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Mixed integer formulations for a routing problem with information collection in wireless networks

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Abstract

We study a routing-collecting problem where a system of stations is considered. A vehicle is responsible for collecting information generated continuously in the stations and to deliver it to a base station. The objective is to determine the vehicle route and the collection operations, both physical and wireless, in order to maximize the amount of information collected during a time horizon. Three mixed integer programming models are introduced and a computational study is reported to compare the performance of a solver based on each one of the models.

Keywords: Routing, Wireless Networks, Mixed Integer Programming

1. Introduction

Technological advances in network architectures add new features and applications to routing problems [6, 27, 17, 31, 26, 29, 23]. In this work, we are interested in the exact solution of a vehicle routing problem (VRP) for information collection in wireless networks. The new characteristic added to this well-know problem is the possibility of picking-up information via wireless transmissions.

A relevant application for this problem is to provide connection for difficult environments [27, 23]. For example, Daknet [27] is a low-cost solution for providing web connection to small and remote villages. Daknet uses existing buses and local transport to carry messages and web connection to modest and isolated villages in which small kiosks are placed allowing people to send email and information in an off-line manner. A very similar case

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is to provide web connectivity for remote military stations which dispose of a set of vehicles to gather and deliver information [23]. Applications for this problem appears whenever one needs to provide vehicle routing strategies with wireless information transmission to the vehicles involved, which is also the case in underwater surveillance [8], environmental monitoring [32] or automated meter reading [30].

In the context considered in this work, a unique base station is connected with the outside and a vehicle is responsible for collecting information from the other stations. The stations are equipped with technology capable of sending information via wireless connection to the vehicle when it is located in another sufficiently close station. Simultaneous transmissions are permitted. Time of transmission depends on the distance between stations, the amount of information transmitted, and other physical factors (e.g. obstacles along the way, installed equipment). Information to be sent outside of the network is continuously generated in each station at a constant rate. The VRP treated in this work looks for the vehicle route as well as for an efficient planning on how to collect information from stations, in order to minimize the total amount of remaining information in the nodes at the end of a finite time horizon T.

It is well-known [25] that a good mathematical model is crucial to solve a problem to optimality using exact approaches such as branch-and-bound and branch-and-cut methods. The focus of this work is the investigation and comparison of Mixed Integer Linear Programming (MILP) formulations for this routing problem with wireless transmission.

1.1. Related works

First, we discuss related studies from the application perspective. Many works in the literature address vehicle routing problems in the presence of wireless transmission with focusing on the study of protocols/policies for routing and data collection. A smaller set of works addresses the development of vehicle routing strategies, most cases, in a two-phase manner (see [14]). When studying such routing and data collection protocols, normally, the authors consider that the vehicle route is set, i.e., the mobility of the vehicle is not controllable. That is frequently the case in applications taking advantage of an existing transportation infrastructure [9, 12, 24, 31]. In some situations, as in [11], the authors assume the vehicle route exist but they suppose vehicles can adjust their position (in order to receive information from some stations).

For applications where vehicle mobility is controlled, some authors address the design of the vehicle routing and the information collection planning in two-phases [15, 16, 21, 28]. In a first step the route is designed by an optimization problem is solved (location, scheduling or routing problem) and in the sequence, based on this route, a collection planning/policy is established. The strategy adopted for the collection planning is used to conduct the vehicle route choice; for example, energy efficiency [15, 21] or single hop guarantee for all nodes [28]. The problem addressed in [5] looks for a vehicle routing that visits a set of nodes of the network needing repairing. However, the route designed is also used to collect information allowing the prediction of node failures. Thus, the vehicle routing changes the network architecture (once the nodes are repaired) as well as the belief of the network.

In [17], the authors were dedicated to the design of the vehicle routing together with the collection planning, but they assume an architecture is defined for the vehicle routing (cycle path or zig-zag path) and study the best placement for such architecture.

To the best of our knowledge, the authors in [8] and [22] are the only ones to study the design of a vehicle route from scratch together with a wireless transmission planning.

In [8], they address an application of underwater wireless sensor networks for submarine monitoring. The authors considered a scenario with a set of surfacing nodes and underwater nodes and look for a routing to an autonomous underwater vehicle (AUV) during a given time period. The AUV must leave and return to a surface node while information generated by the underwater nodes is collected along the selected path. Opposite to our main assumption, the AUV must physically visit each station where information is collected. The value of information collected decreases with time and the strategy adopted by the authors is the maximization of the value of the information delivered to the surface nodes. The authors in [8] proposed an Integer Linear Programming (ILP) formulation able to solve the problem with up to 12 underwater nodes in a time that varies from a few hours to a few days.

The problem treated in [22] is a very close version of the VRP addressed in the present work: all but one assumption are the same. In [22] it is assumed that, once a station starts a transmission to the vehicle, all information accumulated in this station at that moment must be transmitted. This assumption is justified in situations where information is safer once it leaves the stations. However, the authors observed that this imposition contributes for the isolation of some stations. In our problem, transmission of partial information is allowed. Three different strategies adopted in the vehicle routing problem were investigated in [22]: maximizing the total amount of information extracted at the end of the time horizon T; maximizing the average of the information in the vehicle at each time point; and maximizing the satisfaction of each station at the end of the time horizon T. A MILP formulation was introduced with three different objective functions being discussed. The authors presented computational experiments on randomly generated networks with up to 15 stations ($|T| \in \{24, 28\}$), solved in one hour of computation time [1]. The results discussed in [22] showed the problem becomes more difficult as the total amount of information collected increases. The periodicity on remaining information at the stations (after a sequence of vehicle routings is solved) was studied for each strategy. The experiments were used to access how the maximization of one criteria affects the others and impacts the periodicity of the remaining information.

From the modeling perspective, our problem is related with the VRP problem but also with other classical network optimization problems: the classical traveling salesman problem (TSP); the prize collecting traveling salesman problem (PCTSP); and the inventory routing problem (IRP). Since only one uncapacitated vehicle is considered, the problem can be seen as a version of the Traveling Salesman Problem (TSP) [20]. However, opposite to most versions of the TSP, no constraint is imposed neither on the number nor on the frequency of visits to the stations. Notice that, depending on the position of the base station, the

[1] Using the IBM CPLEX Optimizer 12.6.1.0 solver on a server with 15 CPU's Intel ®Xeon (R) E5540@ 2.53Ghz X4, with 16 GB of RAM

vehicle can be conducted to return several times to the base in order to have access to some stations.

In the PCTSP (see [7]), a salesman gets a prize for every city visited. The goal is to find a minimum cost route including enough cities to collect a required amount of prize. As for the PCTSP, our problem assumes that not all the nodes need to be visited and the information collected can be regarded as a prize. Conversely, no routing costs are considered, the same station can be visited more than once, and the information collected (prize) depends on the time of the visits and on the information collected previously. Finally, in a IRP, the routing must be coordinated with the inventory management (see [13]). As for our problem, the same node can be visited multiple times and the amount of inventory that can be picked up or delivered depends on the time of the visit. However, in addition to the absence of routing costs, neither vehicle capacities nor inventory limits on the stations are considered in our problem.

1.2. Our contributions

Although the problem considered here has some similarities with classical problems (TSP, VRP, PCTSP and IRP), it also includes distinct characteristics making the study of the mathematical models a challenge. For classical related problems, several mathematical formulations have been proposed and compared (see, for instance, [18] for the TSP and [19] for the VRP). Adding new aspects to a problem can impact the mathematical modeling choices [10] and, as a consequence, the solution of the problem to optimality, or the design of matheuristics. Our work contributes with the development and comparison of different MILP formulations to the VRP with information collection in wireless networks.

We introduce three MILP formulations to the problem: one based on a time discretization, where each decision is measured in multiples of the time unit; and two formulations using continuous time and based on events. The two formulations based on continuous time differ in the type of events considered. One formulation assumes the events are visits to stations and transfer operations, and the other assumes that events are the vehicle stops. We will see that each such model proposed in this work has pros and cons making it more suitable for a particular instance. To the best of our knowledge these models have never been introduced for a VRP with information collection in wireless networks. Our contribution goes in the same direction as the one presented in [3] in which the authors compare discrete time models with continuous time models for a maritime IRP. However, as we discuss in the later sections, the conclusions are not coincident to the ones in [3] as the problems are different.

The paper is organized as follows. Section 2 formally describes the VRP being solved while notations and assumptions are presented. Three MILP formulations to the problem are presented in Section 3 with the discussion on how the models can be strengthened. The three models are then compared in Section 4 where a deep analysis of the models is presented and some problem assumptions are discussed. Computational experiments are presented in Section 5. Finally, some conclusions and research directions are presented in Section 6.

2. Description of the problem

The wireless network is modeled by a directed graph $D = (V, A)$. The node set V represents the set of stations of the network and the arc set A represents the directed paths connecting pairs of stations in V. The base station is regarded as node 1. Weights t_{ij} and d_{ij} are associated with each arc (path) $(i, j) \in A$ representing, respectively, the travel time from node (station) i to node (station) j and the distance among these nodes (stations). Let $T = \{1, 2, \ldots, m\}$ be the time horizon considered divided in m time periods. At the beginning of the time horizon, each node $j \in V \setminus \{1\}$ contains an amount Q_j of information. For each node $j \in V \setminus \{1\}$, information is generated at a rate of r_j units per time period in T. Thus, the amount of information at node j at each time period $k \in T$, denoted by q_{jk}, depends on the elapsed time from the last extraction (either physically or by radio), i.e.,

$$q_{jk} = \begin{cases} Q_j + kr_j, & \text{if node } j \text{ has not been visited before time period } k, \\ (k - t_{last})r_j, & \text{otherwise, where } t_{last} \text{ is the time period of the last extraction.} \end{cases}$$

Only the base node is properly equipped to send information outside the network; see Figure 1 for an illustration of the problem. A unique vehicle is in charge of collecting information from all stations in $V \setminus \{1\}$ and of transporting it to the base node. There is no capacity limit associated with the vehicle. At the beginning of the time horizon, the vehicle is located at the base node and at the end of the time horizon, it must return to the base node. Multiple visits are allowed to each node in V. Information can only be transferred to the vehicle once it is located in one of the stations in V, i.e., no information transfer is allowed while the vehicle is moving on an arc $(i, j) \in A$. Figure 1 shows the vehicle located at station 5 and information transfer (dotted red lines) occurs from 4, 5 and 7.

Wireless transmission is used to transfer information from a node $j \in V$ to the vehicle located in a node $i \in V$. Wireless transmission is only possible for close enough nodes. Let r_{cov} be the converage radius, i.e., a maximum distance allowing wireless transmission between two points. A node j can wireless transfer its information to (the vehicle in) node i whenever $d_{ij} \leq r_{cov}$. We define the set of nodes that can send information to node i as $Range(i) = \{j \in V : d_{ji} \leq r_{cov}\}$. For the example depicted in Figure 1, $Range(5) = \{4, 7\}$. We make the same physical and technical assumptions as in [22]. Thus, we assume transmission speed inversely proportional to the square of the distance between nodes depending on two additional factors: the amount of information transmitted and physical factors (as equipments used or obstacles between nodes). Let α_{ji} be a parameter representing the physical limitations of sending information among nodes j and i. The amount of information that can be sent per time unit from node j to node i is $\dfrac{1}{\alpha_{ji}(1 + d_{ji}^2)}$ (see Section 2 from [22]). As we have mentioned in the introduction, different from [22], we assume nodes are free to transfer only part of their information to the vehicle.

Simultaneous transmissions are possible but two limitations are imposed. Parameter M denotes the maximum number of nodes that can transfer information simultaneously to the vehicle at each time period. That means at most M nodes can send information to the vehicle simultaneously. For the example depicted in Figure 1, $M \geq 3$. Also, a parameter R

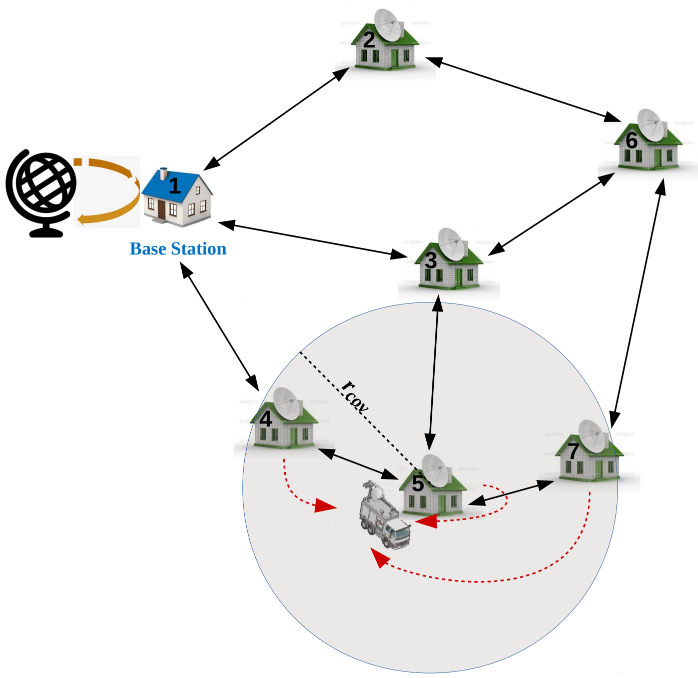

Figure 1: An illustration of the wireless transfer vehicle routing problem with 7 stations.

denotes the maximum amount of information that can be transferred in each time period. During each time period, the total amount of information that the vehicle (located in a node i) receives from all other nodes cannot exceed R units.

A simultaneous information transfer finishes only when each individual wireless transmission finishes. As a consequence, the time of a simultaneous transmission corresponds to the highest time among individual wireless transmissions.

The version of the VRP treated in this paper, called Wireless Transmission VRP (WT-VRP), consists of finding a feasible routing for the vehicle (i.e., a routing leaving at the beginning and returning at the end to the base node) and an efficient planning for collecting information from nodes belonging to $V \setminus \{1\}$. The criterion for measuring the efficiency of a collect planning is the total amount collected which will be maximized in the models described in the next section.

3. Mathematical models

Next, we introduce three MILP formulations to the WT-VRP. First, a time discrete model (Section 3.1) is developed where each decision is a multiple of the time unit. Second, an event model is proposed (Section 3.2) where visits to stations and transfer operations are considered as events. Finally, another event model is presented (Section 3.3) in which the considered events are the vehicle stops.

3.1. Discrete time model

Discrete time models have been used for related problems as maritime inventory routing problems (see [1, 2]). In this model the time horizon is discretized in a number of time periods $T = \{1, 2, \dots, m\}$. Notice that the time horizon discretization is a modeling choice made in the formal presentation of the WT-VRP. We assume that, at each time period in T, the vehicle is either traveling or waiting at a node and this behavior is modeled by the following two sets of binary variables.

For each $(i, j) \in A$ and $k \in T$, let

$$x_{ijk} = \begin{cases} 1 & \text{if the vehicle crosses the arc } (i,j) \text{ (going directly from node } i \text{ to node } j\text{) and arrives} \\ & \text{at the end of time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

For each $j \in V$ and $k \in T$, let

$$z_{jk} = \begin{cases} 1 & \text{if the vehicle is waiting at node } j \text{ during time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

A third set of binary variables controls the wireless transmissions occurring at each time unit. For each pair of vertices $j \in V$, $i \in \mathit{Range}(j)$ and $k \in T$, let

$$\theta_{jik} = \begin{cases} 1 & \text{if node } j \text{ sends information to node } i \text{ during time period } k, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, continuous variables are used to describe the amounts of information at the nodes and the amounts of information transmitted. For each $j \in V$ and $k \in \{0\} \cup T$, let q_{jk} be the amount of information in node j at the end of time period k. For each $j \in V$, $i \in \mathit{Range}(j)$ and $k \in T$, let f_{jik} be the amount of information transmitted from node j to node i during time period k.

The Discrete Time (DT) model follows.

$$\mathit{Minimize} \qquad \sum_{j \in V} q_{jm} \tag{1}$$

$$\text{s.t.} \sum_{(1,j)\in A} x_{1jt_{1j}} = 1, \tag{2}$$

$$\sum_{(j,1)\in A} x_{j1m} = 1, \tag{3}$$

$$\sum_{j\in V} z_{jk} + \sum_{(i,j)\in A} x_{ijk} \leq 1, \qquad \forall j \in V,\ \forall k \in T, \tag{4}$$

$$z_{jk} + \sum_{(i,j)\in A} x_{ijk} = \sum_{(j,p)\in A} x_{jp(k+t_{jp})} + z_{j(k+1)}, \qquad \forall j \in V, \forall k \in T, \tag{5}$$

$$\sum_{j\in Range(i)} \theta_{jik} \leq M z_{ik}, \qquad \forall i \in V, k \in T, \tag{6}$$

$$f_{jik} \leq \frac{\theta_{jik}}{\alpha_{ji}(1+d_{ji}^2)}, \qquad \forall j \in V\ , \forall i \in Range(j)\ ,\ \forall k \in T, \tag{7}$$

$$\sum_{j\in Range(i)} f_{jik} \leq R, \qquad \forall i \in V, k \in T, \tag{8}$$

$$q_{jk} = q_{j,k-1} + r_j - \sum_{i\in Range(j)} f_{jik}, \qquad \forall j \in V,\ k \in T | k > 1, \tag{9}$$

$$q_{j0} = Q_j, \qquad \forall j \in V, \tag{10}$$

$$q_{jk} \geq 0, \qquad \forall j \in V,\ k \in T, \tag{11}$$

$$f_{jik} \geq 0, \qquad \forall j \in V,\ i \in Range(j),\ k \in T, \tag{12}$$

$$\theta_{jik} \in \{0,1\} \qquad \forall j \in V,\ i \in Range(j),\ k \in T, \tag{13}$$

$$z_{jk} \in \{0,1\}, \qquad \forall j \in V,\ k \in T, \tag{14}$$

$$x_{ijk} \in \{0,1\}, \qquad \forall (i,j) \in A, k \in T. \tag{15}$$

The objective function (1) minimizes the total amount of information remaining at the nodes at the end of the time horizon T, i.e., at time period m. Constrains (2)–(5) are the *Routing Constraints* assuring, together with binary conditions (15), each feasible solution of the model corresponds to a valid route of the vehicle. Equations (2) and (3) ensure that the vehicle starts and ends its route at the base node. Inequality (4) ensures that at most one of the following cases can occur at time period k: the vehicle arrives at a node or the vehicle is waiting at a node to receive information. Equations (5) ensure that if either the vehicle arrives at node j or it is waiting at this node at time period k, then, at the next time period $k+1$, either it travels to a neighbor node or it keeps waiting at node j (see Figure 2). As routing variables are time-indexed, equations (4) and (5) garantee the continuity of the vehicle route over the time horizon T. Inequalities (6) and (8) are variable upper bound constraints imposing the *Transfer Constraints*. Constraints (6) guarantees that at most M nodes send information to the vehicle simultaneously. Also, this inequality ensures that, whenever variable θ_{jik} is positive, for some $j \in V$, $i \in Range(j)$ and $k \in T$ (i.e. at time period k, a node j sends information to the vehicle at node i) then z_{ik} must be one (i.e. the vehicle must be located at i at time period k). Similarly, inequalities (7) ensure that the maximum amount of information sent from node j to node i, at a time period k, is obeyed. Additionally, this set of inequalities ensure that if f_{jik} is positive, then the binary variable θ_{jik} must be one. Equations (8) ensures that during each time period the maximum amount of information that can be transferred to a node i cannot exceed R. Constraints (9) and (10) are the *Amount of information Constraints*. Equations (10) set the initial amount of information at each node. Equations (9) are the equilibrium constraints for the amount of information at each node. They impose that the amount of information at a node in time period k is equal to the amount information in the time $k-1$ plus the rate of that node minus the amount information extracted in the previous time period. Finally, inequalities (11)–(15) establish the domain of the variables.

Example 3.1. *Consider an example with six nodes, where node* 1 *is the base station, and with* $m = 30$ *time periods. The information generation rates for nodes* 2 *to* 6 *are given respectively by* $3, 4, 2, 2$ *and* 4*. The parameters* R *and* M *are set to 20 and 3, respectively. We define* $\alpha_{ji} = 1/20$ *for* $j = i$ *and* $\alpha_{ji} = 1/6$*, otherwise. The initial amount of information at each node is zero. The following matrices are considered.*

$$d = \begin{bmatrix} 0 & 4 & 5 & 4 & 7 & 7 \\ 4 & 0 & 2 & 4 & 3 & 6 \\ 5 & 2 & 0 & 2 & 2 & 1 \\ 4 & 4 & 2 & 0 & 5 & 1 \\ 7 & 3 & 2 & 5 & 0 & 2 \\ 7 & 6 & 1 & 1 & 2 & 0 \end{bmatrix} \quad t = \begin{bmatrix} 0 & 4 & \infty & 4 & \infty & \infty \\ 4 & 0 & 2 & \infty & 3 & \infty \\ \infty & 2 & 0 & 2 & \infty & 1 \\ 4 & \infty & 2 & 0 & \infty & 1 \\ \infty & 3 & \infty & \infty & 0 & 2 \\ \infty & \infty & 1 & 1 & 2 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

where d *is the distance matrix,* t *is the travel times matrix, and* W *is the matrix indicating whether* $j \in Range(i)$ $(W_{ji} = 1)$ *or* $j \notin Range(i)$ $(W_{ji} = 0)$*. For example, node 2 can receive information from nodes 2, 3 and 5, while node 3 can receive information from nodes 2, 3, 4, 5, 6.*

The optimal solution obtained with the DT model is depicted in Figure 2. The vehicle leaves the base station (node 1) at the beginning of the time horizon and arrives at node 2 at the end of period 4. It stays in node 2 during time periods 5 and 6, receiving information from nodes 2, 3 and 5. Next, the vehicle moves to node 3 where it spends one time period to receive information from nodes 2, 3 and 6. Then it moves to node 6 to receive information from nodes 3, 4 and 6. At the end of time period 12 the vehicle moves to node 5 where it stays for four periods. It receives information from nodes 3 and 5 during four time periods and from nodes 2 and 6 during two time periods. Then the vehicle moves again to node 6 where it stays for two periods, receiving information from nodes 3, 5 and 6. At the end of time period 22 it moves to node 4 to receive information from nodes 3, 4 and 6 during three time periods. Finally, the vehicle returns to the base station.

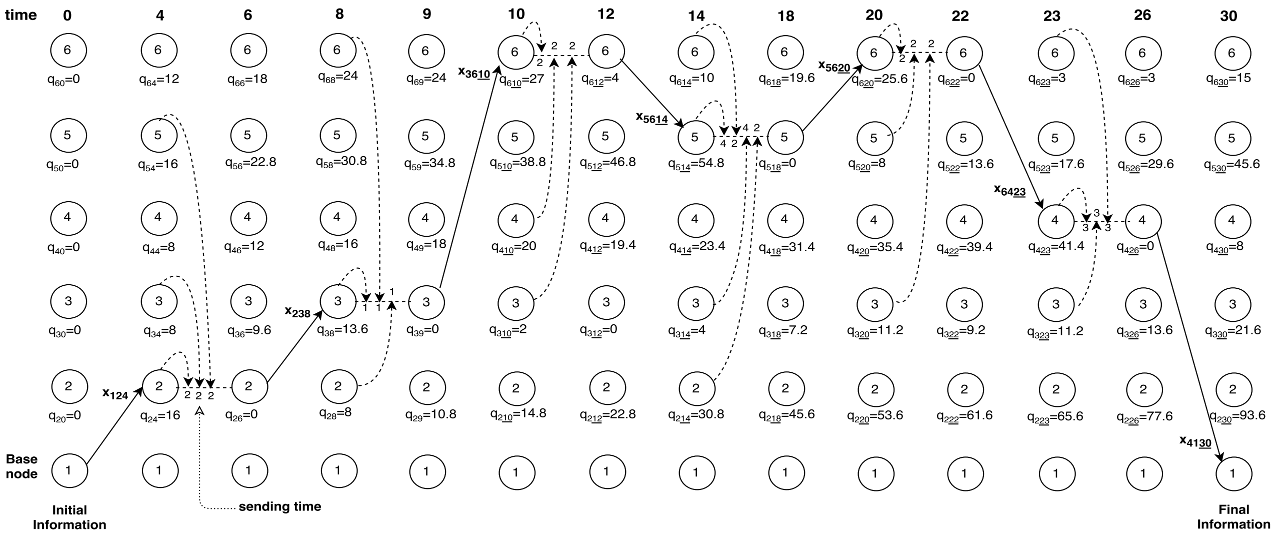

Figure 2: Optimal solution of the instance in Example 3.1 obtained with the Discrete Time model for $m = 30$. Objective function value = 183.8.

Strengthening the model. Here we discuss several enhancements that allow to tighten the model, that is, to derive a new model whose linear relaxation value is closer to the optimum value. In general, exact methods based on the linear relaxation such as branch-and-bound and branch-and-cut will have smaller enumeration trees when the model is tight [33].

The first enhancement is to disaggregate inequalities (6) as follows:

$$\theta_{jik} \leq z_{ik}, \qquad \forall i \in V,\ j \in Range(i),\ k \in T. \tag{16}$$

Notice that inequalities (16) cannot replace inequalities (6) since they do not ensure that at most M nodes send information to the vehicle simultaneously. Hence, the two sets of inequalities are considered in the enhanced model.

Another improvement is to replace inequalities (8) by the following variable upper bound constraints.

$$\sum_{j \in \text{Range}(i)} f_{jik} \leq R\ z_{ik}, \qquad \forall i \in V,\ k \in T. \tag{17}$$

Next we define a set of valid inequalities that impose a limit on the amount transferred, for a subset of time periods. Let

$$t^* = \min_{(i,j)\in A} \{t_{ij}\}$$

denote the minimum traveling time between nodes. The following proposition establishes an inequality based on the fact that, during the subset of time periods $\ell \leq t^* + 1$, only one node can be visited.

Proposition 3.1. *For $l \leq t^* + 1$, the following inequality is satisfied by each feasible solution* *of the DT model.*

$$\sum_{s=1}^{l+1}\sum_{j\in V}\sum_{i\in Range(j)} f_{jis} \leq \sum_{j\in V} Q_j + \max_{\chi\in V}\{r_\chi\} l \tag{18}$$

Proof The proof is straightforward.

The DT model has $\mathcal{O}(\kappa m)$ variables and constraints, where $\kappa = \sum_{i\in V} \mid Range(i) \mid$. When r_{cov} is larger than the greatest distance between two nodes, the size becomes $\mathcal{O}(\mid V \mid^2 m)$, while in the opposite case, when r_{cov} is smaller than the minimum distance, its size becomes $\mathcal{O}(\mid V \mid m)$. Hence, the main components with impact on the number of variables and constraints are the number of nodes (stations), the number of time periods considered, and the number of possible pairs of nodes for wireless transfer.

The DT model provides detailed information on the visits, normally leading to tight models [3]. However, as the model depends on the time discretization, and since a fine discretization may be required to model the traveling times and the transfer operations, it tends to increase with the increase of the time horizon, which makes the model useless for large number of time periods. More concise models can be derived that do not depend on the number of time periods by keeping track of the events.

3.2. Node event model

In this section, we introduce a flow model where only *events* are modeled (see [3]). Two types of events are considered. A first set of events is denoted by Δ^r and includes all the possible physical node visits, which are defined by a pair (i, n) that represents the n^{th} visit of the vehicle to the node i. The second set of events, denoted by Δ^w, include events (j, k) representing the k^{th} information transfer from node j to a neighbour node. For each visit to a given node i, at most one transfer operation can be made from each node in $Range(i)$. Let τ_i, ω_j represent the maximum number of allowed visits to node i and the maximum number of allowed transfers from station j, respectively. Thus, $\Delta^r = \{(i, n) : i \in V, n \in \{1, \ldots, \tau_i\}$ and $\Delta^w = \{(j, k) : j \in V, k \in \{1, \ldots, \omega_j\}$.

A feasible vehicle route is defined by a combination of events from Δ^r and Δ^w.

Next, we define the set of binary variables to be used in this section. For each pair $(i, n), (j, \eta) \in \Delta^r$, with $(i, j) \in A$,

$$x_{inj\eta} = \begin{cases} 1 & \text{if the vehicle goes directly from node visit } (i, n) \text{ to node visit } (j, \eta), \\ 0 & \text{otherwise.} \end{cases}$$

For each $(j, \eta) \in \Delta^r$,

$$w_{j\eta} = \begin{cases} 1 & \text{if the node visit } (j, \eta) \text{ belongs to the vehicle route,} \\ 0 & \text{otherwise.} \end{cases}$$

For each $(j, k) \in \Delta^w$ and $(i, n) \in \Delta^r$,

$$\theta_{jkin} = \begin{cases} 1 & \text{if the } k^{th} \text{ transfer from node } j \text{ to the vehicle occurs at its } n^{th} \text{ stop at node } i, \\ 0 & \text{otherwise.} \end{cases}$$

For each $(i, k) \in \Delta^w$,

$$z_{ik} = \begin{cases} 1 & \text{if at least } k \text{ transfers occur from node } i, \\ 0 & \text{otherwise.} \end{cases}$$

The following set of integer variables is defined. For $(j, k) \in \Delta^w$ and $(i, n) \in \Delta^r$,

ξ_{jkin} : duration, in number of time periods, of the k^{th} transfer from node j to the vehicle, occurred at its n^{th} stop at node i.

For $(i, n) \in \Delta^r$,

γ_{in} : duration, in number of time periods, of the n^{th} visit to node i.
t^r_{in} : time period at which the node visit (i, n) starts.

For $(j, k) \in \Delta^w$,

t^w_{ik} : time period at which the k^{th} transfer from node i can start.

The following set of continuous variables will be also necessary. For $(j, k) \in \Delta^w$ and $(i, n) \in \Delta^r$,

f_{jkin} : amount of information sent during the k^{th} transfer from node j to the vehicle, occurred at its n^{th} stop at node i.

For $(j, k) \in \Delta^w$,

q_{jk} : amount of information in node j at the beginning of the k^{th} information transfer.

The Node Event (NE) model is as follows.

$$minimize \left\{ \sum_{i \in V} Q_i + m \sum_{i \in V} r_i - \sum_{(j,\eta) \in \Delta^w} \sum_{(i,n) \in \Delta^r} f_{j\eta in} \right\} \tag{19}$$

$$\sum_{j \in V \setminus \{1\}} x_{11j1} = 1, \tag{20}$$

$$\sum_{(j,\eta) \in \Delta^r} x_{j\eta 12} = 1, \tag{21}$$

$$\sum_{(j,n) \in \Delta^r | (i,j) \in A} x_{i\eta jn} = w_{i\eta}, \qquad \forall (i,\eta) \in \Delta^r, \tag{22}$$

$$\sum_{(j,n) \in \Delta^r | (j,i) \in A} x_{jni\eta} = w_{i\eta}, \qquad \forall (i,\eta) \in \Delta^r, \tag{23}$$

$$w_{i\eta} \leq w_{i,\eta-1}, \qquad \forall (i,\eta) \in \Delta^r, \eta > 1, \tag{24}$$

$$\sum_{(j,k) \in V^w | j \in Range(i)} f_{jkin} \leq R\gamma_{in}, \qquad \forall (i,n) \in \Delta^r, \tag{25}$$

$$f_{jkin} \leq q_{jk} + r_j \xi_{jkin}, \qquad \forall (j,k) \in \Delta^w, (i,n) \in \Delta^r, j \in Range(i), \tag{26}$$

$$f_{jkin} \leq \frac{\xi_{jkin}}{\alpha_{ji}(1 + d_{ji}^2)}, \qquad \forall (i,n) \in \Delta^r, (j,k) \in \Delta^w, j \in Range(i), \tag{27}$$

$$\sum_{(j,k) \in \Delta^w | j \in Range(i)} \xi_{jkin} \leq M \ \gamma_{in}, \qquad \forall (i,n) \in \Delta^r, \tag{28}$$

$$\xi_{jkin} \leq m\theta_{jkin}, \qquad \forall (j,k) \in \Delta^w, (i,n) \in \Delta^r, j \in Range(i), \tag{29}$$

$$\sum_{k | (j,k) \in \Delta^w} \theta_{jkin} \leq w_{in}, \qquad \forall (i,n) \in \Delta^r, \ j \ \in Range(i), \tag{30}$$

$$z_{jk} \leq \sum_{i \in V} z_{i,k-1}, \qquad \forall (j,k) \in \Delta^w, k > 1, \tag{31}$$

$$z_{jk} = \sum_{(i,n) \in \Delta^r} \theta_{jkin}, \qquad \forall (j,k) \in \Delta^w, \tag{32}$$

$$q_{jk} = Q_j + r_j t_{jk}^w - \sum_{(i,n) \in \Delta^r | i \in Range(j)} \sum_{\ell=1}^{k-1} f_{j\ell in}, \qquad \forall (j,k) \in \Delta^w, \tag{33}$$

$$t_{j\eta}^r \geq t_{in}^r + \gamma_{in} + t_{ij} - (m + t_{ij})(1 - x_{inj\eta}), \qquad \forall (i,n), \ (j,\eta) \in \Delta^r, (j,i) \in A \tag{34}$$

$$t_{i1}^r \geq t_{i1} x_{11i1}, \qquad \forall (i,1) \in \Delta^r, \tag{35}$$

$$t_{j\eta}^r + \gamma_{j\eta} + t_{j1} x_{j\eta 12} \leq m, \qquad \forall (j,\eta) \in \Delta^r, \tag{36}$$

$$t_{jk}^w \leq t_{in}^r + m(1 - \theta_{jkin}), \qquad \forall (i,n) \in \Delta^r, (j,k) \in \Delta^w, \tag{37}$$

$$\xi_{jkin} \leq \gamma_{in} + m(1 - \theta_{jkin}), \qquad \forall (i,n) \in \Delta^r, (j,k) \in \Delta^w, \tag{38}$$

$$x_{j\eta in} \in \{0,1\}, \qquad \forall (j,\eta), (i,n) \in \Delta^r, (j,i) \in A, \tag{39}$$

$$\theta_{jkin} \in \{0,1\}, \qquad \forall (j,k), \in \Delta^w, (i,n) \in \Delta^r, \tag{40}$$

$$q_{jk} \in \mathbb{R}^+, \qquad \forall (j,k) \in \Delta^w, \tag{41}$$

$$
\begin{aligned}
f_{jkin} &\in \mathbb{R}^+, && \forall(j,k), \in \Delta^w, (i,n) \in \Delta^r, && (42)\\
\gamma_{jk} &\in \mathbb{Z}^+, && \forall(j,k) \in \Delta^w, && (43)\\
\xi_{jkin} &\in \mathbb{Z}^+, && \forall(j,k), \in \Delta^w, (i,n) \in \Delta^r, && (44)\\
t^r_{jk} &\in \mathbb{Z}^+, && \forall(i,n) \in \Delta^r, && (45)\\
t^w_{in} &\in \mathbb{Z}^+, && \forall(j,k) \in \Delta^w. && (46)
\end{aligned}
$$

The objective function (19) minimizes the amount of information kept in the nodes at the end of the time horizon. This amount is computed by removing the extracted information from the total information generated through the entire time horizon. Constraints (20)–(24) are the *Routing Constraints*. Equations (20) and (21) ensure, respectively, the vehicle leaves and ends its route in the node base 1. Equations (22) and (23) ensure the flow conservation, stating that if the η^{th} visit to node i occurs, then there must exist an arc entering and leaving that node. Constraints (24) state that if the k^{th} visit to node j occurs, so the previous $k-1^{th}$ visit must have occurred. In the NE model, the set of *Information Transfer Constraints* is defined by constraints (25)–(33). Constraints (25) limit the transfer amount considering the maximum transfer quantity per period. Constraints (26) ensure that the amount that can be transferred cannot exceed the information available at the corresponding node. Constraints (27) limit the transfer amount taking into account the transfer rate. Constraints (28) ensure that during the k^{th} visit of node i, the total duration of all transfers to node i cannot exceed the maximum allowed number of transfers per period, M, times the duration of the visit. Constraints (29) link the variables indicating the duration of the transfer operations (variables ξ_{jkin}) to the binary variables θ_{jkin} indicating whether a transfer occurs. Constraints (30) ensure that an information transfer occurs only if a visit occurs. Constraints (31) state that if the k^{th} transfer occurs so the previous $k-1^{th}$ must have occurred. Constraints (32) relate the binary transfer variables. Constraints (33) define the amount of information at each node at the beginning of each information transfer. Constraints (34)–(38) are the *Time Constraints*. The start time of each k^{th} visit is defined by constraints (34) and (35). Constraints (34) takes into account the start time of the previous visit plus the traveling time between the two locations and the time spent on the last visit (this inequality is redundant whenever $x_{inj\eta} = 0$). Notice that constraints (34) ensure there are no subtours resulting from the solution satisfying the Routing Constraints (20)–(23), which garantees the continuity of the vehicle route until it returns to the base station. Constraints (35) restrict the start time of the first visit. Constraints (36) force all the visits to end early enough so the vehicle can return to the base station before the end of the time horizon. Constraints (37) relate the start time of an information transfer from node j to node i, with the start time of the visit to node i. Constraints (38) ensure that each transfer operation cannot take longer than the duration of the corresponding node visit. Notice that inequalities (37) and (38) are redundant whenever $\theta_{jkin} = 0$. We observe also that inequalities (38) and (33) together with the (maximization) objective function avoid $t^w_{jk} < t^r_{in}$ in the cases where $\theta_{jkin} = 1$. Finally, constraints (39) - (46) define the variables domain.

Example 3.2. *Figure 3 depicts the solution of the instance used in Example 3.1, with the solution representation of the NE model. In this figure, two different set of nodes represent the two different types of events: circles represent events in Δ^r while squares represent events in Δ^w. For instance, we can see that station 6 is visited twice while the other stations are visited once; during the event $(6,2) \in \Delta^r$ (second visit to station 6), occur the events $(3,5),(5,2),(6,4) \in \Delta^w$ corresponding to transfer operations.*

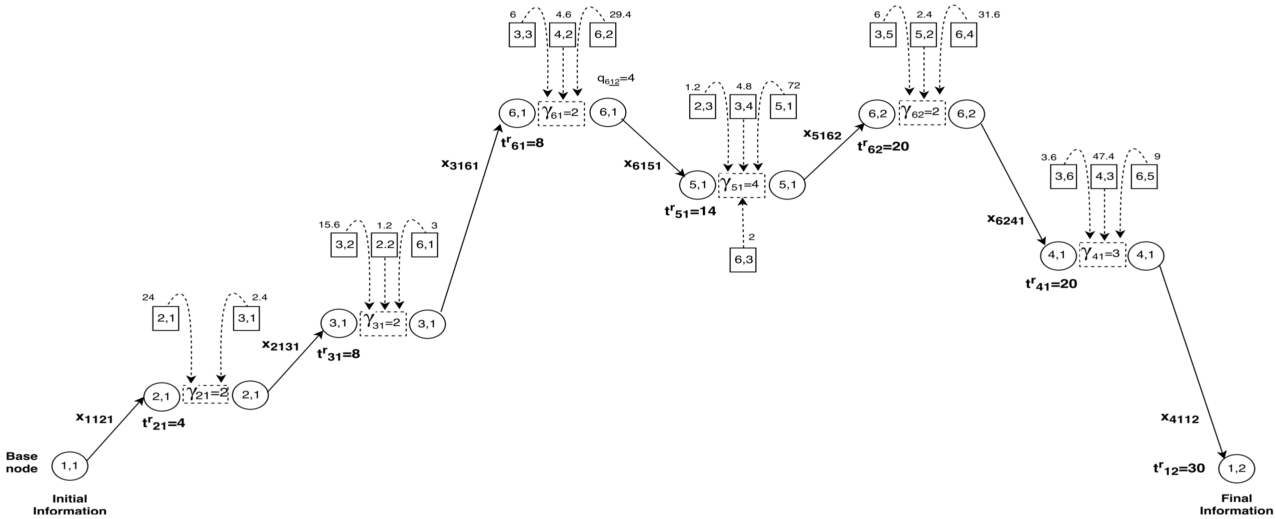

Figure 3: Optimal solution of the instance in Example 3.1 obtained with the Node Event model.

Strengthening the model. The NE model can be strengthened with a valid inequality that bounds the transfer amount $\sum f_{jmin}$ with the maximum transfer rate R times the number of periods the vehicle can receive information, i.e., the number of periods the vehicle is not traveling.

Proposition 3.2. *The following inequality is satisfied by each feasible solution of the NE model.*

$$\sum_{(j,k)\in\Delta^w} \sum_{(i,n)\in\Delta^r : j\in Range(i)} f_{j\eta in} \leq R(m - \sum_{(i,n)\in\Delta^r} \sum_{(j,\eta)\in\Delta^r:(i,j)\in A} t_{ij} x_{inj\eta}). \tag{47}$$

Another inequality can be derived by simply stating that the total duration of the visits plus the total travelling time cannot exceed m.

Proposition 3.3. *The following inequality is valid for NE model.*

$$\sum_{(i,n)\in\Delta^r} \gamma_{in} + \sum_{(i,n)\in\Delta^r} \sum_{(j,m)\in\Delta^r:(i,j)\in A} t_{ij} x_{injm} + \sum_{(i,n)\in\Delta^r} t_{i1} x_{in12} \leq m. \tag{48}$$

A final improvement consists in strengthening the big-M type constraints (29) and (38) by replacing m with the tighter bound (representing an upper bound for the time spent at node i)

$$m_i = m - T_{1i} - T_{i1} \tag{49}$$

where T_{1i} and T_{i1} are the shortest time for the vehicle to travel from the base station to node i, and from node i to the base station, respectively.

Model NE has $\mathcal{O}(\sum_{i\in V}\sum_{j\in Range(i)} \tau_i\omega_j + \sum_{(i,j)\in A} \tau_i\tau_j)$ variables and constraints.

3.3. Vehicle event model

As a single vehicle is considered, the events (node visits and transfer operations) can all be assigned to the vehicle visits. Moreover, a vehicle route includes only a small number of nodes visited. In this section, we define a set of events associated with the vehicle: each event is a vehicle stop. This formulation resembles to the layered formulation used for vehicle routing problems (see [4] and the references therein). Let $N = \{1, \ldots, \hat{N}\}$ denote the set of possible events where $\hat{N}$ is an upper bound on the number of events (for instance, this upper bound can be computed taking into account the distance between stations and considering one time period for information extraction at each visit). The new routing variables indicate the node visited at the k^{th} vehicle stop, indexed by the event $k \in N$.

Next, we define the new set of binary variables. For each $i \in V$, $k \in N$,

$$x_{ik} = \begin{cases} 1 & \text{if the } k^{th} \text{ vehicle event occurs at node } i, \\ 0 & \text{otherwise.} \end{cases}$$

The following integer variables are defined. For each $k \in N$,

$$\begin{aligned} t_k &: \text{time period at which the } k^{th} \text{ event begins,} \\ \gamma_k &: \text{time (in time periods) spent at the } k^{th} \text{ event.} \end{aligned}$$

For each $i, j \in V$, $k \in N$,

ξ_{jik} : duration (in time periods) of the information transfer from node j to node i at event k,

Finally, the following continuous variables are defined. For each $k \in N$ and for each $j \in V$,

$$q_{jk}: \text{ amount of information in node } j \text{ at the beginning of event } k.$$

For each $i, j \in V$, $k \in N$,

$$f_{jik}: \text{ amount of information transmitted from node } j \text{ to node } i \text{ during event } k.$$

The Vehicle Event (VE) model is as follows.

$$Minimize \left\{ \sum_{i \in V} (Q_i + mr_i) - \sum_{i \in V, j \in Range(i), k \in N} f_{jik} \right\} \tag{50}$$

$$\sum_{j:(1,j) \in A} x_{j1} = 1, \tag{51}$$

$$\sum_{k \in N} x_{1k} = 1, \tag{52}$$

$$\sum_{j \in V} x_{jk} \leq 1, \qquad \forall k \in N, \tag{53}$$

$$x_{jk} \leq \sum_{i:(i,j) \in A} x_{i,k-1}, \qquad \forall j \in V, k \in N, \tag{54}$$

$$x_{jk} \leq 1 - \sum_{\ell=1}^{k-1} x_{1\ell}, \qquad \forall j \in V \setminus \{1\}, k \in N, \tag{55}$$

$$t_k \geq t_{k-1} + \gamma_{k-1} + t_{ij}(x_{i,k-1} + x_{jk} - 1), \qquad \forall (i,j) \in A,\ k \in N, \tag{56}$$

$$t_1 \geq \sum_{j:(1,j) \in A} t_{1j} x_{1j}, \tag{57}$$

$$t_k \leq m, \qquad \forall k \in N, \tag{58}$$

$$q_{jk} = Q_j + r_j t_k - \sum_{\ell=1}^{k-1} \sum_{i \in Range(j)} f_{ji\ell}, \qquad \forall j \in V,\ k \in N, \tag{59}$$

$$f_{jik} \leq q_{jk} + r_j \xi_{jik}, \qquad \forall j \in V,\ i \in \text{Range}(j),\ k \in N, \tag{60}$$

$$f_{jik} \leq \frac{\xi_{jik}}{\alpha_{ji}(1 + d_{ji}^2)}, \qquad \forall j \in V,\ i \in \text{Range}(j),\ k \in N, \tag{61}$$

$$\sum_{j \in Range(i)} f_{jik} \leq R\gamma_k, \qquad \forall i \in V,\ k \in N, \tag{62}$$

$$\sum_{i \in Range(j)} \xi_{jik} \leq \gamma_k, \qquad \forall j \in V,\ k \in N, \tag{63}$$

$$\sum_{j \in V, i \in Range(j)} \xi_{jik} \leq M\gamma_k, \qquad \forall k \in N, \tag{64}$$

$$\xi_{jik} \leq m x_{ik}, \qquad \forall i \in V,\ j \in Range(i),\ k \in N, \tag{65}$$

$$f_{jik} \geq 0, \qquad \forall i \in V,\ j \in Range(i),\ k \in N, \tag{66}$$

$$q_{jk} \geq 0, \qquad \forall j \in V,\ k \in N, \tag{67}$$

$$t_k \in \mathbb{Z}^+ \qquad \forall k \in N, \qquad (68)$$
$$\gamma_k \in \mathbb{Z}^+, \qquad \forall k \in N, \qquad (69)$$
$$\xi_{jik} \in \mathbb{Z}^+, \qquad \forall i \in V,\ j \in Range(i),\ k \in N, \qquad (70)$$
$$x_{ik} \in \{0,1\}, \qquad \forall i \in V,\ k \in N. \qquad (71)$$

The objective function (50) minimizes the amount of information kept in the nodes at the end of the time horizon. Constraints (51)–(55) are the *Routing Constraints*. Equations (51) and (52) ensure, respectively, the vehicle leaves and arrives at the base node. Inequalities (53) state that at most one visit labeled k is made. Inequalities (54) ensure that, if the k^{th} visit is made to node j, then the $k-1^{th}$ visit occurred in one of the predecessors of node j. Constraints (55) ensure that all the routing variables are null after the vehicle has returned to the base node. Constrains (56)–(58) are the *Time Constraints*. Inequalities (56) impose that the start time of the k^{th} visit takes into account the start time of the previous visit, the time spent on the last visit and the traveling time between the two locations visited. Constraints (57) restrict the start time of the first visit while constraints (58) force all the visits to start during the time horizon (this includes the last visit which is the return to the base station). As in the previous model, time constraints (56) avoid subtours and, together with the set of Routing Constraints, guarantee the continuity of the vehicle route until it returns to the base station. The *Information Transfer Constraints* are constraints (59)–(65). Constraints (59) define the amount of information at each node at the beginning of each visit. Constraints (60) ensure that the amount that can be transferred cannot exceed the information available at the corresponding node. Constraints (61) limit the transfer amount taking into account the transfer rate, while constraints (62) limit the transfer amount considering the maximum transfer quantity per period. Constraints (63) ensure that, during a visit to node i, the time used to transfer information from each node j to node i, cannot exceed the time the vehicle has spent at node i. Constraints (64) ensure that during each visit, the total transfer time to node i cannot exceed the maximum number of transfers per period, M, times the duration of the visit. Constraints (65) link the transfer variables to the routing variables, ensuring that a node j can transfer information to a node i during the k^{th} visit if the k^{th} visit occurred at node i. Finally, Constraints (66)–(71) define the variables domain.

Example 3.3. *Figure 4 depicts the solution of the instance used in Example 3.1, with the solution representation of the VE model. A node (i,k) in this network representation of a solution is associated with the event $k \in N$ occurring at node $i \in V$. A dotted line from a node (j,k) to a node (i,k) represents an information transfer occurring from j to i during the k^{th} visit. For instance, we can see that the 5^{th} visit of the vehicle occurs at node 6 to receive information from nodes 3, 5 and 6.*

Model VE can also be strengthened by replacing m in inequalities (65) with m_i as defined in (49).

Model VE has $\mathcal{O}(\kappa\hat{N})$ variables and constraints, where $\kappa = \mid \sum_{i\in V} Range(i) \mid$. As for the NE model, the size of the VE model depends on the size of the event set, i.e., from $\hat{N}$.

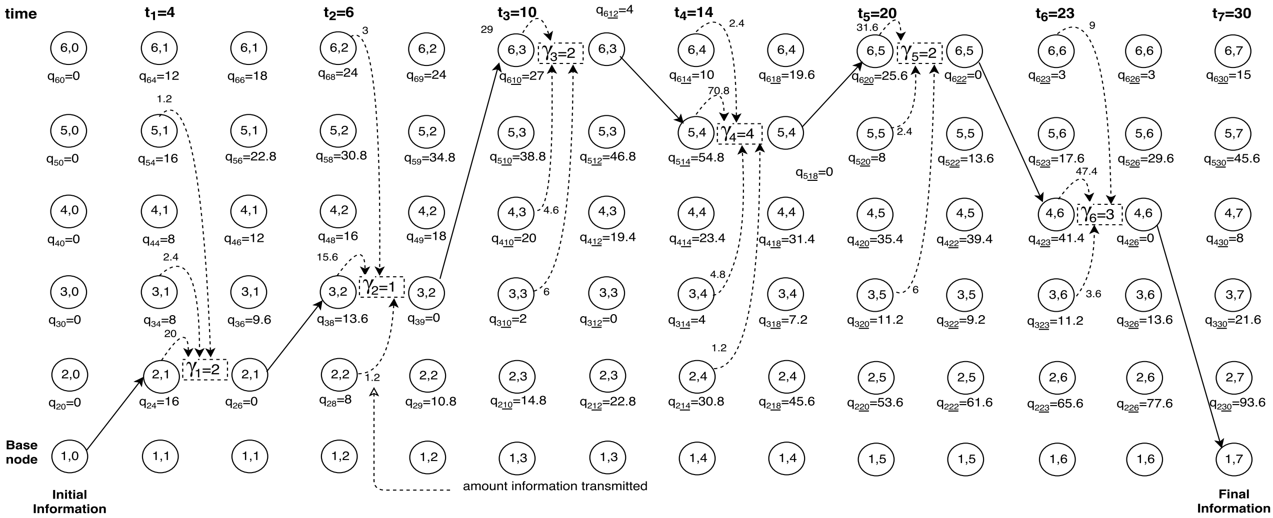

Figure 4: Optimal solution of the instance in Example 3.1 obtained with the Vehicle Event model.

Let $\varphi(\hat{N})$ denote the value of the objective function of the vehicle event model considering a maximum of $\hat{N}$ events occur. The following statements hold true.

(i) φ is non increasing in the maximum number of events, that is, $N_1 > N_2$ then $\varphi(N_1) \leq \varphi(N_2)$.

(ii) $\exists N \in \mathbb{N}$ such that $\varphi(N_1) = \varphi(N)$, $\forall N_1 > N$.

(iii) Let N^* denote the lowest N satisfying statement (ii), that is, the lowest possible value for the number of visits that gives the optimal solution. If $N' < N^*$, then $\varphi(N')$ gives an upper bound on the optimal value $\varphi(N^*)$.

Clearly, N^* is not known. By underestimating it the model becomes easier to solve but the solution cost will increase. Overestimating N^* may lead to longer running times.

4. Model analysis

In this section we analyse the three models introduced in the last section to the WT-VRP. The major difference between the models is related with the time of the visits and the time for the transfer operations. While the DT model is a pure discrete time model, the NE and VE models can be easily used assuming a continuous time. In that case, it suffices to consider the linear relaxation of constraints (43)–(46) in the NE model and constraints (68)–(70) in the VE model. The continuous assumption case can be considered reasonable for most practical cases. For these cases, the DT model can still be useful and provide good approximations by using tiny time discretizations. Clearly, under the continuous time assumption, the DT model can provide different solutions than those provided by the NE and VE models, even when the travelling times between pairs of nodes are integer. From that point, we analyse the models under the discrete time assumption.

While in the DT model, the time at which the vehicle visits each node is considered in the routing variables, in the NE and VE models the time of visits to the nodes is linked to the routing variables through big-M type constraints (34) (for model NE) and (56) (for the VE model). From one hand, variables indexed by the set of discrete times can produce huge models (w.r.t to the number of variables and constraints). On the other hand, Big-M constraints are known to lead to weak models (w.r.t. the value of the linear relaxation).

In the next, we will regard the feasible set of the WT-VRP problem as the intersection of two types of feasible sets: a vehicle routing and scheduling feasible set which defines a vehicle route and schedules the visits to each visited node; and a feasible set for the information transfer operations at each visit. Regarding feasibility, in relation to the vehicle routing and scheduling feasible set, here denoted by route-time assignment, all three models are equivalent (assuming discrete times) since one can easily establish the following result.

Proposition 4.1. *For each feasible route-time assignment* $(\overline{x}, \overline{z})$ *the DT model, if* τ_i *is greater or equal to the number of times node* i *is visited in the route* $(\overline{x}, \overline{z})$*, then there exists a feasible route-time assignment* (x, w, t^r, γ) *(respect.* (x, t, γ)*) in model NE (respect. VE), where the sequence of visits and time of visits coincide.*

Conversely, for each feasible route-time assignment $(\overline{x}, \overline{w}, \overline{t}^r, \overline{\gamma})$ *in the NE model (respect.* $(\overline{x}, \overline{t}, \overline{\gamma})$ *in the VE model) with discrete starting time* $\overline{t}^r$ *(respect.* $\overline{t}$*) and duration* $\overline{\gamma}$ *(respect.* $\overline{\gamma}$*) for the visits, there exists a feasible route-time assignment* (x, z) *in model DE, where the sequence of visits and time of visits coincide.*

However, even under discrete time assumption, the three models are not equivalent regarding information transfer operations. First, we show that provided τ_j, ω_j and $\hat{N}$ are large enough, the NE and VE models provide the same optimal solution for the information transfer operations. Let the k^{th} vehicle stop correspond to the n^{th} visit to node i and let $\ell(j)$ be the number of the transfer operation from node j occurring during that visit. Given a solution (f, ξ, q, γ) to the VE model it suffices to set the value of variables $f_{j\ell(j)in}, \xi_{j\ell(j)in}, q_{j\ell(j)}, \gamma_{jn}$, from the NE model to the value of variables $f_{jik}, \xi_{jik}, q_{jk}, \gamma_k$, respectively, in model VE, and set $t^r_{in}, t^w_{j\ell}$ to t_k. In order to see that the converse assignment also holds (i.e. from an optimal solution to the VE model, define a solution to the NE model), notice that, as explained in Section 3.2, in order to maximize the transfer amount and from constraints (38) and (33), $t^w_{j\ell}$ can be assumed to be equal to t^r_{in}.

We can observe that the DT model is more detailed than both event models since it specifies at what time each node transfers the information, allowing for an exact count of the information available at the beginning of each transfer operation. There is no such detailed information in the NE and VE models. Only the time at which each transfer operation can start and the amounts available to be transferred are defined. The schedule of the transfer operations during a vehicle visit are not provided with the optimal solution obtained with the event models. Consequently, it is not possible, in general, to determine precisely the amount of information available at each node at the precise moment each transfer operation occurs. The amount of information that can be transferred may be underestimated in the NE and VE models, as we show in the example below.

Example 4.1. *Consider an instance with four nodes* $V = \{1, 2, 3, 4\}$*. For simplicity assume only node 2 is accessible to the vehicle and assume that the distance between the base station*

		period				
		2	3	4	5	6
	2	8	10	12	14	16
node	3	6	8	10	12	14
	4	6	8	10	12	14

		period				
		2	3	4	5	6
	2	4	4	0	4	4
node	3	0	4	4	4	0
	4	4	0	4	0	4

Table 1: Description of Exemple 4.1. The left table gives, for each node, the total amount of information generated at the end of each time period. The right table gives, for each node, the transferred amount per period in an optimal obtained by the DT model.

(node 1) and node 2 is 1. Assume $r_j = 2$ for all $j \in V \setminus \{1\}$ and $Q_2 = 4, Q_3 = 2, Q_4 = 2$, Let $m = 7$, $R = 8$ and $M = 2$. Thus, in any optimal solution, the vehicle leaves node one, arrives at node 2 at the the beginning of time-period 2 and stays in node two for time periods 2 to 6 (stays five full time periods), and then it returns to the base station. The following plan (corresponding to a feasible solution of DT), given in Table 1, is optimal since it allows for the maximum amount of information transferred which is 40 units (corresponding to 5 transfer periods times $R = 8$ units). With the VE model an optimal solution is obtained with $t_1 = 2$ and with: $\xi_{2121} = 4, \xi_{3121} = 4, \xi_{4121} = 2$, (due to Constraints (64); and $f_{221} = 14, f_{321} = 12, f_{421} = 8$, (due to Constraints (60); which gives a total of 34 unites transferred (similarly with the NE model).

As illustrated on the above example, the NE and VE models are more restrictive than the DT model. Our event models assume the amount available at each transfer operation to be minimum as possible, i.e., they assume the amount available in the case the transfer operation from each node occurs as soon as possible.

Now, we analyse the other direction. As stated above, the NE and VE models just account for the total amount transferred during a visit. More formally, consider the DT and VE models for a particular visit, say n^{th} visit to node i, occurring from time period t to $t < s < m$, and corresponding to the k^{th} vehicle visit. This corresponds to: a solution for the DT model with $z_{i\ell} = 1$ for $\ell = t, \ldots, s$; a solution for the NE model with $w_{in} = 1$ and $\gamma_{in} = s - t + 1$; and a solution for the VE model with $x_{ik} = 1$ and $\gamma_k = s - t + 1$. Inspecting the models in Section 3, one can observe that, constraints (64) in the VE model (resp. (28) in the NE model) are the aggregation of constraints (6), constraints (61) in the VE model (respectively. (27) in the NE model) are the aggregation of constraints (7), and constraints (62) in the VE model (resp. (25) in the NE model) are the aggregation of constraints (8), using $\xi_{jik} = \sum_{l=t}^{s} \theta_{jil}$, in the VE model (resp. $\xi_{jk'in} = \sum_{l=t}^{s} \theta_{jil}$, where k' denotes the number of the transfer operation from node j to node i in model NE). Thus, for the VE and NE models, the schedule of the transfer operations (given by the values assigned to variables θ_{jik} in the DT model) during a vehicle visit are not provided. In general, given a solution in the transfer operations feasibility set of the event models, it may not be possible to find a feasible discretization (feasible solution for the transfer operations feasibility set of the DT model), as we illustrate in the following example based on the VE model.

Example 4.2. *Consider again the case with four nodes* $V = \{1, 2, 3, 4\}$ *where only node 2 is visited for time periods 2 to 6. Assume* Q_2, Q_3, Q_4, *are large enough to guarantee that Constrains* (60) *are always satisfied. Also, assume* $\kappa_2 = 5, \kappa_3 = 4, \kappa_4 = 3$, $R = 8$ *and* $M = 2$, *where* $\kappa_j = \frac{1}{\alpha_{j2}(1+d_{j2}^2)}$ *is the amount of information that can be sent per time unit from node j to node i (see Section 2). For the VE model, the following solution is feasible:* $t_{21} = 2$; $\gamma_1 = 5$; $\xi_{221} = 5, \xi_{321} = 1, \xi_{421} = 4$; *and* $f_{221} = 25, f_{321} = 4, f_{421} = 11$ *(given by Constraints* (61) *and* (62)*). It is easy to verify that there is no feasible discretization leading to a solution to the DE model (basically due to the fact that node 2 must transfer 5 units at each time period of the visit alternating simultaneous transfer with 3 and 4).*

In fact, deciding whether such discretization exists is a NP-hard problem, as one can easily reduce the partition problem to the problem of deciding whether such discretization exists. Recall that in the partition problem, we are given r positive integers $a_t, t \in K = \{1, \ldots, r\}$ and wish to determine whether there exists a partition $(S, K \setminus S)$ of K such that $\sum_{t \in S} a_t = \sum_{t \in K \setminus S} a_t = \sum_{t \in K} a_t/2$. For the reduction it suffices to consider $V = \{0, \ldots, r\}$, with $Range(r) = \{1, \ldots, r\}$ (all non-base nodes can transfer to node r), $m = 4$, $t_{0r} = 1$, $M = r =\mid Range(r) \mid$, $Q_j = \frac{1}{\alpha_{ji}(1+d_{ji}^2)} = a_j, r_j = 0$, for all $j \in Range(r)$, and assume $\xi_{j,r,1} = a_j, j \in Range(r)$. Setting $R = \sum_{j \in K} a_j/2$, it follows that the partition problem has a positive answer if and only if there is a discretization of the transfer operations from each node $j \in \{1, \ldots, r\}$ to the vehicle visit to node r during the first vehicle stop.

It is also important to remark that, although deciding if such discretization exists is a difficult problem, possibly with no solution, the DT model can have an alternative feasible solution with the same objective value. For illustration, in Example 4.2, an alternative solution for the DT model exists with the same objective value obtained with the feasible solution described for the VE model.

For completeness, we show that it is possible to add a set of constraint to the VE model in order to ensure a feasible discretization of the transfer operations. For the NE model, a similar formulation could be derived and for brevity we omit it here. First, we assume that any transfer operation from a node j during the k^{th} vehicle stop occurs without interruption. After, we discuss how the model can be written for the general case, i.e., accepting interruptions as occurs in the solution depicted in Table 1.

As M is in general small (at most as large as $\mid V \mid$), the idea of the model is to assign each transfer operation to a different label ℓ varying from 1 to M, ensuring that at most M transfer operations occur simultaneously, and to schedule the operations assigned to each label. Consider the additional binary variables λ_{jk}^{ℓ} that indicate whether there is a transfer operation from node j assigned to label ℓ at the k^{th} vehicle stop, and $\sigma_{jj'k}^{\ell}$ that indicates whether the transfer operation from node j occurs before the transfer operation from node j' and both are assigned to label ℓ at the k^{th} vehicle stop. Non-negative variables τ_{jk} indicate the start time of the transfer operation from node j at the k^{th} vehicle stop. The following inequalities are added.

$$\lambda_{jk}^{\ell} + \lambda_{j'k}^{\ell} \leq 1 + \sigma_{jj'k}^{\ell} + \sigma_{j'jk}^{\ell}, \qquad j, j' \in V,\ k \in N, \ell \in \{1, \dots, M\}, \tag{72}$$

$$\sigma_{jj'k}^{\ell} \leq \lambda_{jk}^{\ell}, \qquad j, j' \in V,\ k \in N, \ell \in \{1, \dots, M\}, \tag{73}$$

$$\sigma_{jj'k}^{\ell} \leq \lambda_{j'k}^{\ell}, \qquad j, j' \in V,\ k \in N, \ell \in \{1, \dots, M\}, \tag{74}$$

$$\tau_{jk}^{\ell} + \sum_{i \in V | j \in Range(i)} \xi_{jik} \leq \tau_{j'k}^{\ell} + m_i(1 - \sigma_{jj'k}^{\ell}), \qquad j, j' \in V,\ k \in N,\ \ell \in \{1, \dots, M\}, \tag{75}$$

$$t_k \leq \tau_{jk}^{\ell}, \qquad j \in V,\ k \in N,\ \ell \in \{1, \dots, M\}, \tag{76}$$

$$\tau_{jk}^{\ell} + \sum_{i \in V | j \in Range(i)} \xi_{jik} \leq t_k + \gamma_k, \qquad j \in V,\ k \in N,\ \ell \in \{1, \dots, M\}, \tag{77}$$

$$\lambda_{jk}^{\ell} \in \{0, 1\}, \qquad j \in V,\ k \in N,\ \ell \in \{1, \dots, M\}, \tag{78}$$

$$\sigma_{jj'k}^{\ell} \in \{0, 1\}, \qquad j, j' \in V,\ k \in N,\ \ell \in \{1, \dots, M\}, \tag{79}$$

$$\tau_{jk}^{\ell} \in \mathbb{Z}, \qquad j \in V,\ k \in N,\ \ell \in \{1, \dots, M\}. \tag{80}$$

Inequalities (72) state that if the transfer operations from j and j', at the k^{th} vehicle stop, are assigned to label ℓ then either the transfer from node j precedes the transfer from node j' or vice-versa. Inequalities (73) and (74) state that if variable $\sigma_{jj'k}^{\ell}$ is one than both variables λ_{jk}^{ℓ} and $\lambda_{j'k}^{\ell}$ must be one. Constraints (75) ensure that when the transfer operation from j' occurs after the transfer operation from j ($\sigma_{jj'k}^{\ell} = 1$), then the starting time of the transfer operation from j' must be greater than the starting time of the transfer operation from j plus the transfer time from j. Inequalities (76) and (77) ensure the starting time of each operation occurs within the period of time of the k^{th} vehicle stop. Inequalities (78) - (80) define the domain of the new variables.

While adding (72)-(80) to the VE model ensures a feasible solution, as we have observed before, an optimal solution can use several transfer operations from a same node j during the k^{th} vehicle stop (i.e., interruptions can happen). This would require to create additional copies of variables exactly as it was done for the visits in the NE model described in Section 3.2.

Observation 4.1. *Modeling transfer operations in detail leads to large NE and VE models that cannot be used to solve reasonable size instances, as it can also happens when time is discretized in the DT model.*

Observation 4.2. *Transfer operations can be easily solved when the vehicle route has been established since the number of nodes involved in each stop are usually small. Hence, the provided NE and VE models give an estimation of the amount of the information that can be transferred in each stop. The computational results discussed in Section 5 show that this estimation gives the exact value for those instances where the optimal value is obtained.*

Observation 4.3. *It is important to remark that the NE model can be easily adapted to the multi-vehicle case by adding a new index (indicating the vehicle) to the routing variables.*

$\lvert V \rvert$	m		DT Model	NE Model				VE Model		
				$\tau=1,\omega=2$	$\tau=1,\omega=3$	$\tau=2,\omega=2$	$\tau=2,\omega=3$	$\hat{N}=7$	$\hat{N}=8$	$\hat{N}=9$
10	50	variables	7101	374	524	765	1038	743	849	955
		constraints	4142	597	838	1236	1682	1678	1927	2176
	10	variables	14201	374	524	765	1038	743	849	955
		constraints	8292	597	838	1236	1682	1678	1927	2176
20	50	variables	33701	1769	2543	3709	5200	3655	4177	4699
		constraints	16032	3002	4273	6230	8696	8168	9365	10562
	100	variables	67401	1769	2543	3709	5200	3655	4177	4699
		constraints	32082	3002	4273	6230	8696	8168	9365	10562

Table 2: Number of variables and constraints used in each model for different parameters. For the NE model, the values of τ and ω are, respectively, the maximum number of allowed visits and the maximum number of allowed transfers, assumed the same for each station.

However, adapting the VE model to the multi-vehicle case is not straightforward since the inventory of information available at each node can no longer be associated with the vehicle stop.

5. Computational tests

In this section, we report the computational tests conducted to evaluate the three models presented in the previous section. All the results were performed using a server with 15 CPU's Intel ®Xeon (R) E5540@ 2.53Ghz X4, with 16 GB of RAM. To solve the several MILP models, the IBM CPLEX Optimizer 12.6.1.0 solver was used with a time limit equal to 3600 seconds.

A set of instances was randomly generated as described in [22]. The vertices in V are located on a square grid of length $\ell = 8$. The base station is located on the bottom left vertex and the remaining stations are placed randomly on a square of length $\ell' = 6$ in the upper right of the grid. The distance matrix is given by the euclidean distance between the stations. The graph edges are selected randomly. In order to obtain a certain graph density d, starting from a complete graph, edges are removed randomly, while ensuring connectivity, until the desired graph density is obtained. In this work, we generate instances varying $|V|$ in $\{8, 10, 12, 20\}$ and with $d = 0.4$. We considered the values of $m \in \{72, 120, 240\}$ and the information generation rates r_j are randomly generated in the interval $[1, 5]$. The values of α_{ij} were randomly generated in $\{1/12, 1/13, 1/14\}$ if $i = j$ and in $\{1/5, 1/6, 1/7\}$ otherwise. The following values parameters were set: $r_{cov} = 4$, $R = 20$ and $M = 3$.

First, in Table 2, we compare the size of the three models for the combination of the parameters defining the instances and the models used in our experiments. For model NE, the values τ, ω in the top of the four columns represent, respectively, the maximum number of allowed visits to and the maximum number of allowed transfers from each station. We can observe that DT model is the largest model, while NE model is the smallest one. The VE model is an intermediate model in terms of size. Next, we will describe the results obtained with each one of the three models.

Table 3: Results obtained with the Discrete Time model.

$\|V\|$	d	m	Total	best sol.	Cpu	Gap	Nodes	$\|V\|$	d	m	Total	best sol.	Cpu	Gap	Nodes
10	0.42	72	2392	1676.64	63.69	0	6809	10	0.42	120	3832	2453.21	443.63	0	24423
			2220	1538.64	81.23	0	8589				3564	2216.67	515.01	0	28669
			1744	1128.11	57.55	0	8685				2752	1550.53	748.32	0	206814
			2006	1337.79	96.86	0	11171				3206	1876.88	***	3.64	261685
			1797	1127.11	146.49	0	18012				2853	1581.25	***	0.28	254528
			2073	1413.24	85.64	0	9085				3321	2063.8	515.77	0	23966
			2286	1620.57	106.2	0	11851				3678	2407.19	505.97	0	29222
			1952	1250.58	107.18	0	32751				3104	1726.78	636.26	0	24311
			2339	1671.6	102.15	0	9232				3731	2423.72	483.33	0	25211
			1734	1083.79	99.88	0	11483				2742	1422.04	752.43	0	42202
15	0.4	72	3808	3059.5	271.19	0	8477	15	0.4	120	5477	4086.59	***	2.47	21649
			3205	2468.17	487.51	0	27490				5638	4280.33	***	0.7	25301
			2753	2051.69	277.88	0	12014				4915	3602.7	***	0.7	31131
			3112	2390.6	203.24	0	10785				5140	3758.86	***	3.85	22132
			2738	2059.6	248.73	0	11993				4899	3637.02	***	1.84	22524
			2876	2173.72	***	2.18	360579				5068	3633.39	***	0.45	29757
			3213	2473.69	139.26	0	6694				5616	4138.39	***	0.23	24494
			3669	2912.56	212.09	0	9233				5626	4258.6	***	0.6	28796
			3112	2324.19	259.58	0	6046				6224	4791.89	***	1.76	29075
			2797	2123.41	192.35	0	11900				6234	4811.39	3382.54	0	45284
20	0.4	72	5149	4301.5	1001.32	0	151612	8	0.42	240	5216	2385.33	***	6.39	97091
			4574	3838.6	1033.39	0	16072				4976	2037.34	***	10.34	87142
			4239	3528.7	507.27	0	11944				5700	2895.91	***	4.7	100844
			4924	4167.57	483.79	0	9344				3764	1242.85	***	28.32	74207
			4657	3904.39	963.63	0	10171				4960	2297.1	***	7.09	71848
			5097	4315.39	1094.51	0	21540				4965	2190.77	***	8.42	55845
			4928	4180.5	905.1	0	26529				4977	2127.55	***	15.57	46945
			4175	3448.59	587.1	0	12830				5910	2896.44	***	3.71	110735
			5650	4844.29	410.95	0	8962				5223	2572.06	***	7.5	87638
			5007	4239.89	451.94	0	10770				5448	2700.38	***	5.65	114550

**** time limit: 3600 sec.*

In Table 3, we report the computational results obtained with the DT model. The table is split into two parts accordingly to the time horizons. For each part, the first three columns give the number of stations ($|V|$), the density of the graph, d (since the information generation process may generate graphs with density slightly different from 0.4), and the size of the time horizon m. The fourth column gives the total amount of information generated during the time horizon. The fifth column gives the value of the best feasible solution found, that is, the amount of information remaining in the nodes at the end of time horizon T. The following three columns give: (Cpu) is the running time (in seconds); (Gap) is the final integrality gap reported by the solver at the end of running time; and (Nodes) is the number of nodes of the branch-and-bound algorithm. An instance with $***$ in the (Cpu) and with (Gap) superior to zero is not solved to optimality in the time limit.

The results show that for $m = 72$ all except one instance with $|V| \in \{10, 15, 20\}$ are solved to optimality within the one hour time limit. When m is increased, the number of solved instances decreases. For $m = 240$, even when $|V| = 8$, no instance is solved to optimality with final gap arriving to 28%.

Next, in Table 4 we report the results obtained with the NE model. The four first columns are similar to the ones of Table 3. The following four sets of columns give the same information as the corresponding ones in Table 3; namely the best feasible solution value (best sol), the running time (Cpu), the final gap (Gap) and the number of nodes (Nodes). Again, the values τ, ω in the top of the four multicolumns represent, respectively, the maximum number of allowed visits to and the maximum number of allowed transfers from each station.

The results in Table 4 show that the model can only be solved to optimality for very small values of τ and ω. The average values of the best solutions obtained with the NE Model on the set of instances with $|V| = 10$, $m = 240$ are compared in Figure 5 with the average value of the best solutions found by the VE Model.

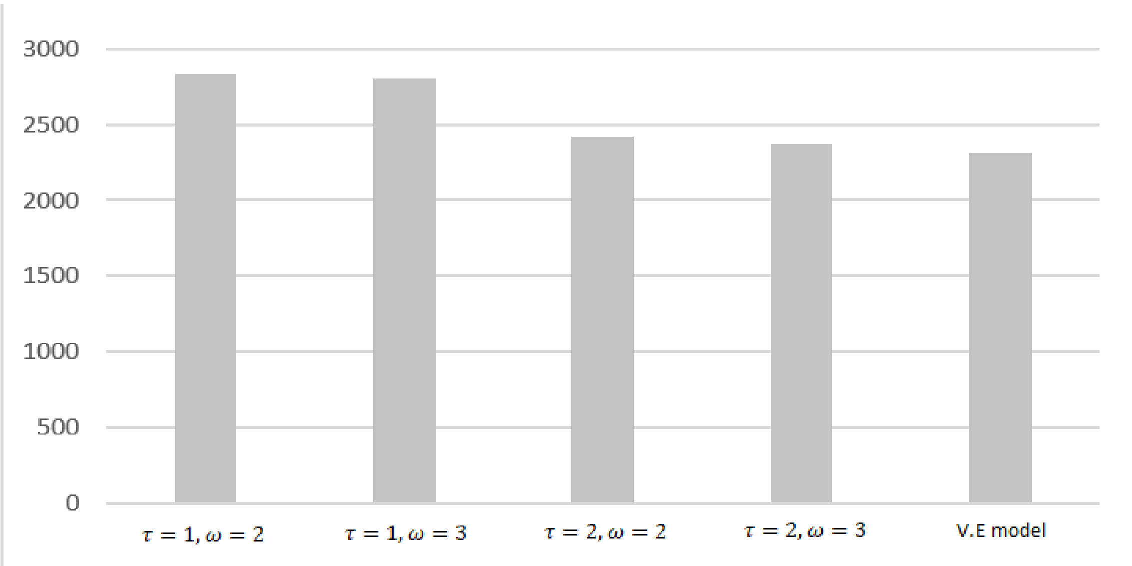

Figure 5: Average values of the best solutions obtained using NE model on the set of with $|V| = 10$, $m = 240$.

Finally, in Table 5, we report the results obtained using the VE Model. The meaning of the columns is the same as for the two previous tables. The last four sets of columns are grouped accordingly to the maximum number of vehicle events $\hat{N}$. The bold numbers mean that the corresponding value N is probably N^*: the objective value did not change for $N > \hat{N}$.

As depicted in Figure 6, for the particular case with $|V| = 15$ and $m = 120$, running times increase when $\hat{N}$ increases. However, as it can be seen from Figure 7 that depicts the average amount of information at the end of time T at all nodes (for the same instances of Figure 6), the solution quality also increases when $\hat{N}$ increases, until the value of N^* is

Table 4: Results obtained with the Node Event model. The values of τ and ω are, respectively, the maximum number of allowed visits and the maximum number of allowed transfers, assumed the same for each station.

$\|V\|$	d	m	Total	$\tau=1\ \omega=2$				$\tau=1\ \omega=3$				$\tau=2\ \omega=2$				$\tau=2\ \omega=3$			
				best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes
10	0.42	72	2392	1724.19	7.28	0	16234	1721.78	40.13	0	150505	1680.88	120.58	0	217636	1677.47	958.92	0	1743486
			2220	1981.11	0.73	0	569	1981.11	1.32	0	806	1551.47	126.29	0	225699	1540.83	2010.69	0	3890514
			1744	1159.59	1.57	0	4860	1155.89	9.39	0	26059	1149.39	44.19	0	109650	1133.84	248.85	0	452336
			2006	1342.00	8.85	0	16471	1337.80	18.82	0	49638	1342.00	167.88	0	224937	1337.80	797.21	0	1096998
			1797	1168.92	6.79	0	17619	1160.55	21.70	0	80846	1151.80	154.88	0	382002	1133.20	958.92	0	2464442
			2073	1424.42	9.12	0	23715	1413.69	22.24	0	62172	1424.42	77.30	0	134214	1413.69	869.02	0	1243046
			2286	1656.60	5.98	0	13413	1656.07	24.14	0	66631	1631.57	129.87	0	262046	1620.87	338.98	0	648962
			1952	1262.28	6.96	0	14370	1250.58	16.63	0	41274	1262.28	35.14	0	70495	1250.58	201.41	0	267979
			2339	1680.40	7.18	0	18216	1671.60	25.66	0	63856	1680.40	167.93	0	293311	1671.60	1592.99	0	2013728
			1734	1085.61	6.61	0	21227	1083.80	28.38	0	127567	1085.61	93.56	0	167879	1083.80	803.70	0	1158081
15	0.4	72	3808	3067.64	92.51	0	139937	3065.00	555.69	0	602120	3067.64	2852.97	0	1742946	3059.85	***	4.96	1292474
			3205	2471.60	159.20	0	217339	2468.19	2335.42	0	876996	2485.19	***	3.41	1605458	2468.19	***	5.95	953355
			2753	2051.69	84.54	0	157423	2051.69	***	1.76	2883359	2051.69	***	6.76	1805974	2051.69	***	10.00	1239171
			3112	2393.79	46.28	0	91822	2390.59	402.13	0	523241	2393.79	1743.38	0	1486567	2390.60	***	3.71	1507746
			2738	2059.60	79.27	0	95944	2059.60	446.75	0	674598	2059.60	2324.37	0	1379926	2061.10	***	4.56	1095142
			2876	2190.50	46.44	0	115243	2181.09	425.26	0	674598	2190.50	2541.83	0	2309950	2177.89	***	7.26	1777379
			3213	2473.69	24.01	0	34690	2473.69	234.56	0	307593	2473.69	1430.90	0	1087154	2473.69	***	6.64	1470692
			3669	3006.35	41.28	0	59057	3006.35	312.73	0	439188	2920.19	803.76	0	601495	2912.56	***	1.69	2028867
			3112	2329.70	25.96	0	30986	2324.19	220.28	0	184423	2329.70	1083.10	0	786336	2324.19	***	5.40	1362645
			2797	2123.41	20.04	0	33926	2123.41	276.62	0	385285	2123.41	1264.57	0	1231338	2123.41	***	6.89	1591442
20	0.4	72	5149	4346.80	262.04	0	370702	4345.20	1853.89	0	1248387	4327.20	535.50	0	408557	4324.10	***	3.17	870060
			4574	3842.70	348.25	0	455598	3839.09	***	3.46	894964	3849.44	***	4.24	1218230	3839.10	***	5.26	727984
			4239	3533.08	2844.35	0	2336663	3528.70	***	3.16	1786727	3538.04	***	5.06	1233783	3528.70	***	6.94	664625
			4924	4168.37	1199.28	0	1055468	4167.57	***	2.13	1697596	4168.37	***	3.78	1149544	4167.57	***	5.09	802513
			4657	3933.00	1618.46	0	1315354	3932.39	***	3.19	1548161	3917.69	***	4.14	1035218	3908.60	***	6.14	609650
			5097	4331.79	1654.64	0	1525459	4329.00	***	2.58	1726026	4331.80	***	3.69	1061453	4332.27	***	6.00	2654966
			4928	4180.50	1876.97	0	1799235	4180.50	***	2.26	1823169	4181.85	***	3.67	1258878	4182.10	***	5.51	807401
			4175	3448.89	361.22	0	658225	3448.60	***	1.95	1605318	3448.89	***	3.88	1192258	3448.60	***	5.90	669661
			5650	4855.25	470.78	0	586843	4849.23	***	2.42	1949539	4855.25	***	1.03	1944104	4849.60	***	4.22	822344
			5007	4268.10	1369.14	0	1501687	4268.10	***	2.36	1992753	4244.66	***	2.77	1166656	4245.20	***	3.92	763593
10	0.42	120	3832	2499.63	18.49	0	49639	2482.61	546.76	0	4453362	2513.68	***	12.05	3085989	2482.61	***	15.38	2298216
			3564	3164.41	0.72	0	351	3164.17	1.44	0	1951	2220.77	***	8.86	3323352	2227.19	***	16.98	812161
			2752	1564.61	11.52	0	45329	1555.61	52.32	0	198699	1564.61	***	13.06	3818080	1555.61	***	19.26	2875421
			3206	1884.73	14.94	0	42682	1876.88	124.04	0	537996	1884.73	***	11.91	3120778	1886.99	***	21.91	3323057
			2853	1590.94	18.80	0	67821	1580.01	96.19	0	334105	1590.94	1046.05	0	2429377	1588.69	***	22.27	2510944
			3321	2079.21	22.91	0	75904	2070.82	183.62	0	626727	2079.21	488.04	0	1097984	2084.44	***	19.09	2200596
			3678	2433.10	15.92	0	55652	2422.67	140.94	0	536561	2419.50	***	5.0	5714557	2412.37	***	14.93	5651898
			3104	1763.53	14.20	0	41969	1738.93	63.17	0	188463	1763.53	522.44	0	1202394	1738.93	***	14.29	2406634
			3731	2434.51	14.67	0	45223	2423.72	87.25	0	369430	2434.51	491.94	0	1157731	2423.72	***	16.34	2433687
			2742	1474.00	30.02	0	157746	1448.57	745.60	0	7864763	1465.39	***	7.63	5368808	1430.59	***	22.96	1962712
15	0.42	120	5477	4090.55	262.69	0	615334	4092.39	***	5.99	5226129	4111.59	***	10.08	1319841	4119.80	***	14.36	483290
			5638	4285.39	377.93	0	951836	4285.39	***	8.84	2432155	4309.71	***	11.39	1330813	4323.85	***	13.79	754754
			4915	3633.30	529.46	0	1260157	3631.40	***	7.45	6187331	3625.08	***	13.34	1146959	3635.98	***	16.15	835953
			5140	3736.86	151.06	0	428057	3736.86	***	2.65	7526906	3736.86	***	9.54	1106788	3745.91	***	13.02	819019
			4899	3614.39	125.35	0	313364	3612.99	***	9.24	2578825	3686.05	***	15.92	1270376	3612.99	***	15.15	850758
			5068	3646.31	147.80	0	365046	3638.19	***	5.67	3102850	3641.10	***	9.51	1152390	3638.20	***	10.93	872539
			5616	4147.30	205.58	0	464785	4142.10	***	5.33	4486528	4149.80	***	9.48	1074275	4153.29	***	11.20	532348
			5626	4262.04	521.38	0	1229788	4256.64	***	7.52	4939164	4269.19	***	10.48	1191714	4262.30	***	11.92	747543
			6224	4775.21	154.36	0	397353	4761.89	***	2.88	5899448	4775.21	***	7.41	1287197	4795.60	***	9.35	877427
			6234	4819.20	126.41	0	309998	4817.36	***	3.68	6165272	4823.20	***	8.44	1160888	4818.60	***	9.10	835388
8	0.42	240	5216	2579.93	4.79	0	16059	2570.42	7.05	0	28015	2463.19	***	9.57	8800389	2365.66	***	42.70	4110411
			4976	2298.28	4.54	0	20935	2231.32	16.38	0	113149	2180.15	***	15.54	8718311	2133.05	***	47.58	5787439
			5700	3122.77	3.77	0	17385	3104.08	5.51	0	25415	2982.94	3443.20	0	10530309	2950.09	***	29.31	6225620
			3764	1444.09	4.34	0	17316	1426.74	8.88	0	63181	1355.23	***	27.22	8650708	1273.45	***	54.24	6069863
			4960	2495.54	3.51	0	15695	2465.88	6.98	0	38090	2371.44	***	4.63	8807297	2327.08	***	16.38	8072843
			4965	3829.64	0.22	0	0	3829.82	0.40	0	0	2285.32	***	14.96	6875154	2229.34	***	56.31	4274151
			4977	2398.77	2.96	0	11783	2368.17	10.72	0	87035	2198.97	***	5.79	8528177	2121.62	***	38.86	4157215
			5910	4704.82	0.31	0	0	4704.82	0.42	0	0	2956.80	***	5.27	7595306	2934.92	***	33.73	4508588
			5223	2661.92	4.39	0	21736	2624.35	16.78	0	145301	2640.83	***	6.89	7775658	2595.61	***	47.25	3309505
			5448	2765.27	6.38	0	33567	2730.17	34.80	0	271530	2770.00	***	8.40	9234889	2730.17	***	40.82	6864844

**** time limit : 3600 sec.*

Table 5: Results obtained with the Vehicle Event model.

$\|V\|$	d	m	Total	$\hat{N}=5$				$\hat{N}=6$				$\hat{N}=7$				$\hat{N}=8$			
				best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes	best sol.	Cpu	Gap	Nodes
10	0.42	72	2392	1713.68	0.29	0	0	**1676.64**	0.41	0	0	1676.64	0.9	0	882	1676.64	2.28	0	2776
			2220	1712.62	0.17	0	18	1549.64	0.22	0	0	**1538.64**	0.58	0	335	1538.64	0.9	0	1401
			1744	1155.89	0.31	0	0	**1128.11**	0.62	0	201	1128.11	0.76	0	0	1128.11	1.77	0	1711
			2006	1372.78	0.3	0	0	**1337.80**	0.44	0	0	1337.80	1.07	0	932	1337.80	1.99	0	2882
			1797	1165.1	0.28	0	0	1157.84	0.41	0	0	**1127.11**	1.04	0	1112	1127.11	2.16	0	3156
			2073	1452.27	0.28	0	0	**1413.24**	0.57	0	229	1413.24	1.03	0	854	1413.24	2.16	0	3019
			2286	1656.07	0.72	0	133	**1620.57**	0.57	0	476	1620.57	1.09	0	1633	1620.57	2.21	0	5228
			1952	**1250.58**	0.43	0	0	1250.58	0.96	0	577	1250.58	1.45	0	1420	1250.58	2.93	0	3380
			2339	1674.43	0.52	0	119	**1671.60**	0.91	0	624	1671.60	1.65	0	1241	1671.60	2.71	0	3736
			1734	1148.44	0.22	0	0	1104	0.45	0	0	**1083.80**	1.11	0	855	1083.8	2.32	0	2585
15	0.4	72	3808	**3059.50**	1.6	0	660	3059.50	2.92	0	3090	3059.50	18.6	0	18411	3059.50	60.74	0	60499
			3205	**2468.19**	1.37	0	472	2468.19	2.55	0	2675	2468.26	21.22	0	18452	2468.17	76.49	0	72026
			2753	2084.17	1.58	0	613	**2051.69**	2.69	0	2869	2051.69	23.97	0	20167	2051.69	55.97	0	55522
			3112	**2390.60**	1.04	0	254	2390.6	1.95	0	1752	2390.6	3.98	0	6506	2390.6	26.41	0	24096
			2738	2066.72	1.28	0	233	**2059.60**	2.43	0	1518	2059.6	6.08	0	7493	2059.69	55.14	0	42246
			2876	2192.54	1.14	0	672	2175.99	2.08	0	2944	**2171.19**	16.71	0	17109	2171.19	35.73	0	46246
			3213	**2473.69**	1.51	0	587	2473.69	2.48	0	3392	2473.69	13.63	0	7894	2473.69	56.47	0	39081
			3669	3004.91	0.9	0	463	**2912.56**	1.58	0	1366	2912.56	4.57	0	6191	2912.56	39.16	0	27267
			3112	**2324.19**	1.63	0	228	2324.19	2.19	0	1676	2324.19	6.05	0	9176	2324.19	36.71	0	29815
			2797	**2123.41**	1.23	0	354	2123.41	2.14	0	2541	2123.41	10.4	0	7976	2123.41	59.36	0	54844
20	0.4	72	5149	4316.39	2.16	0	1705	**4301.50**	5.54	0	9258	4301.50	43.89	0	47785	4301.50	210.99	0	206763
			4574	**3838.59**	2.42	0	1280	3838.59	20.03	0	16696	3838.59	69.96	0	65746	3838.6	586.09	0	565564
			4239	**3528.70**	2.51	0	2004	3528.70	17.8	0	16840	3528.70	61.47	0	57857	3528.70	310.7	0	311865
			4924	**4167.57**	2.07	0	782	4167.57	4.68	0	5245	4167.57	38.34	0	35146	4167.57	175.14	0	176522
			4657	3932.1	2.88	0	1475	**3904.39**	6.35	0	9768	3904.7	69.72	0	59977	3904.4	427.21	0	359717
			5097	4363.2	2.49	0	1803	4329	17.24	0	13732	**4315.39**	63.97	0	64688	4315.39	293.42	0	290607
			4928	4182.1	2.3	0	1886	**4180.5**	20.06	0	20270	4180.5	56.79	0	57610	4180.5	367.51	0	331521
			4175	**3448.59**	2.27	0	1147	3448.6	14.85	0	13423	3448.6	50.02	0	47950	3448.6	288.1	0	288411
			5650	4849.23	2.83	0	1826	**4844.30**	18.16	0	17795	4844.5	57.56	0	56213	4844.3	434.15	0	450813
			5007	4268.09	1.53	0	302	**4239.89**	3.14	0	2577	4239.89	23.74	0	18533	4239.89	112.68	0	90607
				$\hat{N}=7$				$\hat{N}=8$				$\hat{N}=9$				$\hat{N}=10$			
10	0.42	120	3832	**2453.24**	1.09	0	875	2453.21	1.74	0	3326	2453.21	4.88	0	9513	2453.24	22.88	0	30409
			3564	2285.05	0.36	0	0	**2216.67**	0.88	0	783	2216.67	1.68	0	2986	2216.67	10.06	0	18917
			2752	1553.01	0.99	0	518	1553.01	1.41	0	1957	**1550.53**	3.52	0	6987	1550.53	17.23	0	22289
			3206	**1876.88**	1.08	0	1117	1876.88	1.94	0	2423	1876.88	5.06	0	9128	1876.88	22.74	0	27025
			2853	**1580.01**	0.97	0	0	1580.01	1.96	0	3570	1580.01	10.45	0	15240	1580.01	33.26	0	47042
			3321	2070.82	1.15	0	912	**2063.8**	2.01	0	4126	2063.79	13.18	0	20640	2063.8	24.66	0	32398
			3678	**2407.19**	1.37	0	1840	2407.19	2.01	0	5523	2407.19	13.07	0	23340	2407.19	33.2	0	59857
			3104	**1726.78**	1.35	0	1061	1726.78	2.9	0	3004	1726.78	13.12	0	13193	1726.78	29.73	0	31667
			3731	**2423.72**	1.59	0	1285	2423.72	2.44	0	4980	2423.72	6.65	0	13129	2423.72	28.22	0	34909
			2742	1485.29	1.13	0	510	1462.19	1.55	0	2066	1423.4	4.3	0	7205	**1422.2**	20.13	0	24431
15	0.4	120	5477	4073.59	23.21	0	20588	4073.59	73.38	0	92031	4072.7	519.72	0	470363	**4072.61**	2840.06	0	2262717
			5638	4272.14	18.3	0	18636	4272.14	51.74	0	58017	**4271.98**	243.95	0	231793	4271.98	1728.86	0	1673800
			4915	3615.4	5.11	0	8722	**3595.2**	60.25	0	58169	3595.2	172.51	0	128756	3595.2	825.35	0	639380
			5140	**3732.38**	18.51	0	19351	3732.38	40.68	0	50219	3732.38	209.33	0	196023	3732.38	1091.8	0	895180
			4899	**3613.1**	12.04	0	15090	3612.99	34.67	0	44224	3613	157.47	0	168952	3613.2	767.76	0	744047
			5068	**3630.49**	5.73	0	9184	3630.49	58.4	0	48098	3630.5	161.73	0	165570	3630.6	1161.39	0	988048
			5616	**4134.70**	22.83	0	19123	4134.70	56.16	0	56547	4134.70	404.05	0	274153	4134.70	1658.43	0	850429
			5626	**4256.64**	16.9	0	17250	4256.64	48.83	0	67235	4256.64	281.5	0	296837	4256.64	1702.87	0	1579158
			6224	**4761.89**	16.81	0	17767	4761.9	60.87	0	76541	4761.89	282.65	0	314020	4761.89	1270.3	0	1253523
			6234	4811.6	11.97	0	15453	**4811.39**	42.46	0	63248	4811.39	179.98	0	239578	4811.39	832.28	0	978057
				$N=9$				$N=10$				$N=11$				$N=12$			
8	0.42	240	5216	2400.8	1.48	0	1893	**2351.82**	2.8	0	4295	2351.82	12.43	0	15321	2351.82	19.77	0	29151
			4976	2106.46	2	0	3661	2035.21	4.36	0	10006	2015.5	16.15	0	23014	**2003.14**	28.87	0	44288
			5700	2895.17	2.09	0	3473	2872.24	8.23	0	17204	**2865.99**	29.32	0	56577	2865.99	25.92	0	49879
			3764	1262.58	1.48	0	1967	1248.32	3.28	0	6283	1207.12	12.65	0	24800	**1187.08**	20	0	36982
			4960	2290.94	1.88	0	2531	2268.63	2.88	0	6461	**2268.47**	10.65	0	18714	2268.47	25.32	0	51113
			4965	2205.5	1.43	0	1253	2180.72	2.52	0	4830	**2164.96**	4.12	0	8662	2164.96	31.6	0	40639
			4977	2139.89	1.91	0	2953	2086.69	10.05	0	13276	**2042.67**	19.93	0	29649	2042.72	64.89	0	86681
			5910	2897.95	1.13	0	2093	2897.29	2.68	0	5835	**2890.55**	11.4	0	22165	2890.55	36.38	0	70890
			5223	2570.32	1.99	0	2523	2557.37	2.4	0	6838	**2544.78**	14.81	0	26109	2544.78	37.16	0	63157
			5448	2684.69	1.98	0	3319	**2659.17**	4.19	0	9936	2659.17	15.46	0	25578	2659.17	44.29	0	78339

**** time limit : 3600 sec.*

obtained.

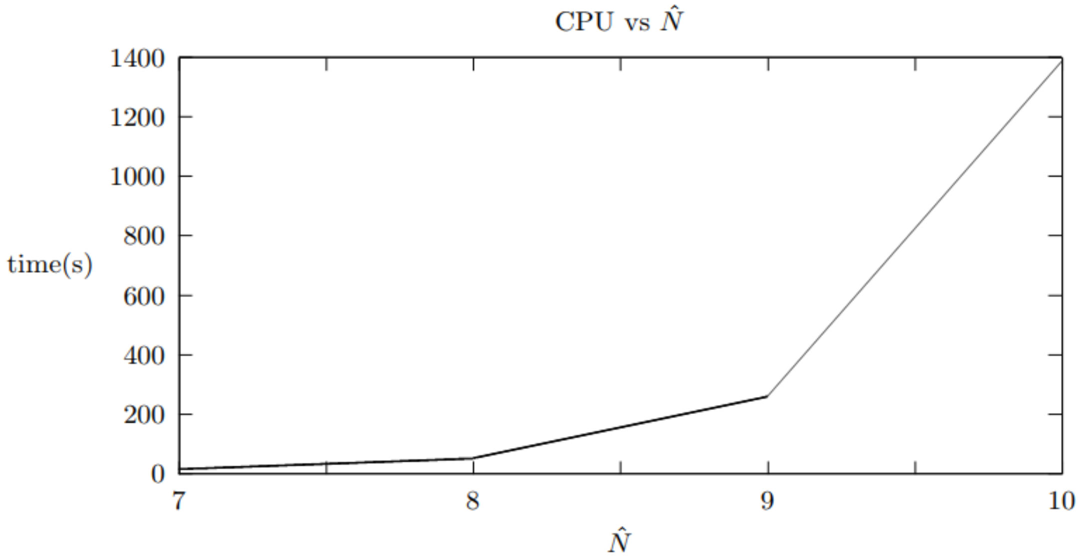

Figure 6: Average running times obtained by the Vehicle Event Model on the instances with $|V| = 15$ and $m = 120$, for different values of $\hat{N}$.

6. Conclusions

A routing-collecting problem (WT-VRP) where a vehicle collects information from a given set of stations is considered. Three mixed integer programming models were introduced. The first model (DT) is based on a time discretization, where each decision is a multiple of the time unit. The second model (NE) is an event model, where the visits to stations and the transfer operations are modeled as events. Finally, the third model (VE) considers the vehicle stops as events. A computational study based on randomly generated instances was conducted to compare the three models. The DT model presents a high number of both variables and constraints while the NE and VE models need to be fed with parameters for the maximum number of events permitted. The results show that the NE model is always the worst and that the best model (DT or VE) depends on the instance. For shorter time horizons the DT model performs well, while for longer time horizons the VE is usually faster. The performance of the VE model is better when the optimal number of vehicle stops can be estimated.

When modeling the WT-VRP, a decision must be made on how times (for node visits and for transfer operations) will be treated. From our discussions in the manuscript, we conclude that depending on the modeling strategy adopted, we can obtain different optimal solutions w.r.t. to transfer operations. The computational experiments show that the impact of this choice in the optimal solutions obtained for random instances is small. Moreover, since for many real applications (as to provide web connectivity for remote military stations [23] or

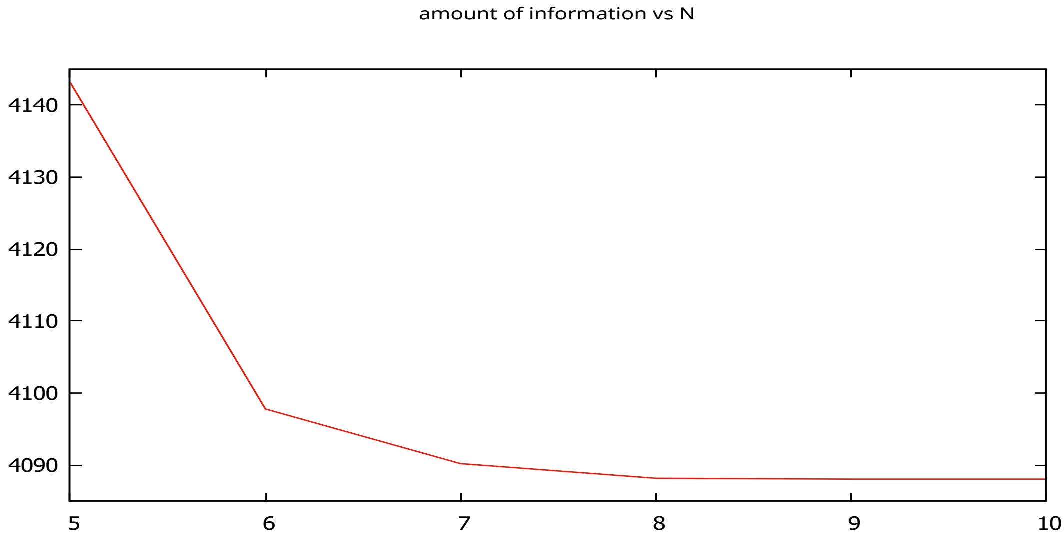

Figure 7: Average amount of information at time T at all nodes using the Event Vehicle Model with $|V| = 15, T = 120$ versus different values of N.

in Daknet [27]) the range of each station is small and the quantity of information to be sent is high, the instances where the optimal solution is affected by the time modeling decision have a low chance to happen. Finally, we also discussed how each model can became more accurate: by tunning the time discretization and increasing the time horizon T considered in the DT model; by adding scheduling constraints to the NE and VE models. However, these accuracies come with a price since all models will became computationally harder to be solved.

For future research, both DT and VE formulations can benefit from a study of the polytope defined by each formulation: the continuous relaxation of each model could be strengthened and the polyhedral study could be the base for the implementation of branch-and-cut methods to the WT-VRP. Moreover, the derivation of fast heuristics is important when the size of instances become large.

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