XV International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS 2019
E. Oñate, D.R.J. Owen, D. Peric, M. Chiumenti & Eduardo de Souza Neto (Eds)

THE BREAKAGE PREDICTION FOR HYDROMECHANICAL DEEP DRAWING BASED ON LOCAL BIFURCATION THEORY

H. LU*, W. HUANG, H. B. YU*** AND S. J. QIN†**

* Key Laboratory of Advanced Forging and Stamping Technology and Science, Ministry of Education
Yanshan University
066004 Qinhuangdao, China
e-mail: honglu@ysu.edu.cn

** National Engineering Research Center for Equipment and Technology of Cold Strip Rolling (NECSR)
Yanshan University
066004 Qinhuangdao, China
e-mail: huangwen@ysu.edu.cn

***Tianjin Heavy Industry Co. Ltd.
First Heavy Machinery Group Co.
300457 Tianjin, China

† Key Laboratory of Advanced Forging and Stamping Technology and Science, Ministry of Education
Yanshan University
066004 Qinhuangdao, China
e-mail: plastic@ysu.edu.cn

Key words: Forming Limit, Bifurcation Theory, Localized Necking Criterion, Characteristic Curve Theory, FE Simulation, Hydromechanical Deep Drawing

Abstract. A criterion of sheet metal localized necking under plane stress was established based on the bifurcation theory and the characteristics theory of differential equation. In order to be capable to incorporate the directional dependence of the plastic strain rate on stress rate, Ito-Goya's constitutive equation which gave a one to one relationship between stress rate component and plastic strain rate component was employed. The hydromechanical deep drawing process of a cylindrical cup part was simulated using the commercial software ABAQUS IMPLICIT. The onset of breakage of the part during the forming process was predicted by combining the simulation results with the local necking criterion. The proposed method is applied to the hydro-mechanical deep drawing process for A2219 aluminum alloy sheet metal to predict the breakage of the cylindrical cup part. The proposed method can be applied to the prediction of breakage in the forming of the automotive bodies.

1 INTRODUCTION

Due to weight reduction, the application of aluminum alloy sheet metal in automobile industry and aerospace field has been increased. As aluminum alloy is low in r value, breakage defect is easy to occur in the forming process. Hydromechanical deep drawing(HDD) process is paid attention as a process to raise its formability in industry. In HDD, the die design, material properties, friction condition, and process parameters such as pre-bulging pressure, forming pressure, and loading path, and etc. have great influence on the forming results. In order to obtain the adapted process condition, it is needed to predict breakage defect accurately.

Breakage is a phenomenon of material seperation and can't be treated using the continuum mechanics. Therefore, the precursors of sheet breakage i.e. local necking and diffusion are taken as the forming limit. The representative work includes Hill's local necking theory[1], Swift's diffusion necking theory[2], the theory proposed by Marciniak and Kuczynski[3] in which the limiting strain of the sheet metal was analyzed under biaxial stretching, and the theory proposed by Stören and Rice[4] which considers the location of the deformation mode.

Although by using some commercial software, forming limit diagram(FLD) can be given, it is produced under proportional loading condition. However, in industry, the loading condition is complicated. In this study, based on the Hill's bifurcation theory[5] and the characteristics theory of differential equation, a criterion of localized necking of sheet metal under plane stress was given. Ito-Goya's constitutive equation, which gave a one to one relationship between stress rate component and plastic strain rate component was used. By combining the simulation results of HDD with the criterion, the occurrence and location of breakage are predicted.

2 CONDITION OF LOCALIZED NECKING

Localized necking which is taken as the precursor of sheet metal breakage, is treated as bifurcation from basic deformation mode to local deformation mode in plastic mechanics and mathematics. Based on Triantafyllidis's analaysis method, bifurcation is considered by general boundary value problem in incremental form. Due to sheet metal forming endures large deformation, the updated Lagrange formulation is used. The incremental equilibrium equation using nominal stress can be expressed by

$$\nabla \cdot \dot{\mathbf{\Pi}} = 0 \quad in \quad V \tag{1}$$

Boundary conditions are expressed by

$$\begin{aligned} \dot{\mathbf{\Pi}} \cdot \boldsymbol{n} &= \dot{\boldsymbol{f}} \quad on \quad S_F \\ \boldsymbol{v} &= \boldsymbol{V} \quad on \quad S_V \end{aligned} \tag{2}$$

where V denotes the region of body in initial configuration, S_F the surface on which the rate of traction force $\boldsymbol{f}$ is prescribed, S_V the surface on which the velocity $\boldsymbol{v}$ is prescribed, $\boldsymbol{n}$ the unit normal to the surface S_F, and $\dot{\mathbf{\Pi}}$ denotes nominal stress rate, which is related to Cauthy stress as shown in Eq. (3).

$$\begin{aligned} \dot{\boldsymbol{\Pi}} &= \overset{\triangledown}{\boldsymbol{\sigma}} + \boldsymbol{w} \cdot \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \boldsymbol{w} - \boldsymbol{L} \cdot \boldsymbol{\sigma} \\ &= \boldsymbol{A} : \boldsymbol{L} \end{aligned} \tag{3}$$

where $\boldsymbol{\sigma}$ denotes Cauchy stress, $\overset{\triangledown}{\boldsymbol{\sigma}}$ the Jaumann rate of the Cauchy stress, $\boldsymbol{L}$ the velocity gradient tensor, $\boldsymbol{w}$ the spin tensor. The components of $\boldsymbol{A}$ are given as Eq. (4):

$$A_{ijkl} = D_{ijkl} + \frac{1}{2}\left(\sigma_{il}\delta_{jk} - \sigma_{jl}\delta_{ik} - \sigma_{ik}\delta_{jl} - \sigma_{jk}\delta_{il}\right) + \sigma_{ij}\delta_{kl} \tag{4}$$

Substituting Eq.(3) into Eq.(1) and (2), the incremental boundary value problem of elasto-plastic material can be written as

$$\begin{aligned} \nabla \cdot (\boldsymbol{A} : \boldsymbol{L}) &= 0 \quad in \quad V \\ (\boldsymbol{A} : \boldsymbol{L}) \cdot \boldsymbol{n} &= \dot{\boldsymbol{f}} \quad on \quad S_F \\ \boldsymbol{v} &= \boldsymbol{V} \quad in \quad S_V \end{aligned} \tag{5}$$

From the first equation of Eqs. (5), we can obtain

$$\nabla \cdot (\boldsymbol{A} : \Delta \boldsymbol{L}) = 0 \quad in \quad V \tag{6}$$

where $\Delta(\cdots)$ denotes the difference of two solution sets. Using $\boldsymbol{A}^L$ as the tensor for the linear comparison solid of $\boldsymbol{A}$[5], we obtain

$$\nabla \cdot \left(\boldsymbol{A}^L : \Delta \boldsymbol{L}\right) = 0 \quad in \quad V \tag{7}$$

Considering the deformation of sheet metal under plane stress, denoting the velocity components corresponding to rectangular coordinate system $x_\alpha\,(\alpha = 1, 2)$ within the sheet plane as $v_\alpha, (\alpha = 1, 2)$, Eq. (7) can be written as following differential equations[6]:

$$\begin{aligned} b_1 \frac{\partial^2 (\Delta v_1)}{\partial x_1^2} &+ b_2 \frac{\partial^2 (\Delta v_1)}{\partial x_1 \partial x_2} + b_3 \frac{\partial^2 (\Delta v_1)}{\partial x_2^2} + \\ b_4 \frac{\partial^2 (\Delta v_2)}{\partial x_1^2} &+ b_5 \frac{\partial^2 (\Delta v_2)}{\partial x_1 \partial x_2} + b_6 \frac{\partial^2 (\Delta v_2)}{\partial x_2^2} + \phi_1 = 0 \\ c_1 \frac{\partial^2 (\Delta v_1)}{\partial x_1^2} &+ c_2 \frac{\partial^2 (\Delta v_1)}{\partial x_1 \partial x_2} + c_3 \frac{\partial^2 (\Delta v_1)}{\partial x_2^2} + \\ c_4 \frac{\partial^2 (\Delta v_2)}{\partial x_1^2} &+ c_5 \frac{\partial^2 (\Delta v_2)}{\partial x_1 \partial x_2} + c_6 \frac{\partial^2 (\Delta v_2)}{\partial x_2^2} + \phi_2 = 0 \end{aligned} \tag{8}$$

whereis $\phi_\alpha\,(\alpha = 1, 2)$ are the function of $\partial\left(\Delta v_\alpha\right)/\partial x_\beta\,(\alpha, \beta = 1, 2)$ and $\partial \sigma_{\alpha\gamma}/\partial x_\beta\,(\alpha, \beta, \gamma = 1, 2)$. The values of $b_i (i = 1, 6)$ and $c_i (i = 1, 6)$ are dependent on stress components, strain components, work-hardening coefficient and tangential stiffness tensor of constitutive e-quation.

When non-zero solution for Eq.(8) exists, it is possible that bifurcation occurs[5]. The occurrence condition of local bifurcation mode is same as that of existence of characteristic curve of the second-order simultaneous differential equations Eq.(8). Denote the angle between the tangent line of the characteristic curve and x_1-axis by α . Using $\lambda = tan\alpha$, the existence condition for characteristic curve of the simultaneous equations Eq. (8) can be expressed by

$$\begin{vmatrix} b_1\lambda^2 - b_2\lambda + b_3 & b_4\lambda^2 - b_5\lambda + b_6 \\ c_1\lambda^2 - c_2\lambda + c_3 & c_4\lambda^2 - c_5\lambda + c_6 \end{vmatrix} = 0 \tag{9}$$

From Eq.(9), we can obtain the following 4th degree algebraic equation related to λ:

$$A_1\lambda^4 + A_2\lambda^3 + A_3\lambda^2 + A_4\lambda + A_5 = 0 \tag{10}$$

where $A_i(i = 1, 5)$ are related to $b_i(i = 1, 6)$ and $c_i(i = 1, 6)$. After gaining the distribution of stress and strain by FEM simulation, the value of $A_i(i = 1, 5)$ can be determined. By combining Eq. (10) and the results of FEM simulation, we can predict the occurrence of local necking(breakage) in sheet metal forming.

3 CONSTITUTIVE EQUATION

The classical J2-flow rule is not suitable for bifurcation analysis. To use the constitutive rule in which there is a one-to-one relationship between stress rate affects and plastic strain rate affects critical point[7]. A constitutive rule which incorporates the characteristics that the direction of deviatoric stress rate affects the direction of plastic strain rate has been proposed by Goya and Ito[8]. In this study, we use the constitutive rule.

$$d\boldsymbol{\varepsilon}^p = \frac{3}{2H'}\left\{K_c d\boldsymbol{\sigma}' + \frac{3}{2}(1 - K_c)\left[\frac{\boldsymbol{\sigma}'}{\bar{\boldsymbol{\sigma}}} \otimes \frac{\boldsymbol{\sigma}'}{\bar{\boldsymbol{\sigma}}}\right] : d\boldsymbol{\sigma}'\right\} \tag{11}$$

where H' denotes the tangential modulus on the stress-strain curve, K_c is the material coefficient, which denotes the direction dependency degree of plastic strain increment on deviatoric stress increment. For plane stress problem, from Eq.(11) we can obtain

$$\begin{pmatrix} \overset{\triangledown}{\sigma}_{11} \\ \overset{\triangledown}{\sigma}_{12} \\ \overset{\triangledown}{\sigma}_{22} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{32} \end{pmatrix} \begin{pmatrix} \dot{\varepsilon}_{11} \\ \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{22} \end{pmatrix} \tag{12}$$

where $\overset{\triangledown}{\sigma}_{\alpha\beta}$ denotes the component of Jaumann rate tensor of Cauchy stress; $\dot{\varepsilon}_{\alpha\beta}$ is the component of strain rate tensor; and α_{ij} denotes the tangential stiffness tensor introduced into the constitutive relation.

4 APPLICATION OF THE LOCAL NECKING CRITERION

The local necking criterion obtained in the previous section is programmed and combined with FEM simulation results. The occurrence of local necking during HDD is investigated.

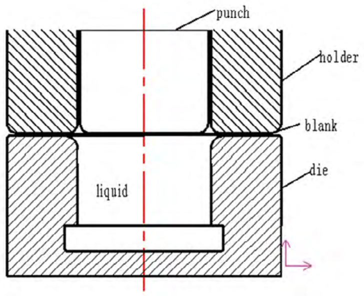


Figure 1: A schematic diagram of the hydromechanical deep drawing

4.1 Finite element model

A schematic diagram of the hydromechanical deep drawing is shown in Fig. 1. The sheet is 200mm in diameter and 1.5mm in thickness. The gap between the die and the holder is constant and is 1.2t(=1.8mm). The diameter of the punch is 100mm and the clearance between the punch and the die is 2mm. The radiuses of the punch, the die and the holder are 10mm respectively. Sheet material A2219 is used. The material parameters are $E = 70.7 \times 10^3 MPa, \nu = 0.3, \sigma_y = 107MPa, C = 634.7MPa, n = 0.3$.

FEM simulation is conducted using commercial software ABAQUS IMPLICIT. The simulation model of HMD is shown in Fig. 2. 3-node and 4-node shell elements S3R and S4R are used. Die and holder are analytical rigid bodies. Since the part and load are axis-symmetrical, 11.25° sheet is used in the model. The friction coefficients between the blank and the holder and the die is 0.01 respectively and the friction coefficient between the blank and the punch is 0.15.

4.2 Loading path

The hydromechanical deep drawing consists of two steps. In the first step, the punch is fixed above the blank. The distance between the punch and the top of blank is 2.25mm. Apply 4MPa chamber pressure linearly onto the bottom of the blank. In the second step, apply maximum chamber pressure of 5MPa and 10MPa respectively onto the bottom of the blank while moving the punch down by 70mm. The pressure-stroke curve is shown as Fig. 3.

4.3 Prediction results and analysis

When the maximum chamber pressure is 5MPa and 10MPa, the stress results of the blank after hydromechanical deep drawing are shown in Fig. 4 .

Bifurcation analysis is conducted by combining the stress and strain results obtained by the simulation of HDD and the local necking criterion. The simulation results are input into self-made Matlab program in which the local necking is predicted according to

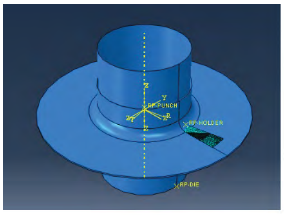


Figure 2: The simulation model of HDD

the criterion obtained in section 2.

When forming pressure is 5MPa, after the HDD, the breakage occurs. The predicted local necking results is shown in Fig. 5. Here coefficient $K_c = 0.3$. When forming pressure is 10MPa, the breakage doesn't occur. The predicted local necking results are compared with experiments[9]. The experiment shows good agreement with the analysis.

5 CONCLUSIONS

- The criterion of localized necking for plane stress has been given by using bifurcation theory. The existing condition of characteristic curve of equilibrium equation in rate form is used. The Ito-Goya's constitutive equation is employed, which gives one-to-one relationship between plastic strain increment component and the stress increment component.

- Combining the proposed criterion that is programmed in Matlab with simulation results of HDD for A2219 aluminum alloy sheet, the occurrence of localized necking is analyzed. The breakage occurs when maximum forming pressure is 5MPa during the analysis. Here, the coeficient K_c is 0.3. The analysis results show good agreement with the experiment.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support by National Natural Science Foundation of China in 2017(Grant no. 51675466). The first and second authors would like to thank the emeritus professor Koichi Ito of Tohoku University of Japan for his instructions and supports.

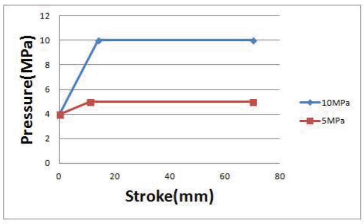


Figure 3: The pressure-stroke curve during forming stage

REFERENCES

[1] HILL, R. On discontinuous plastic states with special reference to localized necking in thin sheets. *Journal of the Mechanics and Physics of Solids* (1952)**1-1**:19–30.

[2] Swift, H. W. Plastic instability under plane stress. J. Mech. Phys. Solids (1952) **1**1-18

[3] Marciniak, Z. and Kuczyński, K. Limit strains in the processes of stretch-forming sheet metal. *Int. J. Mech. Sci.* (1967)**9**: 609-620.

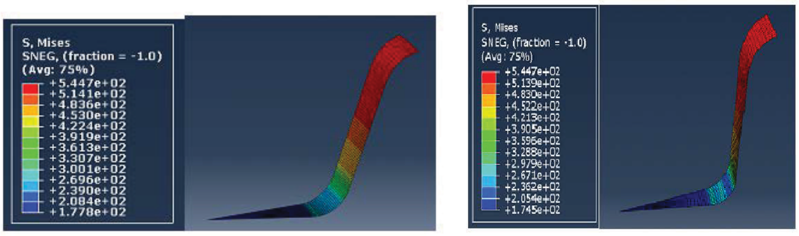
[4] Stören, S. and Rice, J. R. Localized necking in thin sheet. *J. Mech. Phys. Solids*(1975) **23**:421-441.

[5] Hill, R. A general theory of uniqueness and stability in elastic-plastic solids. *J. Mech. Phys. Solids*(1958)**6**:236-249.

[6] Huang, W., Lu, H., Yu, H.B., and et al. The rupture criterion for the sheet metal forming based on local bifurcation theory -the study of enhancing the accuracy of rupture prediction for aluminum alloy sheet metal.*Journal of Plasticity Engineering*(2017)**24(5)**:68-73.

[7] Ito, K., Satoh, K., Goya, M. and Yoshida, T. Prediction of limit strain in sheet metal-forming processes by 3d analysis of localized necking. *Int. J. Mech. Sci.* (2000) **42**:2233-2248.

[8] Goya, M. and Ito, K. An expression of elastic-plastic constitutive law incorporating a stress increment dependence. *Transactions of the Japanese Society of Mechanical Engineering*(1990) **56A(521)**:101-106.



(a) 5MPa

(b) 10MPa

Figure 4: The stress distribution in the blank(maximum values of forming pressure are 5MPa and 10MPa)

[9] Chen, Y. Z. Research on hydromechanical deep drawing of 2219 alumimum alloy cylindrical cup. *Dessertation for the Master Degree in Engineering*(2013) :31-50.

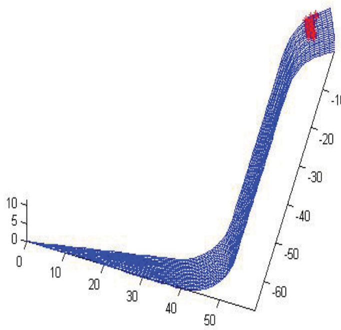


Figure 5: The predicted local necking occurred after HDD(maximum forming pressure is 5MPa)