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Commitment and Conflict in Multilateral Bargaining

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Abstract

We extend the Baron and Ferejohn (1989) model of multilateral bargaining by allowing the players to attempt commiting to a bargaining position prior to negotiating. If successful, commitment binds a player to reject any proposal which allocates to her a share below a self-imposed threshold. Any such attempted commitment fails and decays with an exogenously given probability. We characterize and compare symmetric stationary subgame perfect equilibria under unanimity rule and majority rules. Under unanimity rule, there are potentially many equilibria which can be ordered from the least to most inefficient, according to how how many commitment attempts must fail in order for an agreement to arise. The most inefficient equilibrium exists independently of the number of players, and the delay in this equilibrium is increasing in the number of players. In addition, more efficient commitment profiles cannot be sustained in equilibrium if the number of players is sufficiently large. The expected inefficiency due to delay at the least and at the most efficient equilibrium increases as the number of players increases. Under any (super)majority rule, however, there is no equilibrium with delay or inefficiency. The reason is that competition to be included in the winning coalition discourages attempts to commit to an aggressive bargaining position. We also show that inefficiencies related to unanimity decision making may be aggravated by longer lags between consecutive bargaining rounds. The predicted patterns are by and large consistent with observed inefficiencies in many international arenas including the European Union, WTO, and UNFCCC. The results suggest that the unanimity rule is particularly damaging if the number of legislators is large and the time lags between consecutive sessions are long.

Keywords: bargaining, commitment, conflict, delay, environmental agreements, international negotiations, legislative, majority, multilateral, unanimity
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1 Introduction

"The unanimity rule has meant that some key proposals for growth, competitiveness and tax fairness in the Single Market have been blocked for years." (European Commission press release, Jan 15th 2019)

Efficient and effective governance of global market failures requires a great deal of international agreement. Currently, there are a number of fora where key issues for sustainable development of the planet are negotiated. However, many of these are characterized by a prolonged negotiation process or even a negotiation impasse. First, The United Nations Framework Convention on Climate Change (UNFCCC) has not been able to reach a comprehensive and binding agreement on how to limit carbondioxide emissions (Pizer, 2006).[1] Second, the World Trade Organization's (WTO) Doha Development Round[2] has missed a number of deadlines, and by the present day still has not resulted in a comprehensive agreement (Ehlermann and Ehring, 2005; Bagwell et al., 2016).[3] Alternative plurilateral agreements, such as the WTO TRIPS agreement, have been adopted instead between a considerably smaller number of countries. Finally, the European Commission strives to reach unanimous consent among member states on such issues as common environmental, tax, or refugee policies. No considerable progress has been reached in these fronts. As a consequence, the European Commission, including former Commission president Jean-Claude Juncker, and Pierre Moscovici (the Commissioner for Economic, Financial Affairs, Taxation and Customs) has recently proposed that the unanimity rule should be abandoned in favor of the so called *qualified majority rule*[4] in EU taxation policy. In popular media and among negotiation delegates, the unanimity requirement according to which all parties must reach a consensus for an agreement to arise, has been blamed for its ineffectiveness. Within the WTO and the EU, the criticism seems to have gained momentum as the number of member states in each organization has grown.

Existing models of rational multilateral bargaining, building upon the seminal model by Baron and Ferejohn (1989), do not provide much reason to challenge the unanimity rule. Rather, the basic model under stationary equilibria predicts immediate agreement without delay, independently of whether unanimity or any type of majority is required, and independently of the number of players (Banks and Duggan, 2000). Moreover, unanimity rule is predicted to result in more egalitarian terms of agreement than majority rules in symmetric stationary environments according to the existing models (Eraslan and Merlo, 2017).[5] Models of asymmetric information and/or reputation for obstinacy (Myerson, 1991; Abreu and Gul, 2000; Compte and Jehiel, 2002) may predict delay and inefficiency, but they are typically fairly complicated or even intractable in multilateral settings, and have thus mostly focused on the bilateral case.[6]

[1] See also Nordhaus (2006) and Stern (2008).

[2] The round was launched in 2001 and focused among other things on market access for agricultural goods – especially from the developing to the markets of the developed countries – and the reduction of subsidies in this sector by the developed countries (Meltzer, 2011).

[3] The WTO was not able to meet the December 2002 deadline imposed in paragraph 6 of the Doha declaration on the TRIPS agreement and Public Health since a single member prevented consensus (Ehlermann and Ehring, 2005). Instead a considerably weaker Trade Facilitation Agreement was agreed upon in 2013.

[4] See the European Commission press release, Jan 15th 2019. The *qualified majority rule* applied in the EU decision making requires 55% of member states to vote in favor and these countries must represent at least 65% of the total EU population.

[5] Comparing the majority and unanimity rules, Maggi and Morelli (2006) consider repeated voting without enforcement of outcomes and find that majority rule is more efficient than unanimity when preferences are positively correlated and players are patient.

[6] Ma (2018) provides an analysis of the three-player case. See Eraslan and Evdokimov (2019) for a general review of the literature, and sections 6 and 7 in particular for the literature on asymmetric information. Kiefer (1988) and Kessler (1996) suggest that hazard rates in labor disputes are inconsistent with the incomplete information explanation.

How can the role of unanimity in contributing to delay, impasse and inefficiency then be understood from the perspective of rational interactive decision making, i.e. game theory? In this paper we propose a tractable complete information model of commitment and conflict of multilateral bargaining in an infinite horizon setting which builds on the seminal Baron-Ferejohn model to which it adds the capacity to precommit.[7] (Crawford, 1982; Levenotoğlu and Tarar, 2005; Ellingsen and Miettinen, 2008, 2014; Miettinen and Perea, 2015). At the commitment stage of each round, each player can attempt to commit to rejecting, in the subsequent bargaining stage, any proposal where she receives less than a self-imposed threshold. Any such attempted commitment fails and decays with an exogenously given probability. The commitment status of all players is common knowledge at the bargaining stage of each round. We adopt the simplest such model where each player's probability of failing is independent of the failures of the commitment attempts of other players and, moreover, each individual commitment attempt has an equal chance of failing. Also for the sake of simplicity, we assume that commitments have to be re-established at the commitment stage of each round. As in Ellingsen and Miettinen (2014), commitments are costly to an extent that two strategies which lead to otherwise identical payoffs for a player are ranked lexicographically, with preference given to the strategy that does not require commitment. Once commitment attempts have been made and their success determined, one of the players is randomly drawn to make a proposal to which all others respond by either accepting or rejecting the offer. Agreement arises if, according to an exogenously given consent rule, sufficiently many players give their consent to the proposal.

The key focus of the paper is to compare unanimity rule with various majority rules under these circumstances. We find that, under the unanimity rule, delay and inefficiency are commonplace and often unavoidable. Under a wide range of relevant parameter values, every symmetric stationary equilibrium is associated with delay. Moreover, as the number of players grows larger, the delay and inefficiencies become more severe: both the maximal and the minimal expected conflict duration over all equilibria increase. To the contrary, there is never delay or inefficiency in any equilibrium under the majority or any supermajority rule. Ruling out unanimity and requiring all but one party to agree is enough to restore efficiency in our non-cooperative model. The mechanism underlying this result is that players compete for being included into the winning coalition. Comparing the agreements between unanimity and (super)majority rules, negotiation outcomes are not necessarily more equal under unanimity, and inequality increases with the number of players in the unanimity case. Thus our model provides a clear rationale for favoring less-than unanimity rules (including 'all but one' or other supermajority rules) over unanimity rule, as suggested by many policy makers involved in international negotiations.

The paper contributes to understanding the role of bargaining frictions in multilateral bargaining, and especially the mediating role of the number of negotiating parties in aggravating such frictions. There is a growing need for better understanding of such frictions from an applied perspective, as pointed out by Bagwell et al. (2016, pp. 97) in their survey article on the economics of WTO, for instance. Naturally, our model abstracts from the complexities generated by cross-externalities, asymmetric and incomplete information, and the shadow of alternative pluri- and bilateral treaties which characterize the WTO and UNFCCC negotiations (See Harstad, 2012, for instance). Nevertheless, the present paper provides a tractable and simple framework to understand the role of unanimity in multilateral bargaining where precommitent is feasible.

The paper builds on the complete information strategic pre-commitment literature on bargaining starting with the seminal contributions of Schelling (1956). Crawford (1982) formalized some of Schelling's arguments in a bilateral bargaining framework with both strategic ex ante pre-commitment and ex-post revoking of commitments. He showed that with sufficiently low success probability of individual precommitments, both players make aggressive pre-commitments in the unique equilib-

[7] The seminal ideas were presented by Schelling (1956).

rium which is inefficient since the commitments are mutually incompatible.[8] Ellingsen and Miettinen (2008) show that impasse may be considerably more likely and inefficiency more severe if there is a small cost of commitment. Ellingsen and Miettinen (2014) generalize the results to a dynamic infinite horizon setting.[9] We generalize the Ellingsen and Miettinen (2014) model to a multilateral case.[10]

In our model, the success of the commitment is not based on an attempt to mimic a behavioral type and the time that it takes for an opponent to credibly screen the committing player's true type as in Myerson (1991); Abreu and Gul (2000); Compte and Jehiel (2002). Rather the commitment technology is exogenously given. Our modeling approach is thus considerably simpler. In a multilateral bargaining model with obstinate types, each player would have to track the beliefs about the obstinacy of each of the other players and these would have to be updated both based on the proposals and the rejections made. The optimal actions would then depend on these beliefs. Thus, the dimensionality of the model grows exponentially with the number of players. The simplicity and tractability of the present complete information commitment model makes it scalable from a bilateral to a multilateral bargaining setting.

A key lesson in the existing multilateral bargaining literature is that when players' valuations are heterogeneous, those who need to be compensated the least will be included in the winning coalition. Moreover under unanimity players with lowest valuations, will be monetarily compensated to buy them into the agreement. This theoretical idea receives empirical support in the experiment by Miller et al. (2018). This also illustrates why signaling or mispresenting a private information about valuation (Tsai and Yang, 2010; Eraslan and Chen, 2014), or sending delegates with induced valuations higher or lower than those of the principal (Harstad, 2010) may pay off individually in these settings. Such mispresentation or delegation would result in inefficient delay.

In addition to the present paper, there are other complete information explanations for delay in multilateral settings. Efficient delay and inefficient immediate agreements may arise due to fluctuations over time in the total surplus which is being shared (Merlo and Wilson, 1995, 1998). Opposite to our results, Eraslan and Merlo (2002) show in bargaining with a stochastic surplus that unanimity rule is always efficient but majority rule may lead to inefficiencies: a proposer may be better off buying a majority into an inefficient agreement than passing and waiting for an efficient realization of the pie. In non-stochastic environments and focusing on stationary equilibria, inefficient delay may arise if the principal or proposer negotiates with others sequentially one at a time or in smaller groups (Cai, 2000; Iaryzcower and Oliveros, 2019) or if there are several simultaneous offers at each round and thus free-riding among proposers (Kosterina, 2019). Yildirim (2007, 2018) show that there can be inefficiencies due to endogenous recognition probabilities. Ali (2006) generalizes the analysis of effects of optimism on delay in dynamic bargaining (Yildiz, 2003) to multilateral settings and shows that, unlike in bilateral negotiations, persistent optimism may lead to delay in multilateral settings if unanimity decision making is applied.

The paper is structured as follows. Section 2 presents the model. 3 analyzes the simplest multilateral case of three players. Section 4 has the general analysis and the main results. Section 5 concludes.

[8] A number of authors have investigated the evolutionary dynamics of aggressive commitments in bilateral bargaining. See Ellingsen (1997); Poulsen (2003).

[9] See also Muthoo (1996); Levenotoğlu and Tarar (2005) and Güth et al. (2004).

[10] Binmore (1985) and Morelli (1999) formulate three-player bargaining models where players make demands on prices at which they are willing to join a winning coalition. In Morelli's model there is no proposer advantage; in our model proposer advantage arises when sufficiently many demands fail. In all analysis we restrict attention to the case of exogenous status quo (see Eraslan et al. (2020) for a review on the endogenous status quo case).

2 The model

2.1 The negotiation game

The bargaining game, denoted by G, has infinite horizon. There are n players in the game. In the main analysis we assume that players have identical preferences and technologies. A player's utility is assumed to be linear in the player's share of the surplus. Players are impatient, with per period discount factor δ. The size of the surplus is normalized to one. The rules of the game, the parameters of the model and the rationality of the players are common knowledge.

In each period, indexed by t, actions are taken in two stages – the commitment stage and the bargaining stage. At the commitment stage, players can attempt making short-lived commitments which last at most the current period, i.e. each player i chooses a commitment attempt $x_i \in (0, 1]$ or stays flexible, which we denote by $x_i = 0$. In between the two stages, commitments may decay, each with an independent probability $1 - \rho$. (We will sometimes refer to this event as a player having a "loophole".) The probability that a commitment "sticks" is ρ. The realization of the attempt, the commitment status, is denoted by s_i and, given the technology, it equals x_i with probabiliy ρ and 0 with probability $1 - \rho$.

At the bargaining stage, each player becomes the proposer with probability $1/n$ (each player has an equal recognition probability), in which case that player's commitment loses its strength. I.e. we assume for simplicity that the proposer always has a loophole. With probability $(n - 1)/n$, a player becomes a responder. The proposer proposes a deal $d = (d_1, ..., d_N)$, with $\sum_{i=1}^{N} d_i \leq 1$, where we refer to d_i as the "offer" made to player i. Each player then votes to *accept* or *reject*. Yet, a player i with commitment status $s_i > d_i$ will automatically reject the proposal. The proposed deal is implemented if at least q players (including the proposer) vote to accept. If not, a new period begins with the commitment stage.

If a deal d is implemented in period t, player i's payoff equals $\delta^{t-1} d_i$. We assume that players lexicographically prefer to receive a given share *without* attempting to commit. Substantively, this means that commitment is negligibly costly. Even more drastic commitment costs could be modelled but at the expense of more complicated exposition of the model.

The assumption that each period is divided into a commitment stage and a bargaining stage closely matches Crawford (1982) and Ellingsen and Miettinen (2008, 2014). Indeed, the basic assumption of this literature is that negotiators always have the opportunity to make unilateral commitments before they sit down to engage in bilateral talks. In addition to the number of number of parties in the negotiations, the key difference with respect to the durable commitment model of Ellingsen and Miettinen (2014) is that in the present model commitments endure at most for the present period only. In that sense, the model is reminiscent to Miettinen and Perea (2015). This assumption is made to simplify analysis: there are no commitment states to be kept track of and thus we use simply the stationary subgame perfect equilibrium rather than Markov perfect equilibrium as our solution concept.

2.2 Histories, strategies and equilibrium

A history h_t consists of a sequence of commitment attempts, stochastic commitment status randomizations (determining whether each attempt succeeds or fails) and recognitions of the proposer, proposals and responses at consecutive periods, and the two stages of each period. A behavioral strategy ψ_i of player i is a collection of randomizations of actions of player i, one for each history of the game. A behavioral strategy profile ψ is an n-tuple of strategy profiles, one for each player $i = 1, ..., n$. A subgame conditional on history h_t is denoted by $G(h_t)$ and $\psi | h_t$ is a strategy profile of the game, consistent with history h_t, and thus a strategy in the subgame $G(h_t)$.

A subgame perfect equilibrium is an n-tuple of strategy profiles which is a Nash equilibrium in every subgame, at each history h_t. A stationary subgame perfect equilibrium is a subgame perfect equilibrium where (i) $\psi|h_t$ coincides across h_t at any commitment stage, (ii) proposals at the bargaining stage coincide across histories where the current commitment status profiles coincide, and (iii) voting responses following a proposal depend only on the current proposal s, the identity of the proposer, and the current commitment status profile. In a *Symmetric Stationary Subgame Perfect Equilibrium*, commitment attempts coincide for all players at the commitment stage, and bargaining stage proposal and voting strategies coincide for players with identical commitment statuses. We consider *Stationary Subgame Perfect Equilibria* (SSPE) of the game and when necessary focus on the *symmetric* SSPE. The restriction to symmetric equilibria is clearly indicated whenever we do so.

Let $x^* = (x_1^*, ..., x_n^*)$ be a vector of commitment attempts that is part of a SSPE strategy profile ψ^*. And let $v^* = (v_1^*, ..., v_n^*)$ be the vector of expected utilities associated with that equilibrium. Then the equilibrium strategies specified by ψ^* in the *bargaining stage*, given any commitment status profile $(s_1, ..., s_n)$ satisfies the following conditions (see Eraslan and Evdokimov (2019)):

- Define $\hat{x}_i = max\{\delta v_i^*, s_i\}$ as player i's "price", where s_i is the player's current commitment status. In the negotiation stage, player i votes to accept at any history iff she is offered $d_i \geq \hat{x}_i$.
- If chosen to propose, player i offers $\hat{x}_i$ to the "cheapest" coalition C_i consisting of $q-1$ responders other than i (possibly randomizing if the cheapest coalition C_i is not unique), provided that $1 - \sum_{j \in C_i} \hat{x}_j > \delta v_i^*$. Otherwise, he makes a proposal that fails.

Define $\pi_i(x|\psi^*)$ as the expected (instantaneous) utility that *would* be achieved by player i if, at a given commitment stage, (i) players attempt commiting to x (not necessarily x^*), and (ii) all players followed the equilibrium strategy ψ^* starting at the immediately ensuing bargaining stage. This payoff is pinned down by the conditions already outlined for the bargaining stage and by the additional condition that, in case of failure, player i's EU is given by δv_i^*. (The formal details are cumborsome to express explicitly but will be clearly developed in the subsequent analysis.) Then the commitment attempts x^* satisfy the following:

$$x_i^* = \arg\max \pi_i(x_i, x_{-i}^*|\psi^*)$$

with a lexicographic preference for staying "flexible" ($x_i = 0$), as explained above. Finally, the equilibrium expected utilites v^* satisfy

$$v_i^* = \pi_i(x^*|\psi^*).$$

3 The case of three players

In this section, we illustrate our main results within a simple three-player model. A more general analysis for n players is presented in Section 4. We consider unanimity and majority rules. In both cases, we characterize the set of symmetric SSPE in which all players make the same commitment attempt x^*. In the three player game, these commitments can conceivably be of two types. A "moderate" commitment is such that a deal can be reached even if neither of the two responders has a loophole. An "aggressive" is such that a deal is possible only if (at least) one responder has a loophole. Therefore, aggressive commitments are associated with a positive probability of inefficient delay. We begin by analyzing the unanimity case and first characterizing the "aggressive" equilibrium in subsection 3.1 and then turning to the efficient equilibrium in subsection 3.2. Finally, we analyze the majority rule in subsection 3.3.

3.1 Aggressive equilibrium

Suppose that a symmetric stationary equilibrium with aggressive commitments exists, and denote the (common) expected equilibrium payoff by v^*. Due to stationarity, each player's continuation value is δv^*. Then the largest commitment to which a proposer will concede in the event that exactly one responder's commitment "sticks" in the negotiation stage is given by

$$x^* = 1 - 2\delta v^*. \tag{1}$$

This commitment is targeted to leave both the proposer and the uncommitted responder indifferent between a deal and continuation: they each receive the continuation value δv^*, and the player who is the only one to succeed with her commitment receives the residual, i.e. her commitment $1 - 2\delta v^*$. Since agreement occurs only when at least one responder has a loophole, this equilibrium involves a positive probability of delay and therefore inefficiency. Concretely, the probability of agreement in a given round is $1 - \rho^2$. Conditional on agreement, the sum of payments is one. Therefore, the expected discounted sum of utilities is $\sum_i v^* = \frac{1-\rho^2}{1-\delta\rho^2}$, and so

$$v^* = \frac{1-\rho^2}{3(1-\delta\rho^2)}$$

is the expected equilibrium payoff for each player. The following proposition establishes that this equilibrium always exists in the three-player game under unanimity.

Proposition 1. *In the three-player game under unanimity rule, there always exists an equilibrium with aggressive commitments and delay. Moreover, this is the only symmetric and stationary equilibrium when $\rho < 1/2$.*

In this equilibrium, three things can happen with positive probability in a given round. Suppose w.l.g. that player 1 is proposer. If one responder has a loophole (say, player 3) then player 1 proposes $d = (\delta v^*, x^*, \delta v^*)$ (and symmetrically if player 2 has a loophole). If instead *both* responders have a loophole, player 1 offers $d = (1-2\delta v^*, \delta v^*, \delta v^*)$. Note that we can write $d = (\delta v^* + (x^* - \delta v^*), \delta v^*, \delta v^*)$, reflecting the fact that, in the event of an "extra" loophole, the proposer secures an additional "chunk" of size $(x^* - \delta v^*)$. We will encounter this type of logic again in what follows. Finally, if both responders are committed, player 1 makes any offer where at least one of the responders receives less than the committed share and thus the game proceeds to round $t + 1$. Thus the equilibrium is inefficient and exhibits costly delay. The inefficiency is exhibited in the individual equilibrium payoff, which is strictly below 1/3.[11] Equilibrium payoffs increase and commitments become less aggressive if players discount less (δ increases) or they are less likely to succeed with their commitments (ρ decreases).

Let us now verify that the commitment profile (1) constitutes part of an equilibrium. The discounted present value of utility in equilibrium equals v^*. In a stationary equilibrium all flexible players will renew their attempt to commit to x^* in the follow-up round. Thus the continuation value of a flexible responder at the bargaining stage equals δv^*. Notice first that (1) can be written

[11] The expression closely resembles the unique and inefficient Markov-perfect equilibrium payoff of (Ellingsen and Miettinen, 2014): in the present formula a factor δ is absent in the numerator since there is no discounting between the commitment stage and the bargaining stage and thus there is no exogenously imposed delay in the present model. Moreover, in the present model the proposer's commitment automatically fails and thus the endogenous delay is only driven by the commitment attempts of the two responders and thus the endogenous expected delay coincides in the bilateral model and the three-player version of the multilateral model. Naturally, what is left, once the endogenous delay is accounted for, will be split evenly, ex ante, between the three parties who have identical recognition and commitment success probabilities and who adopt identical strategies.

as $\delta v^* = 1 - \delta v^* - x^*$. Thus the proposer is indifferent between inducing disagreement, on the one hand, and proposing $(\delta v^*, x^*, \delta v^*)$, on the other hand, and thus conceding to the aggressive demand of a single committed responder. On the same grounds, a responder who is uncommitted is indifferent between accepting and rejecting the proposal.

Consider then deviations at the commitment stage. A commitment matters only if the player is drawn to respond (recall that the commitment is automatically relaxed if one is drawn to propose). Since all other players attempt commitment, deviating and not committing would automatically imply that the highest share this player can receive conditional on a deal (and a responder role) equals δv^* which is lower than the share one receives in equilibrium conditional on being responder whose commitment sticks and the other responder having a loophole, and equal in every other contingency. Thus the deviation not to commit $(x = 0)$ does not pay off.

Deviating by choosing a *more* aggressive commitment does not pay off since conceding to such a commitment would result in a payoff lower than δv^* and hence no proposer would ever concede when such a commitment sticks. A deviation down to a *less* aggressive commitment can only increase the payoff if it increases the probability of a deal in the ensuing bargaining stage. In equilibrium, no deal will be reached with probability ρ^2, i.e. in case both responders remain committed to x^*. To induce a deal in this case, a deviating responder would have to choose a less aggressive commitment position y such that $\delta v^* \leq 1 - x^* - y$. Otherwise, conditional on both commitments succeeding, the proposer would want to make a proposal that fails. Thus $y \leq 1 - \delta v^* - x^* = \delta v^*$ and the deviation does not pay off.

3.2 Efficient equilibrium

Suppose now that there exists a symmetric stationary equilibrium in which immediate agreement is certain - i.e. occurs even if both responders' commitments "stick". As above, denote the (common) expected equilibrium payoff by v^*. Then the largest commitment to which a proposer will concede in the event that both one responder's commitments "stick" is given by

$$x^* = \frac{1 - \delta v^*}{2}. \tag{2}$$

This commitment is targeted to leave the uncommitted proposer indifferent between a deal and continuation while allowing two reponders who succeed to each have their commiment shares. Since $x^* - \delta v^* = (1 - 3\delta v^*)/2 \geq 0$, each responder's commitment share is greater than her continuation value δv^*. The proposal where the committed responders receive x^* and the proposer receives δv^* will thus be passed, and the agreement is immediate even in the event that both responders succeed in committing. This equilibrium is thus efficient. The following proposition establishes that this equilibrium exists if and only if $\rho \geq 1/2$.

Proposition 2. *In the three-player game under majority rule, an efficient equilibrium exists if and only if* $\rho \geq 1/2$.

If either of the responders or both fail to commit, however, the proposer offers each such responder the continuation value, thus allowing the proposer to receive $\delta v^* + k(x^* - \delta v^*)$ where k is the number of responders with a loophole. The fact that there are positive probability events where the proposer receives more than her continuation value creates an opportunity to deviate upwards at the commitment stage. Speficically, players may increase their commitment by $(x^* - \delta v^*)$ in an attempt to appropriate the extra "chunk" of size $(x^* - \delta v^*)$ that otherwise accrues to the proposer in the event that their commitment succeeds and the other responder has a loophole.

Such a deviation y will leave the proposer indifferent between continuation and passage in the event that one responder has a loophole, i.e. it satisfies $\delta v^* = 1 - y - \delta v^*$, or $y = 1 - 2\delta v^*$. As before, the deviation matters only if the deviating player is drawn to respond and the commitment succeeds. Conditional on this event, the deviating player will gain $(x^* - \delta v^*)$ if the other responder has a loophole (probability ρ), and lose $(x^* - \delta v^*)$ in case the other responder does not have a loophole (probability $1 - \rho$). Therefore, this deviation does not pay off and the efficient equilibrium exists if and only if $\rho \geq 1/2$.

3.3 Majority rule

Proposition 2 illustrates that aggressive commitment tactics potentially lead to delay, inefficiency and asymmetric deals in multilateral bargaining much the same way as in bilateral bargaining (Ellingsen and Miettinen, 2008, 2014). In fact, in Section 4.4 below will show that the equilibrium with delay is the only symmetric stationary stationary equilibrium if and only if $\rho < 1/2$ and thus inefficiencies necessarily arise in that case. Yet, the result hinges upon a key condition – that of unanimity. Only when mutual consent is needed from all parties can the players force concessions from others in equilibrium. Proposition 3 below shows that all stationary subgame perfect equilibria are efficient under majority rule, as in the original Baron and Ferejohn (1989) model.

Proposition 3. *In the three-player game under majority rule, there exists a unique symmetric SSPE involving no commitments.*

The proof is omitted since a general proof for the $n-$player case is provided in Subsection 4.1. The intuition for this result is simple. Under majority rule, responders compete for being included into the winning coalition. If no player attempts to commit, any player who deviates to a commitment above the common continuation value would simply be left out of any coalition. To see uniqueness, suppose instead that all players commit to some $x^* > 0$. Then $x^* \in (\delta v^*, 1 - \delta v^*)$, as otherwise players would prefer not to commit. The only event in which a player's commitment matters for her payoff is when she is responder and both commitments stick. In this event, she is included in the winning coalition with a probability strictly less than one. Then an arbitrarily small downward deviation to $x^* - \epsilon$ results in her being included with probability one, and has no effect in other events. A typical Bertrand competition argument then shows that commitments are competed down to a level where they have no effect on payoffs, and thus remaining flexible is in fact preferred. Thus the standard Baron & Ferejohn equilibrium outcome is the unique equilibrium outome in this case.

4 General analysis

We now turn to a more general analysis of the n-player game. To start, section 4.1 considers arbitrary $q-$majority rules with $q < n$, and shows that Proposition 3 generalizes, i.e. that there is no inefficienct SSPE in that case. We will then turn to the unanimity rule. As we will see in that case, symmetric equilibria can be ordered in terms of the "aggressiveness" of their associated commitments. In subsection 4.3 we analyze the most aggressive and inefficient symmetric equilibrium, and show that it always exist. In subsection 4.4, we characterize the unique efficient symmetric equilibrium, and show that it exists only if ρ is sufficiently large with respect to the number of players. In subsection 4.5 we consider all other ("intermediate") equilibria, which are more efficient than the most aggressive equilibrium but still not efficient. Finally we analyze how the frequency of negotiation rounds affects the predictions in subsection 4.6.

4.1 Commitment and q-majority rules

Let us begin our analysis by showing that Proposition 3 generalizes: commitments are never used and thus the agreement is immediate when adopt any supermajority rule (apart from unanimity) is used.

Theorem 1. *There is no inefficient SSPE under any q-majority rule with $q < n$.*

Proof. W.l.g. order players such that $0 \leq x_1 \leq ... \leq x_n$. For all i, if $x_i > 0$ then $x_i > \delta v_i$ due to the lexicographic preference. Suppose there exists an i such that $x_i > x_q \geq 0$. Then player i is never included in the winning coalition when he is a responder and his commitment sticks. Therefore, he is better off not committing. Therefore $x_i = x_q$ for all $i \geq q$. Suppose that $x_q > x_{q-1} \geq 0$. Take any $i \geq q$. Since $x_i = x_q > 0$, we have $x_q > \delta v_i$. Then the only event where player i has a strictly positive chance of receiving x_q is when all $i \geq q$ are responders whose commitments stick. Moreover, there exists at least one such player i who is included with a probability smaller than or equal to $1/(n-q+1) < 1$ in this event. By deviating to $x_q - \varepsilon$ for ε arbitrarily small, this player will be included with probability one in that event. A contradiction. Therefore $x_{q-1} = x_q = ... = x_n$. Suppose that $0 \leq x_{q-k-1} < x_{q-k} = ... = x_n$ where $k \in \{1, ..., q-2\}$. Take any $i \geq q-k$. Since $x_i = x_{q-k} > 0$, we have $x_{q-k} > \delta v_i$. Then the only event where player i has a strictly postive chance of receiving x_{q-k} is when among the $n-(q-k-1)$ players making expensive commitents, there are at least $n-q+1$ responders whose commitments stick. Moreover, there exists at least one such player i who is included with a probability smaller than or equal to $1/(n-q+1) < 1$ in that event. By deviating to $x_{q-k} - \varepsilon$ for ε arbitrarily small, this player will be included with probability one in that event. A contradiction. Therefore, it follows $x_n = ... = x_1 = x$. Finally, suppose $x > 0$. Then an analagous argument establishes that at least one player must have an incentive to deviate to $x - \epsilon$. Therefore $x = 0$. □

The intuition for this result is highlighted in Proposition 3 of the three-player example of Section 3. When majority rule is used, there is competition between the responders to be included in the winning coalition. If commitments differ, those who attempt the largest commitments will be left out. If several responders commit in a way such that they are included with positive probability, at least one will have an incentive to "undercut" another's commitment in order to be included more often. This triggers a Bertrand-like competition in commitments. In equilibrium, the commitment cannot exceed the continuation value, which is what each responder receives without commitment anyway (Baron and Ferejohn, 1989). Thus, no aggressive commitments are made. The outcome is efficient and coincides with that of the Baron and Ferejohn (1989) model. The analysis starting from the subsection shows that, contrary to supermajority decision making, commitment strategies are used in any equilibrium and inefficiency due to delay is often unavoidable.

4.2 Symmetric commitment profiles under unanimity decision making

Let us now turn to the analysis of unanimity decision making. In the three player case, we saw that two types of commitments are conceivable. Namely, those which require no loopholes (the modest equilibrium), and those which require one responder to have a loophole (aggressive equilibrium) for an agreement to occur. In both cases the commitment is crafted to make a certain number of players with a loophole (including the proposer) indifferent between passing the agreement and continuation. Conditional on an agreement, a responder whose commitment fails loses a chunk $(x^* - \delta v^*)$, which goes to the proposer. These patterns generalize.

When there are fewer than the targeted number of players with a loophole, any agreement will be vetoed and all players earn δv^*. When there are exactly the required number of loopholes, the proposer earns exactly the continuation value δv^* and each succesful responder earns $(x^* - \delta v^*)$ more than that. When there are more than the required number of loopholes, the proposer receives a chunk $(x^* - \delta v^*)$ from each responder with a loophole beyond the required number. The commitment profiles across equilibrium candidates differ, not only in terms of how many loopholes are required, but also in terms of how large a commitment is made. When more loopholes are required, there are more players earning their commitment value and thus fewer players earning the additional $(x^* - \delta v^*)$. Therefore the commitment must be larger when the number of required loopholes is larger. Since the successes of commitments are indepdendent across players, it also takes longer to reach an agreement and thus the common continuation value δv^* must be smaller, further contributing to the share that the committed reponders are attempting to appropriate. Therefore, the efficiency of the equilibrium is inversely related to the number of loopholes required, on the one hand, and to how large a share of the pie each player commits to, on the other hand.

We will begin by introducing some formalism needed in the following subsections. Let $x^*_{n,h}$ be a symmetric commitment profile when there are n players and at least h loopholes are required in order for an agreement to arise. Denote the symmetric equilibrium value by $v^*_{n,h}$. A successful commitment thus equals $s = x^*_{n,h}$. Suppose that, at the commitment stage of each round, all players attempt committing to $x^*_{n,h}$. Then if, at the bargaining stage, fewer than h responders have a loophole, then all players are put down to their continuation value $\delta v^*_{n,h}$. If, on the other hand, the number of flexible responders is at least h, all agents with a loophole will be offered $\delta v^*_{n,h}$, while those without will be offered $x^*_{n,h}$.

As in the case of three players above, the commitment $x^*_{n,h}$ is chosen so as to make the proposer just indifferent between making a propsal that fails and offering $x^*_{n,h}$ to $(n-h-1)$ committed responders when exactly h responders have a loophole. That is,

$$\delta v^*_{n,h} = 1 - (n-h-1)x^*_{n,h} - h\delta v^*_{n,h}. \tag{3}$$

Note that equation 3 also implies that the h uncommitted responders are indifferent between accepting and rejecting the proposal. Solving for $x^*_{n,h}$ yields

$$x^*_{n,h} = \frac{1-(h+1)\delta v^*_{n,h}}{n-h-1}, \tag{4}$$

showing that, if exactly h reponders have a loophole, then the $n-h-1$ responders each get an equal share of what is left after the proposer and those who have a loophole are paid the continuation value. It is easy to see that when $n = 3$ and $h = 1$ the formula coincides with that in the three-player case above.

In order to learn more about $x^*_{n,h}$ and $v^*_{n,h}$, we need to write explicit expressions of $v^*_{n,h}$. Let us first look at the proposer's expected payoff. Denote the probability that at least h *of* k *responders* will have a loophole by $\eta(k,h)$. That is

Definition 1. Let $f(k,l) = \begin{pmatrix} k \\ l \end{pmatrix} (1-\rho)^l \rho^{k-l}$ be the probability of exactly l loopholes out of k trials. Note that f is the pdf of a binomial probability distribution. Then

$$\eta(k,h) = \sum_{l=h}^{k} f(k,l).$$

Currently, we are interested in the case where $k = n-1$, which is the number of responders. Then we can think of the proposer paying all $(n-1)$ responders $x^*_{n,h}$ and getting back a "chunk" $(x^*_{n,h} - \delta v^*_{n,h})$

from each responder that has a loophole, as long as their number is at least h. Her expected payoff is therefore $\pi_P = (1-\eta(n-1,h))\delta v^*_{n,h} + \eta(n-1,h)\left[1-(n-1)x^*_{n,h} + E(l|h \leq l \leq n-1)(x^*_{n,h} - \delta v^*_{n,h})\right]$ Combined with equation 3, this yields

$$\pi_P = \delta v^*_{n,h} + \eta(n-1,h)\left[E(l|h \leq l \leq n-1) - h\right]\left(x^*_{n,h} - \delta v^*_{n,h}\right)$$

That is, the proposer's expected payoff is equal to the common continuation value (which he gets in case no agreement occurs) plus the additional surplus he expects to secure in case of an agreement. The latter equals the expected number of loopholes in excess of the necessary h, multiplied with a chunk of the pie that each such responder with a loophole loses by failing with the commitment.

Next think about the expected payoff conditional being drawn to respond (which happens with probability $(n-1)/n$). If a responder has a loophole (probability $1-\rho$), she will receive her continuation value no matter whether an agreement will be reached or not. If she does not have a loophole, (probability ρ), then she will receive her commitment if at least h of the *other* $(n-2)$ *responders* have a loophole (probability $\eta(n-2,h)$), and otherwise her continuation value. Therefore her expected utility is given by

$$\pi_R = \delta v^*_{n,h} + \rho\eta(n-2,h)\left(x^*_{n,h} - \delta v^*_{n,h}\right)$$

To sum up, players expected payoff when all players attempt a commitment to $x^*_{n,h}$ in the commitment stage can be written as

$$v^*_{n,h} = \frac{1}{n}\pi_P + \frac{n-1}{n}\pi_R \tag{5}$$

We can now solve equations (3) and (5) to yield an explicit expressions for $v^*_{n,h}$and $x^*_{n,h}$(proof in Lemma 3 in the appendix)

$$v^*_{n,h} = \frac{1}{n}\frac{\eta(n-1,h)}{1-\delta\left(1-\eta(n-1,h)\right)},$$

which reflects the fact that the expected total payoff in the game comes from 1 dollar being paid out randomly at some point, with probability $\eta(n-1,h)$ in each period, i.e. the probability that it will be paid at a given period t is $\eta(n-1,h)(1-\eta(n-1,h))^{(t-1)}$ and so the expected total payoff is $\eta(n-1,h)\sum_{t=1}^{\infty}[(1-\eta(n-1,h))\delta]^{(t-1)}$, which when divided by n gives $v^*_{n,h}$ above.

Clearly, generally $\eta(n-1,h) < 1$ and thus $v^*_{n,h} < 1/n$ and therefore, if these strategies constitute an equilibrium, the equilibrium is inefficient. Moreover, the higher is h, the smaller is $\eta(n-1,h)$ and so is also $v^*_{n,h}$. Thus, symmetric commitment profiles which require more loopholes for a deal to arise are also more inefficient. Yet, $x^*_{n,h}$ increases with h and thus, conditional on succeeding with one's own commitment, the earned share when the deal arises is larger the higher is h. In this sense commitment profiles with higher h generate longer conflict duration, greater inefficiency and greater asymmetries in the shares that the parties receive conditional on reaching an agreement. Thus the commitment profile candidates can be very naturally ordered from the most aggressive one $h = n-2$ to the least aggressive one $h = 0$.

None of what we have said thus far proves that these candidate commitment profiles actually constitute equilibria. In the remainder of this section, we will consider the conditions under which this is the case. We begin by considering the most aggressive, then the least, and finally intermediate equilibria.

4.3 Aggressive equilibrium

Let us begin with considering the most aggressive symmetric commitment profile,

$$x^*_{n,n-2} = 1 - (n-1)\delta v^*_{n,n-2},$$

where each player aims to extract the entire surplus (beyond the continuation value) from the other players. If this is an equilibrium, the expected payoff received by each player equals

$$v^*_{n,n-2} = \frac{1}{n}\frac{\eta(n-1,n-2)}{1-\delta\left(1-\eta(n-1,n-2)\right)},$$

where the probability than an agreement arises in each period, $\eta(n-1,n-2) = (n-1)\rho(1-\rho)^{n-2} + (1-\rho)^{n-1}$ is very small if n and ρ are large. In that case, the expected delay at this profile is long thereby severly undermining efficiency.

In order to check that this is an equilibrium, we must verify that no player wishes to deviate at the commitment stage. Note that the agreement requires that there are at least $h = n-2$ loopholes among the $n-1$ responders. Hence, agreement occurs only in two cases: (i) only one of the commitment attempts succeeds or (ii) none of the commitment attempts succeeds. In both cases, $n-1$ agents will get exactly the continuation value, and the residual, $1-(n-1)\delta v^*_{n,n-2}$, is secured either by the committed responder (note that x^*_{n-2} is exactly the residual) or by the proposer. The key to understanding the effects of deviations, it will be useful to note that deviating to any $y \neq x^*_{n,n-2}$ affects the deviator's payoff only if (a) she is drawn to respond and (b) her commitment sticks. Therefore, the entire analysis that follows focuses on the payoffs achieved only in that event.

Let us first consider an upward deviation to $y > x^*_{n,n-2}$. In that case the proposer will not want to pay the deviator even in the most favorable instance where everyone else has a loophole; so a more aggressive commitment cannot be profitable. Consider then a deviation to less aggressive commitments, $y < x^*_{n,n-2}$. Since the payoff achieved conditional on commitment success and agreement would then be lower, such a deviation can only be beneficial if the probability of an agreement is increased. Therefore, the deviation must have the property that the proposer will concede to it in cases where the deviator's own commitment attempt as well as $k \geq 1$ others succeed. The largest commitment that will be met when (at most) *one* additional person's commitment sticks is such that the proposer is left with her commitment value after giving $x^*_{n,n-2}$ to one succesful responder, y to the deviator and $\delta v^*_{n,n-2}$ to the other responders, i.e.

$$\delta v^*_{n,n-2} = 1 - x^*_{n,n-2} - y - (n-3)\delta v^*_{n,n-2}$$

But substituting $x^*_{n,n-2} = 1-(n-1)\delta v^*_{n,n-2}$ we see that this boils down to

$$y = \delta v^*_{n,n-2}.$$

Therefore in the only event where the commitment matters, the deviator's payoff drops to the continuation value $\delta v^*_{n,n-2}$, whereas if she stays with the equilibrium demand, she will get $x^*_{n,n-2}$ in case all of the other responders have a loophole.

It's clear that the argument can be extended to say that for all $k > 1$, there is also no profitable commitment that would be met if my own and k additional commitments stick, i.e. deviations of the type

$$y = 1 - kx^*_{n,n-2} - (n-1-k)\delta v^*_{n,n-2},$$

since *a fortiori* these commitments would be strictly *smaller* than the continuation value. This is enough to establish

Theorem 2. *Under unanimity rule, the most aggressive symmetric commitment profile, in which at least $n-2$ responder loopholes are required for agreement to be reached, always constitutes a SSPE.*

Let us then analyze the comparative statics of this most aggressive equilibrium. In particular, we are interested in understanding how the duration of the conflict and the fraction of the surplus demanded are affected by the number of players in the game, n, and the the effectiveness of commitments, ρ. Recall that the commitments are independently and identically distributed. A loophole arrives randomly with probability $(1-\rho)$ to each player at each round. Thus comparative statics with respect to ρ is obvious: delay increases in ρ. The number of loopholes at a given round among the $n-1$ responders follows the binomial distribution.[12] The number of required loopholes $n-2$ increases with the number of players n. It is thus intuitive that the duration of the conflict increases with the number of players.

Moreover, since the duration of conflict increases with n, the expected arrival date of the deal also increases with n and thus the the equilibrium payoff, $v^*_{n,n-2}$, and the continuation value, $\delta v^*_{n,n-2}$, which is allocated to each flexible responder when the deal is done, decreases with n. Therefore, the fraction of the pie that the deal allocates to the unique successful player, $x^*_{n,n-2}$, and thus the difference of the final payoffs, $x^*_{n,n-2}-\delta v^*_{n,n-2}$, increases as the number of players increases.

Corollary 1. *Consider the most aggressive commitment equilibrium with profile $x^*_{n,n-2}$.*

- *The duration of conflict increases as n or ρ increases.*
- *The equilibrium payoff, $v^*_{n,n-2}$,decreases as n or ρ increases.*
- *The commitment share, $x^*_{n,n-2}$, increases as n or ρ increases.*

Proof. The duration of conflict is inversely related to the probability of agreement at a given date. This is $\eta(n-1,n-2)=(n-1)\rho(1-\rho)^{n-2}+(1-\rho)^{n-1}$. It is straightforward to show that this is decreasing ρ and that that $\eta(n-1,n-2)>\eta(n,n-1)$ for $n>1$. The other two points follow immediately. □

4.4 Efficient equilibrium

Now consider the opposite extreme, a commitment profile requiring no loopholes, i.e. $h=0$. Since no loopholes are required, this commitment profile constitutes an efficient equilibrium if or when it exists. When $h=0$ and $v^*=1/n$, the optimal commitment characterized in equation (4) yields,

$$x^*_{n,0}=\frac{1}{n-1}\left[1-\frac{\delta}{n}\right]. \tag{6}$$

That is, in the event that all responders succed with their commitment, the $n-1$ responders are sharing what's left after the proposer is permitted to keep $\frac{\delta}{n}$. But when there are loopholes, the proposer will secure a surplus, implying a potential incentive to engage in more aggressive commitments.[13]

Consider now a deviation to a more aggressive commitment $y>x^*_{n,0}$. Recall that such a deviation affects the deviator's payoff only conditional on the deviator being drawn to respond and her commitment succeeding. Since $x^*_{n,0}$ was such that the proposer was permitted to keep exactly the

[12] This approaches the normal distribution with mean $(n-1)(1-\rho)$ and standard deviation $\sqrt{(n-1)(1-\rho)\rho}$ as n tends to infinity.

[13] In fact, it is straightforward to notice that for any efficient equilibrium candidate with $x^*_{n,0}<\frac{1}{n-1}\left[1-\delta v^*_{n,0}\right]$, there is a profitable upward deviation which extracts rents from the proposer without risking the agreement. Therefore, the only effcient equilibrium candidate satisfies (6).

continuation value, $\delta v^*_{n,0}$, the upward deviation must have the property that it will be met only if at least $k \geq 1$ of the *other* responders have a loophole. Then the most aggressive commitment that will be met with exactly k loopholes is $y_k = 1 - (n-2-k)\, x^*_{n,0} - (1+k)\, \delta v^*_{n,0}$. To see this, note that $(n-2-k)$ committed responders will get x^*_0, the deviator will get y_k, and the proposer and k flexible players get $\delta v^*_{n,0}$. Substituting (6) yields the candidate deviation

$$y_k = x^*_{n,0} + k\left(x^*_{n,0} - \delta v^*_{n,0}\right). \tag{7}$$

The equation states that the deviating player is increasing his demand by $k\left(x^*_{n,0} - \delta v^*_{n,0}\right)$, which is the extra residual surplus that can be captured from the proposer if k responders have a loophole. Let us consider the payoff consequences of such a deviation conditional on the deviator's commitment succeeding. In all cases where *fewer* than k of the other $n-2$ *other* responders have a loophole, agreement will fail and the deviating player will *lose* $\left(x^*_{n,0} - \delta v^*_{n,0}\right)$. This occurs with probability $1 - \eta(n-2,k)$. In all cases where *at least* k of the $n-2$ other responders get a loophole, the deviating player will *gain* $k\left(x^*_0 - \delta v^*_{n,0}\right)$. This occurs with probability $\eta(n-2,k)$. So a deviation aiming at $k \geq 1$ loopholes pays off if $\eta(n-2,k)\, k\left(x^*_{n,0} - \delta v^*_{n,0}\right) > (1 - \eta(n-2,k))\left(x^*_{n,0} - \delta v^*_{n,0}\right)$, which boils down to

$$\eta(n-2,k) \leq \frac{1}{k+1}.$$

It follows that a symmetric efficient equilibrium exists iff for all $k \in \{1, ..., n-2\}$, we have $\eta(n-2,k) \leq \frac{1}{k+1}$. This condition can be further simplified. Lemma 2 in the Appendix implies that whenever a deviation targeting $k = 1$ additional loopholes does not pay off, no larger deviation will pay off either. That is, if the condition $\eta(n-2,k) \leq \frac{1}{k+1}$ is satisfied for $k = 1$, then it is satisfied for all $k = \{1, ..., n-2\}$. Thus the efficient equilibrium exists if and only if the probability of having at least one out of $n-2$ loopholes is at most $1/2$, i.e. $1 - \rho^{n-2} \leq 1/2$.[14] Let us denote the maximal n which satisfies this existence condition by $\bar{n}$.

Theorem 3. *An efficient symmetric SSPE requiring no loopholes for agreement to be reached exists iff* $n \leq \hat{n} = 2 - \frac{\ln 2}{\ln \rho}$.

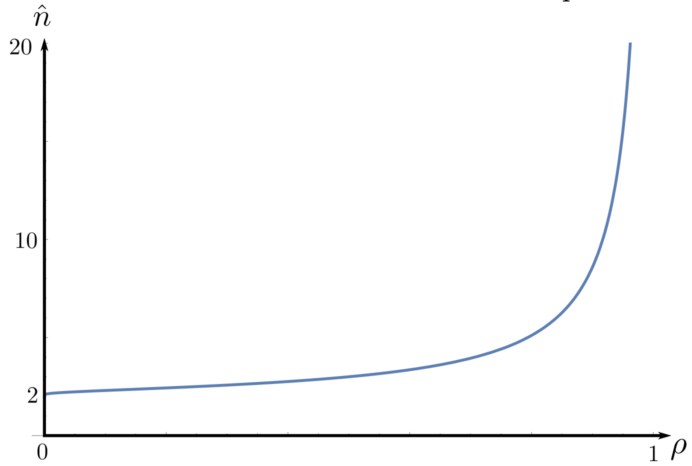
Remark that, as n increases, the probability that at least one loophole arises also increases, so that the efficient equilibrium will eventually be destabilized. Furthermore, the maximum n for which the efficient equilibrium exists, $\hat{n}$, is increasing in ρ and approaches infinity as ρ tends to 1. This is displayed in Figure 1. For $\rho = 0$, $\hat{n} = 2$, so the efficient equilibrium does not exist. However, $\rho = 0$ means that the commitments are ineffective. As ρ tends to 1, $\hat{n}$ tends to infinity. Remark also that $\bar{n}$ rapidly falls as we more down from ρ close to one. For example, $\hat{n}$ is small (less than 10) even for $\rho = 0.9$.

4.5 Intermediate equilibria

So far we have found that the most aggressive equilibrium exists independently of the parameter values. The efficient equibrium exists only if the probablity of a loophole or the number of players is sufficiently small. In between these two extremes, additional equilibria requiring an intermediate number of loopholes, $1 \leq h \leq n-3$, may exist. In this subsection we will characterize the full set of such equilibria and how it depends on the number of players and the probability of a loophole.

[14] Notice that that the condition for the existence of the efficient equilibrium does not depend on δ.

Figure 1: Maximum n such that the efficient equilibrium exists



Consider a profile with symmetric commitments targeting to at least $h \in \{1, n-3\}$ responders with loopholes. Using equations (4), such symmetic commitment is characterized by

$$x^*_{n,h} = \frac{1}{n-1-h}\left[1-(h+1)\delta v^*_h\right],$$

and the associated expected payoff, (3), equals

$$v^*_{n,h} = \frac{1}{n} \cdot \frac{\eta(n-1,h)}{1-\delta(1-\eta(n-1,h))}. \tag{8}$$

As above, we will now study whether and under which conditions this consititutes an equilibrium. In this intermedate case, we need to consider both upward and downward deviations. Both types of deviations affect the deviating player's payoff only if she becomes a responder and her commitment attempt succeeds. So like above we can conduct the analysis conditional on that event.

Let us begin by considering downward deviations, to less aggressive commitments. Such deviations have the property that they may be met even if strictly fewer than h responders have a loophole. The largest commitment y that will be met when there are at least $h-1$ responder loopholes is given by

$$\delta v^*_{n,h} = 1 - y - (n-1-h)x^*_{n,h} - (h-1)\delta v^*_{n,h}.$$

If we substitute the expression for $x^*_{n,h}$, we obtain $y = \delta v^*_{n,h}$. Therefore, just as in the most aggressive equilibrium, the deviator's payoff is reduced to the continuation value, and thus a deviation designed to make agreement possible with one fewer loopholes is not profitable. It is clear that the argument can be extended to say that for all $k > n-1-h$, there is also no profitable commitment that would be met when there are even fewer than $h-1$ loopholes, since *a fortiori* these commitments would be strictly *smaller* than the continuation value. This is enough to establish the following lemma.

Lemma 1. *Consider a symmetric commitment profile where all players commit to $x^*_{n,h}$ for some $h \in \{1, ..., n-1\}$. Then a unilateral downward deviation to any $y < x_{n,h}$ is not profitable.*

Next consider a deviation to a more aggressive commitment $y > x^*_{n,h}$. Such a deviation must have the property that, for some $k \geq 1$, it will be met if at least $h+k \geq h$ of the *other* responders have a loophole. I.e. the deviation is targeted to succeed when at least k loopholes occur *in addition* to the h required in equilibrium. As before, such a deviation makes a difference only in case the deviating player ends up being a responder and her own commitment attempt succeeds. In addition, we can restrict attention to events where at least h of the other $(n-2)$ responder commitments have a loophole since, if there were fewer, the payoff would be $\delta v^*_{n,h}$ in every case. In this contingency, the outcome will depend on how many of the remaining $(n-2-h)$ responders have loopholes. Then by the same reasoning leading up to equation (7), the most aggressive commitment y that will be met with k *additional* loopholes (i.e. $h+k$ in total) is of the form

$$y_k = x^*_{n,h} + k\left(x^*_{n,h} - \delta v^*_{n,h}\right).$$

To see whether such a deviation pays off, note again that we can condition on the event that (a) the deviating responder's commitment sticks and (b) at least h of the other $(n-2)$ commitments attempts fail. Then the tradeoff is as follows. In all cases where *fewer* than k additional loopholes appear (out of $n-2-h$ chances), the deviating player will *lose* $(x^*_{n,h} - \delta v^*_{n,h})$. This occurs with probability $1-\eta(n-2-h,k)$. In all cases where at least k additional loopholes occur, the deviating player will *gain* $k\left(x^*_{n,h} - \delta v^*_{n,h}\right)$. This occurs with probability $\eta(n-2-h,k)$. So a deviation aiming at $k \geq 1$ additional loopholes strictly pays off if and only if $\eta(n-2-h,k)k\left(x^*_{n,h} - \delta v^*_{n,h}\right) > (1-\eta(n-2-h,k))\,(x^*_{n,h} - \delta v^*_{n,h})$ or simply

$$\eta(n-2-h,k) > \frac{1}{k+1}. \tag{9}$$

This establishes that an equilibrium requiring $h \in \{1, n-3\}$ loopholes exists iff the reverse of (9) holds for $k = 1, ..., n-h-2$. To gain intuition, define $g = n-h$. Then, since the equilibrium requires at least $n-g$ loopholes, there are up to $(n-2)-(n-g) = g-2$ chunks of the pie of size $x^*_{n,h} - \delta v^*_{n,h}$ that accrue to the proposer in case more than $n-g$ loopholes appear. The player who deviates upwards is aiming to extract some number k of these chunks from the proposer, at the cost of higher probability of provisional impasse. There are up to $g-2$ variants of upward deviations which are ordered in terms of aggresiveness, k. Each of them generates a benefit of k times the size of the chunk times the probability of at least k additional loopholes $\eta(g-2,k)$. Each deviation labelled by k is also associated with a provisional loss of the probability of less than k additional loopholes $(1-\eta(g-2,k))$ times one chunk of size $x^*_{n,h} - \delta v^*_{n,h}$. The condition checks for each of these deviations, that the deviation does not pay off. It is a necessary and a sufficient condition since we verified above in Lemma 1 that a downward deviation is never profitable. The bargaining stage strategies are optimal in an obvious manner that we are familiar with from the existing literature (see subsection 2.2).

As in our analysis of the efficient equilibrium, the characterization can be simplified further. Lemma2 in the Appendix implies that whenever a deviation targeting $k = 1$ additional loopholes does not pay off, no larger deviation will pay off either. That is, if the condition $\eta(n-2-h,k) \leq \frac{1}{k+1}$ is satisfied for $k = 1$, then it is satisfied for all $k = \{1, ..., n-h-2\}$. Defining $h = n-g$ we have established that for any $g \in \{3, ..., n-1\}$, a symmetric equilibrium requiring at least $(n-g)$ loopholes exists iff $\eta\,(g-2,1) \leq \frac{1}{2}$, which can be written $g \leq 2 - \frac{\ln(2)}{\ln(\rho)}$ or equivalently $\rho > 2^{-\frac{1}{g-2}}$. Combining this insight with Theorems 3 and 2 yields our main result.

Theorem 4. *For any $h \in \{0, ..., n-2\}$, a symmetric equilibrium requiring h loopholes exists iff any of the following equivalent conditions hold:*

- $\eta\,(n-2-h,1) \leq \frac{1}{2}$
- $n \leq \bar{n}(h,\rho) \equiv h + 2 - \frac{\ln(2)}{\ln(\rho)}$
- $\rho > \underline{\rho}(h,n) \equiv 2^{-\frac{1}{n-2-h}}$.

The corresponding commitment profile involves $x^*_{n,h} = \frac{1-(h+1)\delta v^*_{n,h}}{n-h-1}$*, and the expected payoff is* $v^*_{n,h} = \frac{1}{n} \cdot \frac{\eta(n-1,h)}{1-\delta(1-\eta(n-1,h))}$.

In order to appreciate the substance of this result, we should emphasize that it is an if-and-only-if statement. For example, setting $h = 0$ yields that the most efficient equilibrium exists *only* if $n < \bar{n}(0, \rho)$ which of course coincides with the $\hat{n}$ already derived earlier in subsection 4.4. For $n > \bar{n}(0, \rho)$, a profile in which no player attempts to commit in a way that would cause delay does not constitute an equilibrium. Thus, the inefficiency is *not* just a consequence of coordination failure. Even if all players expected others not to commit, each individual player would have an incentive to deviate from such a profile.

Notice also that the expected period of reaching an agreement in an equilibrium with n players and $h > 0$ required loopholes equals $\sum_{t=0}^{\infty} t[(1 - \eta(n-1, h; \rho))]^{t-1}\eta(n-1, h; \rho) = 1/\eta(n-1, h; \rho)$. This is increasing in the number of required loopholes h and the strength of commitment ρ. Yet, it is decreasing in the number of players n, reflecting the fact that any given number of loopholes h is more likely to arise when there are more players. However, it is precisely this effect that makes deviations to more aggressive commitments more profitable, such that an increase in n will eventually destabilize an equilibrium with h required loopholes. For arbitrary n and ρ, the most efficient equilibrium that exists requires at least

$$\underline{h}(n, \rho) \equiv \begin{cases} n-2 & \rho \leq \frac{1}{2} \\ n-2+I\left(\frac{\ln(2)}{\ln(\rho)}\right) & \rho \in \left(\frac{1}{2}, 2^{-\frac{1}{n-2}}\right) \\ 0 & \rho \geq 2^{-\frac{1}{n-2}} \end{cases} \tag{10}$$

loopholes, where $I(\ln(2)/\ln(\rho))$ is the smallest integer larger than $\ln(2)/\ln(\rho)$. Thus, for $\rho < \frac{1}{2}$, only the most inefficient equilibrium exists. As ρ gets larger, additional more efficient symmetric equilibria exist. And for $\rho \geq 2^{-\frac{1}{n-2}}$, all symmetric equilibria, including the fully efficient equilibrium, exist. Notice, yet, that $\underline{h}(n, \rho)$ is increasing in n and, as we discovered at the end of the previous subsection, the efficient equilibrium will eventually destabilize and cease to exist.[15] Therefore, the expected delay before reaching an agreement increases in n.

Theorem 5. *The shortest delay in any equilibrium is increasing in* n.

Proof. We know that for each n, the most efficient equilibrium is either the efficient equilibrium or the profile with $h > 0$ required loopholes such that $\eta(n-2-h, 1) \leq \frac{1}{2}$ and $\eta(n-2-(h-1), 1) = \eta(n-1-h, 1) > \frac{1}{2}$. Find the largest n for which $1-\rho^{n-2} \leq 1/2$. Then $\eta(\bar{n}(0, \rho) - 2, 1) = 1-\rho^{n-2} \leq 1/2$ and the efficient equilibrium exists and yet $\eta(\bar{n}(0, \rho) + 1 - 2, 1) > 1 - \rho^{n-1} > 1/2$. Thus the efficient equilibrium does not exist when $n = \bar{n}(0, \rho) + 1$. Yet, by (10), the equilibrium requiring one loophole does exist in that case. The expected delay clearly increases when moving from $\bar{n}(0, \rho)$ to $\bar{n}(0, \rho)+1$ players in that case. These facts also imply that $\rho^{\bar{n}(0,\rho)-1} < \frac{1}{2} \leq \rho^{\bar{n}(0,\rho)-2}$. Or, $1-2^{-\frac{1}{\bar{n}(0,\rho)-2}} \geq 1 - \rho > 1 - 2^{-\frac{1}{\bar{n}(0,\rho)-1}}$. Thus, for $l, k = 0, 1, ...$ and for $\bar{n}(0, \rho) > 5$,we have that $\binom{\bar{n}(0, \rho) + k}{1 + k + l}(1 - \rho) - \binom{\bar{n}(0, \rho) - 1 + k}{k + l} < \binom{\bar{n}(0, \rho) + k}{1 + k}(1 - \rho) - \binom{\bar{n}(0, \rho) - 1 + k}{k} < \binom{\bar{n}(0, \rho)}{1}(1 - \rho) - \binom{\bar{n}(0, \rho) - 1}{0} \leq \binom{\bar{n}(0, \rho)}{1}(1 - 2^{-\frac{1}{\bar{n}(0,\rho)-2}}) - \binom{\bar{n}(0, \rho) - 1}{0} = \bar{n}(0, \rho)(1 - 2^{-\frac{1}{\bar{n}(0,\rho)-2}}) - 1 < 0$ and thus $= \eta(n, \underline{h}(n+1, \rho)) - \eta(n-1, \underline{h}(n, \rho)) = \sum_{l=\underline{h}(n,\rho)+1}^{n} \rho^{n-l}(1-\rho)^{l-1}\left[\binom{n}{l}(1-\rho) - \binom{n-1}{l-1}\right] <$

[15] $2^{-\frac{1}{n-2}}$ is increasing in n.

0 showing that expected delay increases whenever ρ is such that the efficient equilibrium exists with six or more players. In order to show that the claim holds also for $\bar{n}(0,\rho) \in \{3,4,5\}$, remark first that $\binom{n-1}{l-1} / \binom{n}{l} \geq 1/2$ holds when $l-1 \geq (n-1)/2$. Thus the only remaining non-obvious case is when there are 5 players and ρ is such that the equilibrium requiring one loophole is the most efficient one. In that case, there are four responders and $\binom{5}{2} > \binom{4}{1}$. The probability of an agreement changes from $1-\rho^4$ to $1-\rho^5-5(1-\rho)\rho^4$. The difference equals $-\rho^5+5\rho^5-5\rho^4+\rho^4 < 0$. $\square$

The theorem shows that, although increasing the number of players increases the chances of reaching an agreement at an equilibrium profile with a given number of required loopholes, the strategic incentives also change and more aggressive commitments become more attractive. This destabilizes the most efficient equilibrium. When there is one more player in the game, the most efficient equilibrium also requires one more loophole. This is associated with, not shorter, but longer delay.

The effect of changing ρ has an analogous, but reverse logic. Efficiency decreases and delay increases in a given equilibrium profile as commitment strength ρ increases. Yet, at the same time, increasing commitment strength, also eventually has the effect that more efficient equilibria emerge to the set of equilibria.

4.6 Frequent negotiations

So far we have considered several institutional features of multilateral negotiations, such as which kind of decision rule is being applied, how many parties there are in the negotiations, whether commitment positions can be formulated, and how likely they are to succeed. Let us now analyse another institutional constraint in multilateral negotiations, namely how frequently negotiation rounds take place. Unlike in many face-to-face bilateral negotiations, it is by no means obvious that in international multilateral negotiations, offers could be generated very frequently. In the European Union, a summit where all heads of state gather together takes place once every six months (June and December), the Doha round of the WTO has had nine comprehensive meetings since the start of the round in 2001 (and are by and large inconclusive by the time of writing this manuscript), in climate change negotiations, general meetings (Conference of the Parties, COP) take place once a year (the 25th COP was organized in Madrid in December 2019 and ended without any conclusive agreement on measures or timeline on how to reach the targets set in Paris 2015).

In such institutionally rich settings, it is not entirely obvious how to think about the frequency of negotations. A commonly used approach in the bargaining literature is to consider a continuous time formulation where discounting over two consecutive negotiation rounds is parametrized by the time gap between the rounds, t, such that delay between the rounds is discounted by factor $\delta = exp(-rt)$. Here r is the discount rate reflecting the cost associated with the passage of a naturally given time interval such as a year, and t is the delay between negotiation rounds expressed as a fraction of the natural time interval. For example, if negotiation rounds take place once a year and the yearly institutional interest rate equals 3%, then $\delta = exp(-0.03) \approx 0.97$; if negotiation rounds occur once in every six months $\delta = exp(-0.03/2) \approx 0.985$. A typical question addressed in this setting is what happens when all institutional frictions constraining frequency of rounds are lifted and the rounds rather follow each other in an almost continuous sequence. Formally, what happens when t approaches zero?

In order to formulate that limit, we must first take a stand on what happens to the process of commitments and the exogenous, but stochastic, arrival of loopholes. A straigthforward generalization of the model presented in the previous sections would assume that each player's individual loophole arrivals follow an i.i.d. memoryless Poisson process with an arrival rate of λ. Then within a time period of length t, the probability of zero arrivals, i.e. that no loophole arises for this player, is $\rho = exp(-\lambda t)$. Notice that this implies that the arrival rate of a loopholes among the $n-1$ responders is then just $(n-1)\lambda$. Not surprisingly, the probability of loophole arrival within a given round of negotiations tends to zero as the length of negotiation round, t, tends to zero. Thus the periodic probability of commitment success ρ tends to one. Moreover, among the known properties of Poisson processes is that the probability of two arrivals at exactly the same time is zero.[16] Thus when t tends to zero, two or more loopholes never arrive at the same time.[17]

As established by equation (10), the minimum number of loopholes required for an agreement in the most efficient equilibrium is decreasing in ρ. Since ρ tends to one as t tends to zero, more efficient equilibria will emerge until finally even the efficient equilibrium will exist.[18]

The equilibrium payoff equation in the discrete time formulation of the model (8) can now be adjusted to the continuous time formulation, as a function of n, h, λ, r and t, as follows

$$v^*_{n,h} = \frac{1}{n} \cdot \frac{\eta(n-1,h)}{1 - exp(-rt)(1-\eta(n-1,h))}.$$

Given that at most one loophole arrives in a Poisson process at any given on point in time, two cases are of special interest in the limit where the time period length tends to zero: first the efficient equilibrium,

$$v^*_{n,0} = \frac{1}{n},$$

and second the equilibrium requiring just one loophole,

$$v^*_{n,1} = \frac{1}{n} \cdot \frac{\eta(n-1,1)}{1 - exp(-rt)(1-\eta(n-1,1))} = \frac{1}{n} \cdot \frac{1-(exp(-\lambda t))^{n-1}}{1 - exp(-rt)(exp(-\lambda t))^{n-1}}.$$

In the latter case, applying l'Hôspital's rule yields the limit payoff

$$lim_{t\to 0} v^*_{n,1} = \frac{1}{n} \cdot \frac{\lambda(n-1)}{r + \lambda(n-1)}. \tag{11}$$

Equation (11) reveals that, when offers are generated very frequently, the efficiency losses in the inefficient one-loophole equilibrium increase in the discount rate r and decrease in the loophole arrival rate λ and the number of players n.[19]

Recall that two or more loopholes never arrive at the same time and thus, in the limit, equilibria with of $h \geq 2$ loopholes have value zero. Therefore $v^*_{n,0}$ and $v^*_{n,1}$ are the only equilibrium payoffs bounded away from zero. Since ρ tends to one as t tends to zero, all equilibria with $h = 0, ..., n-2$ exist

[16] This is due to the fact that a Poisson process is orderly and thus simple.

[17] Despite its simplicity and tractability, this approach implicitly assumes some unrealistic features, above all that (i) commitments can be reformulated at an increasing pace after a loophole arrival (at the beginning of the commitment stage following the arrival), and (ii) thus that the loopholes last for an decreasing length of time as t tends to zero. In more realistic formulations, one might decouple the process of re-establishing commitments from the frequency of negotiation rounds, but we leave that for future research.

[18] To confirm this, it is easy to see that staying flexible, deviating down, or deviating up in the most aggressive equilibrium cannot pay off since $v^* > exp(-rt)v^*$for any $t > 0$. In any equilibrium where $h < n-2$ the key condition is thus the upward deviation requiring one more loophole, i.e. the necessary and sufficient condition of Theorem 4.

[19] Notice also that, because the arrival probability of the first loophole is independent of the length of the time period, the expected length of conflict and thus the equilibrium payoff $v^*_{n,1}$ are independent of the length of the time period, too.

but equilibria with $h \geq 2$ have a zero expected payoff and thus commitments are very aggressive $lim_{t \to 0} x^*_{n.h} = 1/(n - h - 1)$. For example the aggressive equilibrium requiring $n - 2$ loopholes always exists and in that equilibrium $lim_{t \to 0} x^*_{n.h} = 1$ much like in the inefficient equilibrium of the bilateral Nash demand game where each party demands the entire pie. In this limit case, the efficient equililibrium always exists and therefore any ineffciency is due to a coordination failure like in the Nash demand game. This is in contrast to the less frequent negotiations in which the efficient equilibrium does not exist.

5 Conclusion

In recent times many prominent policy makers and practitioners have blamed the unanimity rule for the paralyzed decision making in the EU and WTO arenas, for instance, and called for more extensive use of variants of majority decision making (see Jean-Claude Juncker, "State of the Union -speech at the European Parliament 2018, Pierre Moscovici, European Commissioner for Economic and Financial Affairs, Taxation and Customs, European Commission press release, Jan 15th 2019, and Ehlermann and Ehring (2005), for EU and WTO, respectively). In this paper, we have provided an explanation for why the unanimity decision making rule is prone to delay in multilateral bargaining. We also show that any majority rule including the all-but-one supermajority rule circumvents this problem. The stylized predictions of our simplified theoretical model thus fit the empirical patterns in the mentioned international arenas of negotiation where unanimous agreement is required. In both these settings the number of parties is large and has even increased prior to the observed impasses: WTO Doha round failed after the enlargement of the organization in late 1990's and early this Millennium and EU decision making in sensitive areas has stalled ever since the enlargement of 2004. In addition to explaining why supermajority rules result in more efficient outcomes than unanimity, our model also suggests that inefficiencies related to unanimity are likely to be more severe when the number of parties grows larger. Our analysis also suggests that the typical long delay between rounds of negotiations is another potential source of inefficiencies. A policy recommendation which warrants further investigation is thus to organize bargaining rounds more frequently. However, this policy conclusion hinges on the implicit assumption that commitment positions can be re-established at the beginning of the following round independently of how soon the next round arrives. This implicit assumption implies that the frequency narrows the short-term advatage of committed parties over the uncommitted ones thereby undermining the incentive to deviate to a more aggressive commitment which was the source of instability of the more efficient equilibria when length between negotiation rounds is longer.

The virtue of the model is its simplicity and transparency. Yet in order to study the robustness of the results, many extensions are conceivable. One could study asymmetric equilibria. It is of interest to understand whether asymmetric equilibria exist and if so, are outcomes always inefficient when the unanimity rule is used. Another extension would allow commitments to decay stochastically over time as in Ellingsen and Miettinen (2014) in the bilateral case. The complication in the multiparty setting is that chains of deviations are conceivable and each commitment in the chain must take into account the effects of the commitment on the continuation payoffs of the players whose commitments stochastically fail since each optimal commitments must each target to make players in one such set indifferent. The fixed point problem in this multidimensional state-space of commitments is hardly tractable and out of the scope of the present paper. This challenge of bandwagon effects in commitments does not arise in the bilateral case since players can only renew their commitments if their commitment fails and that is precisely the contingency where the opponent's commitment targets to make the player indifferent between accepting or rejecting and countercommitting.

One could also incorporate many institutional features typical for legislative bargaining: endogenize the status quo or recognition probabilities, allow for several policy dimensions, bargain over public and private goods simultaneously. Such extensions could allow understanding how other institutional features interact with the decision making rule and the capacity to commit in generating delay and impasse.

More generally, the paper belongs to an emerging literature which proposes novel complete information explanations for delay in bargaining. Commitments constitute a friction which breaks the "Coase theorem". In addition to international and political economics, these ideas could deliver novel insights in understanding the boundaries of the firm or orgnizational economics and industrial economics more generally or when attempting to understand in labor disputes or the rising costs of legal conflict and the burden on the courts and the legal system.

Appendix

Lemmas

Lemma 2. *If there exists $\hat{h} \geq 1$ such that $\eta\left(k, \hat{h}\right) \leq \frac{1}{\hat{h}+1}$, then $\eta\left(k, \tilde{h}\right) \leq \frac{1}{\tilde{h}+1}$ for all $\tilde{h} > \hat{h}$.*

Proof. Note that $\eta(k,h) = \sum_{l=h}^{k} f(k,l)$, where $f(k,l) = \begin{pmatrix} k \\ l \end{pmatrix} (1-\rho)^l \rho^{k-l}$ is the probability of l 'successes' (loopholes) in a binomial experiment with k trials and success (loophole) probability $(1-\rho)$. It is sufficient to show the following: "If there exists $\hat{h} \geq 2$ such that $\eta\left(k, \hat{h}\right) > \frac{1}{\hat{h}+1}$ then $\eta\left(k, \hat{h}-1\right) > \frac{1}{\hat{h}}$." Suppose there exists $\hat{h} \geq 2$ such that $\eta\left(k, \hat{h}\right) > \frac{1}{\hat{h}+1}$. Suppose $\hat{h} < k(1-\rho)+1$, then $\hat{h}-1 < k(1-\rho)$, implying that $\left(\hat{h}-1\right)$ is below the median of the binomial, and so $\eta\left(k, \hat{h}-1\right) > \frac{1}{2} \geq \frac{1}{\hat{h}}$. Suppose $\hat{h} \geq k(1-\rho)+1$. Since the binomial distribution is discrete log concave, it has the property that $\frac{f(k,h)}{\eta(k,h)}$ is non-decreasing (see An (1997) Proposition 10), which implies $\eta\left(k, \hat{h}-1\right) \geq \frac{f\left(k,\hat{h}-1\right)}{f\left(k,\hat{h}\right)} \eta\left(k, \hat{h}\right)$. Further, it can be shown that $\frac{f\left(k,\hat{h}-1\right)}{f\left(k,\hat{h}\right)} = \frac{\hat{h}}{k+1-\hat{h}} \frac{\rho}{1-\rho}$. Therefore $\eta\left(k, \hat{h}-1\right) \geq \frac{\hat{h}}{k+1-\hat{h}} \frac{\rho}{1-\rho} \eta\left(k, \hat{h}\right) > \frac{\hat{h}}{k+1-\hat{h}} \frac{\rho}{1-\rho} \frac{1}{\hat{h}+1}$. The last expression is increasing in ρ, and we have $(1-\rho) \leq \frac{\hat{h}-1}{k}$ (see above). Therefore, this expression is greater than $\frac{\hat{h}}{\left(k+1-\hat{h}\right)\left(\hat{h}+1\right)} \frac{1-\frac{\hat{h}-1}{k}}{\frac{\hat{h}-1}{k}} = \frac{\hat{h}}{\hat{h}^2-1} > \frac{1}{\hat{h}}$. $\square$

Lemma 3. *Consider a symmetric commitment equilibrium which requires that at least h responders are flexible for the agreement to arise. The decomposed expected equilibrium payoff $\frac{1}{n}\pi_P + \frac{n-1}{n}\pi_R$ equals $\frac{1}{n}\frac{\eta(n-1,h)}{(1-\delta(1-\eta(n-1,h))}$.*

Proof. The equation (5) can be written in the form

$$v_{n,h}^* = \delta v_{n,h}^* + \frac{1}{n}\eta(n-1,h)[E(l|h \leq l \leq n-1) - h] + \frac{n-1}{n}\rho\eta(n-2,h)] \left(x_{n,h}^* - \delta v_{n,h}^*\right).$$

We can plug in the expression for $x_{n,h}^*$ into this equation and solve for $v_{n,h}^*$. This yields

$$v_{n,h}^* = \frac{m(n-1,h)}{(1-\delta)(n-1-h) + \delta n m(n-1,h)}$$

where

$$m(n-1,h) = \frac{1}{n}\eta(n-1,h)[E(l|h \leq l \leq n-1) - h] + \frac{n-1}{n}\rho\eta(n-2,h)$$

and

$$\eta(n-1,h)[E(l|h\leq l\leq n-1)-h]=\sum_{l=h}^{n-1}(l-h)\left(\begin{array}{c}n-1\\l\end{array}\right)(1-\rho)^{l}\rho^{n-1-l}$$

and

$$\eta(n-2,h)=\sum_{l=h}^{n-2}\left(\begin{array}{c}n-2\\l\end{array}\right)(1-\rho)^{l}\rho^{n-2-l}$$

so that

$$\begin{array}{rcl}m(n-1,h) & = & \frac{1}{n}\sum_{l=h}^{n-1}(l-h)\left(\begin{array}{c}n-1\\l\end{array}\right)(1-\rho)^{l}\rho^{n-1-l}+\frac{n-1}{n}\sum_{l=h}^{n-2}\left(\begin{array}{c}n-2\\l\end{array}\right)(1-\rho)^{l}\rho^{n-1-l}\\ & = & \sum_{l=h}^{n-2}\left[\frac{1}{n}(l-h)\left(\begin{array}{c}n-1\\l\end{array}\right)+\frac{n-1}{n}\left(\begin{array}{c}n-2\\l\end{array}\right)\right](1-\rho)^{l}\rho^{n-1-l}+\frac{1}{n}(n-1-h)(1-\rho)^{n-1}\\ & = & \frac{1}{n}\eta(n-1,h)(n-1-h)\end{array}$$

because $\eta(n-1,h)=\sum_{l=h}^{n-1}\left(\begin{array}{c}n-1\\l\end{array}\right)(1-\rho)^{l}\rho^{n-1-l}$ is the expected number of times that a player receives the "bonus" $(x^*_{n,h}-\delta v^*_{n,h})$ in a given period. Therefore,

$$v^*_{n,h} \;=\; \frac{\frac{1}{n}\eta(n-1,h)(n-1-h)}{(1-\delta)(n-1-h)+\delta n\frac{1}{n}\eta(n-1,h)(n-1-h)}=\frac{\frac{1}{n}\eta(n-1,h)}{(1-\delta)+\delta\eta(n-1,h)}.$$

□

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