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# TEMPORAL NATURE OF PLASTICITY IN THE DESIGN OF MATERIALS

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**Abstract.** A full spectrum of materials responses to dynamic loading and observed temporal effects of plasticity from a united viewpoint are analyzed. The relaxation model of plasticity with characteristic time idea is capable to effectively predict the instability of the stress-strain dependencies on strain rates. It is shown that empirical models cannot simultaneously simulate the appearance and disappearance of the yield drop unlike the relaxation model of plasticity presented in the paper.

## 1 INTRODUCTION

The simultaneous growth of the yield point and of the whole stress-strain curve after the beginning of yielding is a typical evidence of the temporal nature of plastic deformation. However, experiments on soft steels [1,2] conducted in a wide range of strain rates exhibit another temporal effect, known as a yield drop phenomenon. The anomalous increase of the peak stress accompanied by a subsequent drop of stresses is ignored by many of existing models of dynamic plasticity (Fig.1.a). The availability of the yield drop phenomena cannot be recognized from the conventional strain rate dependencies of the yield limit. It is necessary to develop approaches that would allow one to consider these effects of strain rate sensitivity of the materials from the united viewpoint.

The structural-temporal approach for determining both brittle strength and yield strength of materials based on the notion of characteristic (incubation) time of the stress relaxation process preceding the macro failure turned out to be an effective alternative approach for describing temporal effects in the high-speed deformation of metals [3,4]. This approach was applied in those strain rate ranges in which the classical Johnson-Cook empirical formula and its modifications do not work [5,6]. These theoretically calculated parameters gave a satisfactory correspondence to the structural-temporal plasticity model with experiment [7,8]. In the work [5], we compared numerical models with the structural-temporal approach considering initial moments of the plastic flow in metals (Fig.1). In this paper, we continue constructing analytical relationships for the entire deformation curve and try to establish connections between parameters of empirical models and those of the relaxation model of plasticity [8,9].

In this paper, responses of some of the materials to the applied dynamic load are predicted based on the plasticity relaxation model [8,9]. Unlike original and modified Johnson-Cook models, one of the advantages of calculating the yield strength by the relaxation model of plasticity is a limited set of material parameters that do not require further modifications in conditions of high strain rates. The main result is the opportunity to describe the principal temporal effects of high-rate plastic deformation of metals from the united viewpoint.

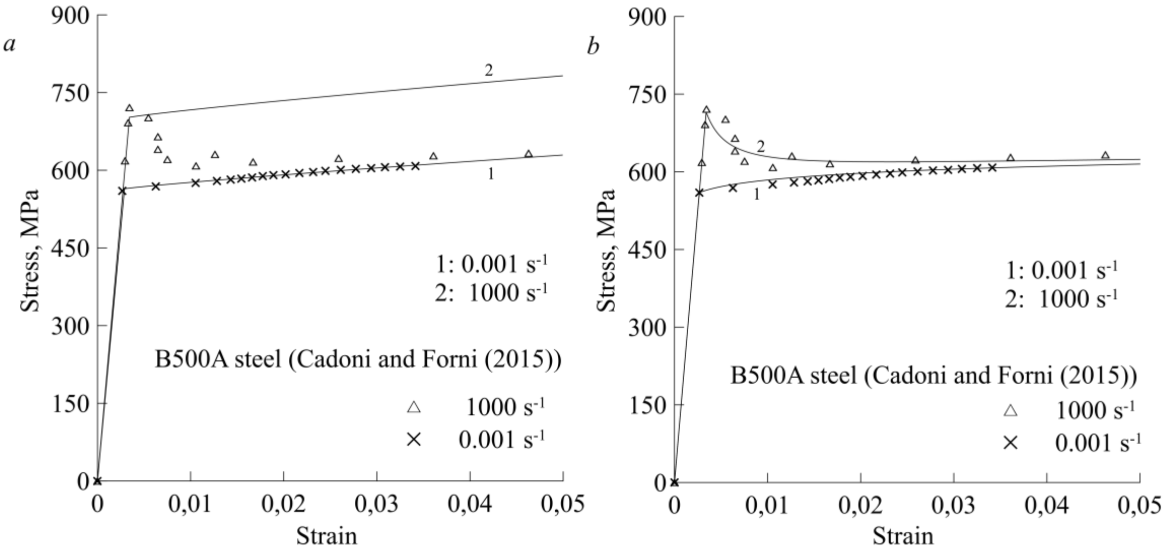


Fig.1. Stress-strain dependences of B500A steel [1] on strain rates 0.001 s$^{-1}$ and 1000 s$^{-1}$, predicted by the Johnson-Cook model (a) and the relaxation model of plasticity (b).

## 2 THE RELAXATION MODEL OF PLASTICITY

Many principal temporal effects of yielding influencing stress-strain relations with increase of the strain rate are taken into account by the incubation time concept. To construct the whole set of stress-strain curves in a wide range of strain rates, we proposed the generalized structural-temporal approach to plastic deformation, known as the relaxation model of plasticity previously introduced by Petrov et al [8,9] and based on the notion of incubation time [3].

Let us introduce a dimensionless relaxation function $0<\gamma(t)\leq 1$, defined as follows

$$\gamma(t)=\begin{cases}1, & \dfrac{1}{\tau}\displaystyle\int_{t-\tau}^{t}\left(\frac{\Sigma(s)}{\sigma_y}\right)^{\alpha}ds\leq 1,\\ \left(\dfrac{1}{\tau}\displaystyle\int_{t-\tau}^{t}\left(\frac{\Sigma(s)}{\sigma_y}\right)^{\alpha}ds\right)^{-1/\alpha}, & \dfrac{1}{\tau}\displaystyle\int_{t-\tau}^{t}\left(\frac{\Sigma(s)}{\sigma_y}\right)^{\alpha}ds>1.\end{cases} \tag{1}$$

The equality $\gamma(t)=1$ in (1) corresponds to the case of purely elastic deformation. Gradual decrease of the relaxation function in the range $0<\gamma(t)<1$ describes a transition to the plastic deformation stage. During the plastic stage of deformation, $t\geq t_*$, the relaxation function $\gamma(t)$

satisfies the condition

$$\frac{1}{\tau}\int_{t-\tau}^{t}\left(\frac{\gamma(t)\Sigma(s)}{\sigma_y}\right)^{\alpha} ds = 1. \tag{2}$$

Here, we suppose that the equality (2) is retained from the yield moment $t=t_*$ (the detailed calculation scheme for the $t_*$ is given in [9]) during the subsequent irreversible deformation process in the material ($0<\gamma(t)<1$). We determine the true stresses in the deformed sample at $t\geq t_*$ in the following form:

$$\sigma(t)=E\gamma^{1-\beta}(t)\varepsilon(t) \tag{3}$$

where $E$ is the Young's modulus and $\beta$ is the scalar parameter ($0\leq\beta\leq1$), which describes the degree of hardening of the material. The case of $\beta=0$ corresponds to the absence of hardening. Considering the stages of elastic and plastic deformations separately, we can obtain from (4) the following stress-strain relation:

$$\sigma(\varepsilon(t)) = \begin{cases} E\varepsilon(t), & \varepsilon(t)/\dot{\varepsilon} < t_*, \\ E\gamma^{1-\beta}\left(\frac{\varepsilon(t)}{\dot{\varepsilon}}\right)\varepsilon(t), & \varepsilon(t)/\dot{\varepsilon} \geq t_*. \end{cases} \tag{4}$$

All parameters $\alpha$, $\tau$, $\beta$ are invariant to loading history and only depend on structural transformations in the material. The relaxation model of plasticity can be used for the arbitrary pulse shape, defined by the $\Sigma(s)$ function.

## 3 THE YIELD DROP PHENOMENON IN THE MATERIALS

Introducing the independent mechanisms controlled by $\alpha$, $\beta$, $\tau$ parameters makes it possible to predict the rate sensitivity effect of the material response and the appearance of the "yield drop" effect in both cases in the perfect plasticity (Fig. 2) and that with hardening (Fig. 3). Further materials without hardening (Material WH) and with hardening (Material H) are considered. The sufficiency of the three parameters to describe the effect of the "yield drop" follows from the fact that the effect of the yield drop contains three quantities that correspond to this phenomenon. These include the width of the yield drop, the upper and lower yield stresses.

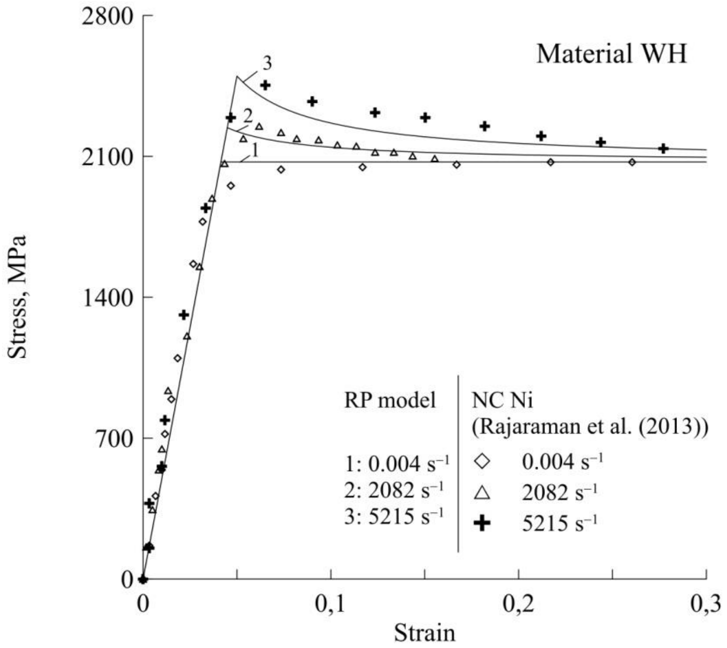


Fig. 2. The responses of nanocrystalline nickel [10] and theoretical stress-strain relations at 0.004 $s^{-1}$, 2082 $s^{-1}$, 5215 s–1, plotted by the relaxation model of plasticity ($\alpha$=1; $\beta$=0; $\tau$=3.3 μs; $\sigma_y$=2072 MPa; $E$= 50 GPa).

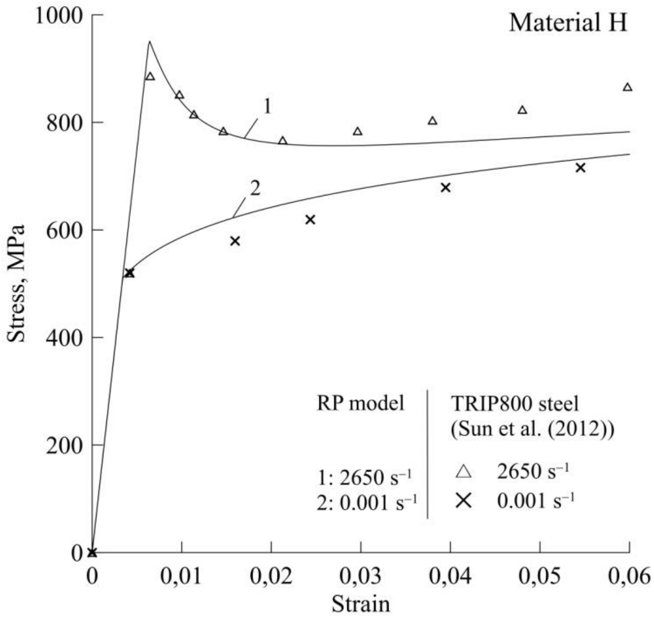


Fig. 3. Stress-strain dependences of TRIP800 steel [2] on strain rates 0.001 $s^{-1}$ and 2650 $s^{-1}$, predicted by relaxation model of plasticity ($E$=150 GPa, $\sigma_y$=510 MPa; $\alpha$=2, $\tau$=2.8μs, $\beta$=0.13).

## 4 DIFFERENCE MATERIALS RESPONSES TO DYNAMIC LOADING

The rate sensitivity in the material can be analyzed using the dependence of the limiting stress (initial point of the irreversible deformation, called the yield stress in metal) on the strain rate and a set of responses of the material to the applied load, characterized by different durations and amplitudes. Different types of the material deformation curves in a wide range of strain rates can be predicted from the viewpoint of the relaxation model of plasticity (Fig. 4 and Fig. 5). Particularly, the increase of loading rate can lead to appearance of anomalies, called "yield drops", at the initial stage of the plastic deformation. In this case, the transition to plastic deformation stage is accompanied by the presence of a pronounced maximum stress with a subsequent stress drop on the deformation dependence. The phenomenon of the maximum stress increasing with the applied load can indicate the existence of the material rate sensitivity. Classical dynamic models of the plastic deformation are only able to account for the cases shown in Fig. 6 when loading rate growth increases the yield stress and the stress-strain dependence remains smooth during transition to the stage of irreversible deformation.

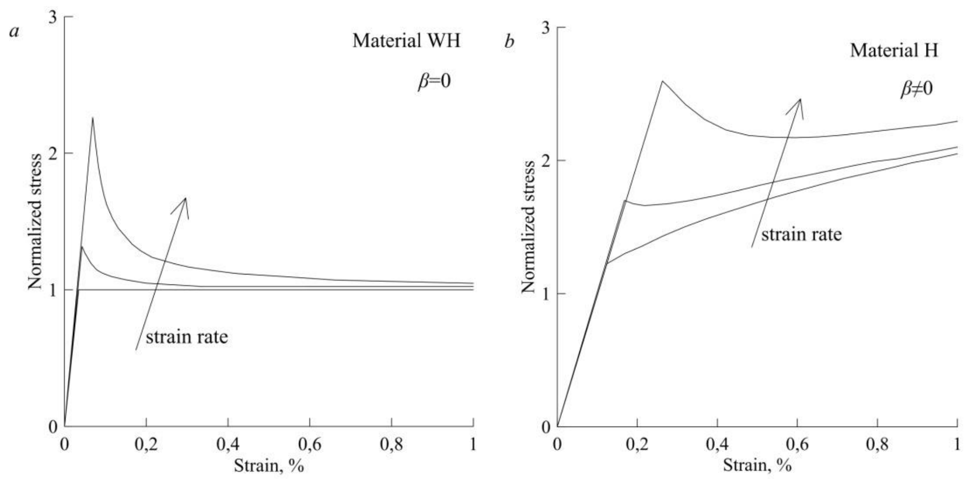


Fig. 4. Prediction of "yield drop" effect with hardening (a) and without hardening (b) using the relaxation model of plasticity.

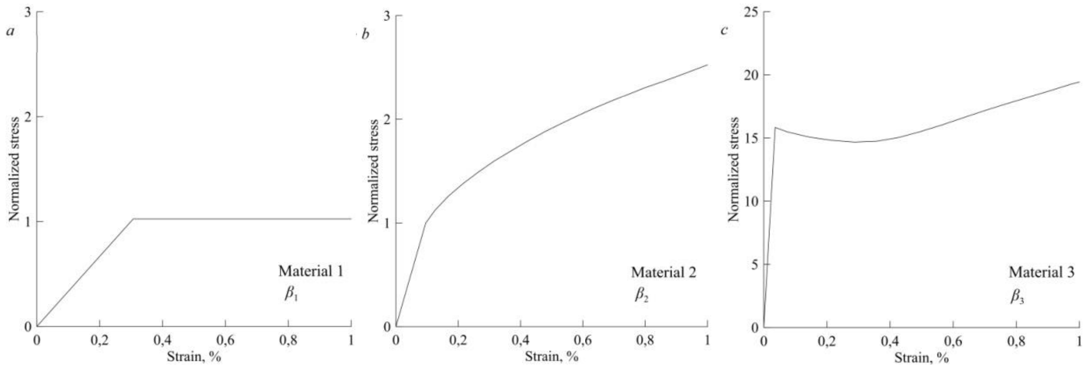


Fig. 5. Full spectrum material responses based on the relaxation model of plasticity: (a) perfect plasticity, (b)

appearance and (c) disappearance of the “yield drop” effect.

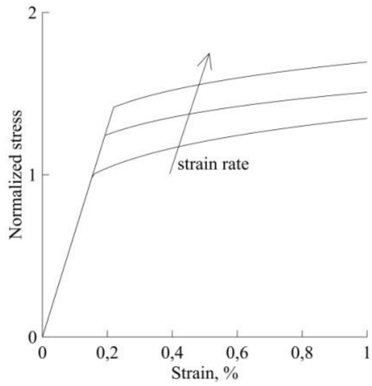


Fig. 6. The increase of the yield stress with disappearance of the “yield drop” effect.

## 5 CONCLUSIONS

Characteristic (incubation time of plastic deformation) time being considered as a material constant can serve a principal parameter of the integral yield criterion that predicts the dynamic behaviour of the yield strength in a wide range of strain rates. The relaxation model of plasticity is able to describe experimentally observed instabilities of stress-strain dependencies, caused by strain rates, including the yield drop phenomenon. It was shown that utilizing this approach the broad spectrum of material responses to dynamic loading can be obtained (Fig. 4 and Fig. 5).

## ACKNOLEGMENTS

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