collocation, source and interpolation points for consider repeated element of cross section of bar. For interpolation of the right hand side of equation (9) as a radial basis function is used the inverted multiquadric function $\varphi(\hat{R}_m) = 1/\sqrt{\hat{R}_m^{\,2} + c^2}$ for shape factor c = 0.1 and L = 6 monomials and is used M = 225 interpolation points $(X_I, Y_I) \in \Omega$.

For given parameters n, κ by using Algorithm 1 the non-dimensional torsional moment $\tilde{M}_T$ as a function of the non-dimensional angle of twist $\tilde{\theta}$ was approximate (Fig. 2). The figure 2 shows the results of the identification $\tilde{M}_T = \tilde{M}_T(\tilde{\theta})$ for κ = 0.023076923, n = {1; 2; 3; 4}. For n = 1 the torsion problem has an elastic character and the calculation results M_T (points) lie close to the linear solution (gray solid line). With the increase of the value of n, the deviation of the nonlinear solutions of linear solutions is growing. For all four examples of discrete results ($\tilde{\theta}$, $\tilde{M}_T$) are approximated using a continuous function $\tilde{M}_T(\tilde{\theta})$ (dashed line in Figure 2).

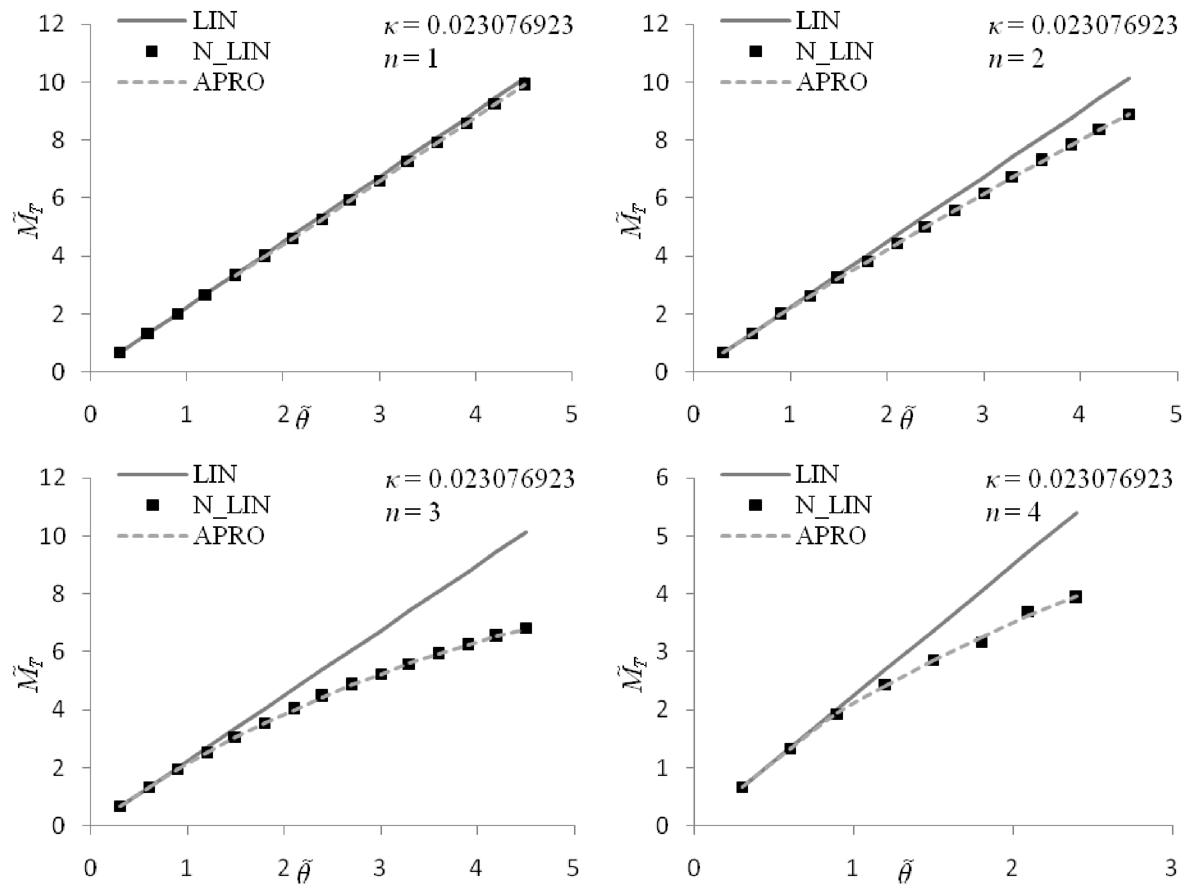

Figure 2: The non-dimensional torsional moment $\tilde{M}_T$ as a function of the non-dimensional angle of twist $\tilde{\theta}$ for square cross section of rod and for two different values of parameters n, κ.

The obtained function $\tilde{M}_T = \tilde{M}_T(\tilde{\theta})$ was used to defined the nonlinear ranges ($\tilde{\theta}_{\min}$, $\tilde{\theta}_{\max}$). For $\tilde{\theta}_{\min} < \tilde{\theta} < \tilde{\theta}_{\max}$ the input data $\{\tilde{M}_{Ti}, \tilde{\theta}_i\}_{i=1}^{Ne}$ to the inverse problem of determination of the non-dimensional material parameters κ and n (Algorithm 2) were generated.
For subsequent values of the coefficients κ and n, in step 3 the value of $\tilde{M}_T(\tilde{\theta}_i, \kappa, n)$ (22) was calculated using Algorithm 1. Derivatives $\frac{\partial \tilde{M}_T(\tilde{\theta}_i, \kappa, n)}{\partial \kappa}$, $\frac{\partial \tilde{M}_T(\tilde{\theta}_i, \kappa, n)}{\partial n}$ have been approximated by using the finite difference for $\Delta\kappa = \kappa_0/200$, $\Delta n = n_0/200$. Identification of material parameters κ and n have been made for $Ne = 10$, for $\tilde{\theta}_i = \tilde{\theta}_{\min} + (i-1)(\tilde{\theta}_{\max} - \tilde{\theta}_{\min})/(Ne-1)$, $i=1,\ldots,Ne$.

The results of the numerical experiment (Table 1) shows that the accuracy and the convergence of the iterative method depends on the initial value of κ_0; n_0 parameters. In one case, an acceptable result was obtained after 11 iterations while in the other for less than 10 iterations. Thus showing that the proposed algorithm is fast converging.

Table 1: The numerical experiment of identification of elastoplastic properties κ, n

$\kappa = 0.023076923$, $n = 2.0$, $\tilde{\theta}_{min}=0.9$, $\tilde{\theta}_{max}=4.5$									
iter	κ	N	$\lVert[\delta\kappa,\delta n]\rVert_2$	κ	N	$\lVert[\delta\kappa,\delta n]\rVert_2$	κ	n	$\lVert[\delta\kappa,\delta n]\rVert_2$
	$K_0 = 0.02$; $n_0 = 2.5$			$\kappa_0 = 0.015$; $n_0 = 3.0$			$\kappa_0 = 0.03$; $n_0 = 3.0$		
0	0.02198	2.05849	2.21E-01	0.02235	1.9772	5.11E-01	0.00241	2.7717	1.15E-01
1	0.0229	2.00407	2.72E-02	0.02294	2.00393	1.34E-02	0.00633	2.5791	9.63E-02
2	0.02278	2.00737	1.65E-03	0.02273	2.0085	2.28E-03	0.01014	2.42563	7.68E-02
3	0.02279	2.00707	1.46E-04	0.02273	2.00851	6.95E-06	0.01339	2.29794	6.39E-02
4	0.02279	2.00705	1.00E-05	**0.022734**	**2.00851**	**3.57E-07**	0.01608	2.20315	4.74E-02
5	**0.02279**	**2.00705**	**6.72E-07**				0.01817	2.13657	3.33E-02
6							0.02184	2.02195	5.73E-02
7							0.0229	2.00368	9.15E-03
8							0.02283	2.00613	1.23E-03
9							0.02284	2.0058	1.68E-04
10							0.02284	2.00579	5.02E-06
11							**0.022839**	**2.00579**	**1.51E-07**
$\kappa = 0.023076923$, $n = 3.0$, $\tilde{\theta}_{min}=0.6$, $\tilde{\theta}_{max}=4.5$									
iter	κ	N	$\lVert[\delta\kappa,\delta n]\rVert_2$	κ	N	$\lVert[\delta\kappa,\delta n]\rVert_2$	κ	n	$\lVert[\delta\kappa,\delta n]\rVert_2$
	$K_0 = 0.02$; $n_0 = 2.5$			$\kappa_0 = 0.03$; $n_0 = 1.5$			$\kappa_0 = 0.03$; $n_0 = 2.0$		
0	0.0342	2.84126	4.21E-01	0.07839	2.63604	5.69E-01	0.0342	2.84126	4.21E-01
1	0.02225	2.976	6.76E-02	0.03224	2.54848	4.95E-02	0.02225	2.976	6.76E-02
2	0.02328	2.99569	9.86E-03	0.02536	2.90776	1.80E-01	0.02328	2.99569	9.86E-03
3	0.02324	2.99758	9.48E-04	0.02326	2.99399	4.31E-02	0.02324	2.99758	9.48E-04
4	**0.023236**	**2.99758**	**7.91E-07**	0.02324	2.99736	1.68E-03	**0.023236**	**2.99758**	**7.91E-07**
5				0.02324	2.9976	1.22E-04			
6				**0.023235**	**2.9976**	**1.30E-11**			

6 CONCLUSIONS

A new inverse method for determining the elastoplastic properties of materials in the torsion problem of prismatic bars is proposed. The algorithm is based on knowledge of some couples of torsional moment and angle of twist $\{\tilde{M}_{T\,i},\tilde{\theta}_i\}_{i=1}^{Ne}$ what permits to obtain non-dimensional material parameters κ and n. In proposed inverse method the Leveberg-Marquadt iteration is used what requires solution direct problem at each iterations. The direct non-linear torsion problem is solved by means of Picard iteration procedure. For prismatic cross section of rod at each iteration step method of fundamental solution and method of particular solution is used. Particular solutions are obtained by means of radial basis function. The propose algorithms are easy o implementation and can be use to complicated geometry because is mesh free. The Leveberg-Marquadt iteration method with MFS is always quickly convergent.

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REFERENCES

[1] Chakrabarty, J. *Theory of Plasticity*. McGraw-Hill Book Company, (1987).
[2] Mendelson, A. *Plasticity: Theory and Application.* MacMillan Company, (1968).
[3] Kliusznikov, W.D. *Mathematical Theory of Plasticity* (in Rusin). Moscow University, (1979).
[4] Mamedov, A. An inverse problem related to the determination of elastoplastic properties of a cylindrical bar. *Int J Nonlin Mech*(1995)**30**:23-32.
[5] Mamedov, A. Determination of elastoplastic properties of a bimetallic and hollow bar. *Int J Nonlin Mech*(1998)**33**:385-392.
[6] Hasanov, A. and Tatar, S. An inversion method for identification of elastoplastic properties of a beam from torsional experiment. *Int J Nonlin Mech*(2010)**45**:562-571.
[7] Hasanov, A. and Tatar, S. Semi-analytic inversion method for determination of elastoplastic properties of power hardening materials from limited torsional experiment. *Inverse Probl Sci En*(2010)**18**:265-278.
[8] Mierzwiczak, M. and Kołodziej, J. A. Application of the method of fundamental solutions and radial basis functions for inverse transient heat source problem. *Comput Phys Commun*(2010)**181**:2035–2043.
[9] Xiong, X.-T., Liu, X.-H., Yan, Y.-M. and Guo, H.-B. A numerical method for identifying heat transfer coefficient. *Appl Math Model*(2010)**34**:1930–1938.
[10] Li, M., Chen, C.S. and Hon, Y.C. A meshless method for solving nonhomogeneous Cauchy problems. *Eng Anal Bound Elem*(2011)**35**:499–506.
[11] Johansson, B.T., Lesnic, D. and Reevea, T. A comparative study on applying the method of fundamental solutions to the backward heat conduction problem. *Math Comput Model*(2011)**54**:403–416.
[12] Johansson, B.T., Lesnic, D. and Reeve T. A method of fundamental solutions for the one-dimensional inverse Stefan problem. *Appl Math Model*(2011)**35**:4367–4378.
[13] Lesnic, D. and Bin-Mohsin, B. Inverse shape and surface heat transfer coefficient identification. *J Comput Appl Math*(2012)**236**:1876–1891.
[14] Huth, J.H. A note on plastic torsion. *J Appl Mech*(1955)**22**:432-434.
[15] Press, W.H., Vetterling, W.T., Teukolsky, S.A. and Flannet, B.P. *Numerical Recipes in Fortran 77*, Second Edition. The art of Scientific Computing, Cambridge University Press (1992).