Economic growth, corruption and tax evasion

Roy Cerqueti
Department of Economic and Financial Institutions
University of Macerata

Raffaella Coppier*
Department of Economic and Financial Institutions
University of Macerata

Abstract

In this paper, we explore tax revenues in a regime of widespread corruption in a growth model. We develop a Ramsey model of economic growth with a rival but non–excludable public good which is financed by taxes which can be evaded via corrupt tax inspectors. We prove that the relationship between the tax rate and tax collection, in a dynamic framework, is not unique, but is different depending on the relevance of the "shame effect". We show that in all three cases - "low, middle and high shame" countries, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease as the tax rate increases. But, for intermediate tax rates, the rate of growth for "low shame" countries is lower than that of "uniform shame" countries which is, in turn, lower than that of "high shame" countries. This happens because the growth rate is more sensitive to variations of t in a honest country rather than in a corrupt country.

Keywords: corruption, evasion, tax revenues, economic growth.
JEL Codes: H21 H26 D73.

*Corresponding author: Raffaella Coppier, Department of Economic and Financial Institutions, University of Macerata, Via Crescimbeni, 20 - 62100 - Macerata.
E-mail: raffaellacoppier@unimc.it

1 Introduction

Tax evasion and fiscal corruption have been a general and persistent problem throughout history with serious economic consequences, not only in transition economies, but also in countries with developed tax systems. In general, tax evasion and corruption can have ambiguous effects on economic growth: tax evasion increases the amount of resources accumulated by entrepreneurs, but it also reduces the amount of public services supplied by the government, thus leading to negative consequences for economic growth. Although there is extensive literature investigating the origins, effects and extent of tax evasion and corruption, from both theoretical and empirical points of view, interaction between them has been partially explored. The analysis of tax evasion in the tax compliance literature dates back at least to the classic paper of Allingham and Sandmo (1972). Since then, a large amount of literature relating to corruption and tax evasion has emerged but, only recently can we find theoretical models which study tax setting and evasion in a context of growth models (e.g. Lin and Yang (2001), Chen (2003) and Ellis and Fender (2006)).

Lin and Yang (2001) extended the portfolio choice model of tax evasion from a static to a dynamic setting, finding that, while growth is decreasing with respect to tax rate in absence of evasion, it is U–shaped with respect to the tax rate in presence of the tax evasion. In contrast to our model, in their work, the public goods are not productive, then diverting resources from the non–productive public sector to the productive private sector, fiscal evasion will be conducive to economic growth.

Chen (2003) integrates tax evasion into an AK model with public capital financed by income tax which can be evaded. In his model, individuals optimize tax evasion, while the government optimizes the tax rate, auditing and fine rate, given the evasion level decided by consumers. In general, these policies have ambiguous effects, but for some parameters the author finds that the growth rate decreases as tax evasion increases.

Ellis and Fender (2006) introduce endogenous corruption into a variant of the Ramsey growth model where a government taxes private producers and uses the resources to either supply public capital or simply consumes the taxes itself (corruption form).

We deal not with bureaucratic but with fiscal corruption which establishes a direct impact of evasion/corruption on tax revenues, and thus on economic growth.

In our work, we develop a Ramsey model of economic growth with a rival but non–excludable public good which is financed by a percentage of taxes. We also assume that tax auditing may be performed by a corruptible tax inspector, who takes a bribe in exchange for not reporting the detected evasion, in accordance with Chander and Wilde (1992), Hindricks, Keen and Muthoo (1999) and Sanyal, Gang and Goswami (2000). Thus, in our model, evasion goes hand in hand with the corruption of the tax inspector. In particular, we analyze the implications of endogenous evasion and corruption at a micro level and then we use the results of our static game as a framework for the growth model. In fact, taxation and tax evasion, in turn, influence both the provision of the public good and capital accumulation, affecting output and economic growth in two opposite ways: on one hand, higher tax evasion implies more capital accumulation and thus more economic growth; on the other hand, higher tax evasion leads to lower tax revenues, less provision of the public good and thus, a lower economic growth rate.

In contrast with some lines of research on tax evasion, we do not consider the issue of optimal

remuneration of tax inspectors by assuming that the inspector is paid a fixed wage[1].
We prove that the relationship between the tax rate and tax collection is not unique but is different depending on the relevance of the "shame effect" and depending on the static or dynamic context of the analysis.
Our work is part of one of two lines of research taken by literature on tax evasion (Feld and Frey, 2007), i.e. the line of research which considers tax morale as the key factor to explaining the fact that, contrary to the results of Allingham and Sandmo (1972), "people who exhibit empirically observed levels of risk aversion normally pay their taxes, although there is a low probability of getting caught and being penalized" (Frey and Torgler, 2007). In particular, we consider a growth model where the aggregate tax evasion is determined by non–pecuniary costs which depend upon the entrepreneurs' attitude to social stigma[2]. In this respect, we analyze a dynamic model, where the aggregate tax evasion is microfounded on non–pecuniary costs.
Several empirical studies highlight the importance of non-economic factors on tax evasion: Alm and Torgler (2006) find that the tax morale can explain more than 20 percent of the total variance of the variable size of the shadow economy (used as a measure of tax evasion); thus, if tax morale is declining, the shadow economy is likely to increase. Richardson (2006) shows, in an empirical analysis based on data for 45 countries, that non–economic determinants have the strongest impact on tax evasion: in particular, tax morale is an important determinant of tax evasion.
According to Kim (2003), we assume that people may fear social stigma (shame effect) only if they are detected as cheaters/corrupted. In this paper, we have extended the static analysis of Cerqueti and Coppier (2009), in a long run context incorporating the presence of a public sector. Indeed, in the short-run, it is a plausible assumption that governments can be completely opportunistic, that is, they provide nothing for the citizenry, not even national defense. But, in the long run, even taxpayers who are initially ashamed of cheating will eventually change their minds and become less ashamed. It is doubtful that the citizenry will have a strong sense of loyalty to an opportunistic government, especially one that offers no productive output to its citizens. Following Barro and Sala-I-Martin (1992), we incorporate a public good into a growth model. Considering in a growth model also the "productive" effect of tax revenues i.e. the provision of public goods, we obtain different results from e.g. Chen (2003) and Lin and Yang (2001) who consider only the negative effect of taxes on capital accumulation. In order to be more precise, we show that in all three cases - "low, middle and high shame" countries, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease as the tax rate increases. As we will see, this result derives from the behavior of tax revenues in a static framework.
The paper is organized as follows. The next section contains a discussion about some stylized

[1] For example Besley and McLaren (1993), Hindriks et al. (1999) and Mookherjee and Png (1995), deal with the issue of optimal remuneration of inspectors. Besley and McLaren (1993) compare three distinct remuneration schemes which provide different incentives to the inspectors: efficiency wages, reservation wages and capitulation wages. Hindriks et al. (1999) consider a model where all the actors are dishonest. Mookherjee and Png (1995) also consider only corruptible agents, but they remove the exogenous matching of the auditor and the evader: they consider it a moral hazard problem, since, for evasion to be disclosed, the inspector has to exert a costly non–observable effort.

[2] For a complete review of the main hypothesis proposed in literature on the different types of non–pecuniary costs that influence tax morale see Dell'Anno (2009).

facts concerning evasion, corruption and growth. In Section 3, we first present the model and then we formalize and solve the game, describing the model in a static framework. In Section 4 we extend the analysis into a dynamic context, endogenizing output and we go on to analyze the relationship between the tax rate, dynamic tax revenues and the income growth rate. We conclude in Section 5.

2 Empirical motivation of the paper: some stylized facts

This section contains some stylized facts about corruption, evasion and their relationship.
The phenomenon of tax evasion is of great relevance when the State collects taxes. In the U.S., the Internal Revenue Service estimates that 17% of income tax liability is not paid (Slemrod and Yitzhaki, 2002). In economies where there is a great extent of corruption, this is related to a high level of tax evasion (see Tanzi and Davoodi, 2000). In this respect, the analysis of tax evasion in a corrupted environment is an important area of research, and empirical economics literature contains some evidence. In particular, two types of corruption may take place: bureaucratic corruption and fiscal corruption. The former concerns the attitude of bureaucrats who ask for a bribe in order to guarantee public services while the latter is related to the dishonesty of tax inspectors, who ask for a bribe in order not to report evasion. Since in this work we deal with the problem of fiscal corruption, we rely henceforth only on the literature on the latter. Chu (1990) mentions that, in a survey undertaken by the city government of Taipei in 1981, 94% of monitored tax administrators are corrupt; in Sanyal, Gang and Goswami (2000), The Police Group in 1985 suggests that 76% of all Indian tax auditors are corrupt and that 68% of taxpayers had paid bribes. Ul Haque and Sahay (1996) state that 20-30% of Nepalese tax revenue goes to bribery, and cite a former prime minister of Thailand as evidencing that corruption is associated to the loss of 50% of tax revenues.
It is worth noticing that evasion is a necessary condition for fiscal corruption. Therefore, the analysis of the phenomenon of evasion may provide several insights into the dynamics of fiscal corruption. The empirical evidence gives that the occurrence of evasion is also driven by the level of tax rate implemented by the State. This aspect is theoretically confirmed when the analysis is carried out in accordance with the classical model of Allingham and Sadmo (1972). Indeed, when the fine imposed on evaders is independent from the tax rate, an increase in the tax rate makes honesty more expensive, while the costs of evasion remain unchanged. In particular, higher tax rates encourage rather than repress tax evasion. In a large majority of the cases, experimental, econometric and survey evidence shows that an increase in the tax rate leads to an increase in tax evasion[3]. Some notable exceptions, however, are Feinstein (1991) and Alm et al. (1993), who find a negative relationship between tax rates and tax evasion. A further supporting argument on the positive relationship between tax evasion and tax rates can be found in Gupta (2005, 2006). The author analyzes data related to Greece, Italy, Portugal and Spain - countries with a well–established tradition of tax evasion - and shows how higher tax evasion would cause a benevolent social planner to optimally increase the tax rates, when implicit taxation is also available as a source of revenue. Our paper also,

[3]See, for example, Friedland et al. (1978); Clotfelter (1983); Baldry (1987); Christian and Gupta (1993); Jaulfaian and Rider (1996); Andreoni et al. (1998) and Pudney et al. (2000).

in a static context, confirms this stylized fact, showing that as the tax rate increases the number of entrepreneurs who will find it worthwhile to be corrupt, i.e. evaders, increases.
A further economic variable to be considered in order to describe corruption is the monitoring level of the State. A remarkable amount of empirical work has validated the deterrence effect of the probability of being caught (auditing) and of penalty severity, although some differences appear regarding the size of the deterrence effects on tax compliance[4]. Moreover, experimental analyses have shown the positive relationship between penalties, audit probability and level of compliance[5].
In Fjeldstad and Semboja (2001) data from Tanzania are analysed. It is proved that compliance is more likely when the probability of being caught is perceived to be high or when sanctions against evasion are severe.
In a static context, our findings agree with the stylized facts described above: indeed, as the probability of being caught increases, the number of entrepreneurs who will find it worthwhile to be honest increases as well, and corruption is less widespread.
Let us now deal with the dynamic context. In this case, the empirical evidence about taxation and growth shows that in cross-country studies a negative link between the tax burden (measured by tax revenue to GDP) and growth for high-income countries emerges. However, the result does not hold for low and middle-income countries, perhaps reflecting measurement problems[6]. Firm–level empirical results, as well as simulation results using computable general equilibrium models, support the contrasting view that higher taxes negatively affect growth[7]. Our model confirms this stylized fact. Indeed, from low tax rates, the calibration we perform shows a decreasing relationship between growth and tax rates.
As we will see, the findings of our paper are in accordance with the stylized facts reported above.

3 The model

We start from Cerqueti and Coppier (2009), and we consider an economy which produces a single homogeneous good, with quantity $y \in [0, +\infty)$. There are three players in the economy: controllers, tax inspectors and entrepreneurs. We consider that the private good is produced by using two production factors, capital k and the public good with quantity $G \in [0, +\infty)$. The provision of the public good allows us to have a rationale for the existence of a government which uses tax revenues to finance the public good. Following Barro and Sala-I-Martin (2004), we consider a rival but non–excludable public good G in order to take the problem of congestion of the public good into account. In particular, G represents public infrastructure such as highways, the water system, police and fire services and courts which are subject to congestion. In this case, the public good available to an individual entrepreneur is the ratio of total public purchases G to the aggregate private capital. Barro

[4]See e.g. Witte and Woodbury, 1985; Dubin and Wilde, 1988; Dubin et al., 1990; Beron et al., 1992; Klepper and Nagin, 1989; Cebula, 1997, 2001; Cebula and Saltz, 2001.
[5]See e.g. Friedland et al., 1978; Becker et al., 1987; Beck et al., 1991; Alm et al., 1992a; Slemrod et al., 2001.
[6]See Blankenau, Nicole, and Tomljanovich (2004).
[7]See Fishman and Svensson (2000) and Feltenstein and Shah (1995).

and Sala-I-Martin (2003) consider $Y_i = AK_i f(\frac{G}{Y})$, but they stressed also that the same results would be obtained under the specification that G had to rise in relation to aggregate private capital K.
We define the number of entrepreneurs as n as the number of tax inspector, K as the aggregate capital of entrepreneurs, A as the productivity parameter summarizing the level of technology and G as the public good.
The j-th entrepreneur use their available per capita quantity of capital $k_j \in [0, +\infty)$ in the production sector. We state the same hypothesis of Lin and Yang (2001), and assume that the capital per person $k_j = k$ is fixed and is equal for each entrepreneur, in a static setting. For simplicity, we hypothesize that capital does not depreciate. Then, the individual production function of the good depends only on the capital, public good and the natural state which may occur: we consider $\alpha \in (0,1)$ and $\delta \in (0,1)$ such that production will be $y = Ak\,(G/K)^{\alpha}$ with a probability $(1-\delta)$, while with a probability δ an adverse natural state will occur, production will not take place and the corresponding production will be $y = 0$.
Following Barro (1990), we assume that the public good is financed contemporaneously by a percentage η, with $\eta \in [0,1]$, of tax revenues E[8]. Tax inspectors cannot invest in the production activity and earn a salary $w \in [0, +\infty)$ which is a portion of tax revenues: in order to be more precise, $w = \frac{(1-\eta)}{n} \cdot E$. The tax inspector, who checks whether tax payment is correct, is able to tell which of the two natural states have occurred for each entrepreneur. It is common knowledge that the tax inspector[9] is corruptible, in the sense that s/he pursues her/his own interest and not necessarily that of the State; in other words, the tax inspector is open to bribery. The tax inspector, in the case of the "good" natural state and in exchange for a bribe $b \in [0, +\infty)$, can offer the entrepreneur the opportunity of reporting that the "bad" natural state has arisen. In this case, the entrepreneur could refuse to pay the bribe or agree to pay the bribe and negotiate the amount with the inspector. The State monitors entrepreneurs' and tax inspectors' behavior through controllers, in order to weed out or reduce corruption and fixes the level of the tax rate $t \in [0,1]$ on the product y. Let $q \in [0,1]$ be the exogenous monitoring level implemented by the State; then q is the probability of being detected, given that corruption has taken place. Following Allingham and Sandmo (1972), we assume that the entrepreneurs incur a punishment rate $c \in [0,1]$ on unreported income[10]. In addition we consider that the entrepreneurs are not homogeneous agents, and to be more precise, the j-th entrepreneur attributes a subjective value c_j to the objective punishment – depending on her/his own "shame effect" – when the corrupt transaction is detected[11]. The entrepreneur, if detected, must pay taxes ty, her/his "shame cost", but s/he

[8] Conversely, Blackburn et. al. (2006) assume that the public good is provided as a fixed proportion of output, while revenues consist of the tax collected by the bureaucrat from high–income households, plus any fines imposed on the bureaucrat detected in a corrupt transaction.

[9] We assume that an entrepreneur is seen by only one inspector and cannot turn to other inspectors to be treated differently.

[10] In different models, Yitzhaki (1974) first considers the penalty as a proportion of the amount of evaded taxes, Caballé and Panadés (2007) show that when penalties are imposed proportionally to the amount of evaded taxes, the rate of capital accumulation cannot increase with the tax rate, while if the penalties are imposed on the amount of unreported income, the amount of income concealed increases with the tax rate.

[11] In fact, if taxpayers values are influenced by cultural factors, then tax morale may be an important determinant of taxpayer compliance and other forms of behavior (see Alm and Torgler, 2006).

is refunded the cost of the bribe paid to the tax inspector[12].

3.1 The game: description and solution

Given the new assumptions, the Cerqueti and Coppier (2009) game becomes the following two-period game (see Figure 1). In what follows, we refer to the entrepreneur payoff by a superscript (1) and to the inspector payoff by a superscript (2): they represent respectively the first and the second element of the payoff vector $\underline{\pi}_{i,j} = (\pi^{(1)}_{i,j}, \pi^{(2)}_{i,j}),\ i = 1, 2$.

Caption of Figure 1: The tree of the game.
INSERT FIGURE 1 ABOUT HERE

(1) At stage one, the tax inspector checks the entrepreneurs' production. If a "bad" natural state occurs, then the tax inspector reports that no tax is owed and in this case, the game ends. Otherwise, if there is a "good" natural state, the tax inspector may ask for the bribe b^d and report that the "bad" natural state has arisen and that the entrepreneur need not pay any tax.

(1.1) If $b^d = 0$ no bribe is asked for, the game ends without corruption and with the following payoff vector:

$$\underline{\pi}_{1,j} = \left(Ak \left(\frac{G}{K} \right)^{\alpha} (1-t), \frac{(1-\eta)E}{n} \right). \tag{1}$$

(1.2) Otherwise, let $b^d > 0$ be the positive bribe asked for by the tax inspector and the game continues to stage two.

(2) At stage two, the entrepreneur decides whether to negotiate the bribe or not.

(2.1) If the entrepreneur refuses the bribe, then the payoff vector is given by $\underline{\pi}_{1,j}$ defined as in (1). Then in this case, the game ends. There is no penalty for the tax inspector.

(2.2) If the entrepreneur decides to agree to pay the bribe, the negotiation starts and the two parties will negotiate the bribe. In this case, the payoffs will depend on whether the inspector and the entrepreneur are detected (with probability q) or not detected (with probability $(1-q)$). There is no penalty for the tax inspector who is detected[13]. In this case, the game ends with corruption and evasion and the expected payoff vector is given by:

$$\underline{\pi}_{2,j} = \left(Ak \left(\frac{G}{K} \right)^{\alpha} (1 - qt - c_j q) - (1-q)b, \frac{(1-\eta)E}{n} + (1-q)b \right) \tag{2}$$

[12] This assumption can be more easily understood when, rather than corruption, there is extortion by the tax inspector, even though, in many countries, the relevant provisions or laws stipulate that the bribe shall, in any case, be returned to the entrepreneur, and that combined minor punishment, (penal and/or pecuniary), be inflicted to her/him.

[13] The results do not depend on the existence of a cost for the tax inspector who is corrupt and detected.

We first determine the equilibrium bribe b^{NB}[14].

Proposition 3.1. *Let $q \neq 1$. Then there exists a unique non negative bribe (b^{NB}), as the Nash solution to a bargaining game, given by:*

$$b^{NB} = \mu \left[Ak \left(\frac{G}{K} \right)^{\alpha} \left(t - \frac{qc_j}{1-q} \right) \right]. \tag{3}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon+\beta}$ is the share of the surplus that goes to the tax inspector and β and ε are the parameters which can be interpreted as the bargaining strength measures of the entrepreneur and the tax inspector respectively.

We assume that the tax inspector and the entrepreneur share the surplus on an equal basis, arriving at the standard Nash case, when $\varepsilon = \beta = 1$. In this case the bribe is:

$$b^{NB} = \frac{1}{2} \left[Ak \left(\frac{G}{K} \right)^{\alpha} \left(t - \frac{qc_j}{1-q} \right) \right]. \tag{4}$$

In other words, the bribe represents 50 percent of the saving which comes from not paying taxes, net of the entrepreneur's "shame cost", if s/he is found out.
In this case, the payoff vector is given by:

$$\underline{\pi}_{2,j} = \left(Ak \left(\frac{G}{K} \right)^{\alpha} \left(1 - \frac{qt + t + c_j q}{2} \right), \frac{(1-\eta)E}{n} + Ak \left(\frac{G}{K} \right)^{\alpha} \cdot \frac{t - qt - c_j q}{2} \right). \tag{5}$$

By solving the static game, we can prove the following proposition:[15]

Proposition 3.2. *Let $0 \leq \frac{t(1-q)}{q} = c^* \leq 1$. Then,*

(a) if $c_j \in [0, c^*)$ the j-th entrepreneur will find it worthwhile to be corrupt and then the game ends with the payoff vector $\underline{\pi}_{2,j}$;

(b) if $c_j \in [c^*, 1]$ the j-th entrepreneur will find it worthwhile to be honest and then the game ends with the payoff vector $\underline{\pi}_{1,j}$.

The threshold c^* can be interpreted as an *honesty threshold.*
This assumption about c^* in Proposition 4.1 holds true when we assume the existence of a minimal threshold of monitoring activity

$$q^{\circ} := \frac{t}{1+t}. \tag{6}$$

Thus, the honesty threshold c^* is well defined when $q \geq q^{\circ}$, e.g. the monitoring level is great enough. We will suppose $q \geq q^{\circ}$ in the remaining part of the paper. Depending on the value of the "shame cost" c_j, two sub–game perfect Nash equilibria can be found:

- If $c_j < c^*$, the entrepreneur finds it worthwhile to start a negotiation with the tax inspector. Thus the surplus to be shared between the entrepreneur and the inspector will keep a negotiation going, the outcome of which is the bribe corresponding to the Nash solution to a bargaining game;

[14] See Appendix A for the proof.
[15] See Appendix B for the proof.

- If $c_j \geq c^*$, what the entrepreneur obtains by evading taxes is not enough to make up for his own expected "shame cost". With this in mind, the tax inspector will not ask the entrepreneur for a bribe. The game, therefore, finishes with the entrepreneur paying taxes. There is no sufficient margin for agreeing on a positive bribe with the tax inspector.

Tax revenues depend on the hypothesis made about the distribution of the "shame cost": if the specific j-th "shame cost" is lower than c^*, the entrepreneur finds it worthwhile to evade all taxes; vice versa, if the j-th entrepreneur's "shame cost" is greater than c^*, then the entrepreneur will be honest.
The cumulative probability density defines the distribution of individual costs $F(c_j)$, where j is the specific entrepreneur. The fraction of corrupted entrepreneurs, i.e. with a "shame cost" $c_j \leq c^*$, is given by $F(c^*)$; analogously, the fraction of honest entrepreneurs, with a "shame cost" $c_j > c^*$, is given by $1 - F(c^*)$.
On an aggregate level, the tax revenues, with a tax rate fixed at t, will be equal to the taxes paid by those who find themselves in a positive natural state (with probability $(1-\delta)$) and who have a "shame cost" which leads them to be honest, and those who are corrupt, but are discovered in the act of corruption:

$$(7) \qquad E(t,q) = Ank\left(\frac{G}{K}\right)^{\alpha} tF(c^*)(1-\delta)q + Ank\left(\frac{G}{K}\right)^{\alpha} t(1-F(c^*))(1-\delta) =$$

$$= Ank\left(\frac{G}{K}\right)^{\alpha} t(1-\delta)[F(c^*)q + 1 - F(c^*)]$$

As we have said, we assume that the amount of public good G is a proportion $\eta \in (0,1)$ of tax revenues, thus $\eta E(t,q) = G$.
By definition, F is a distribution function associated to a random variable whose density function f has support $[0,1]$. The shape of the function f gives good information about the general level of entrepreneurs' honesty. In particular, the symmetry of the function f provides information about the distribution of the entrepreneurs between those with a high or low sense of shame, briefly the honest and the corrupt. If f is a centered symmetric function, then the Country has average level of corruption, and the number of corrupted entrepreneurs balances the number of honest entrepreneurs. The case of f asymmetric to the left can be associated to a Country where most entrepreneurs are corrupt, while f is asymmetric to the right in Countries where most entrepreneurs are honest. Therefore, so that our analysis is complete, we need to discuss the three cases discussed above. Among the distribution function of random variables with support in $[0,1]$, the Kumaraswamy law seems to be the more appropriate choice for F. Indeed, even if the Kumaraswamy distribution is used in a rather mathematical fashion, it has some features that make it suitable for our modeling purposes.
The Kumaraswamy law belongs to the family of the two-parameters distributions, being the Beta distribution being the most famous. The most important feature of the Kumaraswamy random variable is its mathematical tractability, since being an explicit form of its distribution function. Indeed, given $\alpha_1, \alpha_2 \in (0, +\infty)$, the density function f and the distribution function F of a Kumaraswamy random variable are, respectively:

$$(8) \qquad f(c) = \alpha_1\alpha_2 c^{\alpha_1 - 1}(1 - c^{\alpha_1})^{\alpha_2 - 1}, \qquad c \in [0,1];$$

$$F(c) = 1 - (1 - c^{\alpha_1})^{\alpha_2}, \qquad c \in [0, 1]. \tag{9}$$

We assume that the distribution of the costs is a Kumaraswamy law, according to (8) and (9). This choice is reasonable, because the shape of the Kumaraswamy density function changes as the values of α_1 and α_2 vary. Therefore, this probability law is suitable for describing different types of entrepreneurs' ethical behaviors.
More specifically, if $1 < \alpha_2 < \alpha_1$, then the shape of the distribution function is asymmetric to the right, describing entrepreneurs with a "high shame" effect. Conversely, when $1 < \alpha_1 < \alpha_2$, then we have asymmetry to the left, and the entrepreneurs have a "low shame" effect. If $\alpha_1 = \alpha_2 = 1$, then the Kumaraswamy distribution reduces to the uniform distribution.
In order to describe three different types of entrepreneur behavior, we then rely on three cases: symmetry, $\alpha_1 = \alpha_2 = 1$; asymmetry to the left, $\alpha_1 = 1, \alpha_2 = 2$; asymmetry to the right, $\alpha_1 = 2, \alpha_2 = 1$.

- *"Uniform shame" countries.*
  In this case, the entrepreneurs are assumed to be uniformly distributed between the honest and the corrupt. The sense of shame varies accordingly to an uniform distribution, i.e.

  $$F(c) = c, \qquad \forall\, c \in [0, 1]. \tag{10}$$

  By substituting c^* with its expression in (7), a straightforward computation allows us to rewrite $E(t, q)$ as follows:

  $$E(t, q) = (1 - \delta)Ank \left(\frac{G_U}{K}\right)^{\alpha} t \left[1 - \frac{t(1 - q)^2}{q}\right]. \tag{11}$$

  By solving $\eta E(t, q) = G_U$, we find:

  $$G_U = \left[\eta(1 - \delta)AnkK^{-\alpha} t \left(1 - \frac{t(1 - q)^2}{q}\right)\right]^{\frac{1}{1-\alpha}}. \tag{12}$$

- *"Low shame" countries.*
  The number of corrupt entrepreneurs is assumed to be greater than that of the honest. The distribution function F is

  $$F(c) = 1 - (1 - c)^2, \qquad \forall\, c \in [0, 1]. \tag{13}$$

  As in the previous case, we substitute c^* with its expression in (7) and rewrite $E(t, q)$ as follows:

  $$E(t, q) = (1 - \delta)Ank \left(\frac{G_L}{K}\right)^{\alpha} t \left\{q - (q - 1)\left[1 - \frac{t(1 - q)}{q}\right]^2\right\}. \tag{14}$$

  By solving $\eta E(t, q) = G_L$, we have:

  $$G_L = \left[\eta(1 - \delta)AntkK^{-\alpha} \left\{q - (q - 1)\left(1 - \frac{t(1 - q)}{q}\right)^2\right\}\right]^{\frac{1}{1-\alpha}}. \tag{15}$$

- *"High shame" countries.*
  This is the converse case with respect to the previous one. The number of honest entrepreneurs is greater than that of the corrupt. In this case, the costs can be synthesized as follows:

  $$F(c) = 1 - (1 - c^2) = c^2, \qquad \forall c \in [0, 1]. \tag{16}$$

  We rewrite $E(t, q)$ in (7) as follows:

  $$E(t, q) = (1 - \delta)Ank\left(\frac{G_R}{K}\right)^{\alpha} t\left[1 - \frac{t^2(1 - q)^3}{q^2}\right]. \tag{17}$$

  By solving $G_R = \eta E(t, q)$ we have

  $$G_R = \left[\eta(1 - \delta)AntkK^{-\alpha}\left\{1 - \frac{t^2(1 - q)^3}{q^2}\right\}\right]^{\frac{1}{1-\alpha}}. \tag{18}$$

As the tax rate increases, we can detect two different effects in the behavior of tax revenues:

(1) As the tax rate increases, the number of dishonest entrepreneurs who are detected in a corrupt transaction increases and, therefore, the more numerous corrupt entrepreneurs pay higher taxes thus increasing the tax revenues;

(2) As the tax rate increases, the number of honest individuals decreases but they must pay more taxes. Therefore, we have two opposite effects: on the one hand, the decrease in honest entrepreneurs reduces tax revenues; on the other hand, the increase in the tax rate leads to more tax revenues.

The cases of uniform and high shame countries show a U-reversed relationship between tax revenues and the tax rate: in order to be more precise, until a certain threshold value, different for the three cases, the rise in tax rate increases tax revenues. This happens because the positive effect 1) adding up to an effect 2) that in this range is positive. After this threshold, the effect 2) becomes negative as the reduction in the number of honest entrepreneurs is higher than the rise of the tax rate. In this interval, therefore, the rise in the tax rate leads to a decrease tax revenues because the negative effect 2) is greater than the positive effect 1). With regard to the threshold value, we can stress that this value appears to be smaller for "uniform shame" countries and higher for countries with "high shame costs". Therefore, in a static context, a State which wants to maximize tax revenues must take into account the threshold value after which the revenues are decreasing, whereas the threshold value is different depending on the shame effect of the country. For a fixed level of taxe rates, we can see that the tax revenues of the country with "low shame costs" are low, intermediate those of the "uniform shame" country and high for the country with "high shame costs", i.e. $E_R > E_U > E_L$. This happens because the variation in the number of corrupt entrepreneurs (equal and opposite to that of honest entrepreneurs) is, in corresponding to high tax rates, higher for the country with "high shame costs", intermediate for country with "uniform shame costs" and low for the country with "low shame costs". Therefore, for the "low shame" country, in effect (2) the decrease in honest entrepreneurs more than makes up for the increase in the tax rate. As a result, it is not surprising that, for "low shame" countries tax revenues increase with respect to the tax rates. Therefore, for "low shame" countries, there is not a tax rate which maximizes the tax revenues. The solutions G's explicitly derived in (12), (15) and (18) will be denoted hereafter with $G^*(t,q,k)$ with the subscripts U, L, R, to highlight the explicit dependence with respect to t, q and k and maintain an explicit reference to the level of corruption of the country. Because G is a proportion $\eta \in (0,1)$ of tax revenues, thus $G_R > G_U > G_L$.

4 Dynamic Analysis

The game perspective is now expanded to review the consequences of the tax rate on dynamic revenues and on economic growth. The entrepreneur can use her/his payoff $\pi^{(1)}_{i,j}$, $i = 1, 2$, either to consume or invest. We consider a simple constant elasticity utility function:

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma} \tag{19}$$

Each entrepreneur maximizes utility over an infinite period of time, subject to a budget.

$$\max_{c \in \Re^+} \int_0^\infty e^{-\rho t} U(C) dt \tag{20}$$

sub

$$\dot{k} = \pi^{(1)}_{i,j} - C, \qquad i = 1, 2. \tag{21}$$

where C is per capita consumption, ρ is the discount rate in time, and $\pi^{(1)}_{i,j}$ is the payoff for the $j-th$ entrepreneur.
Since the return on the investment for the entrepreneur is different in each of the two equilibria found (with $-\pi^{(1)}_{2,j}$ – and without corruption $\pi^{(1)}_{1,j}$), the problem is solved for the two cases.
By solving the dynamic game, we can prove the following proposition:[16]

Proposition 4.1. *Let $0 \leq c^* \leq 1$ defined as in Proposition 4.1. Then,*

(a) if $c_j \leq c^*$ the growth rate, for the j-th entrepreneur, is

$$\gamma_j^C = \frac{1}{\sigma}\left\{ A \left(\frac{G}{K}\right)^\alpha \left(1 - \frac{qt + t + c_j q}{2}\right) - \rho \right\}; \tag{22}$$

(b) if $c_j > c^*$ the growth rate, for the j-th entrepreneur, is

$$\gamma_j^{NC} = \gamma^{NC} = \frac{1}{\sigma}\left\{ A \left(\frac{G}{K}\right)^\alpha (1-t) - \rho \right\}. \tag{23}$$

Equilibrium depends, therefore, on the individual "shame cost":

[16] See Appendix C for the proof.

- for a given tax rate t, the entrepreneurs with a "shame cost" of $c_j \leq c^*$, will find it worthwhile to be corrupt and so their optimal equilibrium will be with corruption. In such an equilibrium, the j-th entrepreneur will obtain a growth rate of γ_j^C;

- for a given level of tax rate t, entrepreneurs with a "shame cost" $c_j > c^*$ will find it worthwhile to be honest and their optimal equilibrium will be without corruption. In such an equilibrium, the j-th entrepreneur will obtain a growth rate of γ^{NC}.

In equilibrium, $K = kn$ applies. Therefore, we now impose the equilibrium condition $K = nk$ and substitute G with the values found in (12), (15) and (18).
It may be further demonstrated that capital and income also have the same growth rate of consumption and, therefore, equilibrium without corruption, from the dynamic viewpoint, is the equilibrium which allows greater economic growth[17]. In addition, since the tax rate influences the accumulation of capital, the provision of the public good and, as a consequence, economic growth, it will also increase fiscal revenues at a steady state. We would like to remind readers that the static tax revenues are (7):

$$E(t,q) = Ank\left(\frac{G}{K}\right)^{\alpha} t(1-\delta)[F(c^*)q + 1 - F(c^*)]$$

In a steady state, the growth rate of tax revenues should also be constant and, therefore, $E(t,q)$ and k grow at the same rate. Indeed, lower revenues today due to evasion, can bring greater growth through greater capital accumulation and, consequently, greater revenues tomorrow.
At the aggregate level, we will have a growth rate obtained by considering the different growth rates for the corresponding entrepreneurs.
Define ξ the fraction of honest entrepreneurs. In the equilibrium with corruption there will be $(1-\xi)$ entrepreneurs, each with her/his own growth rate γ_j^C, in the equilibrium without corruption there will be ξ entrepreneurs, each with the same growth rate γ^{NC}.
We perform the distinction for the "shame costs" introduced in the previous section.
At the aggregate level, we can prove the following proposition:[18]

Proposition 4.2. *The aggregate growth rate is:*

- *"Uniform shame" countries*
(24)
$$\gamma_U(t,q) = \frac{n}{\sigma}\left\{An^{-\alpha}k^{-\alpha}[G_U^*(t,q,k)]^{\alpha}\left[\frac{c^*t(q-1) - t(q+1)}{2} + 1\right] - \rho - \frac{t^2(1-q)^2}{4q}\right\};$$

[17] In fact, at a steady state, everything grows at the same rate and therefore $\frac{\dot{k}}{k}$ is constant. At equilibrium with corruption we know that $\frac{\dot{k}}{k} = \frac{An^{-\alpha}[G^*(t,q,k)]^{\alpha}k^{-\alpha}[2-t(1+q)]}{2} - \frac{qc_j}{2} - \frac{C}{k}$. Since $\frac{\dot{k}}{k}$ is constant, then the difference between both terms on the right should also be constant, and because A, n, α, c_j, q and t are constant and $G_U^*(t,q,k) \sim G_L^*(t,q,k) \sim G_R^*(t,q,k) \sim k$, then C and k should grow at the same rate. Similarly, since $y = An^{-\alpha}[G^*(t,q,k)]^{\alpha}k^{1-\alpha}$, at a steady state, income grows at the same rate as capital. The same applies in the case of equilibrium without corruption.
[18] See Appendix D for the proof.

- *"Low shame" countries*
  (25)
  $$\gamma_L(t,q) = \frac{n}{\sigma}\left\{An^{-\alpha}k^{-\alpha}[G_L^*(t,q,k)]^{\alpha}\left[\frac{(1-(1-c^*)^2)t(q-1)-t(q+1)}{2}+1\right]-\rho-\frac{t^2(1-q)^2}{4q}\right\};$$

- *"High shame" countries*
  (26)
  $$\gamma_R(t,q) = \frac{n}{\sigma}\left\{An^{-\alpha}k^{-\alpha}[G_R^*(t,q,k)]^{\alpha}\left[\frac{(c^*)^2t(q-1)-t(q+1)}{2}+1\right]-\rho-\frac{t^2(1-q)^2}{4q}\right\}.$$

4.1 Calibration

This section calibrates the model to quantitatively illustrate the long–run effects of the tax rate with different "shame costs". We start by assigning values for structural parameters. Since this paper is motivated by the stylized facts in OECD countries, it is better if we could calibrate parameters using data from these countries.

We would now like to provide a sensitivity analysis of γ with respect to t and q.
We proceed by performing a numerical analysis of the behavior of γ with respect to t and q, since the complexity of the dynamics involved does not allow closed–form results. Hence, a more intuitive description of the real situation is also provided.
We refer to the cases discussed above, with low, high and middle shame countries. We set $\delta = 0.5$, $\sigma = 0.5$, $\rho = 0.03$, $\eta = 0.5$, $n = 10$, $k = 1$ and three different values for the technology parameter $A = 0.5; 1; 2$.
Most choices in the parameter values are basically grounded in the literature, and the sources have been shown in Table 1. Some of them are assumed to have a certain value. A brief discussion about the assumptions on δ, α and n follows.
$\delta \in [0, 1]$ is the probability that the production of the entrepreneur is 0, i.e. the bad state case. We assume that the bad state case occurs with the same probability as the good state one, i.e. $\delta = 0.5$, because we do not want to insert asymmetry in entrepreneurs' endowments.
$\alpha \in [0, 1]$ is the production externality. As long as α is smaller than unity, there is a positive externality of capital accumulation, and the case $\alpha = 1$ is related to no externality of capital accumulation. We also assume symmetry, in this case and an average amount of externality of capital accumulation. Therefore, we set $\alpha = 0.5$.
$n \geq 1$ represents the number of entrepreneurs. Since no guidelines appear in the literature to fix a value of n, several different choices have been implemented in previous studies. We set $n = 10$. We can detect three different effects in the behavior growth rate both of the income and of tax revenues with respect to t.

(1′) As the tax rate increases, the number of honest individuals decreases, but they must pay more taxes. Therefore we have two opposite effects: on the one hand the decreasing of honest entrepreneurs reduces tax revenues and therefore increases capital accumulation and growth; on the other hand the increase in tax rate leads to more tax revenues, less capital accumulation and thus less growth.

(2′) As the tax rate increases, the number of dishonest entrepreneurs increases and, therefore, growth is reduced inasmuch as the number of discovered corrupt

Table 1: Benchmark parameter values.

Definition	Parameter	Value	Source
Coefficient of risk aversion	σ	0.5	Das and Sarkar (in press)
Time preference rate	ρ	0.03	Ljungvist and Sargent (2004)
Probability of a bad state	δ	0.5	Assumption
Share of government investment	η	0.1	Galì and Perotti (2003)
Coefficient of productivity	A	$0.5; 1; 2$	Leung and Tse (2002) Razin and Yuen (1996) Pindyck and Wang (2009)
Productivity of capital	α	0.5	Assumption
Capital per entrepreneur	k	1	Stevens (2003)
Number of entrepreneurs	n	10	Assumption

entrepreneurs increases. The newly corrupt entrepreneur, when discovered (as s/he will be forced to pay taxes but will receive the bribe back), is tantamount to an entrepreneur whose tax burden has increased;

(3′) As the tax rate increases, then the amount of public good G^*, obtained via balance constraints, increases as well, as does the growth rate.

If we take into account the behavior of the growth rate with respect to q, three different effects can be detected.

(1″) As the monitoring level increases, the number of corrupt individuals decreases but they must pay more taxes due higher control levels. Therefore we have two opposite effects: on the one hand, the decrease in corrupt entrepreneurs reduces tax revenues and therefore increases capital accumulation and growth; on the other hand, the increased monitoring level leads to more tax revenues, less capital accumulation and thus less growth.

(2″) As the monitoring level increases, the number of honest entrepreneurs increases and, therefore, the tax revenues increase, reducing capital accumulation and consequently economic growth.

(3″) As the monitoring level increases, *ceteris paribus*, G^* increases. As a consequence, the growth rate increases.

In order to discuss how the growth rate changes as a consequence of variations in the tax rate, we should stress that all three cases - "low, middle and high"- shame countries show the same growth rate behaviour. For this reason, we propose only one figure ("high shame" country) as representative of all cases (see Figure 2). Nevertheless, some differences can be highlighted when the level of monitoring is low or high. In fact, the effect of the tax rate on the growth rate is more relevant for growing values of q, and the growth rate is very sensitive with respect to t when the monitoring value tends to 1. This is reasonable, because in this case all entrepreneurs will find it worthwhile to be honest regardless of their "shame cost" (very limited effect ($2'$)). In fact, exasperate monitoring activity removes the differences between "high" or "low shame" entrepreneurs.

Caption of Figure 2: Growth rate vs tax rate, in "high shame" country.
INSERT FIGURE 2 ABOUT HERE

In all three cases - "low, middle and high shame" countries, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease as the tax rate increases.
In fact, for low tax rates, the increase in growth due to greater amount of public good provided (effect ($3'$)) and smaller capital accumulation (effect ($2'$)), are stronger than the negative effect caused by less capital accumulation by the entrepreneurs (effect ($1'$)). Let us explain the meaning of this behavior: when the tax rate grows, the tax revenues increase due to the fact that the number of corrupted entrepreneurs who have been detected and who pay more taxes increases and that the remaining honest entrepreneurs pay more taxes. Therefore, the increased provision of the public good more than compensates, in terms of economic growth, for the lower capital accumulation by entrepreneurs and thus economic growth increases. Therefore the growth rate follows the trend of increasing tax revenue, i.e. of the increasing amount of public good provided.
Conversely, when the tax rate is high, the decrease in growth due to the smaller amount of public good provided ($3'$) is stronger than the positive effect due to greater capital accumulation by entrepreneurs (effects ($1'$) and ($2'$)). In this case, the smaller amount of taxes paid by the honest and by entrepreneurs who have been discovered, depresses economic growth, via the lower amount of public good provided.
It is important to note that, despite the three cases having the same behaviour, for intermediate tax rates, the rate of growth for "low shame" countries is lower than that of "uniform shame" countries which is, in turn, lower than that of "high shame" countries. This happens because, in a less corrupt population, honest entrepreneurs pay taxes even where there is a high tax rate and therefore accumulate less in the honest countries than in the corrupt ones. In particular, the growth rate is more sensitive to variations of t in a honest country rather than in a corrupt country. There is intuitive reasoning behind this fact: in a honest country, the number of entrepreneurs paying taxes does not decrease remarkably when the tax rate increases, in contrast to what happens in a corrupt country.
As we have said, when the monitoring level is low, the behavior of the growth rate is very similar to when the monitoring level is high, but with different characteristics: even in this case, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease. But, it is important to stress that such a threshold value is different when the monitoring level is low or high. In fact, when the monitoring level tends to 1, the negative effects on growth –due to the entrepreneurs detected in a corrupt transaction having to pay more taxes, depressing capital accumulation ($2'$)– is negligible and therefore the amount of public good provided to entrepreneur increases also for higher tax rate.
The analysis of the behavior of the growth rate with respect to q agrees with previous results. In particular, when a tax rate is fixed, the behavior of the growth rate is very

different depending on whether the tax rate is low or high. For a low tax rate level, the growth rate is roughly constant with the monitoring level: the growth rate does not depend on the monitoring activity. We have an equilibrium situation, where the aggregate negative effects balance the expansion of the growth perfectly. Indeed, in this case, the impulse of public goods due to greater tax revenues balances the minor capital accumulation by the entrepreneurs and viceversa.

When a high tax rate t is fixed, the growth rate increases with respect to the monitoring activity level. The positive effect ($3''$) is greater than the tendency to depression due to ($1''$) and ($2''$). We interpret these findings by noticing that the most important effect of monitoring activity is to increase the amount of the public goods due to greater tax revenues. If we rely on technology parameter A we have that, *ceteris paribus*, when we consider A higher, the growth rate surface behaviors described above are amplified and the values of the growth rates are bigger when A is higher, since the growth rate increases when entrepreneurs invest their resources. The argument of amplification of effects due to a greater value of A applies hereafter, for the entire set of numerical analyses we will perform throughout the paper. Figure 2 refers to the case of $A = 2$.

We now analyze the corner–solutions. In particular, we focus on the extremal values of the monitoring activity level q and of the tax rate t.

- If $t = 0$, then there are no tax revenues for the country. The public good derived from tax revenues is therefore null, economic analysis becomes trivial and quite senseless.

- If $t = 1$, then the entire amount of each entrepreneur's production goes to the State. The presence of a monitoring activity level $q > q^\circ$ prevents the mass of entrepreneurs from becoming corrupt: indeed, in this case $c^* = \frac{1-q}{q} \in (0, 1)$. A closed–form analysis is not suitable, and we prefer to proceed numerically, by adopting the same set of parameters used in the global analysis performed above.

  Caption of Figure 3: Growth rate vs monitoring level, when $t = 1$.
  INSERT FIGURE 3 ABOUT HERE

  Figure 3 shows our findings when $A = 2$. The growth rate increases with respect to q and it is concave in all situations of corruption within the countries. The positive effect ($3''$) is more incisive than the negative impact on growth due to ($1''$) and ($2''$). The most relevant effect of the monitoring activity is the large amount of public goods due to greater tax revenues. Nevertheless, as the monitoring activity level becomes bigger, an inverse tendency is observed, and the growth rate of the country stabilizes. We interpret this inversion by noticing that heavy tax rates and monitoring activities depress capital accumulation and, consequently, growth.

- If $q = 1$, then we have $c^* = 0$ i.e. the entire population is honest. The effects on growth due to (2) and (3) disappear. In this case

$$G^*(t, 1, k) = G^*_U(t, 1, k) = G^*_L(t, 1, k) = G^*_R(t, 1, k) = \left[\eta(1-\delta)An\frac{k}{K^\alpha}t\right]^{\frac{1}{1-\alpha}}$$

and

$$\gamma(t,1) = \gamma_U(t,1) = \gamma_L(t,1) = \gamma_R(t,1) = \frac{1}{\sigma}\left\{An^{1-\alpha}k^{-\alpha}[G^*(t,1,k)]^\alpha(1-t)-\rho\right\}.$$

By applying the first order condition, we find a threshold for the tax rate $t_1^* = \alpha$ such that

$$\left\{\begin{array}{lll} t \le t_1^* & \Rightarrow & \frac{\partial}{\partial t}\gamma(t,1) > 0; \\ t > t_1^* & \Rightarrow & \frac{\partial}{\partial t}\gamma(t,1) < 0. \end{array}\right. \tag{27}$$

If the tax rate is low enough (below the critical threshold t_1^*), then the positive effect on growth due to (4) is more relevant than the negative effect due to (1). This behavior inverts for t greater than t_1^*. The economic key is grounded on two arguments: for low tax rates, capital accumulation is high for low tax rates, even when the State monitors actively. As a consequence, the country's growth increases. Conversely, when the monitoring activity is strong and the tax rate is high, then capital accumulation reduces, and growth reduces as well, even if the amount of public goods increases. We notice that the tax rate threshold t_1^* goes hand in hand with production y, since it coincides with the parameter α.

- If $q = q^\circ$, then we have $c^* = 1$, and the entire population is corrupt. Also in this case, we prefer to proceed via numerical simulation, to provide a more intuitive analysis. The usual parameter set is used. Figure 4 shows our findings for $A = 2$. Since the mass of the population is corrupt, the effect on growth due to $(1')$ vanishes.

Caption of Figure 4: Growth rate vs tax rate, when $q = q^\circ$
INSERT FIGURE 4 ABOUT HERE

In this case, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease as the tax rate increases. In fact, for low tax rates, the increase in growth due to $(3')$ is stronger than the negative effects due to $(2')$. Let us explain the meaning of this behavior: when the tax rate grows, the tax revenues increase due to the fact that the number of corrupted entrepreneurs who have been detected and who pay more taxes increases. Therefore, the increased provision of the public good more than compensates, in terms of economic growth, for the lower capital accumulation by entrepreneurs and thus economic growth increases. In fact, this result is grounded on the fact that, for a low tax rate, the impetus for growth given by greater tax revenues, i.e. the higher amount of public good provided, compensates for the lower economic growth due to lower capital accumulation by entrepreneurs. Conversely, when the tax rate is high, the decrease in growth is due to the joint effect of $(2')$ and $(3')$. Furthermore, as already noted above, the growth rate is more sensitive to the variations of the tax rates as the honesty of the country grows. This explains why a honest country can reach a maximum (minimum) level of growth rate that is higher (lower) than that of a corrupt country (see the global maxima and global minima in Figure 4).

4.2 Bargaining strength

In an asymmetric Nash bargaining solution, the surplus is shared unequally between the tax inspector and the taxpayer, and the equilibrium bribe (b^{NB}) is:

$$b^{NB} = \mu \left[Ak \left(\frac{G}{K} \right)^{\alpha} \left(t - \frac{qc_j}{(1-q)} \right) \right]. \tag{28}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon+\beta}$ is the share of the surplus which goes to the tax inspector and β and ε the bargaining strength of the entrepreneur and the tax inspector respectively.
Thus, the bribe paid to the inspector increases as the inspector's bargaining strength increases, expressed as ε. In fact, by computing this derivative we observe that:

$$\frac{\partial b^{NB}}{\partial \mu} = \left[Ak \left(\frac{G}{K} \right)^{\alpha} \left(t - \frac{qc_j}{(1-q)} \right) \right] > 0 \tag{29}$$

Increasing the bargaining power of the tax inspector increases the bribe which s/he can obtain. In the model, we also see that corruption does not depend on the distribution of the surplus between the inspector and the tax evader, but only on the amount of the surplus τ. In fact, the number of corrupt entrepreneurs is not dependent on the parameters β and ε. On the contrary, such parameters affect any rates of income growth and tax revenue in that a different distribution of power in the area of bargaining affects accumulation by the entrepreneur and, hence, the growth rate.
In particular, in Proposition 4.1, we see that if $c_j > c^*$, the growth rate for the j-th entrepreneur is:

$$\gamma^{NC} = \frac{1}{\sigma} \left[A \left(\frac{G}{K} \right)^{\alpha} (1-t) - \rho \right] \tag{30}$$

and it is not dependent on the parameters β and ε. On the contrary, if $c_j \leq c^*$ the growth rate for the j-th entrepreneur is dependent on the parameters β and ε in that this is the equilibrium where the entrepreneur pays the bribe and the value of this bribe depends on β and ε. The growth rate, if $c_j \leq c^*$, will be:

$$\gamma_j^C = \frac{1}{\sigma} \left[A \left(\frac{G}{K} \right)^{\alpha} [1 - \mu t + q(t + c_j)(\mu - 1)] - \rho \right]. \tag{31}$$

As a result, the aggregate growth rate will also be affected by the bargaining strength of the inspector and the evader. We denote it as $\gamma(\mu)$. In particular, the aggregate growth rate is linear with respect to μ.
We can detect two opposite effects in the behavior growth rate both of the income and of tax revenues with respect to μ:

(1‴) As the bargaining strength of the inspector increases, the entrepreneur must give a greater share of evasion to corruption, i.e. to the tax inspector. In this case, *ceteris paribus*, as the bargaining strength of the inspector increases, a lesser amount of resources will be allocated to investment and generate lower growth;

($2'''$) As the bargaining strength of the inspector increases, the entrepreneur is able to transfer a greater part ($\mu q c_j$) of her/his "shame cost" to the tax inspector. In this case, *ceteris paribus*, as the bargaining strength of the inspector increases, a greater amount of resources will be allocated to investment and generate higher growth.

In order to describe the constant rate of decay of $\gamma(\mu)$, we introduce the subscripts U, L, R and proceed numerically adopting the usual parameter set.

- *"Uniform shame" countries*

$$\gamma'_U(\mu) = \frac{c^*}{\sigma} \cdot An^{-\alpha}[G^*_U(t,q,k)]^\alpha k^{-\alpha} t(q-1) + \frac{q(c^*)^2}{2\sigma}.$$

- *"Low shame" countries*

$$\gamma'_L(\mu) = \frac{(1-c^*)^2}{\sigma} \cdot An^{-\alpha}[G^*_L(t,q,k)]^\alpha k^{-\alpha} t(q-1) + \frac{q(c^*)^2}{2\sigma}.$$

- *"High shame" countries*

$$\gamma'_R(\mu) = \frac{(c^*)^2}{\sigma} \cdot An^{-\alpha}[G^*_R(t,q,k)]^\alpha k^{-\alpha} t(q-1) + \frac{q(c^*)^2}{2\sigma}.$$

In the three cases, the same findings are obtained, and Figure 5 shows our results for $A = 2$.

Caption of Figure 5: Derivative of γ w.r.t. μ: "Uniform shame" country.
INSERT FIGURE 5 ABOUT HERE

We see that the growth rate does not increase at all when the monitoring level is high and the tax rate is low. In contrast, the growth rate increases with respect to the parameter μ when either the monitoring level or the tax rate are high. In this case, the aggregate effect of ($1'''$) is weaker than the positive effect due to ($2'''$). Therefore, as the bargaining strength of the inspector increases, the growth rates increases as well.
This result is compatible with the economic evidence that, when the tax rate and the monitoring activity level are high enough, a proportion of a country's surplus going towards incentivizing the action of tax inspectors has a positive impact on the country's growth rate. The size of such a positive impact varies according to the tax rate which has been applied, the capital productivity, the monitoring level and the marginal utility elasticity. Even if the presence of a part of surplus for the inspectors subtracts resources from the entrepreneurs' investments, the larger amount of taxes paid under a stronger monitoring regime allows for a larger amount of public goods via balance constraints, and this, in turn, permits growth to become higher.

5 Conclusions

The present paper provides a study of the problem of the optimal tax rate, in a dynamic environment where there is widespread corruption. The static analysis of Cerqueti and Coppier (2009) has been extended in a dynamic context, incorporating the presence of a public sector. We introduced endogenous corruption into a variant of the Ramsey growth model where a government taxes private producers and uses the resources to either supply public capital. Incorporating in the growth model also the “productive” effect of tax revenues i.e. the provision of public goods, we obtain different results from e.g. Chen (2003) and Lin and Yang (2001) who consider only the negative effect of taxes on capital accumulation. In fact, in a long run analysis, the result derives from the basic tenet of our model that evasion on one hand stimulates investment, accumulation and thereby growth but, on the other hand, reduces tax revenues and therefore the provision of the public good. In order to be more precise, we show that in all three cases - “low, middle and high shame” countries-, the growth rate increases as the tax rate increases up to a threshold value, after which the growth rate begins to decrease as the tax rate increases. As we showed, this result derives from the behavior of tax revenues in a static framework. Since higher (lower) tax revenues imply a higher (lower) provision of public goods, in our model the growth rate is driven by the amount of public good rather than capital accumulation by entrepreneurs. This is understandable if we consider that not all resources are assigned to capital accumulation: indeed, if evasion is detected by a corrupt inspector, the entrepreneur must give half of evaded taxes to the inspector (bribe) reducing, in this way, the resources devoted to the capital accumulation and therefore to the growth. Since in our calibration we assume that the productivity of public good and of private capital is the same, then the State has a rate of saving higher than entrepreneurs’ one. Therefore, in a comparison between resources detract to entrepreneurs via taxation and used for provision of the public good and resources accumulated by entrepreneurs, the former let the country grow more than the latter.

As a consequence, if the policy maker wants to maximize the growth rate, then s/he must set the rate to the corresponding tax rate. As we said this threshold value is different depending on the monitoring level and the specific “inner honesty” of the country. In particular, given the “inner honesty” of the country, the lower the maximizing tax rate, the lower the monitoring level. On the other hand, given the monitoring level put in place in a specific country, the more honest a country, the greater the maximizing tax rate. This result is different from the U-shaped curve between growth rate and tax rate shown by Lin and Yang (2001), as they simply consider public consumer goods and then economic growth can increase as the tax rate increases because resources are diverted from the unproductive public sector to the productive private sector. In addition, we find that a high probability of auditing increases the growth rate; conversely, Chen (2003) finds that this measure has ambiguous effect on economic growth, due mainly to its indirect effect upon tax compliance and the tax rate.

A Appendix: The Nash Bargaining bribe

Let $\underline{\pi}_\Delta = \underline{\pi}_{2,j} - \underline{\pi}_{1,j} = (\pi_\Delta^{(1)}, \pi_\Delta^{(2)})$ be the vector of the differences in the payoffs between the case of agreement and disagreement about the bribe, between inspector and entrepreneur. In accordance with generalized Nash bargaining theory, the division between two agents will solve:

$$\max_{b\in\Re^+} [\pi_\Delta^{(1)}]^\beta \cdot [\pi_\Delta^{(2)}]^\varepsilon \tag{32}$$

in formula

$$\max_{b\in\Re^+} \left[Ak\left(\frac{G}{K}\right)^\alpha (t - tq - c_j q) - (1-q)b\right]^\beta \left[\frac{(1-\eta)E}{n} + (1-q)b - \frac{(1-\eta)E}{n}\right]^\varepsilon \tag{33}$$

that is the maximum of the product between the elements of $\underline{\pi}_\Delta$ and where $[Ak\left(\frac{G}{K}\right)^\alpha (1-t), \frac{(1-\eta)E}{n}]$ is the point of disagreement, i.e. the payoffs that the entrepreneur and the inspector respectively would obtain if they did not come to an agreement. The parameters β and ε can be interpreted as measures of bargaining strength. It is now easy to check that the tax inspector gets a share $\mu = \frac{\varepsilon}{\varepsilon+\beta}$ of the surplus τ, i.e. the bribe is $b = \mu\tau$. More generally μ reflects the distribution of bargaining strength between two agents. The surplus τ is the saving which comes from not paying taxes, net of " shame cost", which awaits the entrepreneur if s/he is found out: $\tau = Ak\left(\frac{G}{K}\right)^\alpha \left(t - \frac{qc_j}{(1-q)}\right)$.
Then the bribe b^{NB} is an asymmetric (or generalized) Nash bargaining solution and is given by:

$$b^{NB} = \mu\left[Ak\left(\frac{G}{K}\right)^\alpha \left(t - \frac{qc_j}{(1-q)}\right)\right] \tag{34}$$

that is the unique equilibrium bribe in the last subgame, $\forall q \neq 1$.

B Appendix: Solution to the static game

The static game is solved with the backward induction method, which allows identification at the equilibria. Starting from stage 2, the entrepreneur needs to decide whether to negotiate with the inspector. Both payoffs are then compared, because the inspector asked for a bribe.

(2) At stage two the entrepreneur negotiates the bribe if, and only if

$$\pi_{2,j}^{(1)} \geq \pi_{1,j}^{(1)} \Rightarrow \tag{35}$$

$$Ak\left(\frac{G}{K}\right)^\alpha \left[\left(1 - \frac{t(1+q)}{2}\right) - \frac{kqc_j}{2}\right] \geq Ak\left(\frac{G}{K}\right)^\alpha (1-t) \Rightarrow$$

$$\Rightarrow c_j < \frac{t(1-q)}{q} = c^* \tag{36}$$

(1) Going up the decision-making tree, at stage one the tax inspector decides whether to ask for a positive bribe or not.

- Let $c_j < c^*$ defined in (36). Then the tax inspector knows that if s/he asks for a positive bribe, the entrepreneur will agree to negotiate and the final bribe will be b^{NB}. Then at stage one, the tax inspector asks for a bribe if, and only if

$$\pi_{2,j}^{(2)} > \pi_{1,j}^{(2)} \Rightarrow$$

$$\Rightarrow \frac{(1-\eta)E}{n} + Ak\left(\frac{G}{K}\right)^{\alpha}\frac{t(1-q)}{2} - Ak\left(\frac{G}{K}\right)^{\alpha}\frac{qkc_j}{2} > \frac{(1-\eta)E}{n} \tag{37}$$

  that is alway verified. Thus, if $c_j < c^*$, then the tax inspector asks for a bribe b^{NB}, which the entrepreneur will accept.

- Let $c_j \geq c^*$. Then the tax inspector knows that the entrepreneurs will not accept any possible bribe, so s/he will be honest and will ask the entrepreneurs for tax payment.

C Appendix: Solution of the dynamic game

In the equilibrium with corruption, the entrepreneur's profit is:

$$\pi_{2,j}^{(1)} = Ak\left(\frac{G}{K}\right)^{\alpha}\left(1 - \frac{qt+t+c_jq}{2}\right) \tag{38}$$

thus the constraint is:

$$\overset{\bullet}{k} = Ak\left(\frac{G}{K}\right)^{\alpha}\left(1 - \frac{qt+t+c_jq}{2}\right) - C \tag{39}$$

The Hamiltonian function is:

$$H = e^{-\rho t}\frac{C^{1-\sigma}-1}{1-\sigma} + \lambda\left[Ak\left(\frac{G}{K}\right)^{\alpha}\left(1 - \frac{qt+t+c_jq}{2}\right) - C\right] \tag{40}$$

where λ is a constant variable. Assumunig G and K as given, optimization provides the following first-order conditions:

$$e^{-\rho t}C^{-\sigma} - \lambda = 0 \tag{41}$$

and

$$\overset{\bullet}{\lambda} = -\lambda\left\{A\left(\frac{G}{K}\right)^{\alpha}\left(1 - \frac{qt+t+c_jq}{2}\right)\right\} \tag{42}$$

By the first condition, the consumption growth rate is obtained:

$$\gamma_j^C = \frac{1}{\sigma}\left\{A\left(\frac{G}{K}\right)^{\alpha}\left(1 - \frac{qt+t+c_jq}{2}\right) - \rho\right\}. \tag{43}$$

In the equilibrium without corruption, the entrepreneur's profit is:

$$\pi_{1,j}^{(1)} = Ak\left(\frac{G}{K}\right)^{\alpha}(1-t) \tag{44}$$

Thus the constraint is:

$$\overset{\bullet}{k} = Ak\left(\frac{G}{K}\right)^{\alpha}(1-t) - C \tag{45}$$

The Hamiltonian function is:

$$H = e^{-\rho t}\frac{C^{1-\sigma}-1}{1-\sigma} + \lambda[Ak\left(\frac{G}{K}\right)^{\alpha}(1-t) - C] \tag{46}$$

Optimization provides the consumer growth rate:

$$\gamma^{NC} = \frac{1}{\sigma}\left\{A\left(\frac{G}{K}\right)^{\alpha}(1-t) - \rho\right\}. \tag{47}$$

D Appendix: Aggregate growth

Aggregate growth γ is given by the sum of the rates of obtainable growth considered by the number of entrepreneurs who are positioned in that equilibrium. Thus, at the equilibrium with corruption, there will be $(1-\xi)$ entrepreneurs while at the equilibrium without corruption, there will be ξ entrepreneurs.
We impose $K = nk$.
At the equilibrium without corruption, the growth rate γ^{NC} in (47) is independent of reputation costs and will therefore be equal for each entrepreneur with reputation costs $c_j > c^*$; at the equilibrium with corruption, the growth rate γ_j^C in (43) is dependent on reputation costs for which reason each entrepreneur, with a reputation cost of $c_j \leq c^*$, will have a different growth rate. Thus

$$\gamma(t,q) = (1-\xi)\frac{1}{\sigma}\left\{An^{-\alpha}[G^*(t,q,k)]^{\alpha}k^{-\alpha}\left(1-\frac{qt+t}{2}\right) - \rho\right\} - \frac{1}{2\sigma}\left[q\int_0^{c^*} c_j dc_j\right] +$$

$$+\xi\cdot\frac{1}{\sigma}\left\{An^{-\alpha}[G^*(t,q,k)]^{\alpha}k^{-\alpha}(1-t) - \rho\right\}. \tag{48}$$

Substituting $\xi = F(c^*)$, where F has the three expressions in (10), (13) and (16) into (48) and after some simplifications we obtain the aggregate growth rate in the three cases.

References

[1] Allingham, G.M., Sandmo, A., 1972. Income Tax Evasion: A Theoretical Analysis. Journal of Public Economics 1, 323–338.

[2] Alm, J., Jackson, B., McKee, M., 1992a. Deterrence and beyond: toward a kinder, gentler IRS. In: Slemrod, J. (Ed.), Why People Pay Taxes: Tax Compliance and Enforcement. University of Michigan Press, Ann Arbor, pp. 311-329.

[3] Alm, J., Jackson, B., McKee, M., 1993. Fiscal exchange, collective decision institutions and tax compliance. Journal of Economic Behaviour and Organization 22, 285-303.

[4] Alm, J., Torgler, B., 2006. Culture differences and tax morale in the United States and in Europe. Journal of Economic Psychology 27, 224–246.

[5] Andreoni, J., Erard, B., Feinstein, J., 1998. Tax compliance. Journal of Economic Literature 36, 818-860.

[6] Barro, R., Sala-I-Martin, X., 1992. Public Finance in Models of Economic Growth. Review of Economic Studies 59, 645–661.

[7] Barro R., Sala-I-Martin, X. 2004. Economic Growth. Second Edition, The MIT Press.

[8] Barro, R., 1990. Government Spending in a Simple Model of Endogenous Growth. Journal of Political Economy 98(5), 103–25.

[9] Blackburn, K., Bose, N., Haque, M.E., 2006. The incidence and persistence of corruption in economic development. Journal of Economic Dynamics & Control 30, 2447–2467.

[10] Blankenau, W., S. Nicole, and M. Tomljanovich, 2004. Public Education Expenditures, Taxation, and Growth. Journal of Development Economics, 73(2), 583-605.

[11] Beck, P., Davis, J., Jung, W., 1991. Experimental evidence on taxpayers reporting under uncertainty. Accounting Review 64, 468-487.

[12] Becker, W., Buchner, H.J., Sleeking, S., 1987. The impact of public transfer expenditures on tax evasion: an experimental approach. Journal of Public Economics 34, 243-252.

[13] Beron, K., Tauchen, H., Witte, A.D., 1992. The effects of audits and socioeconomic variables on compliance. In: Slemrod, J. (Ed.), Why People Pay Taxes: Tax Compliance and Enforcement. University of Michigan Press, Ann Arbor, pp. 67-89.

[14] Besley, T., Mclaren, J., 1993. Taxes and Bribery: the Role of Wage Incentives. Economic Journal 103, 119–141.

[15] Caballé, J., Panadés, J, (2007). Tax rates, tax evasion, and growth in a multi-period economy. Revista de Economia Publica 183(4), 67–80.

[16] Cebula, R., 1997. An empirical analysis of the impact of government tax and auditing policies on the size of the underground economy: the case of the United States, 1973-94. American Journal of Economics and Sociology 56, 173-185.

[17] Cebula, R., 2001. Impact of income-detection technology and other factors on aggregate income tax evasion: the case of the United States. Banca Nazionale del Lavoro Quarterly Review 219, 401-415.

[18] Cebula, R., Saltz, I., 2001. An empirical note on tax auditing and the size of the underground economy in the United States, 1962-1980. International Review of Economics and Business 48, 119-124.

[19] Cerqueti, R., Coppier, R., 2009. Tax Revenues, Fiscal Corruption and " Shame Costs". Economic Modelling, 26(6), 1239–1244.

[20] Chander, P., Wilde, L., 1992. Corruption in Tax Administration. Journal of Public Economics 49, 333–349.

[21] Chen, B.L., 2003. Tax evasion in a model of endogenous growth. Review of Economic Dynamics 6, 381–403.

[22] Christian, C., Gupta, S., 1993. New evidence on secondary evasion. The Journal of American Taxation Association 15, 72-93.

[23] Chu, C.C.Y., 1990. A model of income tax evasion with venal tax officials: The case of Taiwan. Finance Publiques (Public Finance) 45 3, pp. 392-408.

[24] Dell'Anno, R., 2009. Tax evasion, tax morale and policy maker's effectiveness. Journal of Socio-Economics 38, 988–997.

[25] Dubin, J., Graetz, M.,Wilde, L., 1990. The effect of audit rates on the federal individual income tax, 1977-1986. National Tax Journal 43, 395-409.

[26] Dubin, J., Wilde, L., 1988. An empirical analysis of federal income tax auditing and compliance. National Tax Journal 41, 61-74.

[27] Ellis, C.J., Fender, J., 2006. Corruption and transparency in a growth model. International Tax and Public Finance 13, 115–149.

[28] Feld, L.P., Frey, B.S., 2007. Tax compliance as a result of a psychological tax contract: the role of incentives and responsive regulation. Law & Policy 29(1), 102–120.

[29] Feltenstein, A. and A. Shah, 1995. General Equilibrium Effects of Investment Incentives in Mexico. Journal of Development Economics, 46, 253-69.

[30] Fjeldstad, O.H., Semboja, J., 2001. Why people pay taxes: the case of the development Levy in Tanzania. World Development, 29(12), 2059–2074.

[31] Fishman, R. and J. Svensson, 2000. Are Corruption and Taxation Really Harmful to Growth? Firm Level Evidence- (unpublished; New York: Columbia University).

[32] Frey, B.S., Torgler, B., 2007. Tax morale and conditional cooperation. Journal of Comparative Economics 35, 136–159.

[33] Friedland, N., Maital, S., Rutenberg, A., 1978. A simulation study of income tax evasion. Journal of Public Economics 10, 107-116.

[34] Gupta, R. (2005). Costly state monitoring and reserve requirements. Annals of Economics and Finance, 6(2), 263-288.

[35] Gupta, R. (2006). Asymmetric information, tax evasion and alternative instruments of government revenue. The ICFAI Journal of Monetary Economics, IV(1), 75-89.

[36] Hindriks, J., Keen, M., Muthoo, A., 1999. Corruption, Extortion and Evasion. Journal of Public Economics 74, 394–430.

[37] Jaulfaian, D., Rider, M., 1996. Tax evasion in the presence of negative income tax rates. National Tax Journal 49, 553-570.

[38] Kim, Y., 2003. Income distribution and equilibrium multiplicity in a stigma–based model of tax evasion. Journal of Public Economics 87, 1591–1616.

[39] Klepper, S., Nagin, D., 1989. The anatomy of tax evasion. Journal of Law, Economics and Organization 5, 1-24.

[40] Lin, W.Z., Yang, C.C., 2001. A dynamic portfolio choice model of tax evasion: comparative statics of tax rates and its implication for economic growth. Journal of Economic Dynamics & Control 25, 1827–1840.

[41] Mookherejee, D., Png, P.L., 1995. Corruptible law enforcers: How should they be compensated? Economic Journal 105, 145–159.

[42] Richardson, G., 2006. Determinants of tax evasion: a cross-country investigation. Journal of International Accounting Auditing & Taxation 15, 150–169.

[43] Sanyal, A., Gang, I.N., Gosswami, O., 2000. Corruption, Tax Evasion and the Laffer Curve. Public Choice 105, 61–78.

[44] Slemrod, J., Blumenthal, M., Christian, C., 2001. Taxpayers response to an increased probability of audit: evidence from a controlled experiment in Minnesota. Journal of Public Economics 79, 455-483.

[45] Slemrod, J., Yitzhaki, S., 2002. Tax avoidance, evasion, and administration. Handbook of Public Economics, in: A. J. Auerbach & M. Feldstein (ed.), Handbook of Public Economics, 1423–1470, Elsevier.

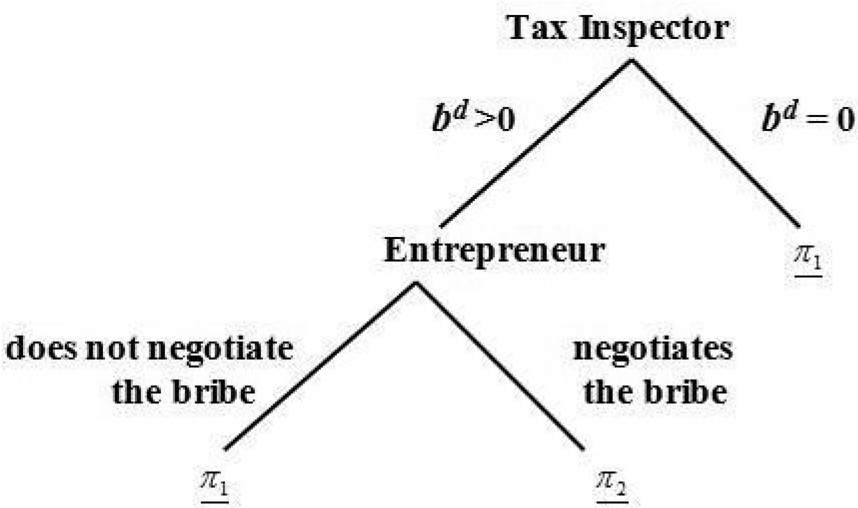
[46] Ul Haque, N., Sahay, R., 1996. Do government wage cuts close budget deficits? Costs of corruption. IMF Staff Papers 43, 754-778.

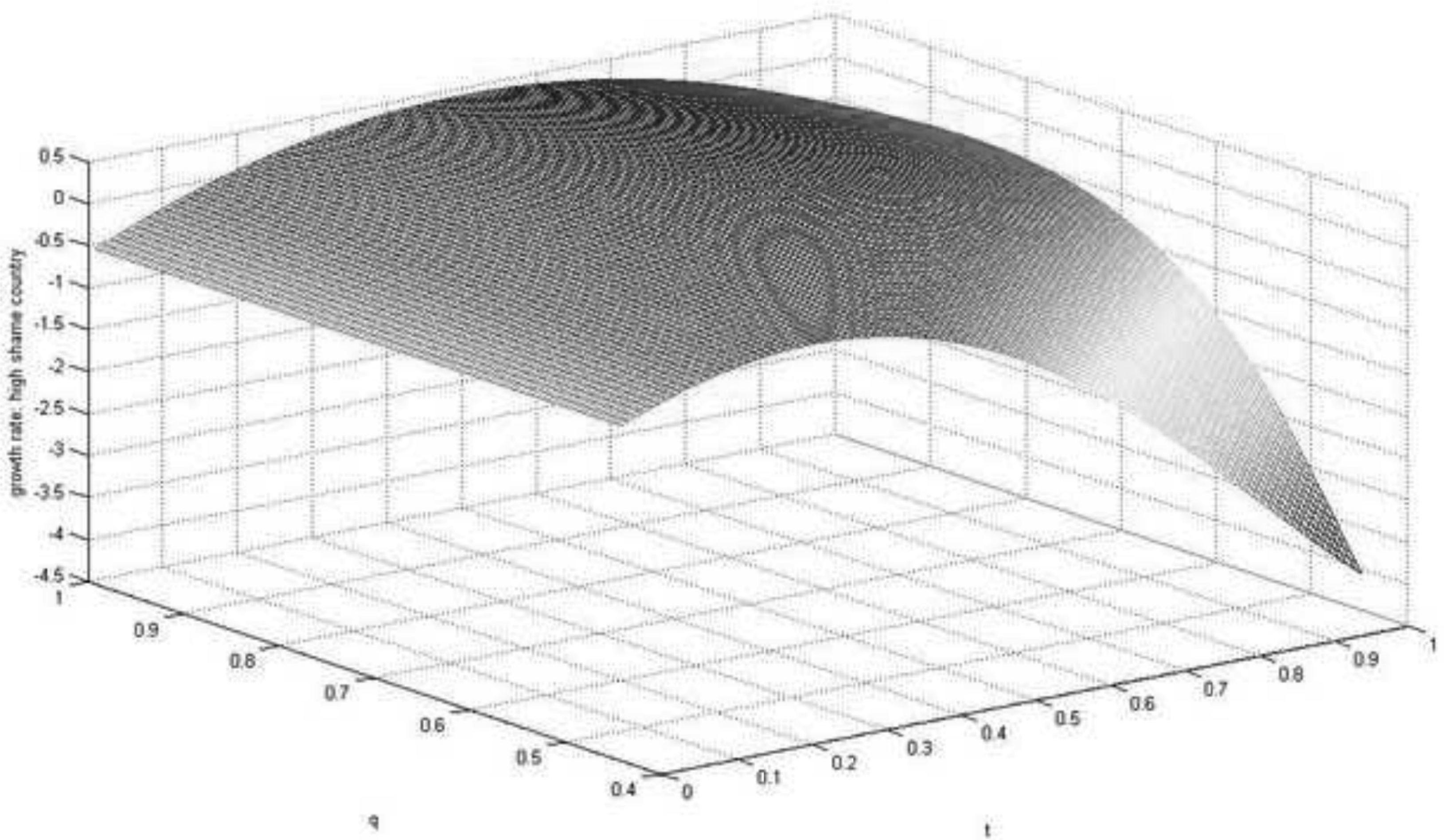
[47] Tanzi, V., Davoodi H. R., 2000. Corruption, Growth, and Public Finances. IMF Working Paper N 182.

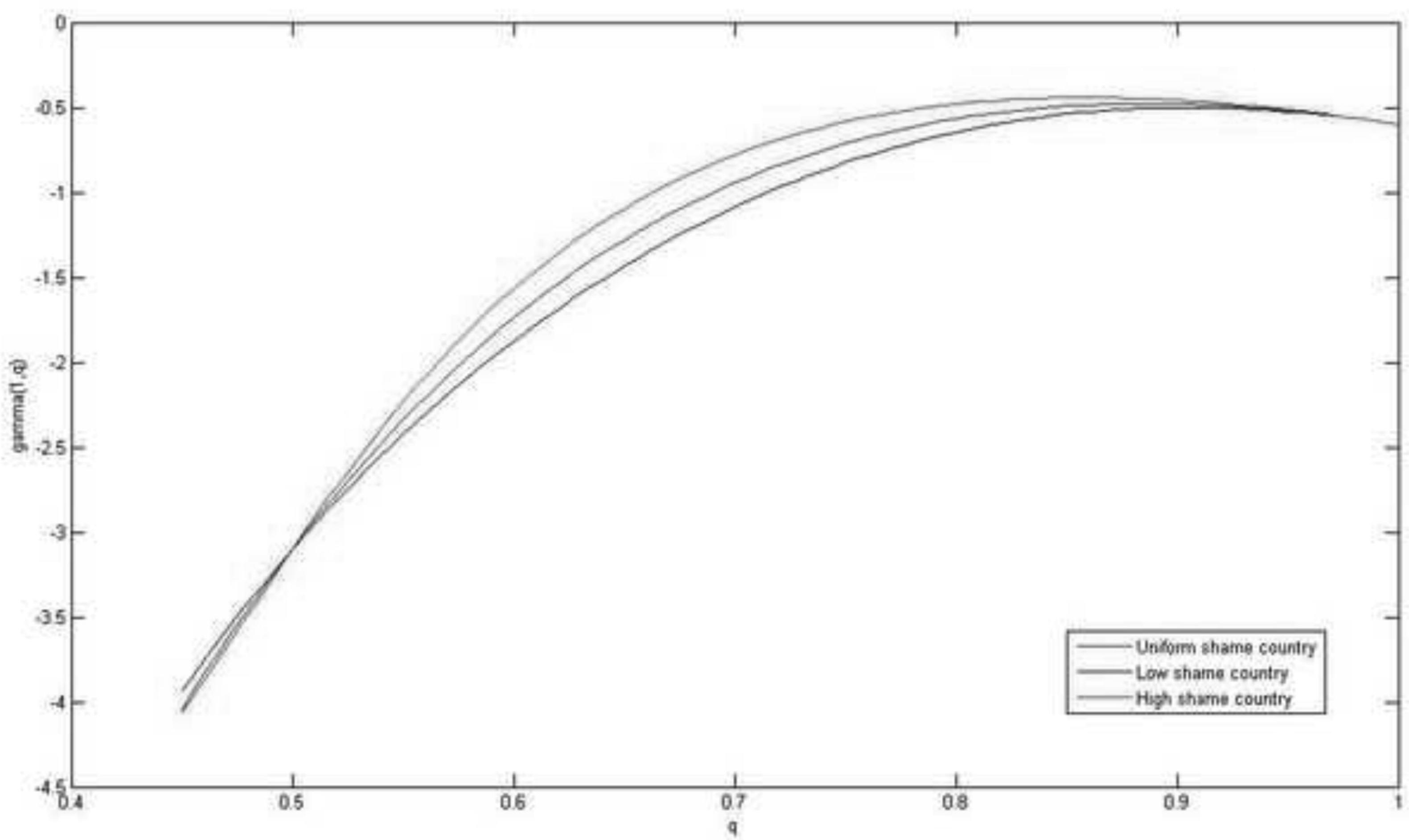
[48] Witte, A.D., Woodbury, D., 1985. The effect of tax laws and tax administration on tax compliance: the case of US individual income tax. National Tax Journal 38, 1-13.

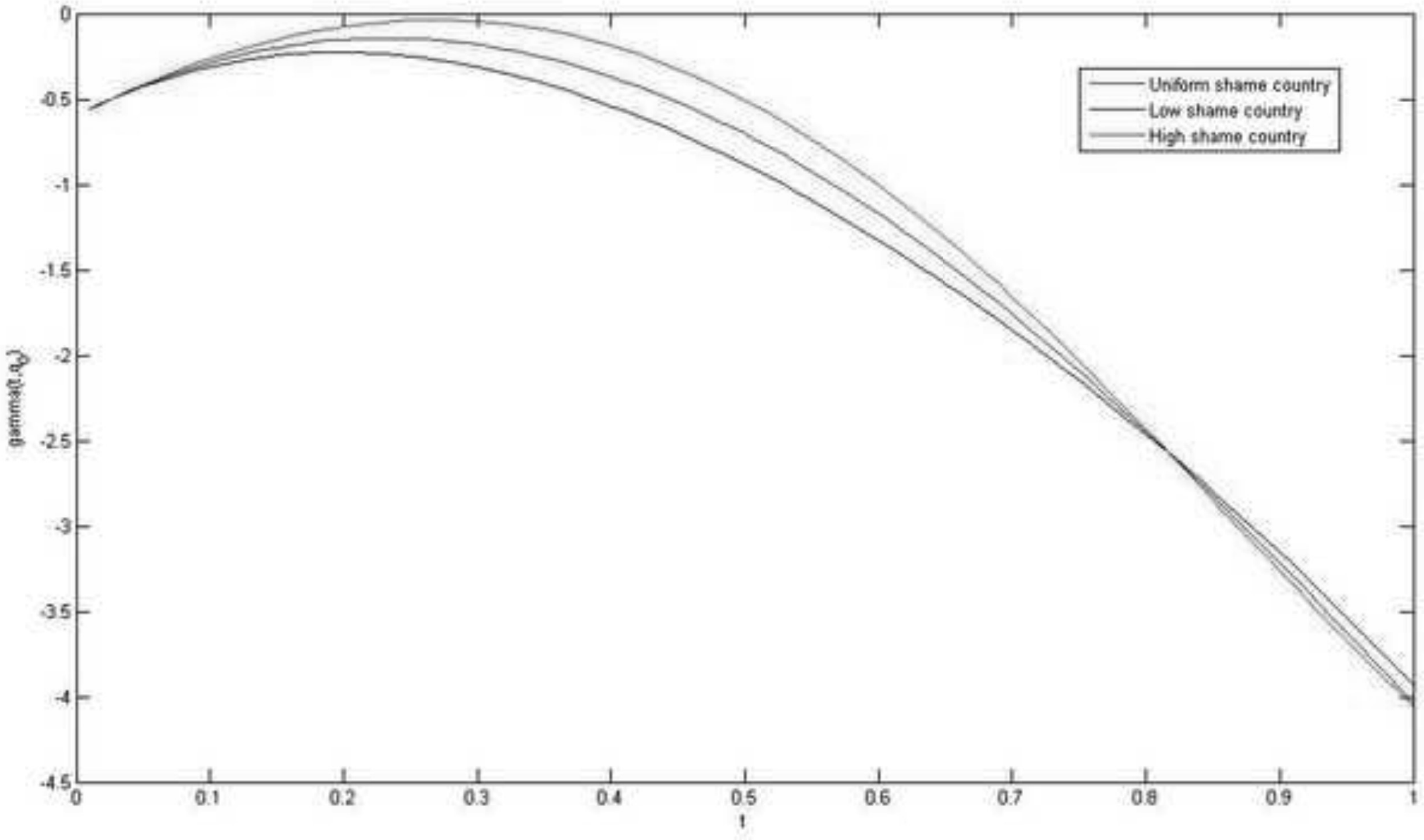
[49] Yitzhaki, S., 1974. Income tax evasion: A theoretical analysis. Journal of Public Economics 3(2), 201–202.

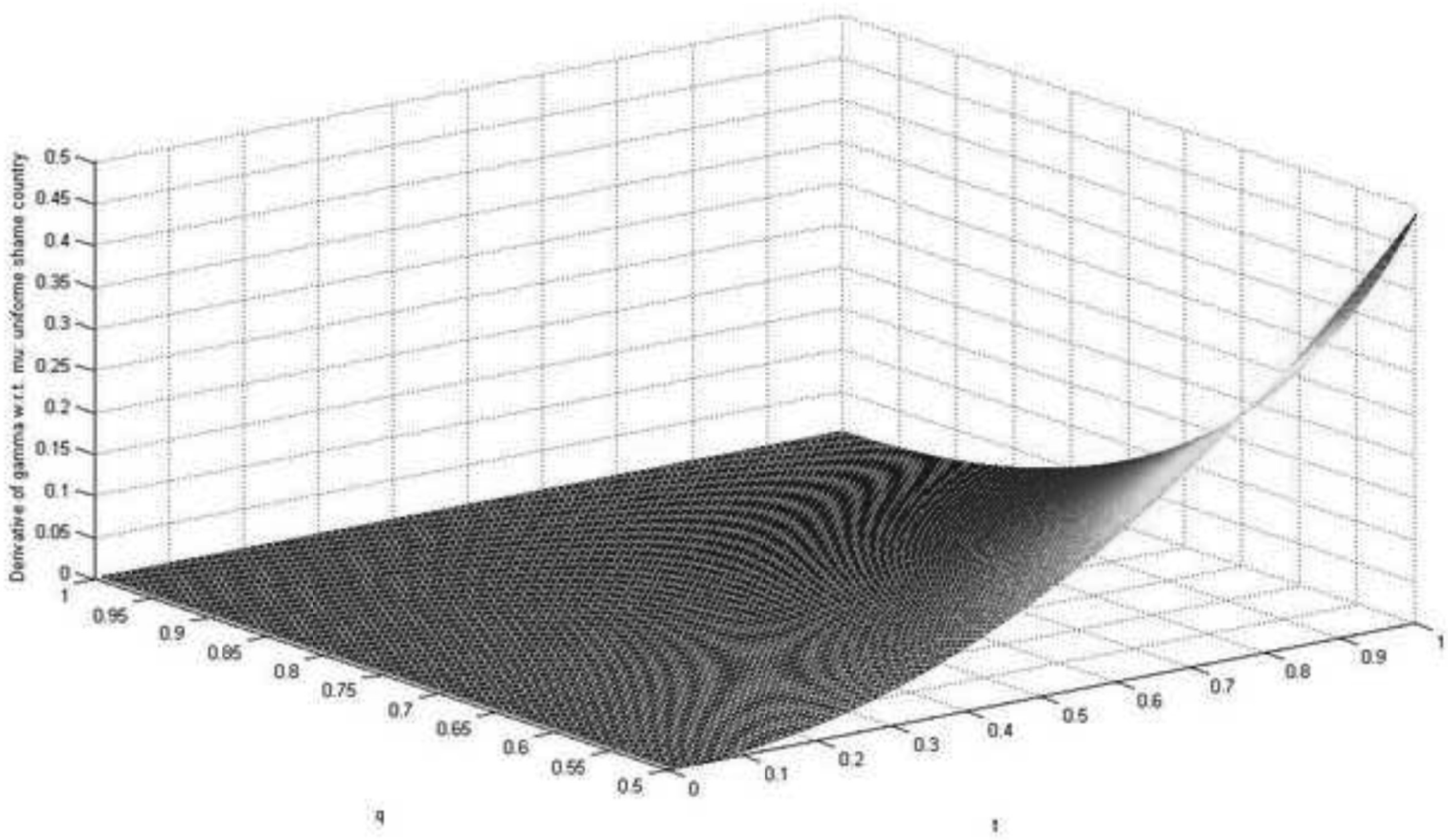
Figure(s)


Tax Inspector
$b^d > 0$
$b^d = 0$
Entrepreneur
$\underline{\pi_1}$
does not negotiate the bribe
negotiates the bribe
$\underline{\pi_1}$
$\underline{\pi_2}$


growth rate: high shame country
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
1
0.9
0.8
0.7
0.6
0.5
0.4
q
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


gamma(1,q)
q
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
0.4
0.5
0.6
0.7
0.8
0.9
1
Uniform shame country
Low shame country
High shame country


Uniform shame country
Low shame country
High shame country
gamma(t,q$_0$)
t
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Derivative of gamma w.r.t. mu: uniforme shame country
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
q
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t