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## Propagation Mechanism of Non-uniform Distribution of Stress and Strain in Tension Test

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**Key Words:** Elastic-plastic FEA, Tension test, Propagation Mechanism, Stress distribution, Strain distribution, Non-uniformity

### Summary.

Elastic-plastic finite element analysis is carried out on tension test to check the uniformity of stress and strain that highly influences the result of numerical analysis in forming processes. Usually stress is calculated by dividing the force by the cross sectional area of the specimen and strain is calculated by the change in a limited gauge length. Basic assumptions are that stress and strain distributions be uniform in the cross section and between two gauge points and the cross section lie between two gauge points. Result of numerical analysis shows that stress and strain distributions are not necessarily uniform especially for those material with poor work hardening ratio. Initiation and propagation of mechanism of this non-uniformity is analyzed to propose a method for a precise measurement of stress and strain.

### 1 INTRODUCTION

Tension test is one of the most common methods for acquiring mechanical properties of materials. However there is a report on the non-uniformity of stress and strain distributions in tension test [1]. Previous work [2] made it qualitatively clear that non-uniformity of stress and strain arises depending upon the intensity of work hardening ratio of the material. Further research work on the initiation and propagation of these non-uniformities is a key to the precise measurement. Initiation and propagation mechanism may highly depend upon the geometrical nonlinearity and material nonlinearity. At the beginning of tension test the specimen undergoes elastic deformation and non-uniform distributions of stress and strain may be laid in the specimen. Clarification of the mechanism by elastic-plastic finite element analysis leads to a proposal of precise measurement of stress and strain.

## 2 ANALYSIS

The geometry of specimen supplied to tension test is given in Fig.1. It is a JIS No-5 sheet specimen. Taking into consideration plane-symmetry only a quarter of the specimen is subjected to elastic-plastic finite element analyses using a code “ELFEN” developed at University of Swansea [3]. It is a plane stress analysis because the specimen is thin. The number of elements is 1616. Deformations on X- and Y-axes are constrained in Y- and X-directions respectively, and uniform displacement is given on each node in the gripping portion on the right.

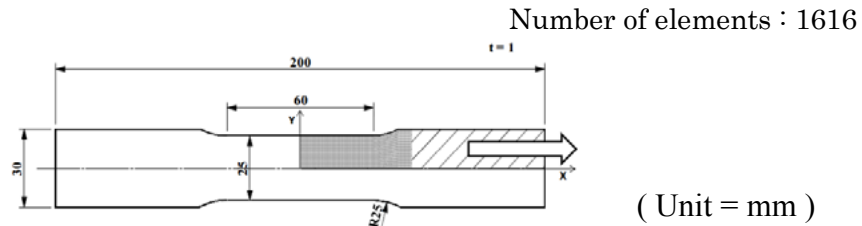


Figure 1: Geometry of sheet specimen for tension test and mesh division

The stress-strain relationship in plastic region is given by equation (1) where  $\sigma$  and  $\epsilon$  are stress and plastic strain and F and n are constants.

$$\sigma = F\epsilon^n \quad (1)$$

Fig.2 shows the stress-strain curve used in the analyses. It represents the feature of typical medium carbon steel at room temperature. The Young’s modulus, the Poisson’s ratio and the initial yield stress are 200GPa, 0.3, 400MPa. For the present case the value of n is 0.17.

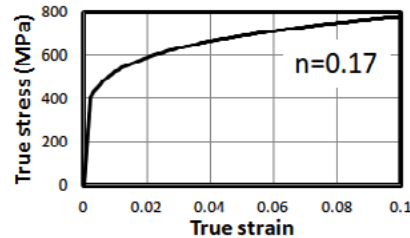


Figure 2: Stress-strain curves for analyses

## 3 Results

Examples of distribution of axial stress and strain are given in Fig.3 for the case when the maximum axial strain on Y-axis is 5%. Stress distribution is almost uniform especially on Y-axis, whereas there is a strong non-uniformity in axial strain distribution.

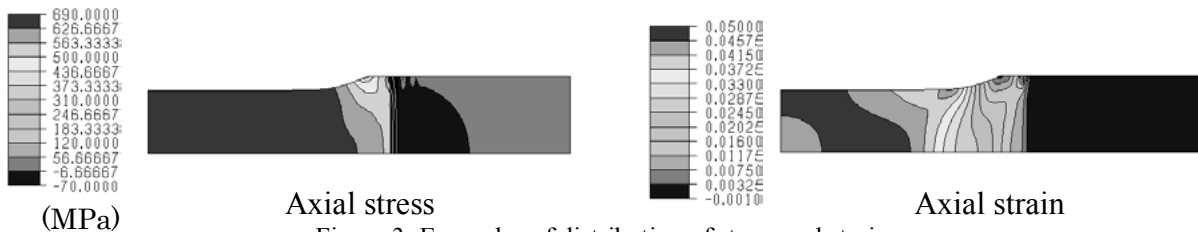


Figure 3: Examples of distribution of stress and strain

Fig.4 shows transition patterns of axial strain from elastic stage to plastic stage. On the elastic stage the range of distributed axial strain is small but it expands after the deformation moves to the plastic stage. Usually the axial strain is measured by an extensometer and the measured strain is an average axial strain between two gauge points. Therefore this result suggests that measured strain does not necessarily represent the strain on the plane where axial stress is measured.

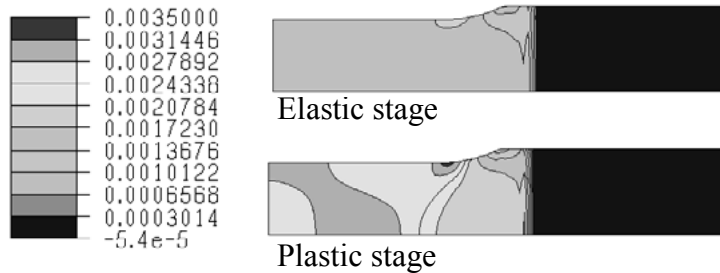


Figure 4: Examples of transition of axial strain from elastic to plastic stages

Stress concentration is induced in the vicinity of shoulder portion due to the specimen geometry, and shear stress and strain increase as the deformation stage moves from elastic to plastic stage as it is shown in Fig.5. The parallel portion tends to contract in width direction uniformly but the gripping portion does not due to the deformation constraint. Shear stress and strain are induced to compensate this difference in width contraction because the specimen is a single body. Shear stress and strain expand from the shoulder portion toward the direction against the centre axis with an angle of approximately 45 degrees.

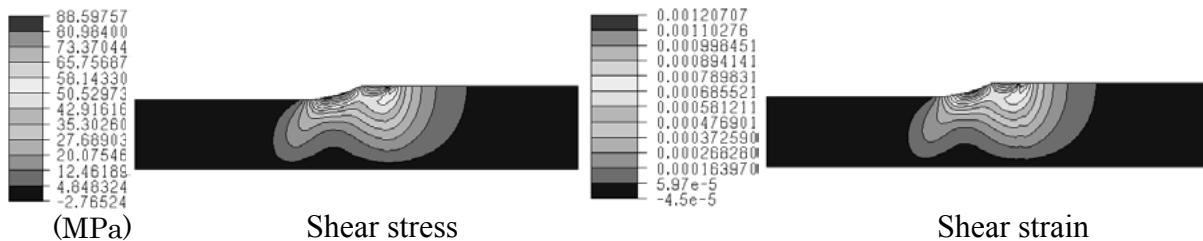


Figure 5: Examples of distributions of shear stress and strain on plastic stage

Axial stress distribution is mostly uniform as it is shown in Fig.6.

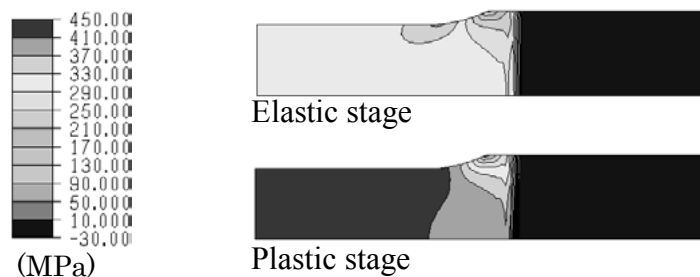


Figure 6: Examples of distributions of axial stress on elastic and plastic stages

Influence of work hardening ratio on the distributions of axial stress and strain is evaluated. A new stress-strain curve with smaller n-value of 0.05 illustrated in Fig. 7 is adopted and the numerical results are compared with those with n-value of 0.17. Fig.8 shows a comparison of two results. Stress distribution on Y-axis is still uniform but the non-uniformity of axial strain distribution on X-axis is larger. This result suggests that for a material with small n-value measured axial strain by an extensometer is less precise.

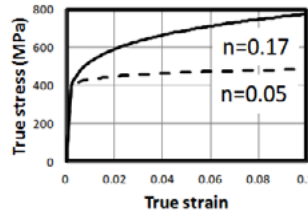


Figure 7: Two stress-strain curves for comparing stress and strain states

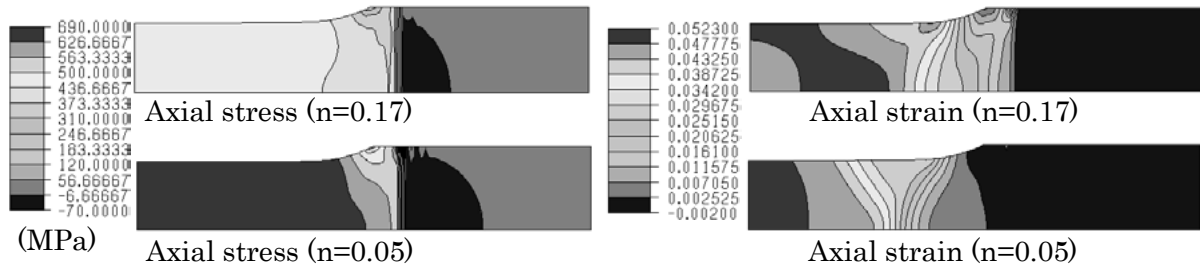


Figure 8: Influence of n-value on distributions of axial stress and strain

#### 4 Validity check

In order to check the validity of the predicted results tension tests are carried out. In order to measure the strain distribution on the sheet specimen image processing technology (IPT)<sup>[4]</sup> is adopted. This technology was developed by associate professor Dr.Yoshinori Yoshida of Gifu University. Because of the restrictions of experimental condition the geometry of specimen is slightly different from that of JIS-5 as it is illustrated in Fig.9. For analyzing the data by image processing technology a lattice work is drawn on the specimen and strain gauges are placed on selected points where change of axial strain is observed through finite element analysis. The material is SNCM439 of which chemical compositions are shown in Table 1. It has a low work-hardening ration after proper thermal treatment. In the present case the value of n is 0.05.

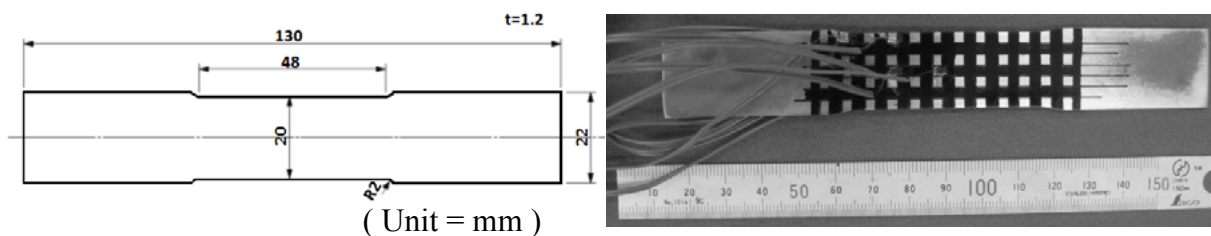


Figure 9: Geometry of sheet specimen and lattice work and placement of strain gauges for validity check

Table 1: Chemical compositions of SNCM439

	C	Si	Mn	P	S	Ni	Cr	Mo
[%]	0.4	0.15	0.7	0.02	0.02	1.8	0.7	0.2

Fig.10 shows the measured stress-strain curve using a pair of strain gauges placed on the centre points on front and rear surfaces of specimen. By using this curve simulation of tension test is carried out on the specimen illustrated in Fig.9. The results are compared with the measured results in Fig.11.

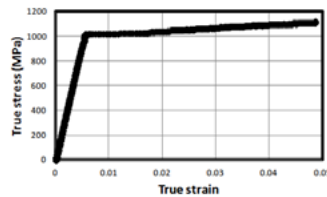


Figure 10: Stress-strain curve of SNCM439 used for validity check of the predicted results by FEA

The distribution pattern of axial strain in Fig.11 is an example when the axial strain on the centre point is 2%. It is clearly indicated in Fig.11a that there is a peak of axial strain in the vicinity of point of X=16mm. The value of axial strain by finite element analysis (FEA) is plotted against X-axis in Fig.11b to compare the measured results by strain gauges (SG) and image processing technology (IPT). The axial stress distribution on the Y-axis remains almost uniform and approximation is satisfactory in calculating axial stress by dividing the tensile force by the cross sectional area.

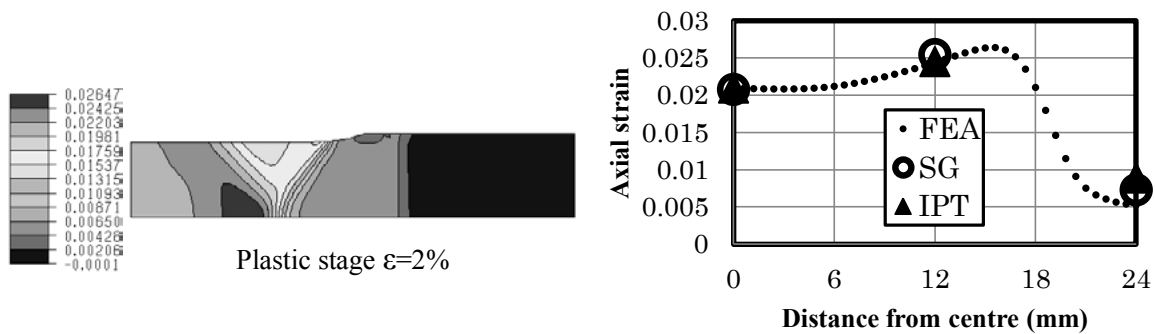


Figure 11a: Example of axial strain distribution

Figure 11b: Comparison of axial strains

Figure 11: Example of distribution pattern of axial strain in thin sheet tension test

As it is shown in an example in Fig.11 axial strain distribution is non-uniform in the parallel portion of the specimen. Usually axial strain is measured by using a device having a limited gauge length such as an extensometer. Therefore the measured strain is an average strain of the material lying between two gauge points and it does not necessarily correspond to the strain of the point or plane where stress is measured. For such a case stress-strain curve does not represent the mechanical behavior of the material, and one must be careful in adopting stress-strain curve in the numerical analysis, especially in adopting stress-strain curve of the material with poor work hardening ratio.

## 5 Discussion

In order to check the applicability of the propagation mechanism of non-uniformity to different specimen geometry tension test is carried out. Fig.12 shows the geometry of specimen and examples of axial strain distribution on elastic and plastic stages. Displacement constraints on X- and Y-axis are the same as those for sheet specimen but displacement is given only on the nodes on the rim of the gripping portion. It is an approximation of the displacement given in X-directions by a screw. The distributions of axial strain in Fig.12 are similar to those of sheet specimen both on elastic and plastic stages. Slight difference is observed on the plastic stage especially in the vicinity of the shoulder portion because of the difference in displacement boundary condition, but the non-uniform distribution of axial strain is similar to that of sheet specimen and a similar propagation mechanism of non-uniformity is applicable to a round specimen.

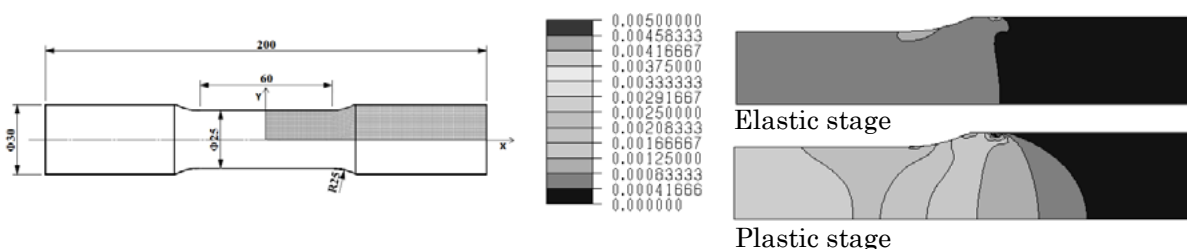


Figure 12: Geometry of round specimen for tension test and examples of distributions of axial strain

## 6 CONCLUSIONS

Elastic-plastic finite element analyses were carried out to examine the distribution of stress and strain in tension test. The result showed that non-uniformity of stress and strain distributions expands from the shoulder portion toward the parallel portion on the plastic stage. The cause of non-uniform distribution of strain is the discrepancy in width contraction between the parallel portion and the zone near the gripping portion, i.e. the existence of dead metal in the vicinity of the gripping portion and the geometrical nonlinearity of the specimen generate this discrepancy. This phenomenon does not highly depend on specimen geometry, but it depends on the work hardening ratio of the material; the poorer the work hardening ratio is, the higher the non-uniformity is. In order to measure a strain precisely, it is recommended to use a device having a short gauge length.

## REFERENCESIONS

- [1] Goh Katsuki, Masayoshi Akiyama, Precision of measured strain in simple tension test, ISOPE, (2009):756-763
- [2] Goh Katsuki, Masayoshi Akiyama, Precision of stress-strain curve obtainable by tension test, COMPLAS-X, (2009):451
- [3] <http://www.rockfield.co.uk/>
- [4] Y. Yoshida, N. Yukawa, T. Ishikawa, Notched Plate Tensile Test Using Image Analysis and Its Application for Deformation Analysis of Shearing, Steel Research International, (2011): 586-592

## UNLOADING AND RE-LOADING FEATURES OF PRE-STRAINED STEEL AT LOW TEMPERATURE

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**Key words:** Elastic response, Plastic pre-strain, Unloading, Re-loading, Inverse loading, Spring back.

**Summary** *In the present work the response of medium carbon steel is investigated experimentally and information on the features of initial loading, unloading, re-loading and inverse loading processes is collected. The experiment is a simple tension and compression test using a round specimen and stress-strain curves are drawn after giving strain histories to specimens. The value of Young's modulus is measured from the stress-strain curve to evaluate the influence of pre-strain. Taking the result into consideration the intensity of predicted spring back of a thin sheet after bending process is evaluated.*

### 1 INTRODUCTION

Numerical analyses have been used as powerful assisting tools in many industries. Elastic-plastic finite element analysis is one of those tools. However it is said that there is often a discrepancy between the predicted and real product geometries. One of the reasons is the modeling of the material behaviour especially the elastic response after loading. Usually metallic materials are deemed a linear elastic bodies and the elastic constants are constant regardless of the plastic strain history. Namely the Young's modulus and Poisson's ratio remain constant after the materials undergo plastic strain history. However there are some research works in which it is stated that Young's modulus considerably decreases after small plastic deformation <sup>[1],[2]</sup>. Further research work to be carried out from now on is collecting quantitative data on the intensity of the decrease in Young's modulus because there is a discrepancy among the data in previous works. In addition the responses of unloading, re-loading and inverse loading processes must be separately collected for the modeling of material behaviour that more precisely predicts the geometry of products after processing processes. In the present paper consecutive tension and compression tests are carried out to grasp the basic responses of medium carbon steel, and a method is proposed to predict the final geometry of product taking an example of elastic-plastic bending process.

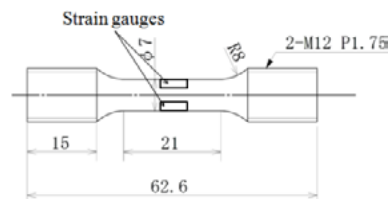
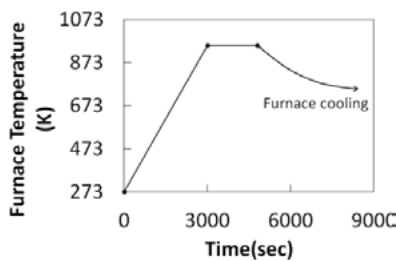
## 2 EXPERIMENT

The material adopted for the tension test is a medium carbon steel of which chemical compositions are shown in Table 1.

**Table 1:** Chemical compositions of material

C	Si	Mn	P	S
0.42~0.48	0.15~0.35	0.60~0.90	<0.030	<0.035

Fig.1 shows the outline of experiment. Parent steel bar was subjected to annealing process to follow the thermal history in Fig.1 and round specimens were manufactured by turning. Two strain gauges were placed at the centre prior to tension and compression tests. The tests were carried out in a framework to ensure a concentric testing of a specimen. After initial loading cycles of unloading and re-loading were repeated in a pitch of about 1% plastic strain until inverse loading was given to the specimen. The average value of the signals of two strain gauges was calculated to draw a stress-strain curve.



**Figure 1a:** Thermal history give to parent bar

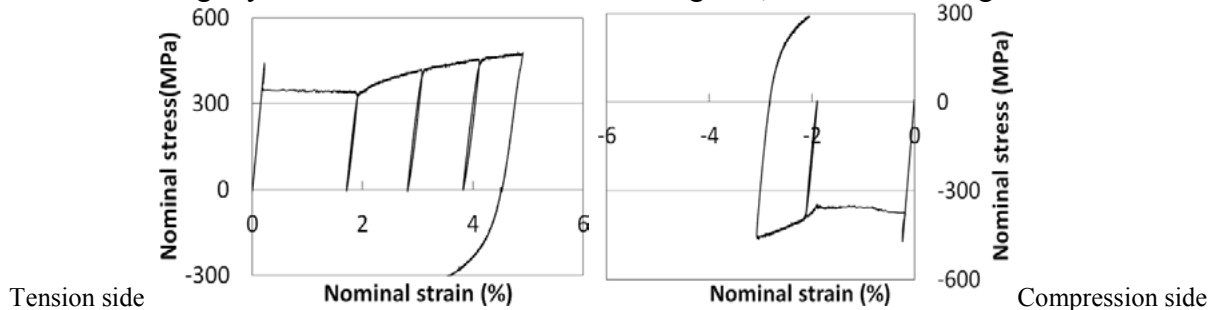
**Figure 1b:** Specimen geometry

**Figure 1c:** View of testing

**Figure 1:** Experimental procedure

## 3 RESULTS

Examples of cycles of nominal stress and nominal strain history on tension side and compression side are shown in Fig.2. The unloading line and inverse loading line are not linear because of the influence of the Bauschinger's effect. The tangent of re-loading line seems to be slightly smaller than that of initial loading line, i.e. initial Young's modulus.



**Figure 2:** Examples of nominal stress and strain history on tension and compression sides