XII International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS XII
E. Oñate, D.R.J. Owen, D. Peric and B. Suárez(Eds)

APPLICATION OF THE MESHLESS PROCEDURE FOR THE ELASTOPLASTIC TORSION OF PRISMATIC RODS

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Key words: Torsion, Prismatic bar, Method of Fundamental Solutions, Homotopy Analysis Method.

Abstract.

In this paper torsion of prismatic bars considering elastic-plastic material behavior is studied. Based on the Saint-Venant displacement assumption and the Romberg-Osgood model for the stress-strain relation, the boundary value problem for stress function is formulated. In reality an area of cross section of a bar has two regions: elastic with linear governing equation and plastic with non-linear governing equation.

In the solution procedure, the meshless procedure based on the Homotopy Analysis Method HAM connected with the Method of Fundamental Solutions (MFS) and Radial Basis Functions (RBF) is applied. The considered nonlinear partial differential equation (PDE) is transform into a hierarchy of linear inhomogeneous PDEs. The accuracy of the obtained approximate solution is controlled by the number of components of the calculate solution, while the convergence of the process is monitored by an additional parameter of the method. The advantage of the proposed meshless approach is that it does not require the generation of a mesh on the domain or its boundary, but only using a cloud of arbitrary located nodes.

1 INTRODUCTION

The torsion analysis of bars by end couples has a long history, and can be traced back to Saint-Venant, who gave a final conclusion to the problem of elastic uniform torsion. The Saint-Venent semi-inverse method is used not only for the elastic torsion but very often for the elastic-plastic torsion analysis (see for example books: [1] chapter 3, [2] chapter 11, [3] chapter 4). The main interest from designer point of view is torsional rigidity. Its can be easily obtained from relations between torsional moment and angle of twist per unit length. If elasto-plastic material properties of bar are known this relation is obtained by solution some non-linear boundary value problem.

Now there is many methods for solution this non-linear boundary value problem. Nadai [4] was the first to propose a solution for an elastic–plastic pure-torsion problem, and he calculated a fully plastic torque based on his sand heap analogy. In this analogy sand is piled onto a horizontal table having the shape of the cross section of a bar. The slope of the resulting heap cannot exceed the angle of internal friction, which corresponds to the shear yield stress. Sadowsky [5] extended this analogy to sections with holes. Nadai [6] developed an approximate solution for an elastic–plastic torsion by combining the membrane analogy and the sand heap analogy. The analytical solution for the elastic–plastic problem was first proposed by Sokolovsky [7]; he prepared and used independent governing equations for elastic and plastic regions. He also developed a solution for torsion of an oval section of a bar of an elastic/perfectly plastic material using an inverse method. Christopherson [8] obtained a numerical solution for an elastic–plastic problem for an I-section using finite deference method (FDM) and relaxation method. The sand hill analogy for multiply connected cross sections was considered in paper [9]. The analytical solutions of rectangular sections, which have elastic–plastic material property, were developed by Smith and Sidebottom [10] based on the Rayleigh–Ritz expansion and the principle of stationary complementary energy. Hodge [11-13] used non-linear programming for elastic–plastic torsion problem for perfectly plastic material. In Ref. [14] the comparison between FDM and non-linear programming method in solution of elastic–plastic torsion problems was presented. Unloading phenomenon was subsequently confirmed by Stout and Hodge [15] who employ the finite-difference technique based on a stress function. In all mentioned above works the material is assumed to be perfectly plastic (no strain hardening) thus giving rise to the fully plastic state associated with the classical theory of plastic analysis at the limit where the yield regions covers the whole cross-section of the bar. Torsion of bars with material hardening was considered in series of papers by means of finite element method (FEM). Yamada et al. [16] studied elastic–plastic uniform torsion first time by FEM. Baba and Kajita [17] used a two-node, four-degree-of-freedom beam element for the uniform torsion analysis and a four-node, 12-degree-of-freedom rectangular section element for warping analysis of the sections. Bathe and Chaudhary [18] in frame of FEM used a warping displacement function for torsional stiffness representation of elasto-plastic beams. May and Al-Shaarbaf [19] used a standard three-dimensional 20-node isoparametric quadratic brick element in the elastic–plastic analysis of uniform and nonuniform torsion of members subjected to pure and warping torsion. The FDM for the solution of a torsional springback in square bar with non-linear work-hardening material was given in Ref. [20]. The authors used the deformation theory of plasticity with a Ramber–Osgood type stress–strain relationship. The problem of torsional springback was also considered in Refs. [21–23]. Billinghurst et al. [24] used the mitre method to obtain elastic–plastic solutions for various cross sections. In paper [25] solution for torsion of heat treated rod of an elastic/perfectly plastic material using an semi-inverse method. The method of fundamental solutions for elasto-plastic torsion of prismatic rods is presented in paper [26].

As results from above review the problem of elasto-plastic torsion was considered by many authors usualy by means of mesh methods such as FEM or FDM. Exception is paper [26] where first time one of the meshless method was applied for this problem.

The purpose of the present paper is application of In the solution procedure, the meshless procedure based on the Homotopy Analysis Method HAM connected with the Method of Fundamental Solutions (MFS) and Radial Basis Functions (RBF) for elasto-plastic torsion problem. The considered nonlinear partial differential equation (PDE) is transform into a hierarchy of linear inhomogeneous PDEs. The accuracy of the obtained approximate solution is controlled by the number of components of the calculate solution, while the convergence of the process is monitored by an additional parameter of the method. The advantage of the proposed meshless approach is that it does not require the generation of a mesh on the domain or its boundary, but only using a cloud of arbitrary located nodes.

2 FORMULATION OF THE PROBLEM

The governing equation of elastic torsion of a prismatic bar has form:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -2 \cdot \theta \cdot G \text{ for } (x, y) \in \Omega_E , \tag{1}$$

where $\psi(x, y)$ is a Prandtl stress function, θ is a angle of twist per unit length, G the shift modulus.

Since the lines of shear stress at each point of the boundary of the section must be directed along the tangent to the boundary, the lateral surface of the bar being stress free, the boundary curve Γ must be a line of constant stress function. For simply connected cross section, we may take

$$\psi = 0 \text{ for } (x, y) \in \Gamma . \tag{2}$$

According the Saint-Venant torsion theory only stress τ_{xz}, τ_{yz} are not equal zero:

$$\tau_{xz} = \frac{\partial \psi}{\partial y} , \; \tau_{yz} = -\frac{\partial \psi}{\partial x} . \tag{3}$$

For elasto-plastic torsion there is few different models of plastic behavior. Generalized form of the Ramberg-Osgood stress-strain law has been used in deformation theory of Nadai:

$$\varepsilon = \frac{\sigma}{E}\left[1 + \alpha\left(\frac{\sigma}{\sigma_y}\right)^{n-1}\right] \tag{4}$$

where σ_y is the yield stress, $\alpha = 0.02$ (for example).

If all of the stress components are normalized by the yield stress σ_y and strains are normalized with respect to the corresponding tensile yield strain $\varepsilon_y = \dfrac{\sigma_y}{E}$ then

$$\bar{\varepsilon} = \bar{\sigma} + \alpha\bar{\sigma}^n \tag{5}$$

where $\bar{\varepsilon} = \varepsilon / \varepsilon_y$, $\bar{\sigma} = \sigma / \sigma_y$.
For 3-D case we have

$$\varepsilon_{ij} = (1+\nu)\bar{S}_{ij} + \frac{1-2\nu}{3}\sigma_{kk}\delta_{ij} + \frac{3}{2}\sigma_e^{n-1}\bar{S}_{ij} \tag{6}$$

where $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$ is the second invariant of the stress deviator, $\frac{1}{3}\sigma_{kk}$ is the hydrostatic component of stress and the effective stress σ_e in form:

$$\sigma_e^2 = \frac{3}{2}\bar{S}_{ij}\bar{S}_{ij} \,. \tag{7}$$

Since only two components of stress, namely $\tau_{13} = \tau_{xz}$ and $\tau_{23} = \tau_{yz}$ have been assumed to be non-zero (3), that $\sigma_{kk} = 0$, $\varepsilon_{ij} = (1+\nu)\bar{S}_{ij} + \frac{3}{2}\sigma_e^{n-1}\bar{S}_{ij}$ and $\sigma_e = \sqrt{3\left(\tau_{xz}^2 + \tau_{yz}^2\right)}$.
Assuming that $\tau_{xz} = \dfrac{\partial\psi}{\partial y}$, $\tau_{yz} = -\dfrac{\partial\psi}{\partial y}$:

$$\bar{\sigma}_e = \frac{\sqrt{3}}{\sigma_y}\left(\frac{\partial\psi}{\partial x}^2 + \frac{\partial\psi}{\partial y}^2\right)^{1/2} \tag{8}$$

Puting these relation into compatibility eqaution after simplified we obtain the differential equation which is supposed governed torsion loading in the plastic region:

$$-\theta\cdot E = \left\{(1+\nu) + \frac{3}{2}(n-1)\alpha(\bar{\sigma}_e)^{n-2}\right\}\left(\frac{\partial}{\partial x}\left(\bar{\sigma}_e\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\bar{\sigma}_e\frac{\partial\psi}{\partial y}\right)\right) + \left\{(1+\nu) + \frac{3}{2}(2-n)\alpha(\bar{\sigma}_e)^{n-1}\right\}\nabla^2\psi \tag{9}$$

For the dimensionless variable: $\Psi = \dfrac{\psi}{L\sigma_y}$, $X = \dfrac{x}{L}$, $Y = \dfrac{y}{L}$, $\widetilde{G} = \dfrac{G}{\sigma_y}$, $\widetilde{\theta} = \theta\cdot L$ we can written the stress components as:

$$\tau_{xz} = \frac{\partial\Psi}{\partial Y} \,,\ \tau_{yz} = -\frac{\partial\Psi}{\partial X} \tag{10}$$

and therefore eq. (7) has form

$$\sigma_e = \sqrt{3}\left(\frac{\partial \Psi}{\partial X}^2 + \frac{\partial \Psi}{\partial Y}^2\right)^{1/2} \tag{11}$$

and the plastic condition

$$\frac{\partial \Psi}{\partial X}^2 + \frac{\partial \Psi}{\partial Y}^2 = \frac{1}{3}. \tag{12}$$

For elastic region the governing equation (1) has form

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -2\tilde{\theta}\tilde{G} \text{ for } (X,Y) \in \Omega_E, \tag{13}$$

and for plastic region the governing equation (9) has form

$$\left\{1 + 3^{\frac{n+1}{2}} \kappa \cdot \left[\left(\frac{\partial \Psi}{\partial X}\right)^2 + \left(\frac{\partial \Psi}{\partial Y}\right)^2\right]^{\frac{n-1}{2}}\right\} \nabla^2 \Psi + \tag{14}$$

$$+ 3^{\frac{n+1}{2}} \kappa (n-1) \left[\left(\frac{\partial \Psi}{\partial X}\right)^2 + \left(\frac{\partial \Psi}{\partial Y}\right)^2\right]^{\frac{n-3}{2}} \left[\left(\frac{\partial \Psi}{\partial X}\right)^2 \frac{\partial^2 \Psi}{\partial X^2} + 2\frac{\partial^2 \Psi}{\partial X \partial Y}\frac{\partial \Psi}{\partial X}\frac{\partial \Psi}{\partial Y} + \left(\frac{\partial \Psi}{\partial Y}\right)^2 \frac{\partial^2 \Psi}{\partial Y^2}\right] = -2\tilde{\theta} \cdot \tilde{G}$$

$$\text{for } (X,Y) \in \Omega_P,$$

where: $\kappa = \frac{\alpha}{2(1+\nu)}$.

The Torsion moment for dimensionless variable is express by:

$$\tilde{M}_T = 2\iint \Psi \, dXdY \tag{15}$$

where $\tilde{M}_T = M_T / (L^3 \sigma_y)$, $M_T = 2\iint \psi \, dxdy$.

3 APPLICATION OF THE MESHLESS METHODS FOR SOLUTION OF DIRECT AND INVERSE PROBLEM

In numerical experiment the square cross section of bars can have axis of symmetry. In such case it is convenient consider some repeated element of cross section (Fig. 1a). On axis of symmetry Γ_2 in repeated element one have boundary condition with normal derivative and other part of boundary Γ_1 Dirichlet boundary conditions.

Let's, consider the following nonlinear PDE:

$$\mathbf{A}\Psi = f(X,Y) \text{ in } (X,Y) \in \Omega. \tag{16}$$

with the boundary conditions:

$$\Psi = 0 \text{ in } (X,Y) \in \Gamma_1. \tag{17}$$

$$\frac{\partial \Psi}{\partial n}=0 \text{ in } (X,Y)\in \Gamma_2 .$$

where $f(X,Y)=-2\widetilde{\theta}$, $\mathbf{A}\Psi=\nabla^2\Psi$ for $(X,Y)\in\Omega_E$,

$$\mathbf{A}\Psi=\left\{1+3^{\frac{n+1}{2}}\kappa\cdot\left[\left(\frac{\partial\Psi}{\partial X}\right)^2+\left(\frac{\partial\Psi}{\partial Y}\right)^2\right]^{\frac{n-1}{2}}\right\}\nabla^2\Psi+ \qquad \text{for } (X,Y)\in\Omega_P ,$$

$$3^{\frac{n+1}{2}}\kappa\cdot(n-1)\left[\left(\frac{\partial\Psi}{\partial X}\right)^2+\left(\frac{\partial\Psi}{\partial Y}\right)^2\right]^{\frac{n-3}{2}}\left[\left(\frac{\partial\Psi}{\partial X}\right)^2\frac{\partial^2\Psi}{\partial X^2}+2\frac{\partial^2\Psi}{\partial X\partial Y}\frac{\partial\Psi}{\partial X}\frac{\partial\Psi}{\partial Y}+\left(\frac{\partial\Psi}{\partial Y}\right)^2\frac{\partial^2\Psi}{\partial Y^2}\right]$$

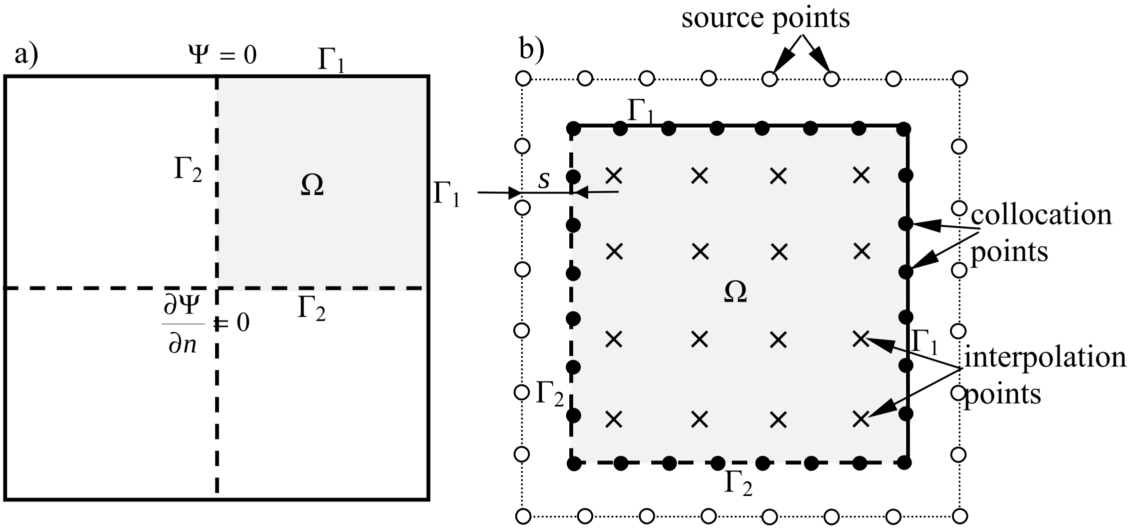


Figure 1: The consider repeated element Ω of cross section of bar a) and distributed collocation, source and interpolation points b).

In the solution procedure, the homotopy analysis method (HAM) is applied to convert the considered nonlinear partial differential equation (PDE) into a hierarchy of linear inhomogeneous PDEs. In order to apply the HAM, it is required to construct a linear problem

$$\nabla^2\widetilde{\Psi}=\nabla^2\Psi_0 \text{ in } (X,Y)\in\Omega . \tag{18}$$

$$\widetilde{\Psi}=\Psi_0 \text{ in } (X,Y)\in\Gamma_1 .$$

$$\frac{\partial\widetilde{\Psi}}{\partial n}=\frac{\partial\Psi_0}{\partial n} \text{ in } (X,Y)\in\Gamma_2 .$$

where Ψ_0 is a preselected zero*th*-order solution.

Then a homotopy is constructed to deform the linear problem (18) into the nonlinear problem (16-17) as follows:

$$(1-\lambda)\left(\nabla^2\widetilde{\Psi}-\nabla^2\Psi_0\right)-\lambda h\left(\mathbf{A}\widetilde{\Psi}-f\right)=0 \text{ in } (X,Y)\in\Omega . \tag{19}$$

$$(1-\lambda)\left(\widetilde{\Psi}-\Psi_0\right)-\lambda h\widetilde{\Psi}=0 \text{ in } (X,Y)\in\Gamma_1 .$$

$$(1-\lambda)\left(\frac{\partial\tilde{\Psi}}{\partial n}-\frac{\partial\Psi_0}{\partial n}\right)-\lambda h\frac{\partial\tilde{\Psi}}{\partial n}=0 \text{ in } (X,Y)\in\Gamma_2.$$

where $\lambda\in[0,1]$ is the homotopy parameter and $h<0$ is auxiliary parameter, to control the convergence. In the application of the HAM, this homotopy deformation is usually assumed to be very smooth such that the solution $\tilde{\Psi}(X,Y,\lambda)$of problem (19) can be expanded by:

$$\tilde{\Psi}(X,Y,\lambda)=\Psi_0(X,Y)+\frac{\lambda}{1!}\Psi_1(X,Y)+\frac{\lambda^2}{2!}\Psi_2(X,Y)+\ldots \tag{20}$$

Problem (19) is reduced to Eq.(18) when $\lambda=0$ and reduced to Eq.(16-17) when $\lambda=1$. This suggests that $\tilde{\Psi}(X,Y,1)$ should be the solution of the original nonlinear PDE system in Eq.(16-17) as follows:

$$\Psi(x,y)=\tilde{\Psi}(X,Y,1)=\Psi_0(X,Y)+\frac{1}{1!}\Psi_1(X,Y)+\frac{1}{2!}\Psi_2(X,Y)+\ldots \tag{21}$$

Now, we are in apposition to obtain a sequence of linear inhomogeneous PDEs by substituting Eq.(20) into Eq.(19) and collecting the coefficients of the powers of λ to have

$$\nabla^2\Psi_i(X,Y)=R_{i-1}(X,Y) \text{ in } (X,Y)\in\Omega \tag{22}$$

with boundary conditions:

$$\Psi_i(X,Y)\big|_{\Gamma_z'}=\delta_i(0-\Psi_{i-1}), \text{ in } (X,Y)\in\Gamma_1. \tag{23}$$

$$\frac{\partial\Psi_i}{\partial n}(X,Y)\Big|_{\Gamma_z'}=\delta_i\left(0-\frac{\partial\Psi_{i-1}}{\partial n}\right), \text{ in } (X,Y)\in\Gamma_2.$$

where $i=2, 3, \ldots, \delta_1=1, \delta_j=0$ for $i>1$,

$$R_1(X,Y)=h(\mathbf{A}\Psi_0-f)$$

$$R_i(X,Y)=i\left(\nabla^2\Psi_{i-1}+h\frac{\partial^{i-1}\mathbf{A}(\tilde{\Psi}_{i-1})}{\partial\lambda^{i-1}}\Bigg|_{\lambda=0}\right), \; i=2, 3, \ldots$$

If the solution of Ψ_i can be solved successively, the desired solution of Eq.(16-17) can be obtained by using Eq.(21). From Eq. (21), it is clear that the *i*th-order solution is defined as $\Psi_0+\Psi_1/1!+\ldots+\Psi i/i!$

Differential deformation $\Psi_0(X,Y)$ is known from solve the auxiliary problem (elastic torsion problem):

$$\nabla^2\Psi_0(X,Y)=-2\tilde{\theta},\; (X,Y)\in\Omega, \tag{24}$$

$$\Psi_0(X,Y)=0,\; (X,Y)\in\Gamma_1'. \tag{25}$$

$$\frac{\partial\Psi_0(X,Y)}{\partial n}=0,\; (X,Y)\in\Gamma_1'. \tag{26}$$

Approximate solution of problem. (24-26) in the Method of Fundamental Solutions has the following form:

$$\Psi_0=-\frac{\tilde{\theta}}{2}\left(X^2+Y^2\right)+\sum_{n=1}^{N}c_n\ln\left((X-\tilde{X}_n)^2+(Y-\tilde{Y}_n)^2\right), \tag{27}$$

where $\{\tilde{X}_n, \tilde{Y}_n\}_{n=1}^{N}$ are coordinates of source points which are places outside of considered region Ω (see Fig. 1b), and $\{c_n\}_{n=1}^{N}$ are constants which are determined from fulfil the boundary condition (25, 26) in collocation sense.

Subsequent deformations $\Psi_i(X,Y)$ are determined by solving the inhomogenous boundary value problem (16-17) by using the Method of Fundamental Solutions (MFS) and Radial Basis Functions (RBF). The approximation solution has form:

$$\Psi_i = \Psi_i^h + \Psi_i^p. \tag{28}$$

where homogenous solution:

$$\Psi_i^h = \sum_{n=1}^{Ns} c_n \ln\left(\left(X - \tilde{X}_n\right)^2 + \left(Y - \tilde{Y}_n\right)^2\right). \tag{29}$$

and particular solution:

$$\Psi_i^p = \sum_{m}^{Ni} \alpha_m \tilde{\psi}(r_m) + \sum_{k}^{L} \beta_k \tilde{\varphi}_k(X,Y) \tag{30}$$

where $r_m = \sqrt{(X - X_m)^2 + (Y - Y_m)^2}$, $\tilde{\psi}(r_m)$ is particular solution for RBF, $\{\tilde{\varphi}_k(X,Y)\}_{k=1}^{L}$ are particular solutions for monomials.

Constants $\{\alpha_m\}_{m=1}^{Ni}$ and $\{\beta_k\}_{k=1}^{L}$ are determined from interpolation of the right hand side $R_{i-1}(X,Y)$:

$$\begin{aligned} &\sum_{m}^{Ni} \alpha_m \phi(r_{jm}) + \sum_{k}^{L} \beta_k \varphi_k(X_j, Y_j) = R_{i-1}(X_j, Y_j) \quad j = 1,\dots,Ni \\ &\sum_{j}^{Ni} \alpha_m \varphi_k(X_j, Y_j) = 0 \quad k = 1,\dots,L \end{aligned} \tag{31}$$

where $r_{jm} = \sqrt{(X_j - X_m)^2 + (Y_j - Y_m)^2}$, $\phi(r_m)$ is Radial Basis Function, $\{\varphi_k(X,Y)\}_{k=1}^{L}$ are monomials.

Constants $\{c_n\}_{n=1}^{Ns}$ are determined from fulfil the boundary condition (23) in collocation sense:

$$\sum_{n=1}^{Ns} c_n \ln\left(\left(Xc_j - \tilde{X}_n\right)^2 + \left(Yc_j - \tilde{Y}_n\right)^2\right)\Bigg|_{\Gamma_z'} = \delta_i(0 - \Psi_{i-1}) - \Psi_i^p, \text{ in } \{Xc_j, Yc_j\}_{j=1}^{Nc1} \in \Gamma_1. \tag{23}$$

$$\sum_{n=1}^{Ns} c_n \frac{\partial\left[\ln\left(\left(Xc_j - \tilde{X}_n\right)^2 + \left(Yc_j - \tilde{Y}_n\right)^2\right)\right]}{\partial n}\Bigg|_{\Gamma_z'} = \delta_i\left(0 - \frac{\partial \Psi_{i-1}}{\partial n}\right) - \frac{\partial \Psi_i^p}{\partial n}, \text{ in } \{Xc_j, Yc_j\}_{j=1}^{Nc2} \in \Gamma_2.$$

where $\{Xc_j, Yc_j\}_{j=1}^{Nc}$ are coordinates of collocation points which are places on the boundary Γ_1 and Γ_2 respectively.

The accuracy of the obtained approximate solution is controlled by the number of components of the Taylor series (25), while the convergence of the process is monitored by *h* parameter of the method.

4 NUMERICAL EXAMPLE

The numerical experiment are effect for a rod of square cross section for κ = 0.023076923 and n = {1, 2, 3}. All numerical experiments are carried out for Nc = 176 number of collocation points $(X_c, Y_c) \in (\Gamma_1 \cup \Gamma_2)$, and for Nz = 60 number of source points. Source points $(\tilde{X}_n, \tilde{Y}_n)$ are located on the fictitious contour similar to the boundary of the area at a distance s = 0.2 from it. For interpolation of the right hand side of equation (22) as a radial basis function is used the inverted multiquadric function $\varphi(\hat{R}_m) = 1 / \sqrt{\hat{R}_m^{\ 2} + c^2}$ for shape factor c = 0.1 and L = 6 monomials and is used M = 225 interpolation points $(X_I, Y_I) \in \Omega$.

For given parameters n, κ the non-dimensional torsion moment $\tilde{M}_T$ as a function of the non-dimensional angle of twist $\tilde{\theta}$ was determined (Fig. 2). The figure 2 shows the results of the identification $\tilde{M}_T = \tilde{M}_T(\tilde{\theta})$ for κ = 0.023076923, n = {1; 2; 3}. For n = 1 the torsion problem has an elastic character and the calculation results $\tilde{M}_T$ (points) lie close to the linear solution (solid line). With the increase of the value of n, the deviation of the nonlinear solutions of linear solutions is growing.

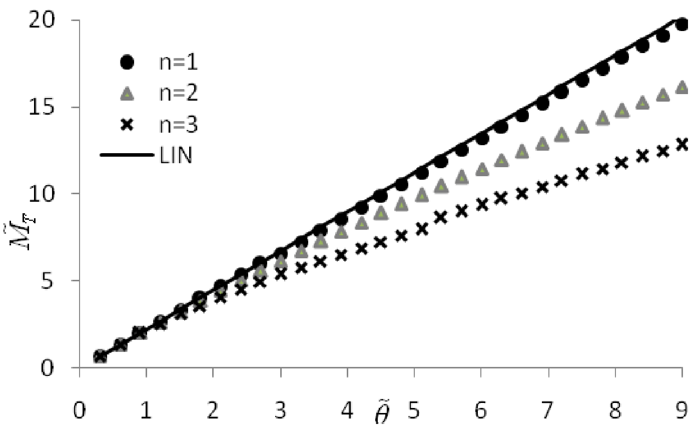
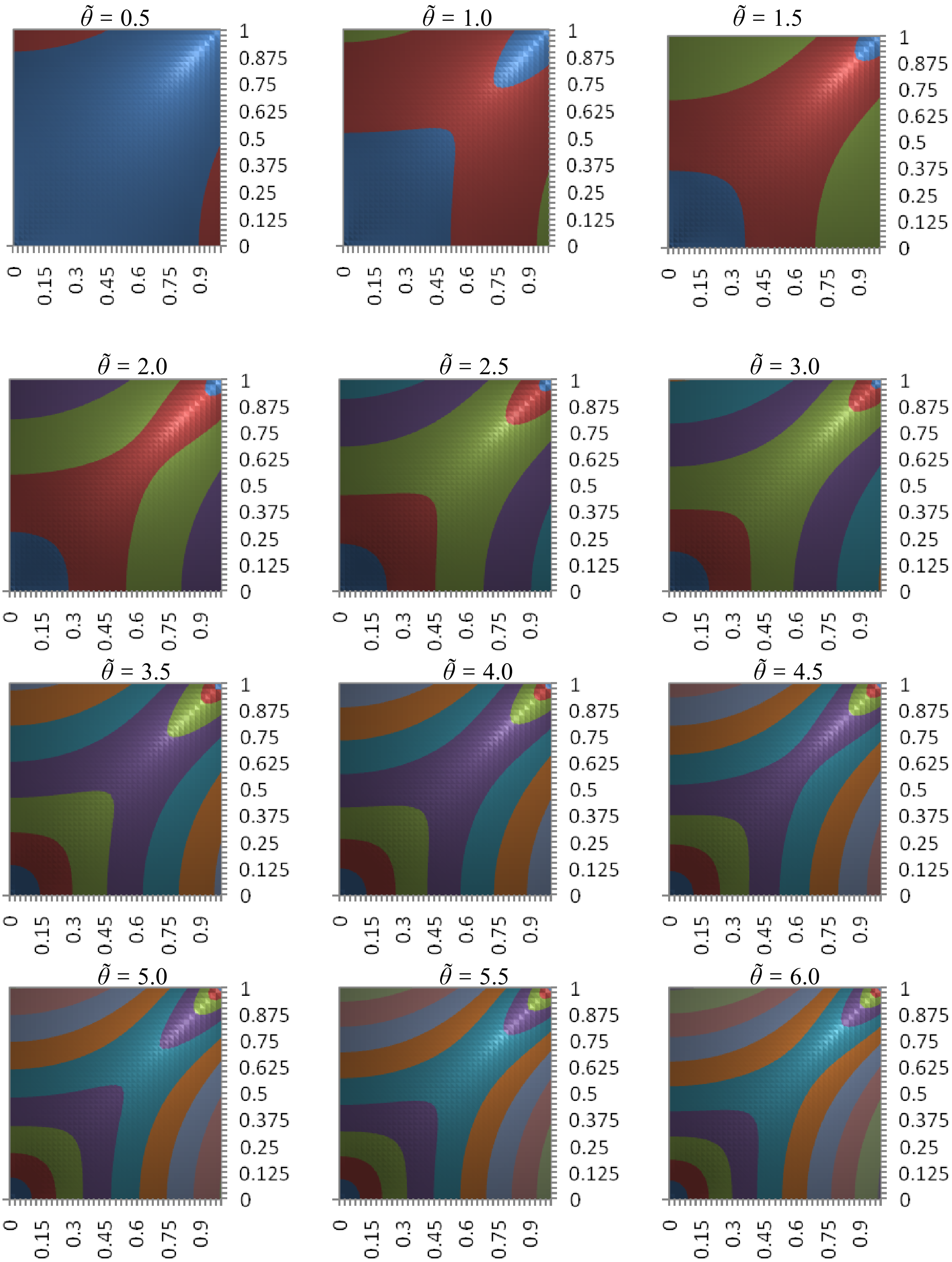


Figure 2: The non-dimensional torsion moment $\tilde{M}_T$ as a function of the non-dimensional angle of twist $\tilde{\theta}$ for square cross section of rod and for two different values of parameters for κ = 0.023076923, n = {1; 2; 3}.

In numerical experiment in order to obtain the convergence for n = 1 the auxiliary parameter was taken h = - 1, for n = 2, h = - 0.5 and for n = 3, h = - 0.05. The figure 3 shows the non-dimensional shear stress σ_e (16) for κ = 0.023076923, n = 3 and for different value of the non-dimensional angle of twist $\tilde{\theta}$ = i 0.5, where i = 1, ..., 15. Already for $\tilde{\theta}$ = 0.5 in considered domain the plastic area appeared. With the increase in dimensional angle of twist $\tilde{\theta}$ from $\tilde{\theta}$ = 1 to $\tilde{\theta}$ = 7.5, the area of plasticity clearly increasing.


$\tilde{\theta}$ = 0.5
$\tilde{\theta}$ = 1.0
$\tilde{\theta}$ = 1.5
$\tilde{\theta}$ = 2.0
$\tilde{\theta}$ = 2.5
$\tilde{\theta}$ = 3.0
$\tilde{\theta}$ = 3.5
$\tilde{\theta}$ = 4.0
$\tilde{\theta}$ = 4.5
$\tilde{\theta}$ = 5.0
$\tilde{\theta}$ = 5.5
$\tilde{\theta}$ = 6.0
0 0.125 0.25 0.375 0.5 0.625 0.75 0.875 1
0 0.15 0.3 0.45 0.6 0.75 0.9

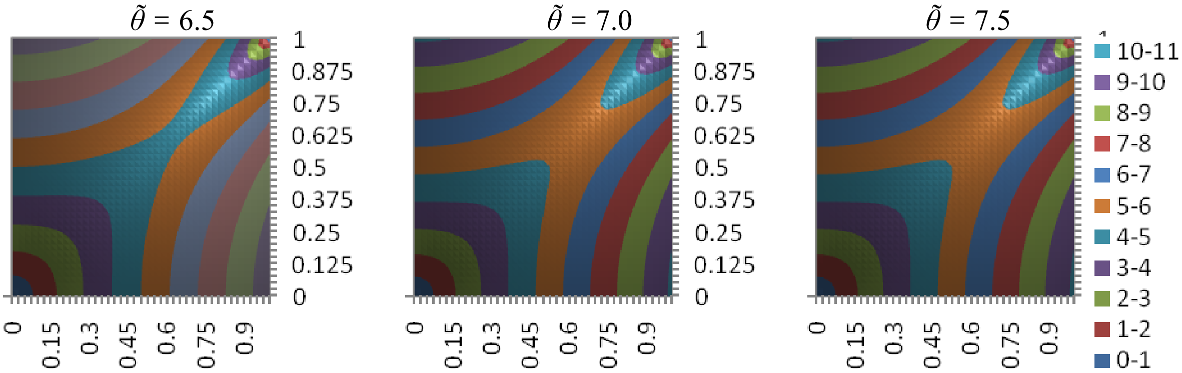


Figure 3: The non-dimensional shear stress σ_e (16) for κ = 0.023076923, n = 3 and for $\tilde{\theta}$ = i 0.5, i=1, .., 15

6 CONCLUSIONS

The non-linear torsion problem is solved by means of homotopy analysis method. For prismatic cross section of rod at each iteration step method of fundamental solution and method of particular solution is used. Particular solutions are obtained by means of radial basis function. The propose algorithm is easy to implementation and is mesh free. The accuracy of the obtained approximate solution is controlled by the number of components of the calculate solution. The convergence of the process is monitored by an additional parameter . For strong linearity (n = 3) in order to obtain the convergence, the small value of the additional parameter h was needed.

Acknowledgements

The work has been supported by 21-418/2013 DS grant.

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