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FE^2 EVALUATION OF STRESS TRIAXIALITY / LODE ANGLE DEPENDENCIES OF VOID GROWING PROCESSES

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Abstract. This document provides information and instructions for preparing a Full Paper to be included in the Proceedings of *COMPLAS 2019 Conference.*

1 INTRODUCTION

We can find many pieces of research focusing on the behavior of voids at the microscopic structure to predict the ductile failures of metals. It is now recognized that stress triaxiality and Lode parameter strongly affect the damage evolutions, in other words, growing behaviors of micro-voids [1][2][3]. In these researches, experimental material tests have been made to correlate these parameters with the failure processes. However, generally speaking, these stress states quantities are not uniform in the specimen and also in the cross-sections, and are not measurable directly, so they are usually evaluated from deformed shapes or FEM simulations. In addition, it is also difficult to keep the quantities during the loading periods of experiment [2], because these parameters always change depending on the deformations of each material points. It should be also noted that the "damage" quantities, such as damage parameters or void ratio, are difficult to define or evaluate from the experiments [4].

As for the "damage" evolution mechanisms in metals, we can find several investigations, which simulates the micro void's behaviors [5][6]. These reseaches are motivated by the original literatures on damage mechanics, and try to understand the mechanisms of damage occurring inside metals by arranging a void inside the whole strucuture of the specimems and giving many variations of loading conditions. In the research, an arbitrary stress triaxiality and Lode parameter are given to the unit cell, and the relationship between stress state and void damage is investigated [7]. In that, it is common to reproduce an arbitrary stress state by displacement-controlled loading.

In this study, we evaluate the dependencies of void growing processes on the stress state by loading a unit cell model while controlling the stress state using FE^2 simulation . This method can relate the macroscopic behavior of a structure and the corresponding behavior in the microscopic region, where the material inhomogeneities are defined [8]. We would simulate mechanical behavior of microscopic models with spherical voids against arbitrary stress triaxiality and Lode parameters by changing the macroscopic loading conditions using three-dimensional FE^2 procedure, and evaluate the dependency of voids growth upon the macroscopic stress triaxiality & Lode angle.

2 Multi-scale modeling for Ductile Fracture simulations

In this section, we describe the formulation and modeling method for multi-scale analysis to simulate void growing processes. First, we describe the large deformation / elasto-plastic two-scale boundary value problem based on homogenization methods derived for general composite materials and define the BVP handled in this research [9]. Next, we explain the way of expressing stress states. Finally, Problem settings and the finite element models for Micro and Macro scales are defined to study the dependence on stress states.

2.1 Two-scale boundary value problem

the micro-scale BVP is defined by the representative volume element(RVE) virtual work equation, here stated in its spatial version,

$$\int_{\mathcal{Y}} \boldsymbol{\tau}^0(\boldsymbol{x}, \boldsymbol{y}) : \nabla_y \boldsymbol{\eta}^1 \frac{dy}{J_Y} = 0 \quad \forall \boldsymbol{\eta}^1 \in W^{1,p}_{\text{per}}(\mathcal{Y}) \tag{1}$$

along with a selected constitutive equation that relates the microscopic deformation to the microscopic Kirchhoff stress $\boldsymbol{\tau}^0(\boldsymbol{x}, \boldsymbol{y})$. Here, dy denotes the differential volume of the current configuration of the RVE, i.e., $dy = J_Y dY$ and $W^{1,p}_{\text{per}}$ is the Sobolev apace of variations of kinematically admissible displacements of the RVE. The functions of $W^{1,p}_{\text{per}}$ are periodic on the boundary of the RVE.

On the other hand, the macro-scale BVP is governed by the following variational equations in spatial description:

$$\int_{\mathcal{B}} \tilde{\boldsymbol{\tau}}(\boldsymbol{x}) : \nabla_x \boldsymbol{\eta}^0 \frac{dx}{\tilde{J}} - g_{\text{ext}}(\boldsymbol{\eta}^0) = 0 \quad \forall \boldsymbol{\eta}^0 \in W_0^{1,p}(\mathcal{B}) \tag{2}$$

$$\tilde{\boldsymbol{\tau}}(\boldsymbol{x}) = \frac{1}{|\mathcal{Y}|} \int_{\mathcal{Y}} \boldsymbol{\tau}^0(\boldsymbol{x}, \boldsymbol{y}) \frac{dy}{J_Y} \tag{3}$$

where $|\mathcal{Y}|$ is the volume of the RVE in its current configuration and $W_0^{1,p}(\mathcal{B})$ is the space of kinematically admissible displacement fields over the macroscopic continuum. Here, the microscopic volumetric change is given as $dy = J_Y dY$, whereas for the macroscopic counterpart we have $dx = \tilde{J} dX$.

2.2 Characterization of the stress state

Stress states can be described by three principal stresses. There are some values representing the stress state.

$$p = -\boldsymbol{\sigma}_m = -\frac{1}{3}\mathrm{tr}([\boldsymbol{\sigma}]) = -\frac{1}{3}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3) \tag{4}$$

$$q = \overline{\boldsymbol{\sigma}} = \sqrt{\left(\frac{3}{2}[\boldsymbol{S}] : [\boldsymbol{S}]\right)} = \sqrt{\frac{1}{2}[(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)^2 + (\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_3)^2 + (\boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_1)^2]} \tag{5}$$

$$r = \left[\frac{27}{2}\det([\boldsymbol{S}])\right]^{1/3} = \left[\frac{27}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_m)(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_m)(\boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_m)\right]^{1/3} \tag{6}$$

Furthermore, stress states can be described by Lode angle θ and stress triaxiality T defined by three stress invariants. The stress triaxiality T can be defined by the following equation as an index for measuring the triaxiality at an arbitrary point of a material.

$$T = \frac{-p}{q} = \frac{\boldsymbol{\sigma}_m}{\overline{\boldsymbol{\sigma}}} \tag{7}$$

$\boldsymbol{\sigma}_m$ and $\overline{\boldsymbol{\sigma}}$ is the average stress and the effective stress. The value of T increases with increasing triaxiality. The Lode angle θ is defined as the angle between the deviatoric stress on the deviatoric stress surface and the adjacent pure shear line. Lode angle is defined by the following equation.

$$\cos(3\theta) = \left(\frac{r}{q}\right)^3 \quad (-30^\circ \leq \theta \leq 30^\circ) \tag{8}$$

Furthermore, Lode angle θ is normalized,

$$\overline{\theta} = 1 - \frac{6\theta}{\pi} \quad (-1 \leq \overline{\theta} \leq 1) \tag{9}$$

Normalized Lode angle $\overline{\theta}$ is called as Lode parameter. From its parameter, it is possible to know whether the current stress state is tension or pure shear or compression. In this paper, the lode angle is defined only in the region where the relationship with $\sigma_1 \geqq \sigma_2 \geqq \sigma_3$ holds.

2.3 Modeling of an initial void

Using models and material properties for multi-scale method are shown in **Fig. 1** and **Table 1**. A micro-scale cell model containing a spherical space is prepared for representing the growth behavior of micro-voids. 3D models is used for controling triaxial stress states. A 3D model was used to control the triaxial stress state. Similar to the assumption in Gurson's damage model , an isotropic spherical initial void was contained in the center of the micro-model. In this paper, the percentage of the void volume to the volume of the whole microstructure model was defined as the void fraction f to quantify the damage level.

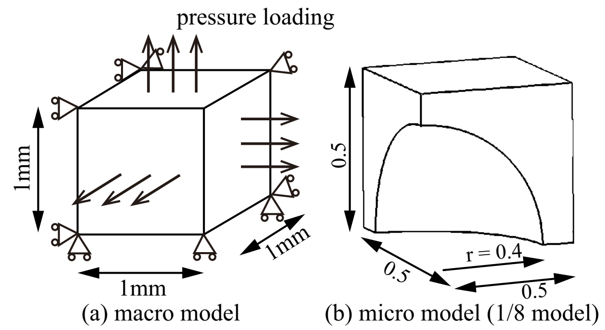

Fig. 1. The chosen model (Macro-model and Micro-model)

Table 1: Macroscopic material parameters

Young's modulus	E [GPa]	200
Poisson's ratio	ν	0.3
Initial yield stress	σ_Y [MPa]	200
Hardening parameter 1	H [MPa]	100
Hardening parameter 2	δ	3
Maximum yield parameter 3	σ_Y^∞ [MPa]	800

3 Results

3.1 simulations of 3-D void growth using Multi-scale simulation

Triaxial stress states is given to the Macro-Micro models shown in **Fig. 1** by Multi-scale methods. **Fig. 2** shows that the relationship between the macroscopic stress and the macroscopic strain, and the void fraction in the micro-structure. Also, the deformation of the microstructure and the corresponding plastic strain distribution at each feature point in **Fig. 2** are shown in **Fig. 3**.

- In the region before the maximum macro-stress, the whole microstructure in a micro-model is deformed as shown in (a) and (b) of **Fig. 3**. Work hardening processes can be also confirmed in the macroscopic stress-strain relationship. In addition, the void fraction in the micro-model increases linearly, which is consistent with the behavior assumed in general damage models.

- In the region after the peak macro-stress, deformation in a micro-model is concentrated in the central part of the void as shown in (c) of **Fig. 3**. This localized deformation in microstructures makes softening behavior in macro stress-strain relationships. In addition, the void fraction increased rapidly compared to before the peak stress, and the rapid damage evolution can be observed as in the damage model.

By modeling the 3-D micro-void in multi-scale simulations, we can describe the drop of macroscopic stress level and the softening behavior of the true stress / true strain

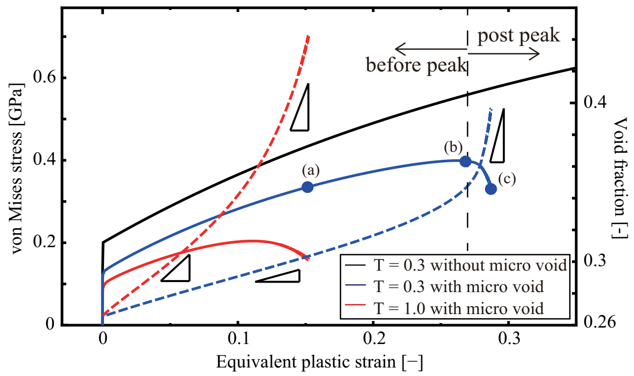

Fig. 2. Macroscopic stress, the void fraction versus Macroscopic strain for $\overline{\theta} = 1,\ T = 0.3, 1.0$

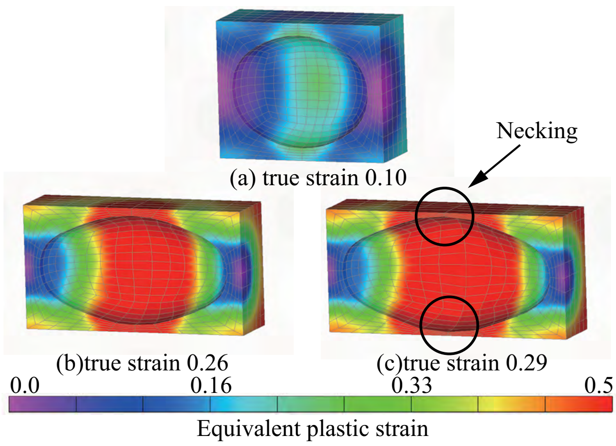

Fig. 3. Contour plots of equivalent plastic strain for void model in micro-structure at each feature points in **Fig. 2**

relationship due to necking, compared with the simulation without a void. By multi-scale analysis assuming void in microstructure, we can obtain qualitatively the similar result as mechanical response assumed by continuum damage mechanics such as void growth in microstructure, stiffness reduction and softening behavior in the macroscopic response.

3.2 Stress triaxiality and Lode angle dependencies of void growing processes

As we mentioned in the previous section, the multi-scale analysis can express void growth behavior. In this section, we would change the stress state and organize the void growth process in each stress state.

Fig. 2 shows the stress-strain relationship in $\overline{\theta} = 1,\ T = 0.3$ and 1.0. From this figure, it can be seen that the maximum stress is reached at low strain levels as the triaxiality increases. It can be considered that this is because the void deforms in three directions when the triaxiality is high, the hole can be expanded more easily as the result. The figure also shows that the void increasing rate before peak stress is larger when the triaxiality

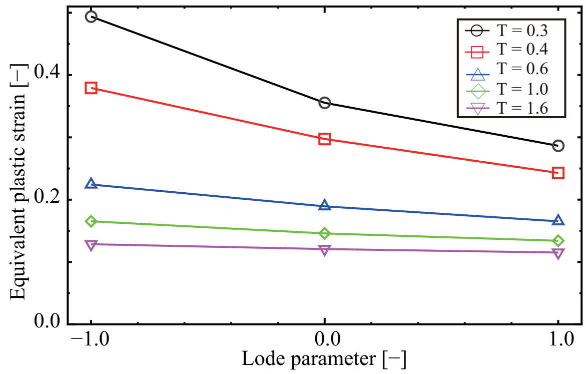

Fig. 4. the evolution of equivalent plastic strain at the same void fraction in different stress triaxiality and Lode angle

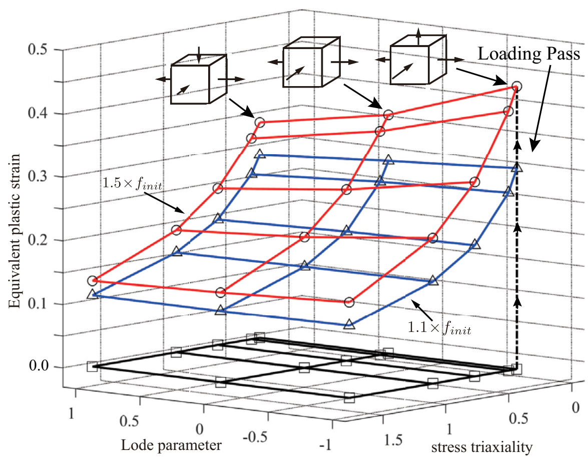

Fig. 5. 3-D damage progress surface in the space of stress triaxiality, the Lode parameter and the equivalent plastic strain of some damage levels

is higher. This result is also reported in the experiment by Yamashita et.al [10], and it can be said that this analysis can properly consider the effect of stress triaxiality.

The equivalent plastic strain when reaching the same void fraction is plotted for each stress state in **Fig. 4**. From this result, it can be confirmed that the equivalent plastic strain required to reach the same void fraction decreases as the stress triaxiality increases. In other words, it can be said that voids are easily to expand in highly stress triaxiality states. As for Lode angle, it can be seen that the void tends to expand as $\theta = 1$, ie, closer to the tensile stress state. Furthermore, it can be read that void growth largely depends on the Lode angle in the low triaxial region of $T = 0.4$ or less. From the above, it can be said that the growing processes of the void depend on the stress triaxiality in the high triaxial region, and conversely, the Lode angle in the low triaxial region.

Furthermore, in order to make it easier to understand visually the dependencies of void growth on the stress state, **Fig. 5** is a replot of **Fig. 4** in three dimensions. This is a damage progress surface formed by connecting the equivalent plastic strains at the same void fraction. The dotted line is the loading pass. Moreover, the damage surface is made on the basis of the point where the void fraction reached 1.2 and 1.4 times from the initial void fraction. From this figure, it can be seen that the void is most easily to grow in high triaxial and tensile stress conditions, that is, the damage in materials progresses rapidly. This tendency is consistent with the results of the previous research, and in this analysis it can be said that the material behavior under the triaxial stress state can be correctly represented [7].

4 CONCLUSIONS

In this study, dependencies of damage evolution on stress triaxiality and Lode angle is organized using multi-scale analysis. The conclusions can be summarized as follows.

- By using multi-scale analysis with a micro-model containing voids, void damage behaviors can be expressed. Moreover, void growing processes under triaxial stress are expressed using 3D void model.
- The multi-scale simulation is able to express the situation that the void growth and the accompanying damage evolution depend on the stress state. It is confirmed that the damage was most easy to develop at high triaxiality and Lode angles corresponding to tensile stress conditions. It is also confirmed that the influence of the Lode angle is more pronounced in the low triaxiality region.

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