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How to overcome limitations of analytic solutions when determining the direction of a gravitational wave using experimental data: an example with the Schenberg detector

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How to overcome limitations of analytic solutions when determining the direction of a gravitational wave using experimental data: an example with the Schenberg detector

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Abstract. It has been commonly assumed that analytic solutions can efficiently provide the direction of a gravitational wave (GW) once sufficient data is available from gravitational wave detectors. Nevertheless, we identified that such analytic solutions (based on the GW matrix reconstruction) present unforeseen theoretical and practical limitations (indeterminacies) and that for certain incoming directions they are unable to recover the latter. We present here important indeterminacy cases as well as a mathematical procedure that reduces such indeterminacies. Also, we developed a method that requires the least computational power to retrieve GW directions and which can be applied to any system of detectors able to reconstruct the GW matrix. As a test for the method, we used simulated data of the spherical, resonant-mass GW detector Schenberg, which involves five oscillating modes and six transducer readouts. The results show that this method canceled indeterminacies out satisfactorily.

1. Introduction

Determining the direction of a gravitational wave (GW) source is necessary, for instance, for confrontations between a candidate GW signal and its electromagnetic counterpart. In Figure 1 the wave's direction is z' while the detector's frame is xyz.

Gravitational waves are expected to transfer different amounts of energy to the five degenerate quadrupolar modes of the spherical antenna as a function of the incoming GW direction. In order to monitor the modes' oscillations, transducers are placed on the sphere's surface[1]. The electric readout of the k-th transducer is here described in the frequency domain by the function $I_k(f)$. We will assume that the sphere is monitored by $k = 6$ transducers. The relation between their readouts and the m-th mode of the sphere, which is described by the mode channel $h_m(f)$, is given by

$$h_m(f) = T_{mk}(f)I_k(f), \qquad k = 1, .., 6,\ m = 0, 2, ..., 4\ , \tag{1}$$

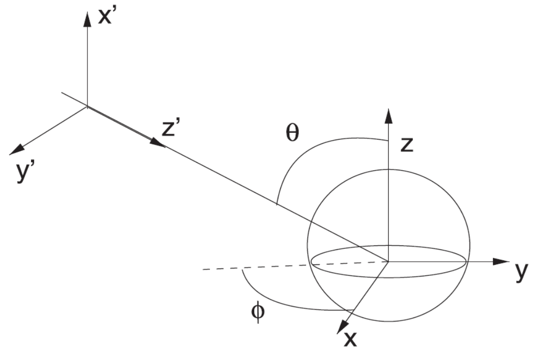

Figure 1. Position of the GW frame (primed) relative to the proper detector frame (unprimed).

where $T_{mk}(f)$ is proportional to the inverse transfer function $\mathcal{T}^{-1}(f)$ of the detector.

We can write the GW matrix in the lab frame in terms of the mode channels:

$$\mathbf{h} = \begin{pmatrix} h_0/\sqrt{3} + h_3 & h_4 & h_1 \\ h_4 & h_0/\sqrt{3} - h_3 & h_2 \\ h_1 & h_2 & -2h_0/\sqrt{3} \end{pmatrix}. \quad (2)$$

In the context of low latency pipelines it is assumed that analytic expressions (based on the GW matrix reconstruction) can efficiently provide the GW direction. In this work we indicate that this is not true in general and we propose means to overcome this limitation. Details of the results presented here can be found in [2].

2. Analytical limitations

We found that analytic expressions present theoretical and practical limitations (indeterminacies) which were not previously mentioned in the literature. Under realistic conditions, we identified that for certain incoming directions such solutions are unable to recover them. Examples of these directions are given by the red zones of Figure 2

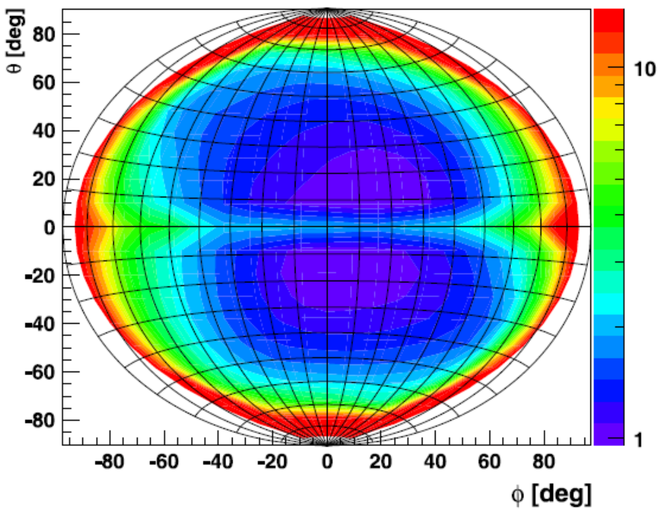

Figure 2. Aitoff projection of the average direction reconstruction errors (in degree) as functions of the injected signal directions (under SNR$\sim$ 80). The plot shows the results when using the direction vector V_{2-3}. In the colored legend on the right of the plots (log scale in degree), values lower than two (shades of blue) indicate areas with small direction reconstruction errors in the GW direction reconstruction, while higher values (in red) indicate high direction reconstruction errors and thus indeterminacy zones. θ and ϕ are ranging from 90 to +90.

The knowledge of the five mode channels enables one to obtain the relevant quantities of the GW, particularly its incoming direction. We found that the analytic solutions depend on the

GW direction in a particular way. In Table 1 we present a summary of the different equations and their dependence on the GW direction, (θ, ϕ).

Table 1. Summary of the analytic solutions and their dependence on the incoming gravitational wave direction (θ, ϕ). The subindices $_{(1)}$ and $_{(2)}$ denote the two options to compute θ_r

Pair of lines	$1-2$	$1-3$	$2-3$
$\tan\phi_r$	$\frac{\sin\theta\cos\theta\sin\phi}{\sin\theta\cos\theta\tan\phi}$	$\frac{\sin^2\theta\sin^2\phi}{\sin^2\theta\cos\phi\sin\phi}$	$\frac{\sin^2\theta\cos\phi\sin\phi}{\sin^2\theta\cos^2\phi}$
$\tan\theta_{r\ (1)}$	$\frac{1}{\sin\phi_r}\frac{\cos\theta\sin\theta\sin\phi}{\cos\theta^2}$	$\frac{1}{\sin\phi_r}\frac{\sin^2\theta\sin^2\phi}{\sin\theta\cos\theta\sin\phi}$	$\frac{1}{\sin\phi_r}\frac{\cos\phi\sin\theta^2\sin\phi}{\cos\theta\cos\phi\sin\theta}$
$\tan\theta_{r\ (2)}$	$\frac{-1}{\cos\phi_r}\frac{\cos\theta\cos\phi\sin\theta}{\cos\theta^2}$	$\frac{-1}{\cos\phi_r}\frac{\cos\phi\sin\theta^2\sin\phi}{\cos\theta\sin\theta\sin\phi}$	$\frac{-1}{\cos\phi_r}\frac{\cos\phi^2\sin\theta^2}{\cos\theta\cos\phi\sin\theta}$

3. The proposed method

In order to avoid the limitations illustrated in Figure 2 derived from equations in Table 1 we decided to use the direction vector $\mathbf{V}$, which satisfies $(\mathbf{h} - \lambda\mathbf{I})\mathbf{V} = 0$. By using this vector we were able to reduce the indeterminacy zones to only three circles in the Aitoff projection. The vector $\mathbf{V}$ fully describes the incoming GW direction but it is more usual to give this information in terms of (θ, ϕ):

$$\cos\theta_r = V_3/|\mathbf{V}|\,, \tag{3}$$

with three options for the determination of ϕ,

$$\cos\phi_r = V_1/\sqrt{V_1^2 + V_2^2}\,, \tag{4}$$

$$\sin\phi_r = V_2/\sqrt{V_1^2 + V_2^2}\,, \tag{5}$$

$$\tan\phi_r = V_2/V_1\,. \tag{6}$$

The advantage of these solutions is that θ_r and ϕ_r are computed independently. Therefore there is no error propagation to θ_r.

We implemented our method in the context of a low latency pipeline [3] and in the presence of noise[4] applying a weighted average of the direction vector for each sample. We tested our method injecting a simulated burst into the mathematical model of the GW detector Schenberg [5, 6].

We characterize the direction reconstruction error using the angular distance

$$\delta s = \arccos\left(\sin(\theta)\sin(\theta_r)\cos(\phi - \phi_r) + \cos(\theta)\cos(\theta_r)\right)\,, \tag{7}$$

which is the angle between the direction (θ, ϕ) of the injected signal and the reconstructed direction (θ_r, ϕ_r) of the signal in the presence of noise.

We found that it is possible to cancel out indeterminacies in the presence of noise using the following method: for each direction vector $\mathbf{V_{1-2}}_t$, $\mathbf{V_{1-3}}_t$ and $\mathbf{V_{2-3}}_t$, we apply the averaging technique where each sample direction is weighted by its signal-to-noise ratio (SNR). From the three resulting averaged direction vectors $\langle\mathbf{V_{1-2}}\rangle$, $\langle\mathbf{V_{1-3}}\rangle$ and $\langle\mathbf{V_{2-3}}\rangle$, we select the direction vector that presents the lowest value of its components' standard deviation product:

$$\sigma^3(V) \equiv \sigma(V_x)\sigma(V_y)\sigma(V_z)\,. \tag{8}$$

Then using its averaged components $\langle V_x \rangle$, $\langle V_y \rangle$ and $\langle V_z \rangle$, we compute the direction in terms of the spherical coordinates (θ_r, ϕ_r).

We tested our method with the injected burst signal in the Schenberg detector and the red zones in Figure 2 turned blue, as shown in Figure 3, eliminating indeterminate incoming GW directions.

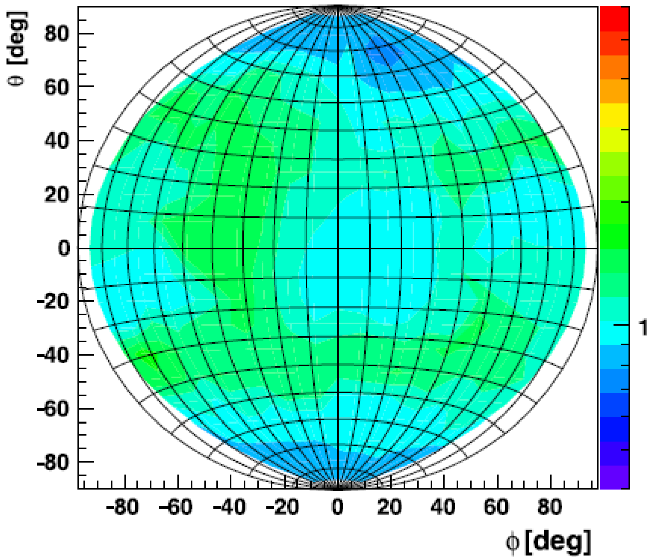

Figure 3. Aitoff projection of the average angular distances $\langle \delta s \rangle$ (in deg) as a function of the direction of the injected signal (SNR$\sim$ 80). We used our proposed method to reconstruct the GW direction in the presence of noise. The minimum is at 0.8 deg (shades of blue) and the maximum is at 1.7 deg (green).

4. Conclusions

Our method proved efficient in facing indeterminacies in the evaluation of the direction of a GW and it is suited for a low latency pipeline. It was tested with the mathematical model of the resonant-mass GW detector Schenberg, and it can be applied to any array of detectors that use the GW matrix to retrieve a wave's incoming direction.

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