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Roots and Effects of Financial Misperception in a Stochastic Dominance Framework

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Roots and Effects of Financial Misperception in a Stochastic Dominance Framework

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Abstract

This work deals with the issue of investors' irrational behavior and financial products' misperception. The theoretical analysis of the mechanisms driving erroneous assessment of investment performances is explored. The study is supported by the application of Monte Carlo simulations to the remarkable case of structured financial products. Some motivations explaining the popularity of these complex financial instruments among retail investors are also provided. In particular, investors are assumed to compare the performances of different projects through stochastic dominance rules. Unreasonably and in contrast with results obtained by the application of the selected criteria, investors prefer complex securities to standard ones. In this paper, introducing a new definition for stochastic dominance which presents asymmetric property, we provide theoretical and numerical results showing how investors distort stochastic returns and make questionable investment choices. Results are explained in terms of framing and representative effects, which are behavioral finance type arguments showing how decisions may depend on the way the available alternatives are presented to investors.

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1 Introduction

The importance of making choices based on the expected performances of economic and financial variables is out of question and the correct way to do it represents the focus of an endless scientific debate. In this respect, *decision theory* can be seen as the field that describes and formalizes the process of making a choice among several possible uncertain alternatives.

Surprisingly, the decision process is not always driven by rationality, and several papers provide evidence related to the occurrence of unreasonable choices and preference inversions. Camerer (1989) reports that 31.6% of subjects reverses preferences, when the same choice is presented in an identical manner twice, while Starmer and Sugden (1989) estimate that the percentage of preference reversal is 26.5% and, according to Wu (1994) it falls in a range of 5–45%. Hey and Orme (1994) find that around 25% of decisions are inconsistent when an individual faces twice the same choice problem and she/he can declare indifference. Moreover, Hey (2001) provides experimental evidence that the variability of the subjects' responses is generally higher than the difference in the predictive error of various deterministic decision theories. Starting from this point, Blavatskyy (2007) argues the necessity of abandoning the expected utility theory for a new decision-making paradigm that considers random errors performed by the deciders.

Furthermore, in a rational financial market, given an identical set of opportunities, two different individuals should operate the same choices, once the decision rule is fixed. Unexpectedly, the decision rule is often violated and choices toward the objectively-measured less attractive situations are often registered. This evidence plays a central role in financial theories which are grounded on psychological and human judgment of investors.

In this paper we provide theoretical results, supported by numerical analysis, on how investors distort stochastic returns and, unreasonably, prefer complicated financial derivatives to standard ones. Our arguments are applied to the analysis of structured financial products which have features particularly appropriate for our study.

Ritter (2003) highlights the causes of investors' irrationality through some specific *effects*. Some are of particular interest. The *framing effect* explains how decisions depend crucially on the way in which the available alternatives are represented. Tversky and Kahneman (1981), and Bazerman (1983) present the same experiment, in a slightly different fashion. People are grouped in two similar populations, $\mathcal{P}_1$ and $\mathcal{P}_2$, and a choice between a gamble A or a secure plan B is proposed to each individual. A change in the frame of A and B implies a reversal of preferences, in terms of percentages of choices in $\mathcal{P}_1$ and $\mathcal{P}_2$. The *representativeness effect* is first hypothesized by Reichenbach (1934) who describes the attitude to overweight the importance of the most recent experiences. In a larger sense, individuals who are affected by this effect guess that the salient global properties of a financial or economic variable can be explored by performing a local analysis of the related phenomenon.

The psychological biases of investors should be tackled in quantitative models based on decision

criteria. Kahneman and Tversky (1979) conduct psychological experimental researches to show that investors distort subjectively probabilities and overweight rare events. They provide the constitutive framework of *prospect theory*, for which Daniel Kahneman is one of the Nobel laureates in 2002[1].
A further important theoretical consequence of Kahneman and Tversky' researches (1979) relies on the formalization of the mechanism leading to irrational investors' choices. In this context, the issue of performance measures is of crucial importance.
The decision tool we deal with is the stochastic dominance (see Fishburn, 1964; Hadar and Russel, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970), since it undoubtedly represents one of the most general way to compare financial products. The stochastic dominance criteria rely on the distribution functions of the random amounts and take into account the whole set of their characteristics (i.e.: fat tails, kurtosis, asymmetry, etc.). To the best of our knowledge, the most recent monograph on stochastic dominance is provided by Sriboonchitta et al. (2010), who present also several applications to economics and finance, but we mention also Levy (2006).
Stochastic dominance can be *violated* in a *direct* or *indirect* way. In the former case, investors prefer project A instead of project B, even if A is dominated by B. In the latter case, project B is underpriced with respect to a dominated project A or, alternatively, the transitivity property fails, i.e. C dominates D and D dominates E, but E is preferred to C. The indirect violations are more frequent than the direct ones, as argued in Birnbaum (1997) and references therein.
There is no need to specify that violations of stochastic dominance can be viewed as the foundation of the process of misperception and associated irrational choices of investors. In this sense, prospect theory can appropriately describe the biasing mechanisms through violations of the first order stochastic dominance, as in the original Kahneman and Tversky's setting. To solve the problem related with first order stochastic dominance violation, *cumulative prospect theory* is introduced (Tversky and Kahneman, 1992). In the new setting, the Authors extend their previous theory to the case of random amounts with a continuous set of realizations. In this framework, the probability distortion is interpreted as acting on the cumulative probability distribution rather than on probabilities. Cumulative prospect theory improves prospect theory because it offers a broader analysis instrument through two key transformations -one for the values of outcomes and the other for objective probabilities. Stott (2006) reviews the main transformations proposed in the literature along with their properties. Chateneuf and Wakker (1999) give an axiomatization of preferences for decisions under risk, i.e. when probabilities are transformed. In doing so, they follow the line traced by Wakker and Tversky (1993), who propose the axiomatization of the cumulative prospect theory. Under a more practical point of view, some contributions are worth noting. Dierkes et al. (2010) evidence the usefulness of the cumulative prospect theory framework to empirically explore the dependence of individuals' investment strategies on the time horizons of different financial op-

[1] Amos Tversky passed away in 1996, but his valuable contribution is acknowledged in Kahneman's Nobel lecture.

portunities.

In a behavioral finance perspective, the violation of stochastic dominance is not necessarily a weakness for describing the decision-making mechanisms, because people often choose dominated lotteries. On the other hand, empirical evidence shows that investors usually violate higher order stochastic dominance rather than the first order one, because of their sensitivity to risk.

The subjective preferences of investors and human intervention in comparing amounts are responsible of the violation of the stochastic dominance. The cumulative prospect theory does not admit first order stochastic dominance violation, but higher orders may continue to be violated.

Our paper deals with the violation of stochastic dominance in the case of structured products. We elaborate a theoretical model to describe how the cumulative probability distributions of random returns are distorted by investors. In particular, we advance two proposals: the random sums may be perturbed by a deterministic trend or a lump sum. The former case concerns a global rereading of random amounts, while the latter relies on a investors' local misunderstanding of some realizations. In order to formalize the inversion of stochastic dominance, we follow the route traced by Levy and Wiener (1998) and Levy and Levy (2002, 2004)[2], and introduce a new definition of the decision criteria at hand. The theoretical results are validated via numerical simulations.

Some contributions in the literature are worth mentioning. Tversky and Kahneman (1986), Birnbaum and Navarette (1998) and Leland (1998) deal with the problem of direct violations and provide also an explanation of the motivation driving these violations. In particular, they propose a behavioral-finance argument grounded on the scarce transparency of how the stochastic dominance rules should be used to order a set of financial projects.

Following Tversky and Kahneman (1986), Birnbaum and Navarette (1998) and Leland (1998), we propose a possible motivation for the misrepresentation of structured products through a behavioral-finance type argument. In particular, we show that the presentation by investment banks (or insurance companies) of optimistic prospectuses related with complex products to investors may represent the ground of the erroneous perception mechanism. In doing this, we merge framing and representativeness effects in an unified framework. This merging constitutes the basis of the misperception mechanism (see also Wang and Fishbeck, 2004 for the analysis of the incidence of framing effects on decisions, with a particular reference to prospect theory). It is worth noting the asymmetric behavior of the financial institutions in framing in a more attractive fashion only the complex products. This aspect must be considered when defining the new stochastic dominance rule, that should present asymmetric properties.

Some papers are particularly close to our perspective. Berger and Smith (1998) focus on the prospect theory based on framing effect. In particular, the authors propose an experiment on three simultaneous framing tactics and develop arguments on their main effects and interactions. The problem is

[2]The authors introduce the concepts of *prospect stochastic dominance* and *Markowitz stochastic dominance*.

presented in a rather qualitative fashion and perhaps it can be well inserted in the field of marketing. Breuer and Prest (2007) deal with the behavioral finance-type analysis of the structured products, and show that these financial instruments are preferred by investors who underestimate the volatility of the underlying securities. Bernard et al. (2011) discuss the overpricing of some structural products due to too optimistic framing of the prospectuses proposed by investment banks to retail investors. The work of Bernard et al. (2011) is framed in the standard mean-variance theory, which is much more restrictive than stochastic dominance. Moreover, the authors analyze a misperception driven by a lump sum and neglect the case of a global misrepresentation of random returns. In this respect, we will show that the presence of misperceived isolated gains does not allow to change investor's mind and drive the decision process.

To sum up, this paper contributes to the existing literature in some directions. First, we deal with the problem of misperception of structured financial products, which is quite neglected in financial studies. Despite the scarce attention paid to this topic, it represents a really important issue since structured products are very popular mainly among retail investors and households (Bernard et al. 2011; Carlin, 2009) . Second, a behavioral finance-type discussion on our results, based on the main features of structured financial products, is addressed. In particular, the misperception concerning structured products follows from a merge of framing and representativeness effects, in that it can be interpreted as a consequence of reticent prospectuses proposed by financial institutions to investors. In a more general context, we argue that investments banks and insurance companies are morally responsible of some irrational investors' behaviors. Third, we analyze some peculiar types of distortion of random amounts. In particular, in accordance with some classical models of time series analysis, we assume that random sums may be perturbed via a deterministic trend or a lump sum. Fourth, we refer to stochastic dominance decision rules and, in this respect, we propose a new definition of asymmetric stochastic dominance by considering the investors' misperception of random amounts.

The rest of the paper is organized as follows. Section 2 is devoted to the statement of the theoretical results regarding the violation of stochastic dominance and provides a new definition of stochastic dominance. Section 3 is related with the main features of structured financial products and presents techniques and results of the numerical analysis. In Section 4 our findings are discussed, together with some concluding remarks.

2 Theoretical framework

In this section the statement of theoretical results regarding the violation of stochastic dominance in a cumulative prospect theory framework is presented. In doing so, a new definition of stochastic dominance rules is provided.

Consider a random variable X on a probability space $(\Omega, \mathcal{F}, P)$ with a cumulative function F_X. Fix

$r \in \mathbb{R}$ and define

$$A_X^{(n)}(r) = \begin{cases} F_X(r), & \text{if } n = 1; \\ \\ \int_{-\infty}^{r} A_X^{(n-1)}(t)dt, & \text{if } n \geq 2. \end{cases} \tag{1}$$

In order to be self-contained, it is worth to recall the definition of stochastic dominance.

Definition 1. *Consider $n \in \mathbb{N}$ and two random amounts X and Y.*
X dominates stochastically of order n Y ($X >_n Y$, hereafter) if and only if

$$\begin{cases} A_X^{(n)}(r) \leq A_Y^{(n)}(r), & \forall\, r \in \mathbb{R} \\ \text{and} \\ \exists\, r^* \in \mathbb{R} \text{ such that } A_X^{(n)}(r^*) < A_Y^{(n)}(r^*). \end{cases} \tag{2}$$

Fix $n \in \mathbb{N}$ and consider two random amounts X and Y, such that $X >_n Y$. If an investor has a particular subjective perception of profit and losses related to Y, then the n-th order stochastic dominance may be violated and the investor may prefer Y instead of X. Substantially, the random amount Y is perceived as a random amount Y_p, that can be viewed as a perturbation of Y and such that $Y_p >_n X$. The distortion of Y is attained by introducing a rule $\mathcal{R}$ which transforms Y in Y_p. More formally, $\mathcal{R}$ is a two variable function which transforms the support and density function of Y into the support and density function of a different random variable, named Y_p.
In the theoretical approach, the definition of a subjective concept of stochastic dominance criteria, based on the perturbing rule $\mathcal{R}$, is introduced.

Definition 2. *Consider $n \in \mathbb{N}$, two random amounts X and Y and a perturbing rule $\mathcal{R}$.*
Y $\mathcal{R}$-dominates stochastically of order n X ($Y >_{n\mathcal{R}} X$, hereafter) if and only if $Y_p >_n X$, where Y_p is the perturbation of the random amount Y generated by the rule $\mathcal{R}$. Conversely, $X >_{n\mathcal{R}} Y$ if and only if $X >_n Y_p$.

It is worth noting that Definition 2 is based on the misperception of just one of the two projects to be compared. This asymmetry represents the main difference between the proposed new concept and the definitions given by prospect and Markowitz stochastic dominance (Levy and Wiener, 1998; Levy and Levy, 2002; 2004) that are based on particular transformations of both the investments to be compared. The proposed approach is motivated by the necessity to take into account decision rules which are based on the misperception of only one of the investments under scrutiny. As we will see, this is suitable with the structured products case.
It is also worth noting that, analogously to the stochastic dominance rule, relation $>_{n\mathcal{R}}$ is a semiorder among the random amounts.
This paper, in line with the classical literature on time series analysis, treats two remarkable cases: Y_p is obtained by perturbing Y with a lump sum or a deterministic trend. The related rules will be denoted hereafter as $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively.

2.1 Lump sum

Define a random mass Z as follows:

$$Z = \begin{cases} \bar{z}, & \text{with probability } \pi; \\ 0, & \text{with probability } 1-\pi. \end{cases} \tag{3}$$

where $\bar{z} \in \mathbb{R}^+$, $\pi \in (0,1)$ and Z independent from Y[3].

The random sum Y_p is defined through the rule $\mathcal{R}_1$ as follows: $Y_p = Y + Z$.

The following result holds true.

Proposition 3. *For each $n \in \mathbb{N}$, we have*

$$A_{Y_p}^{(n)}(r) = \begin{cases} (1-\pi)A_Y^{(n)}(r), & \text{if } r < \bar{z}; \\ \pi + (1-\pi)A_Y^{(n)}(r), & \text{if } r \geq \bar{z} \text{ and } n = 1; \\ \frac{\pi(r-\bar{z})^{n-1}}{n-1} + (1-\pi)A_Y^{(n)}(r), & \text{if } r \geq \bar{z} \text{ and } n \geq 2. \end{cases} \tag{4}$$

Proof. The result is straightforward, by invoking the induction principle and by definition of Y_p. □

Proposition 4. *Fix $n \in \mathbb{N}$ and assume that $X >_n Y$. It doesn't exist $\bar{z} \in \mathbb{R}^+$ and $\pi \in (0,1)$ such that $Y >_{n\mathcal{R}_1} X$.*

Proof. The cases $n = 1$ and $n \geq 2$ are separately treated.

- Assume $n = 1$.

Since $X >_1 Y$, we have

$$A_X^{(1)}(r) \leq A_Y^{(1)}(r), \ \forall\, r \in \mathbb{R} \qquad \text{and} \qquad \exists\, r^* \in \mathbb{R} \,|\, A_X^{(1)}(r^*) < A_Y^{(1)}(r^*).$$

By definition of Y_p through the lump sum Z, we can write:

$$A_{Y_p}^{(1)}(r) = \begin{cases} (1-\pi)A_Y^{(1)}(r), & \text{if } r < \bar{z}; \\ \pi + (1-\pi)A_Y^{(1)}(r), & \text{if } r \geq \bar{z}. \end{cases} \tag{5}$$

When $Y >_{1\mathcal{R}_1} X$, then the following system is satisfied:

$$\begin{cases} (1-\pi)A_Y^{(1)}(r) \leq A_X^{(1)}(r), & \text{if } r < \bar{z}; \\ \pi + (1-\pi)A_Y^{(1)}(r) \leq A_X^{(1)}(r), & \text{if } r \geq \bar{z}. \end{cases} \tag{6}$$

Second equation of system (6) brings to

$$\pi \leq \frac{A_X^{(1)}(r) - A_Y^{(1)}(r)}{1 - A_Y^{(1)}(r)}, \qquad \text{with } r \geq \bar{z}, \tag{7}$$

that is satisfied only when $\pi = 0$, since the second term of (7) is not positive. Therefore, it doesn't exist $\bar{z} \in \mathbb{R}^+$ and $\pi \in (0,1)$ such that $Y >_{1\mathcal{R}_1} X$

[3]It is worth noting that the limiting cases for π are trivial: when $\pi = 0$, then $Z = 0$, while $\pi = 1$ implies that $Z = \bar{z}$.

- Assume $n \geq 2$.

By Proposition 3 we have that the following system has to be fulfilled, in order to have $Y >_{n\mathcal{R}_1} X$:

$$\begin{cases} \pi \geq \max\left\{ \sup\limits_{r \geq \bar{z}} \left[\frac{A_X^{(n)}(r) - A_Y^{(n)}(r)}{\frac{(r-\bar{z})^{n-1}}{n-1} - A_Y^{(n)}(r)} \right] ; 1 - \inf\limits_{r < \bar{z}} \left[\frac{A_X^{(n)}(r)}{A_Y^{(n)}(r)} \right] \right\} \\ \bar{z} > \sup\limits_{r > \bar{z}} \left\{ r - [(n-1)A_Y^{(n)}(r)]^{\frac{1}{n-1}} \right\}. \end{cases} \tag{8}$$

Since

$$\sup_{r > \bar{z}} \left\{ r - [(n-1)A_Y^{(n)}(r)]^{\frac{1}{n-1}} \right\} = +\infty,$$

then the second condition of (8) cannot be true, and this completes the proof. □

Proposition 4 states that the distortion of Y through a lump sum cannot be responsible of the inversion of stochastic dominance between X and Y. This result will be discussed in a more detailed way at the end of the section.

2.2 Deterministic trend

The rule $\mathcal{R}_2$ is introduced by the definition of an increasing function $t : \mathbb{R} \to \mathbb{R}$, that can be viewed as a deterministic trend affecting the realizations of the random sum Y. The intervention of the function t on Y drives the definition of the perturbed random amount Y_p through the cumulative probability function of Y:

$$F_{Y_p}(r) = F_Y(r - t(r)). \tag{9}$$

The following result is trivial, but it is useful to formalize it in order to provide a precise analysis of the first order stochastic dominance in this context.

Proposition 5. *Assume that $X >_1 Y$. If the function t is such that $A_Y^{(1)}(r - t(r)) \leq A_X^{(1)}(r)$, for each $r \in \mathbb{R}$, and there exists $r^* \in \mathbb{R}$ such that $A_Y^{(1)}(r^* - t(r^*)) < A_X^{(1)}(r^*)$, then $Y >_{1\mathcal{R}_2} X$.*

For the stochastic dominance of order greater than 1 no general results can be explicitly formalized. We then restrict the analysis only to the most frequent cases, well-known in the financial econometric literature: constant trend and linear trend.

- *Constant trend*

In this case $t(r) = t \in \mathbb{R}$, for each $r \in \mathbb{R}$.

Proposition 6. *For each $n \in \mathbb{N}$, we have*

$$A_{Y_p}^{(n)}(r) = A_Y^{(n)}(r - t) \tag{10}$$

Proof. We use the induction principle.

By definition, the thesis is true when $n = 1$.

Now, assume that (10) holds for $n-1$. Then we have:

$$A^{(n)}_{Y_p}(r) = \int_{-\infty}^{r} A^{(n-1)}_{Y_p}(s)ds = \int_{-\infty}^{r} A^{(n-1)}_{Y}(s-t)ds = \int_{-\infty}^{r-t} A^{(n-1)}_{Y}(s)ds = A^{(n)}_{Y}(r-t).$$

□

By using Proposition 6 we derive a sufficient condition to reverse the stochastic dominance criterion.

Proposition 7. *Fix $n \in \mathbb{N}$ and assume that $X >_n Y$. Moreover, assume that*

$$\begin{cases} A^{(n)}_{Y}(r-t) \leq A^{(n)}_{X}(r), \qquad \forall\, r \in \mathbb{R} \\ \text{and} \\ \exists\, r^* \in \mathbb{R} \text{ such that } A^{(n)}_{Y}(r^*-t) < A^{(n)}_{X}(r^*). \end{cases} \tag{11}$$

Then $Y >_{n\mathcal{R}_2} X$.

Proof. By Proposition 6 and by Definition 2 we obtain the thesis. □

- *Linear trend*

In this case there exists an $\alpha \neq 0$ such that $t(r) = \alpha r$, for each $r \in \mathbb{R}$.

Proposition 8. *For each $n \in \mathbb{N}$, we have*

$$A^{(n)}_{Y_p}(r) = \frac{1}{(1-\alpha)^{n-1}} A^{(n)}_{Y}(r - \alpha r). \tag{12}$$

Proof. Also in this case, the result is proved by using the induction principle.

For $n = 1$, the result is trivially true.

Assume that (12) holds for $n-1$. Then

$$A^{(n)}_{Y_p}(r) = \int_{-\infty}^{r} A^{(n-1)}_{Y_p}(s)ds = \frac{1}{(1-\alpha)^{n-2}} \int_{-\infty}^{r} A^{(n-1)}_{Y}(s-\alpha s)ds =$$

$$\frac{1}{(1-\alpha)^{n-2}} \int_{-\infty}^{(1-\alpha)r} \frac{A^{(n-1)}_{Y}(s)ds}{1-\alpha} = \frac{1}{(1-\alpha)^{n-1}} A^{(n)}_{Y}(r-\alpha r),$$

and the result is proved. □

Proposition 8 implies the following sufficient condition for the inversion of the stochastic dominance relation.

Proposition 9. *Fix $n \in \mathbb{N}$ and assume that $X >_n Y$. Moreover, assume that*

$$\begin{cases} \frac{1}{(1-\alpha)^{n-1}} A^{(n)}_{Y}(r - \alpha r) \leq A^{(n)}_{X}(r), \qquad \forall\, r \in \mathbb{R} \\ \text{and} \\ \exists\, r^* \in \mathbb{R} \text{ such that } \frac{1}{(1-\alpha)^{n-1}} A^{(n)}_{Y}(r^* - \alpha r^*) < A^{(n)}_{X}(r^*). \end{cases} \tag{13}$$

Then $Y >_{n\mathcal{R}_2} X$.

Proof. The proof comes from Definition 2 and Proposition 8. □

Theoretical results show that misperception is attained when the perturbation is due to the introduction of a trend, while no stochastic dominance violation takes place when a lump sum is introduced. This findings provide interesting information since a trend affects the entire set of realizations of a random amount, while a lump sum is related only to an impulsive shock. Therefore, when agents have a global misperception irrationally prefer the worst project. Conversely, the presence of a misperceived adjunctive isolated gain in the dominated project is not able to invert agents' mind and drive the decision process. This result contrasts with that obtained by Bernard et al. (2011), who considers the distortion through a lump sum as the responsible for the reversal of preferences, due to the use of a local decision tool like the modified Sharpe ratio.
In a certain sense, these results were expected. Indeed, stochastic dominance involves the entire probability distributions of stochastic returns. Hence, a local distortion -a lump sum- cannot be responsible of inversion of preferences, while a global distortion -trend- can do the job.

3 Structured financial products

This section is devoted to the analysis of the structured products which, as we will see, represent a paradigmatic example of our theoretical findings.
Structured products, also known as market-linked products, are generally pre-packaged investment strategies based on derivatives written on a single security, a basket of securities, options, indices, commodities, debt issuances and/or foreign currencies and, to a lesser extent, swaps. The variety of products shows that there is no single and uniform definition for the term structured product.
The U.S. Securities and Exchange Commission[4] (SEC) defines structured securities as "securities whose cash flow characteristics depend upon one or more indices or that have embedded forwards or options or securities where an investor's investment return and the issuer's payment obligations are contingent on, or highly sensitive to, changes in the value of underlying assets, indices, interest rates or cash flows". To this definition it may be added that a common feature of some structured products is represented by a *principal guarantee* function, which offers protection of principal if the security is held to maturity.
Structured products, usually issued by investment banks or affiliates, are created to meet specific needs that cannot be satisfied by standardized financial products and are proposed as an alternative to direct investments, as part of the asset allocation process, to reduce the risk exposure of a portfolio, or to take advantage of current market trends. These investment tools are available at mass retail level, since one of their attractions is the ability to customize a variety of assumptions into one instrument.

[4]SEC, Rule 434 regarding certain prospectus deliveries

The main disadvantages of structured products may include: credit risk (structured products are unsecured debt from investment banks); lack of liquidity (structured products rarely trade after issuance and anyone looking to sell a structured product before maturity should expect to sell it at a significant discount[5]); high complexity (only few investors truly understand how the structured product will perform relative to simply owning the underlying). At the latter point we can tie further considerations related to the lack of pricing transparency. Since there is no uniform standard for pricing, it is hard to compare the net-of-pricing attractiveness of alternative structured products and investor cannot know for sure what are the implicit costs of the instrument. In addition, asymmetric information exists since investment banks that build and manage complex products understand them far better than investors who buy them. Overall, the more complex is a product, the more a retail investor is willing to pay. Carlin (2009) concludes that retail investors often make purchases without knowing exactly what they are buying and may also be unaware about overpricing.

The market-linked products considered in this paper and representing the subject of the numerical application are *globally-floored*, in the sense that provide a guaranteed minimum return. Hence, absent default risk, the final return will never be less than a prespecified floor which applies to the entire life of the contract. Furthermore, using real world examples, *capped* contracts will be considered. Capping the maximum ensures that the payoff is never too extreme and that the value of the contract is not too outrageous.

The basic reasons why this type of structured contracts is considered here are essentially due to their popularity among retail investors and their weird characteristics, which are not fully understood and have not been widely studied in literature. In particular, via stochastic dominance, we will show that the locally capped contract is dominated by the globally capped but, surprisingly, the former is more popular than the latter. This popularity is highlighted by the fact that the percentage of globally floored contracts listed in the AMEX, as April 2008, is about 45% for locally capped contracts and about 10% for globally capped ones. To explain this irrational behavior, the prospect theory perspective and the behavioral finance type discussion for misperception discussed in the previous section are implemented.

3.1 Globally floored - locally/globally capped contracts

In many cases, the redemption amount paid out on guaranteed minimum return product depend exclusively on the periodical performance of the underlying asset (i.e. the *reference portfolio*). It is not uncommon to lock in gains at specific dates, capping the periodical returns at a given maximum. At maturity, the payoff is given by a combination of periodic returns that are added[6] to

[5]Because of this, structured products tend to be more of a buy-and-hold investment decision rather than a means of getting in and out of a position with speed and efficiency.

[6]It is worth to notice that there are two common types of globally-floored locally-capped contracts, the compound contracts, where returns in each year are compounded, and the simple contracts where the returns are periodically

the guaranteed redemption amount and paid out to investors.

The final payoff, Y_T, for this globally floored - locally capped contract is given by (Boyle and Tset, 1990, Boyle et al., 2009):

$$Y_T = Y_0 \left\{ (1+F) + \max \left[0, \sum_{k=1}^{nT} \min \left(c, \frac{S_{t_k}}{S_{t_{k-1}}} - 1 \right) - F \right] \right\} \tag{14}$$

with T representing the maturity of the contract; Y_0 the amount paid at the inception date, $t = 0$; F the global floor (guaranteed minimum return at maturity); c the local cap (maximum allowed periodical return) and $S_{t_k} \in [0, T]$ the price of the reference portfolio at the prespecified dates t_k.

The maximum and minimum payoffs of this contract are given respectively by $Y_0 (1 + nT \cdot c)$ and $Y_0 (1 + F)$.

This market-linked product is characterized by high complexity since its final pay-off is path-dependent and cannot be easily replicated. From the point of view of the seller, aimed at minimizing market risks, their main exposure is to volatility since this contract is very subtle in its dependence on the assumed volatility model. The classical references to this phenomenon, which is not the focus of this paper, are Avellaneda et al. (1995), Lyons (1995) and Wilmott (2000).

Conversely, a particular and simpler case is given when the redemption amount of a contract is made up of a guaranteed minimum return (i.e. the floor rate) and a bonus return which varies according to the performance of the reference portfolio between the issue and maturity dates. In this case, the bonus return is calculated as a percentage of the difference by which the price of the reference portfolio on the maturity date exceeds its price on the issue date. If the price falls, no bonus return is paid out and the rate of return of the structured products is given by the floor rate. Briefly, investors can profit from a rise in the price of the reference portfolio but, if this price drops, do not have to bear the loss, since bearers only participates in the relative performance of the reference portfolio up to a certain maximum value.

The issuer promises a final payoff proportionate to the change in the reference portfolio's price. In cases where the price of the reference portfolio decreases, the issuer guarantees a minimum redemption amount. At the same time, the issuer limits the investor's participation in the instrument's performance by setting an upper limit (i.e., the global cap).

The standard final payoff, X_T, for a globally floored - globally capped contract is given by (Boyle and Turnbull, 1989; Bernard et al., 2011):

$$X_T = X_0 \left\{ (1+F) + \max \left[0, \min \left(C, \frac{S_T}{S_0} - 1 \right) - F \right] \right\}, \tag{15}$$

with T representing the maturity of the contract; X_0 the amount invested at the inception date, $t = 0$; F the global floor (guaranteed minimum return at maturity); $C > F$ the global cap (maximum allowed return); S_0 and S_T the price of the reference portfolio at the inception date and maturity,

added together to give the final payoff.

respectively.

The initial investment of X_0 yields a rate of return in the closed interval $[F, C]$. The cap becomes operational only if the return of the reference portfolio in the interval $[0, T]$, computed as $\frac{S_T}{S_0} - 1$, exceeds the global cap rate C.

By no arbitrage, the current price of this contract can be replicated by a portfolio of three securities: a long position in a zero coupon bond which reaches maturity on T and has a face value of $X_0\,(1+F)$; a long position on $\frac{X_0}{S_0}$ European call options on the underlying reference portfolio with strike $S_0(1+F)$ and maturity T; a short position on $\frac{X_0}{S_0}$ European call options on the underlying reference portfolio with strike $S_0(1+C)$ and maturity T. It is worth noting that the maximum pay-off of this contract is given by $X_0\,(1+C)$, while the minimum pay-off is $X_0\,(1+F)$. Hence, to avoid arbitrage, it must be:

$$(1+C) > e^{rT} > (1+F)\,, \tag{16}$$

with r representing the risk-free rate.

3.2 Simulation results

The complex (globally floored - locally capped) and simple (globally floored - globally capped) contracts described in the previous section show quite different properties. In particular, the first one is characterized by a path-dependent final pay-off and there is no closed form solution to compute its initial price. On the contrary, the simpler contract can be decomposed into a portfolio consisting of a zero coupon bond and two standard call options. Thus, it may even be priced using standard Black and Scholes' formula.

Since the application of this work is based on a comparison of the two above described structured products, we believe the two contracts should be priced and analyzed using the same numerical methodology, in order to achieve more reliable results.

As per usual, in case of path dependency property required to determine the final payoff (i.e. Asian options, barrier options, cliquet options and many other exotics), one of the most commonly used approach is Monte-Carlo (MC) method since, as opposed to other numerical approaches, offers a greater flexibility and becomes increasingly attractive compared to other methods of numerical integration as the dimension of the problem increases.

The first building block of this application based on the globally floored - globally capped and globally floored - locally capped (from now on simple contract and complex contract, respectively) is represented by the fair determination of the global and local cap, C and c respectively. To find these quantities, using Black-Scholes-Merton's assumptions[7], we take the expectation under the risk

[7]Despite the stringent assumptions of the Black-Scholes-Merton framework, Stoimenov and Wilkens (2005) and Wilkens and Stoimenov (2007) used the celebrated approach to evaluate structured products issued in Germany.

neutral distribution of the payoff, discounted at the risk free rate of return, assuming the price and other contract parameters as given, and solving for the global and local caps that make the price of the two contracts equal to an assigned level $\widehat{P}$.

We apply here the standard lognormal model for the reference portfolio, S_t, so that the accumulation factors $\frac{S_t}{S_{t-1}}$ are independent and identically lognormally distributed. The complex contract involves the sum of lognormal random variables that, with a sufficiently large number of simulations, can provide arbitrarily close to the true value of the contract (Boyle et al., 1997).

Using the assumption of no arbitrage, the expectation is taken with respect to a transformation of the original probability measure (i.e. the risk-neutral measure; see Duffie, 2001, for a survey on the subject).

The applied approach consists of the following steps.

- Select a given vector of parameters, $\xi = \left[\begin{array}{cccccccc} T & X_0 = Y_0 & F & r & \sigma & \delta & S_0 & \widehat{P} \end{array} \right]$, where S_0 is the current stock price, r is the riskless interest rate, δ and σ are respectively dividend yield and volatility of the reference portfolio, T is the contract's maturity, F is the minimum guaranteed return, X_0 is the amount invested, $\widehat{P}$ is the price of the contract at time zero.

- Select an integer q and a q-dimensional vector, Γ, whose elements are the values to be assigned to the unknown global cap, C.

- Simulate sample paths of the underlying state variables (e.g., underlying reference portfolio price) over the relevant time horizon, T, according to the risk-neutral measure. As in Black-Scholes-Merton's model, we assume the reference portfolio price follows a log-normal diffusion. Independent replications of the terminal stock price under the risk-neutral measure can be generated from the formula:

$$S_T^{(i)} = S_0 e^{(r-\delta-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z_i} \quad i = 1, ...m \tag{17}$$

where the Z_i are independent samples from the standard normal distribution and $m = 10,000$.

- Evaluate, for each element j^{th} of the $q-$dimensional vector $\Gamma = \left(\Gamma^{(j)}\right)_{j=1,...,q}$, the discounted cash flows of the option component on each sample path, as determined by the structures of the simple contracts:

$$\widehat{P}_{simple}^{(j)} = \frac{1}{m}\sum_{i=1}^{m} e^{-rT} X_0 \left((1+F) + \max\left(0, \min\left(\Gamma^{(j)}, \frac{S_T^{(i)}}{S_0} - 1\right) - F\right)\right), \quad j = 1, ..., q. \tag{18}$$

More realistic assumptions (i.e. stochastic volatility and interest rate; jumps in the stochastic process describing the underlying dynamics, credit risk, etc.) could be considered, but this is not the focus of this work.

- Average the discounted cash flows over sample paths and find $j^* \in \{1, ..., q\}$ such that $\Gamma^{(j^*)}$ minimize the distance of the risk neutral price $\widehat{P}^{(j)}_{simple}$ from the given price $\widehat{P}$. Lastly, denote $\widehat{P}_{simple} = \widehat{P}^{(j^*)}_{simple}$.

Similarly, we find the local cap level, c, of the complex contract, implementing an appropriate discretization for (17). Using the given vector of parameters, ξ, and the q-dimensional vector, $\gamma = \left(\gamma^{(j)}\right)_{j=1,...,q}$, whose elements are the values to be assigned to the unknown local cap, c, we find $j_* \in \{1, ..., q\}$ such that $\gamma^{(j_*)}$ minimize the distance of the risk neutral price $\widehat{P}_{complex}$ from the given price $\widehat{P}$:

$$\widehat{P}_{complex} = \widehat{P}^{(j_*)}_{complex} = \frac{1}{m}\sum_{i=1}^{m} e^{-rT} Y_0 \left\{ (1+F) + \max\left[0, \sum_{k=1}^{nT} \min\left(\gamma^{(j_*)}, \frac{S^{(i)}_{t_k}}{S^{(i)}_{t_{k-1}}} - 1\right) - F\right]\right\}. \tag{19}$$

To perform the numerical analysis we choose the same vector of parameters, $\xi = \left[\begin{array}{cccccccc} T & X_0 = Y_0 & F & r & \sigma & \delta & S_0 & \widehat{P} \end{array}\right]$, for both simple and complex contracts. The contracts share the same underlying, S, whose dynamics are described by (17), with parameters $r = 0.05$, $\delta = 0.02$, $\sigma = 0.15$ and $S_0 = 10$. Simple and complex contracts have the same maturity ($T = 5$ years), same initial investment ($X_0 = Y_0 = 1,000$) and guaranteed minimum return ($F = 0.1$). The initial price of both contracts is $\widehat{P}_{simple} = \widehat{P}_{complex} = 920$. The complex contract is based on a quarterly sum cap ($n = 4$) and its estimated fair cap producing an initial value $\widehat{P}_{complex} = 920$ is $\widehat{c} = 0.0867$. The fair global cap level, producing an initial value $\widehat{P}_{simple} = 920$, is $\widehat{C} = 0.3053$. The parameters selected for the application correspond to standard market assumptions. However, results hold also for different vectors of parameters, ξ.

Given the vector of parameters, ξ, the second building block of the analysis relies on the simulation of the final payoffs for the simple contract, X_T, and complex contract, Y_T. In addition, here, we implement stochastic dominance rules in order to classify, according to their final performances, the two contracts representing the subject of the analysis.

Using (17) and (15), with $\Gamma^{(j^*)} = \widehat{C}$ and $m = 10,000$ replications, we obtain the empirical probability distribution for the globally capped contract, X_T. Using an opportune discretization for (17) and (14) , with $\gamma^{(j_*)} = \widehat{c}$ and $m = 10,000$, we get also the empirical probability distribution for Y_T.

INSERT FIGURE 1 ABOUT HERE.

Caption of Figure 1: Simulated PDFs of the globally and locally capped contracts.

Figure 1 shows the empirical probability distribution functions of the payoffs of the simple (globally capped, left side), X_T, and complex (locally capped, right side), Y_T, contracts. The complex contract has a very high probability of yielding the minimum guaranteed return, $F = 10\%$, with a probability mass of 62.32%. The probability of attaining a return greater than 70% is 0.79%, while the probability of the maximum attainable return in the right tail, $\widehat{c}nT = 173,4\%$, is practically

equal to 0.

It is worth to notice that the sum of 20 quarterly returns will be equal to $173,4\%$ if and only if all consecutive quarterly returns exceed the local cap level, $\widehat{c} = 0.0867$, so that the probability of this event is virtually quite impossible.

On the other hand, the simple contract has a distribution characterized by two probability masses: one at the minimum guaranteed return $F = 10\%$, with probability 51.65%, and the other at the maximum attainable return, $\widehat{C} = 30.53\%$, with probability 30.56%. Thus, investors have a probability of only 17.79% of obtaining an intermediate return between these two extremes. As it can be seen, returns are almost uniformly distributed in the central part of the distribution (as shown in the graph on the left side of Figure 1).

In order to evaluate the random amounts of the two considered contracts via the stochastic dominance criteria discussed in previous section, two distinct prospects for the simple and complex contracts, with cumulative density functions respectively given by $\widehat{F}_X$ and $\widehat{F}_Y$, are determined.

Given (1) and definition 2, we have that X dominates stochastically Y of order $n \in \mathbb{N}$ ($X >_n Y$) if and only if:

$$\begin{cases} \widehat{A}_X^{(n)}(r) \leq \widehat{A}_Y^{(n)}(r), \qquad \forall\, r \in \mathbb{R} \\ \text{and} \\ \exists\, r^* \in \mathbb{R} \text{ such that } \widehat{A}_X^{(n)}(r^*) < \widehat{A}_Y^{(n)}(r^*). \end{cases} \tag{20}$$

In order to verify (20), we check whether X dominates Y of order n by invoking the Law of Large Number on a rather large number of simulations. Firstly, we run the algorithm described in the first building block to obtain $j = 1, ..., K$ ($K = 10,000$) prospects, named $X^{(j)}$ and $Y^{(j)}$. In practice, we compute:

$$\Psi_K^{(n)} = \frac{1}{K} \sum_{j=1}^{K} 1\left(\widehat{D}_j^{(n)}(r) \leq 0,\ \forall r \in \mathbb{R} \text{ and } \exists r^* : \widehat{D}_j^{(n)}(r^*) < 0\right) \tag{21}$$

where:

$$\widehat{D}_j^{(n)}(r) = \widehat{A}_{X^{(j)}}^{(n)}(r) - \widehat{A}_{Y^{(j)}}^{(n)}(r) \tag{22}$$

and $1(\cdot)$ denotes the indicator function.

Results can be summarized as follows: global X is not dominated neither dominates local Y of the first order; in 55% of cases we have $X >_2 Y$ and in 100% of cases we obtain $X >_3 Y$.

Applying (20) to the simulated cumulative density functions of simple and complex contracts we obtain results that partially contradict what really happens on real markets and highlight the problem of investors' irrational choices. In particular, even though market data suggest that the complex contract, Y, is more popular than the simpler contract, X, we find that the stochastic dominance criteria predict that consumers should prefer the simpler contract, X.

The results of the numerical analysis clearly point out that investor which are risk-averse and expected utility maximizers tend to prefer contract X to contract Y. Therefore, the popularity of

the locally capped contracts with respect to the globally capped ones is counterintuitive and can be explained only through the investors' irrational behavior. Since first order stochastic dominance requires that investors prefer higher returns to lower ones, implying an utility function with non negative first derivative, on the ground of the obtained results, we conclude that for this type of investors we cannot say which is the preferred product. On the contrary, the analysis of second order stochastic dominance, positing diminishing marginal utility (sufficient for risk aversion), and third order stochastic dominance, implying the empirical attractive feature of increasing absolute risk aversion (non-negative third derivative), provide significant results in favor of the simple contract.
It is worth to notice here that the interpretation of the results may be even stronger where it is considered that many empirical studies find that structured products are more overpriced the harder they are to evaluate (Wilkens and Stoimenov, 2007; Carlin, 2009; Ruf, 2011; only to cite a few) and that in our numerical analysis we assume thats simple and complex contracts are both fairly priced.
The third building block of the application is based on the implementation of the misperception algorithm describing the mechanism that push retail investors to chose complex products, instead of simple ones. As remarked in the previous section, the misperception algorithm is applied only to one of the two considered projects since we assume that the complex product, Y, is misperceived. Given the theoretical results illustrated in the previous section, we will consider rule $\mathcal{R}_2$ in the particular case of constant trend, $t(r) = t \in \mathbb{R}$, $\forall\ r \in \mathbb{R}$ to perturbate the empirical cumulative distribution function of the contract Y and to map Y into Y_p.
The misperception algorithm allows us to find the minimum constant trend, t^*, such that $F_Y\left(r - t^*\right) \leq F_X\left(r\right)$ for each $r \in \mathbb{R}$. The optimal t^* will provide an inversion of the investors' preferences according to first and second order stochastic dominance.
The approach consists in the following steps:

- Given $j = 1, ..., K$ prospects obtained in the second building block, select an integer v and a v-dimensional vectors, $\Delta = (\Delta^{(l)})_{l=1,...,v}$, whose elements are the values to be assigned to the unknown constant trend, t.
- Perturbate Y through Δ and set the vector $Y_p = \left(Y_p^{(l)}\right)_{l=1,...,v}$.
- Run the algorithm described in the first building block to obtain $X^{(l)}$, with $l = 1, ..., v$.
- Evaluate, for each element l^{th} of the $v-$dimensional vector Δ, the stochastic dominance criteria as determined by the perturbation of the complex contract, Y. In particular, in order to verify the $\mathcal{R}_2$-stochastic dominance, we compute:

$$\Phi_v^{(n)} = \frac{1}{v}\sum_{l=1}^{v} 1\left(\widehat{\mathcal{R}_2 D}_l^{(n)}\left(r\right) \leq 0,\ \forall r \in \mathbb{R} \text{ and } \exists r^* : \widehat{\mathcal{R}_2 D}_l^{(n)}\left(r^*\right) < 0\right) \tag{23}$$

where:

$$\widehat{\mathcal{R}_2 D}_l^{(n)}\left(r\right) = \widehat{A}_{Y_p^{(l)}}^{(n)}(r) - \widehat{A}_{X^{(l)}}^{(n)}(r). \tag{24}$$

- Find the minimum value $l^* \in \{1, ..., v\}$ such that $Y_p^{(l)} >_n X^{(l)}$.

Running the procedure for $n = 1, 2$, we find that: with $t^* = 0.453$ the perturbated complex contract dominate for the first order stochastic dominance the simple one, $Y >_{1\mathcal{R}_2} X$, in the 71% of the K cases; with $t^* = 0.4672$ we have $Y >_{1\mathcal{R}_2} X$, in the 100% of the K cases and with $t^* = 0.0316$ the perturbated complex contract dominate for the second order stochastic dominance the simple contract, $Y >_{2\mathcal{R}_2} X$.
Results imply that if investors erroneously perceive that the probability mass (62.32%) of the complex contract return is concentrated in correspondence of 46.72% in such a way that the most likely final wealth is given buy $Y_T = 1613.92$, they select the complex contract in light of the first stochastic dominance criteria.

4 Results and concluding remarks

In this paper the issue of misperception is tackled, with a particular reference on why and how it takes place. A theoretical analysis, supported by simulations, is presented. The misperception mechanism is implemented to explain the popularity of some complex structured financial products. In particular, the preference accorded by investors to locally-capped contracts with respect to globally-capped ones may be motivated by the framing effect, which is a trick commonly exploited by financial institutions to pursue profit targets.
Some points need to be emphasized, in order to highlight the main findings of this paper.

- Simulation results allows to state an hypothesis to formalize the misperception mechanism for locally-capped contracts. It is undoubtedly true that a violation takes place, since the complex contracts are unreasonably more popular than the simple ones. An explanation of the reasons why global caps are misperceived is obtained by analyzing the main features of this type of financial product and developing a behavioral finance argument.

  We recall that the presence of a guaranteed minimum return and the easy access to these products make them suitable for retail investors and households. The behavior of this type of agents is generally driven by their confidence on the financial institutions proposing the investments. In this respect, the intervention of investment banks and insurance companies in retail investors' decisions plays a key role. Financial institutions pursue profit targets and therefore may present financial products in a rather fraudulent fashion, exploiting the framing effect on investors' choices. Structured financial products, with a particular focus on the locally-capped contracts, can be seen as a case of misperception due to framing. Financial institutions usually show to potential investors some prospectuses on the future performances of the instument with local cap. Clearly, not all the possible scenarios can be shown, because

of the randomness of the evolution of the underlying. Scenarios may vary depending on two variables: the expected returns at the expiration date and their probabilities of occurrence. A fair and honest proposal should reflect either optimistic and pessimistic outcomes, and the sample of scenarios should be opportunely weighted with their related occurrence probabilities. Almost all developed Countries regulates this aspect, by introducing some devoted regulations. For instance, the Federal Act on Collective Investment Schemes (the so-called CISA) entered into force in Switzerland on January 1st, 2007, establishing some transparency and simplification criteria introduced to guarantee a conscious understanding of structured products. CISA provides also a guideline to the contents of the prospectuses that financial institutions should submit to investors' attention. A further example can be found in the Prevention of Fraud (Investments) Act, issued in UK in 1959 and aimed at protecting investors by the introduction of penalties for fraudulently inducing investors to invest their money. In a larger sense, this rule induces banks and insurance companies to propose structured financial products without reticences on their negative admissible future performances. Unfortunately, the reality is quite different and the state securities acts are systematically violated. It is also worth noticing that the mispricing of complex structured products is commonly accepted to be one of the roots of the actual financial crisis since they are subject also to credit risk and their true implicit risk is not fully disclosed to investors. The prospectuses that investment banks propose to their clients are often too optimistic and the framing effect may push investors to purchase these products, even if they are reasonably unsuitable for a wide part of households. Some details on this are presented in Bernard et al. (2011), where some samples of real prospectuses for locally-capped contracts are reported. Furthermore, in Illinois the lawyers Burke and Stoltmann are preparing arbitration claims to recover losses against Wall Street brokerage firms for the selling of structured products[8]. They deeply investigate this phenomenon and conclude that such investments were not clearly presented to investors and inappropriately pitched as 100% safe and secure.

To conclude, we think that financial institutions are sometimes morally responsible for the negative consequences due to investors' misperception, in the sense that they frame in an unclear way the main characteristics of such investments to pursue profit targets. Bernard et al. (2011) report some real examples extracted from the prospectus proposed by several investment banks.

- The introduction of a new definition of stochastic dominance is needed to capture the selectiveness of the distortions in evaluating different projects. With this respect, we propose stochastic dominance rules based on the misperception of one of the investments to be compared. This asymmetry is grounded on the evidence, discussed partially above, that investment banks may

[8]More information on http://www.structuredproductfraud.com/

emphasize some positive aspects and be reticent on other remarkable characteristics of the investments. In this case, it is rather obvious that just one of such investments will be misperceived by the client. In this direction, it would be an intriguing development of this work the replication of the original Kahneman and Tversky's experiment. One may subgroup people in two identical populations $\mathcal{P}_1$ and $\mathcal{P}_2$ and propose two gambles, say G_1 and G_2, with G_1 stochastically dominating by G_2. Letting G_2 always fairly presented, one could present G_1 fairly to $\mathcal{P}_1$ and frame G_1 with optimistic scenarios for population $\mathcal{P}_2$. By repeating the experiment for a large number of optimistic prospectuses, an empirical analysis of the stochastic dominance inversion by comparing the behaviors of $\mathcal{P}_1$ and $\mathcal{P}_2$ may be performed. One can also get information on the risk attitudes of the population $\mathcal{P}_2$. Moreover, if the misperception is hypothesized to depend on a deterministic trend rule on G_1 as in our approach, it is possible to deduce the minimal levels of trend parameters allowing the violation of n-th order stochastic dominance.

- Misperception is theoretically attained when the complex contracts are distorted by the introduction of a trend, while no stochastic dominance violation takes place when a lump sum is introduced. This finding provide an interesting information on the nature of the human judgment implemented by investors, when the performance measure is the stochastic dominance. A trend affects the entire set of realizations of a random amount, while a lump sum is related to an impulsive shock. Therefore, investors having a certain global misperception irrationally prefer the worst project. Conversely, the presence of an isolate gain in the dominated investment is not able to invert subjects' mind and drive the decision process. A future theoretical research in this direction concerns the introduction of different misperception rules.

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Figure
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