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Actuator and sensor fault estimation based on a proportional-integral quasi-LPV observer with inexact scheduling parameters ⋆

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Abstract:
This paper presents a method for actuator and sensor fault estimation based on a proportional-integral observer (PIO) for a class of nonlinear system described by a polytopic quasi-linear parameter varying (qLPV) mathematical model. Contrarily to the traditional approach, which considers measurable or unmeasurable scheduling parameters, this work proposes a methodology that considers inexact scheduling parameters. This condition is present in many physical systems where the scheduling parameters can be affected by noise, offsets, calibration errors, and other factors that have a negative impact on the measurements. A $\mathcal{H}_\infty$ performance criterion is considered in the design in order to guarantee robustness against sensor noise, disturbance, and inexact scheduling parameters. Then, a set of linear matrix inequalities (LMIs) is derived by the use of a quadratic Lyapunov function. The solution of the LMI guarantees asymptotic stability of the PIO. Finally, the performance and applicability of the proposed method are illustrated through a numerical experiment in a nonlinear system.

Keywords: qLPV system, Inexact scheduling parameters, PI observer, fault diagnosis.

1. INTRODUCTION

Timely diagnosis of faults is particularly essential in order to increase the safety and reliability of a system. Fault diagnosis algorithms and their applications have received considerable attention and have been the subject of intensive research during the last decades (Li et al., 2018). Model-based fault diagnosis (FD) techniques have been widely recognized as powerful approaches that have been successfully applied in many practical systems, such as unmanned aerial vehicles (López-Estrada et al., 2016), electric vehicles (Djeziri et al., 2013), DC motors (Casavola and Gagliardi, 2015), among others.

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It is well known that an effective model-based FD system requires a mathematical model that captures the nonlinear dynamics inherent in most of the physical systems. In that sense, convex systems such as linear parameter varying (LPV) and quasi-LPV (qLPV) systems have proved to represent complex nonlinear systems by a set of linear-time varying models interpolated by weighting functions. These convex models consider scheduling functions based on exogenous measurable parameters (Hamdi et al., 2019). However, in general, these functions include the input, output, and states of the system (Casavola and Gagliardi, 2015). Note that when the nonlinear sector approach (Ohtake et al., 2003) is applied to a nonlinear model, both qLPV systems and Takagi-Sugeno (TS) Systems are equivalent (Rotondo et al., 2016).

Most of the proposed approaches for FD based on convex systems consider that the scheduling variables are perfectly measurable. In practical applications, the scheduling variables are not measurable or are measured with uncertainties due to measurement noise, offset, low-resolution sensors, bad calibration, indirect measurements, and other factors (Zhang et al., 2016). In this case, it is necessary to consider the inexact scheduling variables in order to design a reliable and effective FD system. For instance, in Chandra et al. (2017), a sliding mode observer scheme to reconstruct actuator and sensor when the scheduling parameters are imperfectly known was presented. In Hassanabadi et al. (2017), a polytopic proportional-integral (PI) unknown input observer to address the problem of actuator fault estimation for singular delayed LPV systems was proposed. Zhu and Zhao (2017) proposed a methodology for simultaneous fault detection and control of switched LPV systems with inexact scheduling parameters. However, despite the few works reported in the literature, problem remains open and is of both practical and theoretical importance.

In this work, the design of a PI observer for qLPV systems is proposed in order to estimate system states, actuator and sensor faults. The proposed approach considers inexact scheduling variables, the faults are considered as time-variants and the performance criterion is chosen to describe robustness to sensor noise and measurement uncertainty on the scheduling variables by solving a set of linear matrix inequalities (LMIs), which are obtained through a Lyapunov formulation.

The rest of the paper is organized as follows: the problem formulation and preliminaries are given in Section 2. In Section 3, the design of a proportional-integral qLPV observer for the fault estimation in actuators and sensors considering inexact scheduling functions is presented. Simulation results are given in Section 4. Finally, the paper finishes with the conclusions in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a qLPV system subject to sensor noise, actuator and sensor faults described by the equations:

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u(t) + F_a(\alpha)f_a(t), \\ y(t) &= Cx(t) + F_s f_s(t) + Dw(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{n_u}$, $y(t) \in \mathbb{R}^{n_y}$, $f_a(t) \in \mathbb{R}^{n_{f_a}}$, and $f_s(t) \in \mathbb{R}^{n_{f_s}}$ are the state, input output, actuator faults, and sensor faults vectors, respectively. $w(t) \in \mathbb{R}^{n_w}$ is the sensor noise. $A(\alpha) \in \mathbb{R}^{n\times n}$ denotes the state matrix, $B(\alpha) \in \mathbb{R}^{n\times n_u}$ is the input matrix, $C \in \mathbb{R}^{n_y\times n}$ is the output matrix, $F_a(\alpha) \in \mathbb{R}^{n\times n_{f_a}}$ and $F_s \in \mathbb{R}^{n_y\times n_{f_s}}$ are the actuator and sensor fault distribution matrices, respectively, $D \in \mathbb{R}^{n_y\times n_w}$ is the disturbance matrix, and α_i is weighting functions. Assume that the matrices $F_a(\alpha)$ and F_s are full rank. Suppose that the numbers of scheduling variables is q and the scheduling variables are independent with each other. Then, if the bounds of scheduling variables are known and measurable, the system can be described by a polytopic qLPV system with 2^q vertices, such that the system matrix set $\mathcal{S} = (A(\alpha), B(\alpha), F(\alpha))$ can be expressed as:

$$\mathcal{S} = \left\{ \Omega | \Omega = \sum_{i=1}^{2^q} \alpha_i \Omega_i; 0 \leq \alpha_i \leq 1; \sum_{i=1}^{2^q} \alpha_i = 1 \right\} \tag{2}$$

where $\Omega_i = (A_i, B_i, F_{a,i})$ and the value of matrix set for each vertex is known. Since the scheduling variables are measurable online, the value of weighting functions α_i for each vertex can be determined online.

Note that, in the case of perfectly measured scheduling variables, the weighting factor can be used directly in the design of components of a control system, such as observers or controllers. However, in the case of inexact scheduling variables there exist mismatches between the real and the measured weighting factors that can deteriorate or destabilize the observer or controller. In this case, it is necessary to use a robust approach that considers these mismatches in order to guarantee stability and good performance. In this work, the weighting factors are:

$$\alpha_i = \lambda_i(t)\hat{\alpha}_i, \tag{3}$$

where $\hat{\alpha}_i$ are the uncertain weighting factors due to an inaccurate measurement of the scheduling variables; $\lambda_i(t)$ is the uncertain factor, whose minimum and maximum values are given by $\underline{\lambda}_i$ and $\overline{\lambda}_i$, respectively, such that:

$$\begin{aligned} A(\alpha) &= \sum_{i=1}^{2^q} \alpha_i A_i = \sum_{i=1}^{2^q} \lambda_i(t)\hat{\alpha}_i A_i \\ &= \sum_{i=1}^{2^q} \hat{\alpha}_i (A_i + (\lambda_i(t) - 1)A_i) = \sum_{i=1}^{2^q} \hat{\alpha}_i \left(A_i + \Delta A_i(t)\right); \end{aligned} \tag{4}$$

with $\sum_{i=1}^{2^q} \hat{\alpha}_i = 1$, and following the above procedure yields:

$$B(\alpha) = \sum_{i=1}^{2^q} \hat{\alpha}_i \left(B_i + \Delta B_i(t)\right); \quad \text{and} \tag{5}$$

$$F_a(\alpha) = \sum_{i=1}^{2^q} \hat{\alpha}_i \left(F_{a,i} + \Delta F_{a,i}(t)\right); \tag{6}$$

with:

$$\Delta A_i(t) = (\lambda_i(t) - 1)A_i, \quad \Delta B_i(t) = (\lambda_i(t) - 1)B_i, \tag{7}$$

$$\Delta F_{a,i}(t) = (\lambda_i(t) - 1)F_{a,i}. \tag{8}$$

Then, the system (1) can be rewritten as an uncertain system as:

$$\begin{aligned} \dot{x}(t) &= \left(A(\hat{\alpha}) + \Delta A(\hat{\alpha})\right) x(t) + \left(B(\hat{\alpha}) + \Delta B(\hat{\alpha})\right) u(t) \\ &\quad + \left(F_a(\hat{\alpha}) + \Delta F_a(\hat{\alpha})\right) f_a(t), \\ y(t) &= Cx(t) + Ff_s(t) + Dw(t). \end{aligned} \tag{9}$$

In order to estimate simultaneously the actuator and sensor faults, the qLPV system (9) is transformed by using a new state $z(t) \in \mathbb{R}^{n_y}$ that is a filtered version of $y(t)$ (Youssef et al., 2017), defined by $\dot{z}(t) = -E\left(z(t) - y(t)\right)$, where E is a stable matrix. The augmented system can be represented as follow:

$$\begin{aligned} \dot{X}(t) &= (\bar{A}(\hat{\alpha}) + \Delta\bar{A}(\hat{\alpha}))X(t) + (\bar{B}(\hat{\alpha}) + \Delta\bar{B}(\hat{\alpha}))u(t) \\ &\quad + (\bar{F}(\hat{\alpha}) + \Delta\bar{F}(\hat{\alpha}))f(t) + \bar{H}w(t), \\ Y(t) &= \bar{C}X(t), \end{aligned} \tag{10}$$

where $X(t) = \left[\, x(t) \;\; z(t) \,\right]^T \in \mathbb{R}^{\bar{n}}$ and $f(t) = \left[\, f_a(t) \;\; f_s(t) \,\right]^T \in \mathbb{R}^{n_f}$ with $\bar{n} = n + n_y$, $n_f = n_{f_a} + n_{f_s}$ and

$$\bar{A}(\hat{\alpha}) = \begin{bmatrix} A(\hat{\alpha}) & 0 \\ EC & -E \end{bmatrix}, \quad \Delta\bar{A}(\hat{\alpha}) = \begin{bmatrix} \Delta A(\hat{\alpha}) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{B}(\hat{\alpha}) = \begin{bmatrix} B(\hat{\alpha}) \\ 0 \end{bmatrix}, \qquad \Delta\bar{B}(\hat{\alpha}) = \begin{bmatrix} \Delta B(\hat{\alpha}) \\ 0 \end{bmatrix},$$

$$\bar{F}(\hat{\alpha}) = \begin{bmatrix} F_a(\hat{\alpha}) & 0 \\ 0 & EF_s \end{bmatrix}, \; \Delta\bar{F}(\hat{\alpha}) = \begin{bmatrix} \Delta F_a(\hat{\alpha}) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{H} = \begin{bmatrix} 0 \\ ED \end{bmatrix}, \qquad \bar{C} = \left[\, 0 \;\; I_{n_y} \,\right].$$

In the augmented system (10) the sensors faults appear as actuator faults and the matrix $\bar{F}(\hat{\alpha})$ is full column rank.

The faults $f(t)$ are assumed to be time-varying signals whose kth time derivatives are bounded by f_0. The following notation is used:

$$\begin{aligned}\dot f(t) =& f_1(t),\\ \dot f_1(t) =& f_2(t),\\ &\vdots\\ \dot f_{k-1}(t) =& f_k(t),\\ f_k(t) \leq & f_0.\end{aligned} \tag{11}$$

3. DESCRIPTION OF THE PROPORTIONAL-INTEGRAL OBSERVER

Under the assumption that the uncertain system (10) is locally observable, the proposed PI observer estimates simultaneously the state and the faults in spite of inexact scheduling variables, and is given by the following:

$$\begin{aligned}\dot{\hat X}(t) =& \bar A(\hat\alpha)\hat X(t) + \bar B(\hat\alpha)u(t) + \bar F(\hat\alpha)\hat f(t)\\ &+ K_P(\hat\alpha)\left(Y(t) - \hat Y(t)\right) + \varphi_x(\hat\alpha),\\ \hat Y(t) =& \bar C\hat X(t),\\ \dot{\hat f}(t) =& K_I(\hat\alpha)\left(Y(t) - \hat Y(t)\right) + \hat f_1(t) + \varphi_f(\hat\alpha),\\ \dot{\hat f}_j(t) =& K_I^j(\hat\alpha)\left(Y(t) - \hat Y(t)\right) + \hat f_{j+1}(t) + \varphi_{f_j}(\hat\alpha),\end{aligned} \tag{12}$$

for $j = 1, \ldots, k-1$, where $K_P(\hat\alpha)$, $K_I(\hat\alpha)$ and $K_I^j(\hat\alpha)$ represent the proportional and integral gains, respectively, which are to be designed. The signals $\varphi_x(\hat\alpha)$, $\varphi_f(\hat\alpha)$ and $\varphi_{f_j}(\hat\alpha)$ are introduced in order to compensate the effect due to inexact scheduling variables, as shown later in Theorem 1.

Based on (11), the augmented form of the LPV model (10) and the PI observer (12) are given, respectively, by:

$$\begin{aligned}\dot{\bar X}(t) =& (\mathcal{A}(\hat\alpha) + \Delta\mathcal{A}(\hat\alpha))\bar X(t) + (\mathcal{B}(\hat\alpha) + \Delta\mathcal{B}(\hat\alpha))u(t)\\ &+ Gw(t) + Rf_k(t),\\ \bar Y(t) =& \mathcal{C}\bar X(t),\end{aligned} \tag{13}$$

and

$$\begin{aligned}\dot{\hat{\bar X}}(t) =& \mathcal{A}(\hat\alpha)\hat{\bar X}(t) + \mathcal{B}(\hat\alpha)u(t) + \mathcal{K}(\hat\alpha)(\bar Y(t) - \hat{\bar Y}(t)) + \varphi(\hat\alpha),\\ \hat{\bar Y}(t) =& \mathcal{C}\hat{\bar X}(t),\end{aligned} \tag{14}$$

where

$$\bar X(t) = \left[\, X(t)\ f(t)\ f_1(t)\ \ldots\ f_{k-1}(t)\,\right]^T,$$
$$\hat{\bar X}(t) = \left[\, \hat X(t)\ \hat f(t)\ \hat f_1(t)\ \ldots\ \hat f_{k-1}(t)\,\right]^T,$$
$$\varphi(\hat\alpha) = \left[\, \varphi_x(\hat\alpha)\ \varphi_f(\hat\alpha)\ \varphi_{f_1}(\hat\alpha)\ \ldots\ \varphi_{f_{k-1}}(\hat\alpha)\,\right]^T,$$

with $\bar X(t) \in \mathbb{R}^{n_k}$, $\varphi(\hat\alpha) \in \mathbb{R}^{n_k}$, $n_k = \bar n + k \times n_f$ and $\bar e(t) = \bar X(t) - \hat{\bar X}(t)$, $\bar e_y(t) = \bar Y(t) - \hat{\bar Y}(t)$,

$$\mathcal{A}(\hat\alpha) = \begin{bmatrix} \bar A(\hat\alpha) & \bar F(\hat\alpha) & 0 & 0 & \ldots & 0\\ 0 & 0 & I_{n_k} & 0 & \ldots & 0\\ 0 & 0 & 0 & I_{n_k} & \ldots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \ldots & 0\end{bmatrix}, \mathcal{B}(\hat\alpha) = \begin{bmatrix}\bar B(\hat\alpha)\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix},$$

$$\mathcal{K}(\hat\alpha) = \left[\, K_P(\hat\alpha)\ K_I(\hat\alpha)\ K_I^1(\hat\alpha)\ \ldots\ K_I^{k-1}(\hat\alpha)\,\right]^T,$$

$$G = \left[\, \bar H\ 0\ 0\ \ldots\ 0\,\right]^T, R = \left[\, 0\ 0\ 0\ \ldots\ I_{n_f}\,\right]^T,$$

$$\Delta\mathcal{A}(\hat\alpha) = \begin{bmatrix} \Delta\bar A(\hat\alpha) & \Delta\bar F(\hat\alpha) & 0 & 0 & \ldots & 0\\ 0 & 0 & 0 & 0 & \ldots & 0\\ 0 & 0 & 0 & 0 & \ldots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \ldots & 0\end{bmatrix},$$

$$\Delta\mathcal{B}(\hat\alpha) = \left[\, \Delta\bar B(\hat\alpha)\ 0\ 0\ \ldots\ 0\,\right]^T,$$

$$\mathcal{C} = \left[\, \bar C\ 0\ 0\ 0\ \ldots\ 0\,\right].$$

In order to facilitate the observer design, using (7) and (8) the uncertainties are bounded as follows:

$$\|\Delta\mathcal{A}_i\| \leq \zeta_{1,i}, \tag{15}$$
$$\|\Delta\mathcal{B}_i\| \leq \zeta_{2,i}, \tag{16}$$

with positives scalars $\zeta_{1,i}$ and $\zeta_{2,i}$. therefore, the following is valid:

$$\begin{aligned}||\Delta\mathcal{A}(\hat\alpha)|| =& ||\sum_{i=1}^{2^q}\hat\alpha_i\Delta\mathcal{A}_i|| = \sum_{i=1}^{2^q}\hat\alpha_i||\Delta\mathcal{A}_i|| \leq \sum_{i=1}^{2^q}\hat\alpha_i\zeta_{1,i},\\ ||\Delta\mathcal{B}(\hat\alpha)|| \leq& \sum_{i=1}^{2^q}\hat\alpha_i\zeta_{2,i},\end{aligned} \tag{17}$$

The dynamics of the augmented state estimation error, denoted as $\dot{\bar e}(t)$, is represented in the following form:

$$\begin{aligned}\dot{\bar e}(t) =& \left(\mathcal{A}(\hat\alpha) - \mathcal{K}(\hat\alpha)C\right)\bar e(t) + \Delta\mathcal{A}(\hat\alpha)\bar X(t) + Wv(t)\\ &+ \Delta\mathcal{B}(\hat\alpha)u(t) - \varphi(\hat\alpha),\end{aligned} \tag{18}$$

where $v(t) = \left[\, f_k(t)\ w(t)\,\right]^T$ and $W = \left[\, R\ G\,\right]$.

The error dynamics (18) of the augmented system is associated with the state vector $\bar X(t)$, the input $u(t)$, the noise $w(t)$, and the function $\varphi(t)$. Since the main criterion for selecting the gain $K(\hat\alpha)$ is to make the estimation error system stable such that the estimation error would converge to zero in absence of uncertainties, we define a new variable as:

$$\xi(t) = L\bar e(t), \tag{19}$$

with a constant matrix L. Then, the challenges and objectives are to tune the observer gain $K(\hat\alpha)$ and discontinue function $\varphi(t)$, such that the dynamical estimation error in (18) is asymptotically stable in absence of uncertainty and noise, and the effect from the external input $v(t)$ to the signal $\xi(t)$ is constrained as:

$$\|\xi(t)\|_2 < \gamma\|v(t)\|_2, \tag{20}$$

where $\|\cdot\|_2$ denotes the 2-norm of a $\mathcal{L}_2$-bounded signal and γ is the $\mathcal{H}_\infty$ performance index.

The following theorem provides the conditions for the asymptotic stability and the $\mathcal{H}_\infty$ performance of the estimation error in (18).

Theorem 1. Given the LPV system (13) and the state observer (14), the estimation error (18) is asymptotically stable with $\mathcal{H}_\infty$ performance and attenuation level $\gamma > 0$, if there exist a matrix $P > 0$, matrices M_i and positive scalars ψ_6 and ψ_2, such that following LMI constraints are feasible:

$$\begin{bmatrix} \Lambda_i & PW & P & \zeta_{1,i} & L^T\\ * & -\gamma^2 I & 0 & 0 & 0\\ * & * & -\psi_6 I & 0 & 0\\ * & * & * & -\psi_2 I & 0\\ * & * & * & * & -I\end{bmatrix} < 0, \tag{21}$$

with: $\Lambda_i = \mathcal{A}_i^T P + P\mathcal{A}_i - \mathcal{C}^T M_i^T - M_i\mathcal{C}$, $\bar{e}_y(t) = \bar{Y}(t) - \hat{\bar{Y}}(t)$, if $|\bar{e}_y(t)| \geq \epsilon$, then

$$\varphi(\hat{\alpha}) = \sum_{i=1}^{2^q} \hat{\alpha}_i \psi_3 \zeta_{1,i}^2 \frac{\hat{\bar{X}}(t)^T \hat{\bar{X}}(t)}{2\bar{e}_y(t)^T \bar{e}_y(t)} P^{-1}\mathcal{C}^T \bar{e}_y(t) + \sum_{i=1}^{2^q} \hat{\alpha}_i \psi_5 \zeta_{2,i}^2 \frac{u(t)^T u(t)}{2\bar{e}_y(t)^T \bar{e}_y(t)} P^{-1}\mathcal{C}^T \bar{e}_y(t), \quad (22)$$

if $|\bar{e}_y(t)| < \epsilon$, then

$$\varphi(\hat{\alpha}) = 0, \quad (23)$$

where:

$$\psi_5 = \frac{\psi_2\psi_6}{\psi_4 - \psi_2\psi_6(\psi_4 + 1)}, \psi_3 = \frac{\psi_4}{\psi_2},$$

ψ_4 and ϵ are positive scalars arbitrarily fixed.

Then, the PI observer parameters are computed as:

$$\mathcal{K}_i = P^{-1}M_i. \quad (24)$$

Proof The performance criterion (19) is equivalent to: (Zhang et al., 2016)

$$\mathcal{J}(t) := \dot{V}(t) + \xi(t)^T\xi(t) - \gamma^2 v(t)^T v(t) < 0, \quad (25)$$

where $V(t)$ is the Lyapunov function which is selected as $V = \bar{e}(t)^T P \bar{e}(t)$, with $P > 0$, such that:

$$\mathcal{J}(t) := \dot{\bar{e}}(t)^T P \bar{e}(t) + \bar{e}(t)^T P \dot{\bar{e}}(t) + \xi(t)^T\xi(t) - \gamma^2 v(t)^T v(t).$$

Then, by considering (18), the following inequality is obtained:

$$\begin{aligned} \mathcal{J}(t) := \bar{e}(t)^T \big((\mathcal{A}(\hat{\alpha}) - \mathcal{K}(\hat{\alpha})\mathcal{C})^T P + P(\mathcal{A}(\hat{\alpha}) - \mathcal{K}(\hat{\alpha})\mathcal{C})\big)\bar{e}(t) \\ + \bar{e}(t)^T PWv(t) + v(t)^T W^T P\bar{e}(t) \\ + \bar{X}(t)^T \Delta\mathcal{A}(\hat{\alpha})^T P\bar{e}(t) + \bar{e}(t)^T P\Delta\mathcal{A}(\hat{\alpha})\bar{X}(t) \\ + u(t)^T \Delta\mathcal{B}(\hat{\alpha})^T P\bar{e}(t) + \bar{e}(t)^T P\Delta\mathcal{B}(\hat{\alpha})u(t) \\ - 2\bar{e}(t)^T P\varphi(\hat{\alpha}) + \bar{e}(t)^T L^T L\bar{e}(t) - \gamma^2 v(t)^T v(t) < 0. \quad (26) \end{aligned}$$

Lemma 1. (Ichalal et al., 2010) For matrices X and Y with appropriate dimensions, the following properly holds for any positive scalar ψ:

$$X^T Y + Y^T X \leq \psi X^T X + \psi^{-1} Y^T Y.$$

Hence, by applying Lemma 1:

$$\begin{aligned} &\bar{X}(t)^T \Delta\mathcal{A}(\hat{\alpha})^T P\bar{e}(t) + \bar{e}(t)^T P\Delta\mathcal{A}(\hat{\alpha})\bar{X}(t) \\ &\leq \psi_1^{-1}(P\bar{e}(t))^T P\bar{e}(t) + \psi_1 \bar{X}(t)^T \Delta\mathcal{A}(\hat{\alpha})^T \Delta\mathcal{A}(\hat{\alpha})\bar{X}(t) \end{aligned}$$

considering (15), the following relationship is established:

$$\begin{aligned} &\psi_1^{-1}(P\bar{e}(t))^T P\bar{e}(t) + \psi_1 \bar{X}(t)^T \Delta\mathcal{A}(\hat{\alpha})^T \Delta\mathcal{A}(\hat{\alpha})\bar{X}(t) \\ &\leq \psi_1^{-1}\bar{e}(t)^T P^2 \bar{e}(t) + \psi_1\zeta_1(\hat{\alpha})^2 \bar{X}(t)^T \bar{X}(t), \quad (27) \end{aligned}$$

where $\bar{X}(t) = \bar{e}(t) + \hat{\bar{X}}(t)$, then, the expression (27) becomes:

$$\psi_1^{-1}\bar{e}(t)^T P^2 \bar{e}(t) + \psi_1\zeta_1(\hat{\alpha})^2 \bar{X}(t)^T \bar{X}(t) = \psi_1^{-1}\bar{e}(t)^T P^2 \bar{e}(t) + \psi_1\zeta_1(\hat{\alpha})^2 \Big(\bar{e}(t)^T\bar{e}(t) + \hat{\bar{X}}(t)^T\bar{e}(t) + \bar{e}(t)^T\hat{\bar{X}}(t) + \hat{\bar{X}}(t)^T\hat{\bar{X}}(t)\Big).$$

Using again the Lemma 1, the last expression can be rewritten as follows:

$$\begin{aligned} &\psi_1^{-1}\bar{e}(t)^T P^2 \bar{e}(t) + \psi_1\zeta_1(\hat{\alpha})^2 \Big(\bar{e}(t)^T\bar{e}(t) + \hat{\bar{X}}(t)^T\bar{e}(t) \\ &+ \bar{e}(t)^T\hat{\bar{X}}(t) + \hat{\bar{X}}(t)^T\hat{\bar{X}}(t)\Big) \leq \psi_1^{-1}\bar{e}(t)^T P^2 \bar{e}(t) \\ &+ \psi_2^{-1}\zeta_1(\hat{\alpha})^2 \bar{e}(t)^T\bar{e}(t) + \psi_3\zeta_1(\hat{\alpha})^2 \hat{\bar{X}}(t)^T\hat{\bar{X}}(t), \quad (28) \end{aligned}$$

where $\psi_2^{-1} = \psi_1(1 + \psi_4^{-1})$ and $\psi_3 = \psi_1(1 + \psi_4)$. Using the previous procedure one gets:

$$\begin{aligned} &u(t)^T \Delta\mathcal{B}(\hat{\alpha})^T P\bar{e}(t) + \bar{e}(t)^T P\Delta\mathcal{B}(\hat{\alpha})u(t) \\ &\leq \psi_5^{-1}(P\bar{e}(t))^T P\bar{e}(t) + \psi_5 u(t)^T \Delta\mathcal{B}(\hat{\alpha})^T \Delta\mathcal{B}(\hat{\alpha})u(t) \\ &\leq \psi_5^{-1}\bar{e}(t)^T P^2 \bar{e}(t) + \psi_5\zeta_2(\hat{\alpha})^2 u(t)^T u(t). \quad (29) \end{aligned}$$

If $e_y(t)$ is zero, since each subsystem is observable, the estimation error is zero. If $e_y(t)$ is non-zero, in order to cancel the effect of the uncertainties on the dynamics of the output system, $\varphi(t)$ is selected as in the equation (22) and by substituting the expression (22) in (26):

$$\begin{aligned} 2\bar{e}(t)^T P\varphi(\hat{\alpha}) = &2\bar{e}(t)^T P\psi_3\zeta_1(\hat{\alpha})^2 \frac{\hat{\bar{X}}(t)^T\hat{\bar{X}}(t)}{2\bar{e}_y(t)^T\bar{e}_y(t)} P^{-1}\mathcal{C}^T\bar{e}_y(t) \\ &+ 2\bar{e}(t)^T P\psi_5\zeta_2(\hat{\alpha})^2 \frac{u(t)^T u(t)}{2\bar{e}_y(t)^T\bar{e}_y(t)} P^{-1}\mathcal{C}^T\bar{e}_y(t \end{aligned}$$

$$2\bar{e}(t)^T P\varphi(\hat{\alpha}) = \psi_3\zeta_1(\hat{\alpha})^2 \hat{\bar{X}}(t)^T\hat{\bar{X}}(t) + \psi_5\zeta_2(\hat{\alpha})^2 u(t)^T u(t), \quad (30)$$

with $\bar{e}_y(t) = \mathcal{C}\bar{e}(t)$ and $\bar{e}_y(t)^T = \bar{e}(t)^T\mathcal{C}^T$.

Such that the performance criteria $\mathcal{J}(t)$ is:

$$\mathcal{J}(t) \leq \sum_{i=1}^{2^q} \hat{\alpha}_i \Big\{\bar{e}(t)^T\Gamma_i\bar{e}(t) + \bar{e}(t)^T PWv(t) + v(t)^T W^T P\bar{e}(t) - \gamma^2 v(t)^T v(t)\Big\} \leq 0$$

$$\sum_{i=1}^{2^q} \hat{\alpha}_i \left\{ \begin{bmatrix} e(t)^T & v(t)^T \end{bmatrix} \begin{bmatrix} \Gamma_i & PW \\ * & -\gamma^2 I \end{bmatrix} \begin{bmatrix} e(t) \\ v(t) \end{bmatrix} \right\} \leq 0, \quad (31)$$

where

$$\Gamma_i = (\mathcal{A}_i - \mathcal{K}_i\mathcal{C})^T P + P(\mathcal{A}_i - \mathcal{K}_i\mathcal{C}) + \psi_6^{-1}P^2 + \psi_2^{-1}\zeta_{1,i}^2 + L^T L. \quad (32)$$

with $\psi_6^{-1} = \psi_1^{-1} + \psi_5^{-1}$.

The analysis prove that (31) holds if:

$$\begin{bmatrix} \Gamma_i & PW \\ * & -\gamma^2 I \end{bmatrix} < 0. \quad (33)$$

Given that (33) is nonlinear, a change of variable $M_i = P\mathcal{K}_i$ is performed in order to obtain a LMI representation. Finally, the Schur complement is considered to obtain the LMI given in Theorem 1, which can be easily solved with specialized software. This completes the proof. □

In the practical implementation the magnitude of $\varphi(\hat{\alpha})$ increases without limit due to the fact that the estimation error $e_y(t)$ tends to zero. This problem can be overcome considering that $e_y(t)$ not converge asymptotically to zero, but to keep it in a small neighborhood of zero depending on the magnitude of ϵ, ϵ is a small positive scalar, as is considered in Theorem 1.

It necessary to mention that the $\mathcal{H}_\infty$ performance index γ indicates the effect of the disturbance $v(t)$ to the signal $\xi(t)$. It is desired that the performance index γ is as small as possible.

4. SIMULATION EXAMPLE

In this section, a Van de Vusse reactor is used to illustrate the method proposed for fault estimation. A Van de Vusse reactor is known to be a highly nonlinear process (Rabaoui et al., 2017). In this reactor, a product A is converted into the desired product B in an isothermal continuous stirred tank reactor (CSTR) which has exothermic reaction instability with a prolonged cooling jacked temperature above 305K and the product B is also converted to a product C: $A \xrightarrow{k_1} B \xrightarrow{k_2} C$. In addition to this consecutive reaction, a high order parallel reaction occurs and A is converted into product D, $2A \xrightarrow{k_3} D$. Where the concentration of product A and B is denoted by C_A and C_B respectively, the following mass balance equation is obtained:

$$\begin{aligned}\frac{dC_A}{dt} &= -k_1C_A - k_3{C_A}^2 + (C_{Af} - C_A)\frac{F_u}{V},\\ \frac{dC_B}{dt} &= k_1C_A - k_2C_B + (-C_B)\frac{F_u}{V},\end{aligned} \quad (34)$$

where C_{Af} the concentration of the reactant A in the inlet flow, f_u is the inlet flow rate and V is the constant volume of the CTRS. Considering $x_1 = C_A$ and $x_2 = C_B$ as state variables and $u = f_u/V$ as the input, the state-space model is given by:

$$\begin{aligned}\dot{x}_1(t) =& -k_1x_1(t) - k_3{x_1}^2(t) + (C_{Af} - x_1(t))[u(t) + f_a(t)];\\ \dot{x}_2(t) =& k_1x_1(t) - k_2x_2(t) + (-x_2(t))[u(t) + f_a(t)];\\ y_1(t) =& x_1(t) + f_s(t) + w(t);\\ y_2(t) =& x_2(t) + w(t);\end{aligned} \quad (35)$$

where C_{Af} the concentration of the reactant A in the inlet flow, f_u is the inlet flow rate and V is the constant volume of the CTRS. The kinetic parameters are chosen to be $k_1 = 100/h$, $k_2 = 50/h$, $k_3 = 10/h$, $C_{Af} = 10mol/h$ and $V = 1L$. f_a is the actuator fault, f_s is the sensor fault and $w_1(t)$, $w_2(t)$ are the measurement noises.

The nonlinear model (35) can be converted into a qLPV representation by embedding the nonlinearities within the varying parameters such as:

$$\begin{aligned}\dot{x}(t) =& A(\alpha(\rho(t)))x(t) + B(\alpha(\rho(t)))u(t) + F_a(\alpha(\rho(t)))f_a(t)\\ y(t) =& Cx(t) + F_sf_s(t) + Dw(t)\end{aligned} \quad (36)$$

where the matrices are defined as:

$$A(\alpha(\rho(t))) = \begin{bmatrix} -k_1 - k_3\rho_1(t) & 0 \\ k_1 & -k_2 \end{bmatrix};$$

$$B(\alpha(\rho(t))) = \begin{bmatrix} C_{Af} - \rho_1(t) \\ \rho_2(t) \end{bmatrix};\ F_a(\alpha(\rho(t))) = B(\alpha(\rho(t)));$$

$F_s = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, with the scheduling parameters $\rho_1(t) = x_1(t)$ and $\rho_2(t) = x_2(t)$ vary in a hyper-cube such that: $0.1 \leq \rho_1(t) \leq 1$ and $0.2 \leq \rho_2(t) \leq 1.2$. Finally, we can rewrite the proposed form (36) in the following polytopic form:

$$\begin{aligned}\dot{x}(t) =& \sum_{i=1}^{4} \alpha_i \left[A_ix(t) + B_iu(t) + F_{a,i}f_a(t)\right];\\ y(t) =& Cx(t) + F_sf_s + Dw(t);\end{aligned} \quad (37)$$

where convex weighing functions are defined as:

$$\alpha_1(\rho(t)) = \frac{\rho_1(t) - \underline{\rho}_1}{\overline{\rho}_1 - \underline{\rho}_1}\frac{\rho_2(t) - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2},\ \alpha_2(\rho(t)) = 1 - \alpha_1(\rho(t)),$$

$$\alpha_3(\rho(t)) = \frac{\overline{\rho}_1 - \rho_1(t)}{\overline{\rho}_1 - \underline{\rho}_1}\frac{\rho_2(t) - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2},\ \alpha_4(\rho(t)) = 1 - \alpha_3(\rho(t)),$$

and the vertex matrices are described as follows:

$$A_1 = A_2 = \begin{bmatrix} -k_1 - k_3\overline{\rho}_1 & 0 \\ k_1 & -k_2 \end{bmatrix};$$

$$A_3 = A_4 = \begin{bmatrix} -k_1 - k_3\underline{\rho}_1 & 0 \\ k_1 & -k_2 \end{bmatrix};$$

$$B_1 = \begin{bmatrix} C_{Af} - \overline{\rho}_1 \\ -\overline{\rho}_2 \end{bmatrix};\ B_2 = \begin{bmatrix} C_{Af} - \overline{\rho}_1 \\ -\underline{\rho}_2 \end{bmatrix};$$

$$B_3 = \begin{bmatrix} C_{Af} - \underline{\rho}_1 \\ -\overline{\rho}_2 \end{bmatrix};\ B_4 = \begin{bmatrix} C_{Af} - \underline{\rho}_1 \\ -\underline{\rho}_2 \end{bmatrix};$$

$F_{a,1} = B_1$; $F_{a,2} = B_2$; $F_{a,3} = B_3$; $F_{a,4} = B_4$.

4.1 Actuator and sensor fault estimation

Solving the LMI (21) in Theorem 1, the unknown gains of the PIO are obtained. The constants are selected as $E = 80$, $\epsilon = 10^{-5}$, $\psi_4 = 10$ and since the main objective is to estimate the state variables and the faults, the value of matrix L is selected as

$$L = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{bmatrix}.$$

In this case, the uncertain factors are delimited as follows:

$$\lambda_i \in [0.9 \ \ 1.1], \text{ for } i = 1,\ \dots,\ 4. \quad (38)$$

In consequence, it is possible to compute $\zeta_{1,i}$ and $\zeta_{2,i}$ from (15) and (16).

The simulation results are carried out with level attenuation $\gamma = 7.64$, the obtained constants $\psi_6 = 4.82 \times 10^4$, $\psi_2 = 1.38 \times 10^5$ and matrices K_i ($i = 1, \dots, 4$), are used to construct the PIO (14) which is implemented in simulation. The initial conditions are:
$\bar{X}_0 = [\,0.5\ 0.4\ 0.51\ 0.41\ 0\ 0\ 0\ 0\,]^T$ and
$\hat{\bar{X}}_0 = [\,0.1\ 0.2\ 0.1\ 0.2\ 0\ 0\ 0\ 0\,]^T$.

The input flow rate F_u is considered variable such that the input signal $u(t)$ will expressed by:

$$u(t) = \begin{cases} 5 + 2\sin(0.07t) & for\ t \geq 0 \\ 0 & otherwise \end{cases}, \quad (39)$$

and the actuator and sensor fault is assumed as a variant time signal and its second-derivative norm-bounded such that:

$$f_a(t) = \begin{cases} 0.2\sin(2t - 2) & for\ 1 \leq t \leq 4.142 \\ 0 & otherwise \end{cases} \quad (40)$$

$$f_s(t) = \begin{cases} 0.05\sin(3t - 18) & for\ 6 \leq t \leq 9.157 \\ 0 & otherwise \end{cases} \quad (41)$$

The measurement noise $w(t)$ in the output, is a centered Gaussian noise with variance 0.0001. The scheduling variables are also affected by the measurement noise, in order to consider the uncertainty in the weighting functions, these weighting functions with uncertainty are shown in Fig. 1. In this case, the measurements of the scheduling variables are considered free of faults.

Fig. 2 shows states estimation and Fig. 3 shows time-varying sinusoidal actuator and sensor faults and their estimates. These simulation results demonstrate the applicability of the method for estimating actuator and sensor faults for qLPV systems with weighting functions with uncertainty.

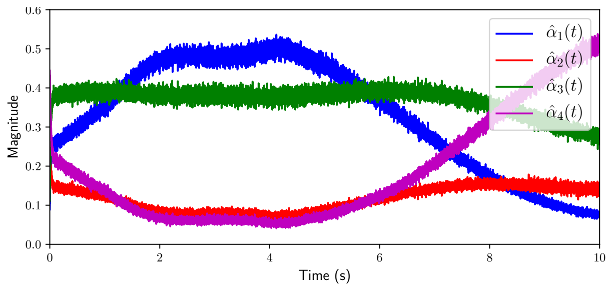

Fig. 1. Inexact weighting factors

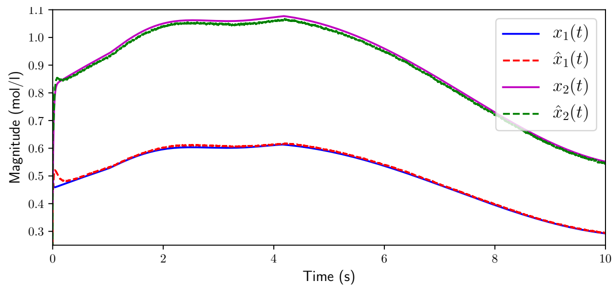

Fig. 2. State variables and their estimates

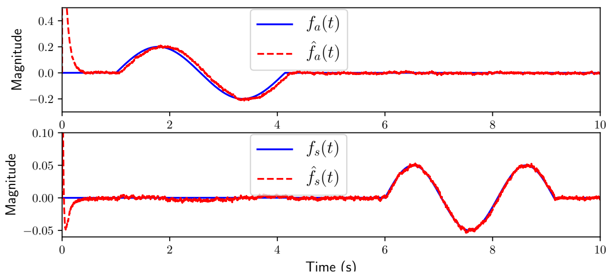

Fig. 3. Faults and their estimates

5. CONCLUSION

In this work, a polytopic PIO for state, actuator and sensor faults estimation was proposed. It was considered that the qLPV system was affected by noise measurement in the scheduling variables and output of the system. The used strategy was based on the $\mathcal{H}_\infty$ performance criteria to be robust against sensor noise and uncertainty induced by inexact scheduling variables. Furthermore, it was demonstrated that the proposed approach is suitable to estimate system states and actuator and sensors faults by a qLPV Proportional-Integral observer. Finally, a numerical example of the Van de Vusse reactor model was presented to show the effectiveness and applicability of the proposed approach. Future work will be done to extend the method to fault tolerant control. Note that a reconfigurable controller can be designed in order to maintain stability, acceptable dynamic performance and steady state of the system, in the event of a fault, based on the the proposed fault estimation method.

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