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MATERIAL POINT METHOD FOR DETERIORATING INELASTIC STRUCTURES

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Abstract. The material point method (MPM) is one of the latest developments in particle in cell methods (PIC). The structure is discretized into a number of material points that hold all the state variables of the system [1] such as stress, strain, velocity, displacement etc. These properties are then mapped to a temporary background grid and the governing equations are solved. The momentum conservation equations (together with energy and mass conservation considerations) are solved at the grid nodes. The state variables of the particles are then updated by transferring the solutions from the grid nodes back to the material points. Since the background grid is used only to solve the governing equations at the end of each computational step it can be reset to its undistorted form and thus mesh distortion and element entanglement are avoided.

In this work an explicit MPM accounting for elastoplastic material behavior with degradations is proposed. The stress tensor is decomposed into an elastic and a hysteretic – plastic part [5] where the hysteretic part of the stresses evolves according to a Bouc-Wen type hysteretic rule [2]. The inelastic constitutive material law provides a smooth transition from the elastic to the inelastic regime and accounts for the different phases during elastic loading, unloading, yielding and stiffness and strength degradation. Heaviside type functions are introduced that act as switches, incorporate the yield criterion and the terms for stiffness and strength degradation as in the Bouc-Wen model of hysteresis [2]. The resulting constitutive law relates stresses and strains with the use of the tangent modulus of elasticity, which now includes the Heaviside functions and gathers all of the governing inelastic degrading behavior.

1 INTRODUCTION

In the Material Point Method, the domain is discretized into a set of material points or particles. Each particles represents a fraction of the volume of the material and carries all the properties and the state of the material (mass, stress, density strain etc.). In addition to the material points a background grid is employed. This is in most cases structured, but it can also be arbitrary. The grid is static and does not deform and it is reset to its original form after each computational step. It is used to solve the governing equations of motion. The properties of the material points are mapped to the background grid using shape functions similar to FEM. After the solution is obtained in the background grid nodes, the updated quantities are mapped back to the material points.

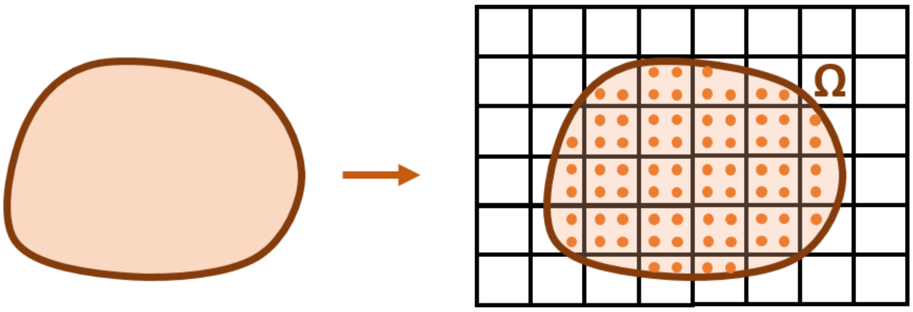

Figure 1: MPM discretization.

In this work cubic B-Splines shape functions are used [3]. They have been shown to reduce quadrature errors and the grid crossing errors that occur when a material point crosses between two elements of the background grid if the gradients of the shape functions are discontinuous [4].

2 THE MATERIAL POINT METHOD

In the MPM algorithm the following steps are considered: firstly, the element of the background grid that each material point lies in, is identified and the corresponding shape functions are evaluated. The material point masses M_p and momenta $(Mv)_p$ are mapped to the background grid and the nodal masses m_i and momenta $(mv)_i$ are calculated:

$$
\begin{aligned}
m_i &= \sum_{p=1}^{N_p} M_p N_i \\
(mv)_i &= \sum_{p=1}^{N_p} (Mv)_p N_i
\end{aligned}
\tag{1}
$$

where N_i are the corresponding shape functions. The nodal internal forces F_i^{int} are calculated on the background grid based on the material point stresses and using the gradient of the shape functions:

$$
F_i^{\text{int}} = -\sum_{p=1}^{N_p} \frac{M_p}{p_p} \sigma_p \nabla N_i \tag{2}
$$

where p_p is the density of the material point *p*. The total nodal force vector F_i, on the background grid is calculated and the appropriate boundary conditions are applied:

$$F_i = F_i^{ext} + F_i^{\text{int}} \tag{3}$$

The momenta at the background grid nodes are updated:

$$(mv)_i = (mv)_i + F_i dt \tag{4}$$

The properties are mapped back to the material points and their positions and velocities are updated as follows:

$$\begin{aligned} v(x_p) &= v(x_p) + \left(\sum_{i=1}^{N} \frac{F_i N_i}{m_i} \right) dt \\ x_p &= x_p + \left(\sum_{i=1}^{N} \frac{(mv)_i N_i}{m_i} \right) dt \end{aligned} \tag{5}$$

The final step is to calculate the strain increments and from those the stress increments. Using the Modified Update Stress Last (MUSL), that has been shown to conserve energy better, the grid nodal momenta are recalculated based on the new particle velocities and the particle strain increments $\Delta\varepsilon_p$ are calculated based on the new nodal velocities:

$$\begin{aligned} (mv)_i &= \sum_{p=1}^{N_p} M_p v(x_p) N_i \\ v_i &= \frac{(mv)_i}{m_i} \\ \Delta\varepsilon_p &= \sum_{i=1}^{N} v(x_p) \nabla N_i dt \\ \Delta\sigma_p &= [D]\Delta\varepsilon_p \end{aligned} \tag{6}$$

where $[D]$ is the plane stress elasticity matrix:

$$[D] = \begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{E\nu}{(1-\nu^2)} & 0 \\ \frac{E\nu}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \tag{7}$$

3 PLASTICITY MODEL WITH DEGRADATIONS

The mechanical analogue of the Bouc – Wen [2] hysteretic model for a Single Degree of Freedom system is presented in Figure 2.

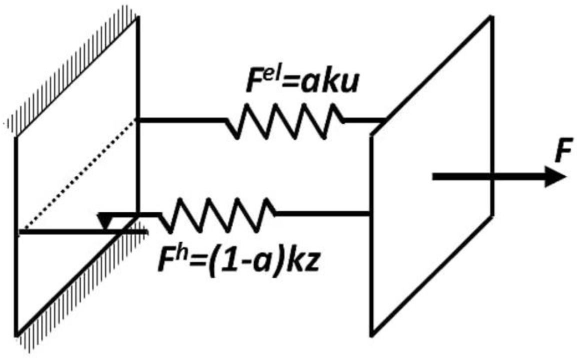

Figure 2: Mechanical analogue of Bouc-Wen model.

The model can be visualized as the parallel combination of two components, one being a linear spring with reduced stiffness ak, where a is the ratio of the post yield stiffness to the initial elastic one. The second element consists of a linear spring and a slider that are connected in series. If the force that acts on the system is smaller than the yield force, then the system behaves elastically with its initial stiffness. If, however, the yield force is exceeded, then the force in the second element stays constant and equal to the yield force and the linear spring provides the additional hardening.

The differential equations of the Bouc-Wen model for the single degree of freedom system are:

$$F = F^{el} + F^{h} = \alpha K u + (1-\alpha) K z$$
$$\dot{z} = \left[1 - \left| \frac{z}{z_y} \right|^n \beta + \gamma \operatorname{sgn}(z\dot{u}) \right] \dot{u} \tag{8}$$

where z is the hysteretic parameter, z_y is the maximum value of the hysteretic parameter, sgn is the signum function, K is the stiffness of the spring and a is the ratio of the post yield stiffness to the initial elastic one. The total force is uncoupled into an elastic one and a hysteretic one.

The Bouc – Wen model as explained in the previous paragraph is generalized herein regarding the stress tensor which is now decomposed into an elastic a hysteretic part as:

$$\{\sigma\} = [\alpha]\{\sigma^e\} + ([I] - [\alpha])\{\sigma^h\} \tag{9}$$

where $[a]$ is a diagonal matrix that hold the ratio of the post yield stiffness to the elastic one and $[I]$ is the identity matrix. The elastic part of the stresses relates to the strains with the use of the classic constitutive matrix $[D]$:

$$\{\sigma^e\} = [D]\{\varepsilon\} \tag{10}$$

The hysteretic part of the stresses follows a Bouc-Wen type hysteretic rule and thus:

$$\{\dot{\sigma}^h\} = [D]([I] - H_1 H_2 [R])\{\dot{\varepsilon}\} \tag{11}$$

where H_1, H_2 are Heaviside type functions and $\left[R\right]$ is the interaction matrix. Its formulation stands from the theory of classical plasticity and can be found in [5] and [7] and contains no hardening related terms [6]. The Heaviside type functions are given as:

$$H_1 = \left|\frac{\Phi}{\Phi_0}\right|^n \quad , \qquad H_2 = 0.5 + 0.5 sign\left(\{\varepsilon\}^T \{\dot{\sigma}\}\right) \tag{12}$$

where Φ is the yield criterion. These functions essentially smooth the transition from the elastic to the inelastic regime and control loading and unloading behaviour. The rate form of equation (9), using equation (11), can be written as:

$$\{\dot{\sigma}\} = [\alpha][D]\{\dot{\varepsilon}\} + ([I]-[\alpha])[D]([I]-H_1H_2[R])\{\dot{\varepsilon}\} \tag{13}$$

Two additional parameters are added into the model to account for stiffness degradation and strength deterioration. These parameters where first introduced by Baber and Wen [8] and equation (13) now becomes:

$$\{\dot{\sigma}\} = [\alpha][D]\{\dot{\varepsilon}\} + \frac{1}{n_s}([I]-[\alpha])[D]([I]-v_sH_1H_2[R])\{\dot{\varepsilon}\} \tag{14}$$

where:

$$\begin{aligned} n_s &= 1 + c_n e^h \\ v_s &= 1 + c_v e^h \end{aligned} \tag{15}$$

Regarding the parameters in the previous relations, c_n and c_v are the model parameters that need to be identified, while e^h is the accumulated hysteretic energy due to plastic energy dissipation. It is calculated from the hysteretic stresses as:

$$e^h = \int \{\sigma^h\} d\{\varepsilon\} \tag{16}$$

Finally, the constitutive equation can be written as:

$$\begin{aligned} \{\dot{\sigma}\} &= \left[E^t\right]\{\dot{\varepsilon}\}, \\ \left[E^t\right] &= \left[[\alpha][D] + \frac{1}{n_s}([I]-[\alpha])[D]([I]-v_sH_1H_2[R])\right] \end{aligned} \tag{17}$$

where $\left[E^t\right]$ can be considered as a tangent matrix effectively controlling the smooth transition from the elastic to the inelastic regime, loading and unloading, as well as accounting for stiffness and strength degradation. This matrix can now substitute the classic elasticity matrix in (6) and extend the MPM to account for plasticity and degradations.

4 NUMERICAL EXAMPLES

In order to verify the proposed model within the MPM framework a cantilever beam is considered. Material is steel with *E=210GPa*, and yield strength of s_y*=240MPa*. The dimensions of the beam are *1m* by *0.2m*. In this analysis the beam was discretized with 320 material points using 4 points per element. The discretized beam and the employed background grid can be seen in Figure 3. A sinusoidal force is applied at the free end of the beam during a time of 5 secs with a maximum value ok 375kN and a period of 2π to simulate one full cycle. The results are plotted in Figure 4 regarding the stress strain diagram of the material point closest to fixed end both with and without degradations. In addition, the Von Mises stresses of the beam at its maximum displaced position are plotted in Figure 5. Results show that the formulation is able to capture accurately the main features of plasticity together with degradation phenomena.

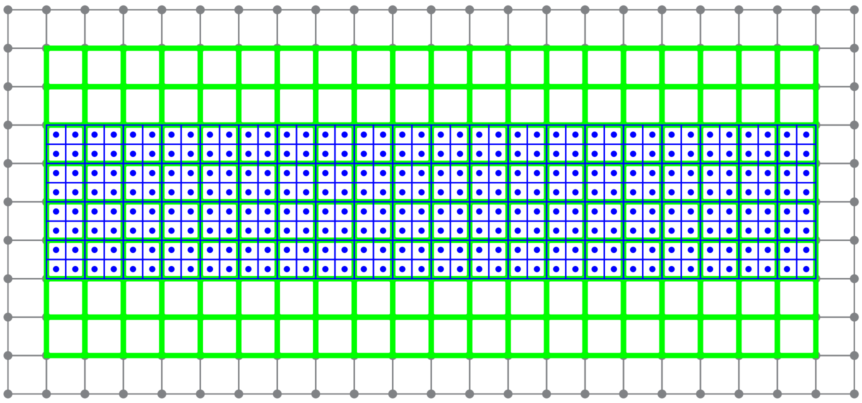

Figure 3: MPM discretization of the beam with 320 material points.

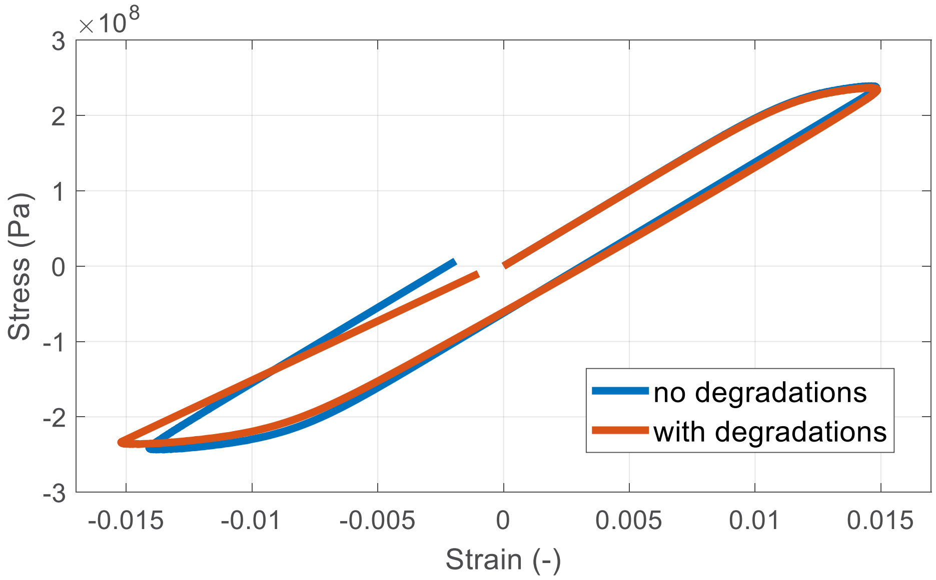

Figure 4: Stress strain diagram for the material point closest to the fixed end.

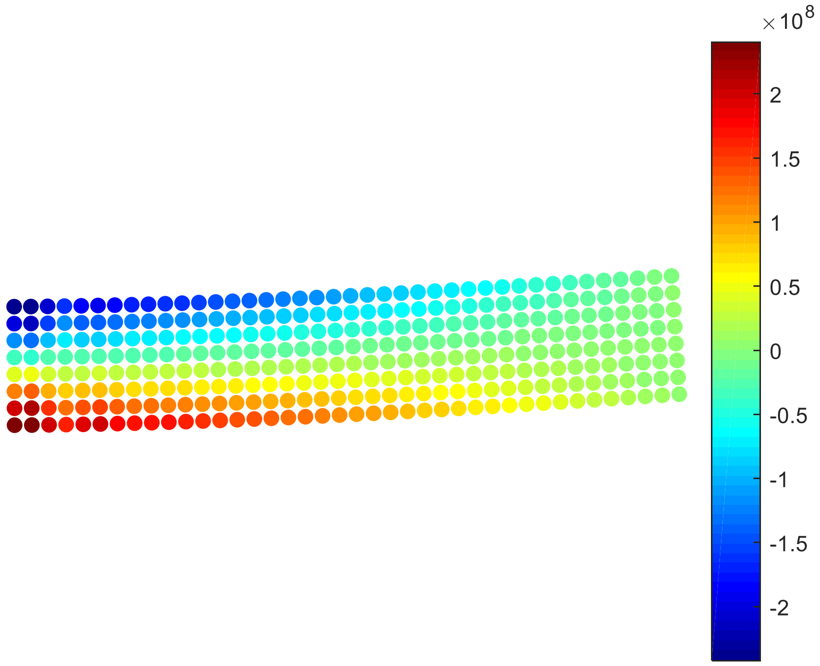

Figure 5: Von Mises stresses.

5 CONCLUSIONS

- The Material Point Method is used in an explicit formulation scheme to model plasticity with degradation phenomena.
- Use of higher order cubic B-Splines effectively minimizes the grid crossing errors and improves the accuracy of the MPM method.
- The hysteretic - plasticity model for nonlinear analysis accounts for smooth transition from the elastic to the inelastic regime.
- The model accounts for stiffness degradation and strength deterioration. and has been incorporated into the MPM framework by modifying the tangent modulus of elasticity.
- Numerical examples are presented that verify the proposed model ability to simulate plastic and damage phenomena.

6 AKNOWLEDGMENTS

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REFERENCES

[1] Sulsky, D., Chen Z. and Schreyer H.L., *A particle method for history-dependent materials*. Comput. Methods, Appl. Mech. Eng. 118,179 (1994).
[2] Bouc R. *Forced vibration of mechanical system with hysteresis*. Proc. 4th Conf. on Nonlinear Oscillation, Prague
[3] Andersen S., Andersen L., *Analysis of spatial interpolation in the material-point method*.

Computers and Structures 88 (2010), 506-518.
[4] Steffen M, Kirby RM, Berzins M. *Analysis and reduction of quadrature errors in the material point method (MPM)*. International Journal for Numerical Methods in Engineering;76:(2008), 922–48
[5] Triantafyllou S., Koumousis V., *Bouc-Wen Hysteretic Plane Stress Element*, Journal of Engineering Mechanics, 138 (2012), 235-246
[6] Casciati F. *Stochastic dynamics of hysteretic media*. In: Krée P., Wedig W. (eds) Probabilistic Methods in Applied Physics. Lecture Notes in Physics, vol 451. Springer, Berlin, Heidelberg, (1995)
[7] Sofianos, C. and Koumousis, V., Hysteretic model for the explicit material point method. 6th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, COMPDYN 2017, Rhodes Island, Greece, (2017)
[8] Baber, T. T., Yi-Kwei Wen.. *Seismic response of hysteretic degrading structures*, Publ. Turk. Natl. Comm. on Earthquake Eng., (1980) Volume 7:457-464.