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The dc power observed on the half of asymmetric superconducting ring in which current flows against electric field

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The observations the dc voltage on asymmetric superconducting ring testify that one of the ring segments is a dc power source. The persistent current flows against the total electric field in this segment. This paradoxical phenomenon is observed when the ring or its segments are switched between superconducting and normal state by non-equilibrium noises. We demonstrate that the dc voltage and the power increase with the number of the identical rings connected in series. Large voltage and power sufficient for practical application can be obtained in a system with a sufficiently large number of the rings. We point to the possibility of using such a system for the observation of the dc voltage in the normal state of superconducting rings and in the asymmetric rings made of normal metal.

1. INTRODUCTION

The Meissner effect [1] is first experimental evidence that superconductivity is macroscopic quantum phenomenon. The paradoxicality of this effect is accentuate in the publications of Jorge Hirsch: "*Strangely, the question of what is the force propelling the mobile charge carriers and the ions in the superconductor to move in direction opposite to the electromagnetic force in the Meissner effect was essentially never raised nor answered to my knowledge, except for the following instances:* [2] (*H. London states: "The generation of current in the part which becomes supraconductive takes place without any assistance of an electrical field and is only due to forces which come from the decrease of the free energy caused by the phase transformation", but does not discuss the nature of these forces*), [3] (*A.V. Nikulov introduces a quantum force to explain the Little-Parks effect in superconductors*)" [4]. The Little - Parks effect [5] is also paradoxical. According to the generally accepted explanation [6], the quantum oscillations of the resistance in magnetic field [5] are observed due to the non-zero velocity v of Cooper pairs and the persistent current $I_p = sn_sqv$ of these pairs with a density n_s and the charge $q = 2e$.

An electric current induced in a resistive circuit will rapidly decay in the absence of an applied voltage. But the Little - Parks effect [5] testifies that the persistent current i.e. the electric direct current circulating in a ring does not decay any long time. The persistent current of Cooper pairs is observed at a non-zero resistance $R_l > 0$ [5] in the temperature region $T \approx T_c$ corresponding to the superconducting resistive transition where $0 < R_l < R_n$ because of thermal fluctuations which switch rings segments between the superconducting and normal state [6]. According to the measurements [5] the value of the persistent current changes periodically in magnetic field B with the period $B_0 = \Phi_0/S$ corresponding to the flux quantum $\Phi_0 = 2\pi\hbar/q$ inside the ring with the area $S = \pi r^2$. The measurements of the magnetic susceptibility in the fluctuation region near T_c [7] corroborate that the direction of the persistent current also changes with magnetic field: this current is diamagnetic at $n'\Phi_0 < \Phi < (n' + 0.5)\Phi_0$, paramagnetic at $(n' + 0.5)\Phi_0 < \Phi < (n' + 1)\Phi_0$ and equal zero at $\Phi = n'\Phi_0$ and $\Phi = (n' + 0.5)\Phi_0$.

It is known that the conventional circular electric current I_{sw} induces the potential difference

$$V = 0.5(R_{gr} - R_{sm})I_{sw} \quad (1)$$

on the halves of the ring with different resistance $R_{gr} > R_{sm}$. Could the persistent current induce a similar voltage? The observations [5, 7] $I_p \neq 0$ at $R_l > 0$ allow to answer on this question experimentally. The dc voltage the value and sign of which change periodically in magnetic field similar to the value and direction of the persistent current [7] was observed first time more than fifty years ago at the measurements of an asymmetric dc SQUID, i.e. a superconducting loop with two Josephson junctions [8]. The authors [8] did not attach much importance to the effect they observed. They concluded that the emitted electromagnetic radiation of broadcasting stations is responsible for the observation of the dc voltage and interpreted the effect as a consequence of a rectification process which takes place in their superconducting loop [8]. They have confirmed that the visible dc voltage disappears when all parts of the equipment are shielded carefully [8]. The authors were realizing that the periodicity in the voltage as a function of the applied magnetic field is observed due to the periodicity in the current circulating in the loop. But they did not take into account that the observation of the circulating current flowing against electric field in one of loops segments is paradox. This paradox was observed later at measurement near superconducting transition of aluminum asymmetric ring [9]. The authors [8] interpreted the dc voltage V as the result of the rectification of ac current induced by electromagnetic

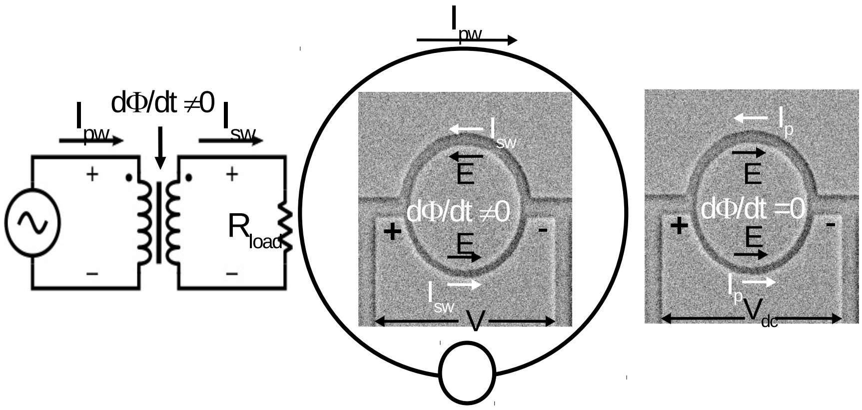

FIG. 1: The transformer diagram is shown on the left. The current I_{pw} of the primary winding creates magnetic flux varying in time $d\Phi/dt$ in order to induce the Faraday electromotive force and the current I_{sw} in the secondary winding and the voltage $V_{load} = R_{load} I_{sw}$ in the load with the resistance R_{load}. The current I_{pw} of the primary winding can induce the current I_{sw} also in the asymmetric ring shown in the center. In this case the wide half may be considered as the secondary winding and the narrow half as the load. On the right is the case of the dc voltage observed on an asymmetric ring with the persistent current at a magnetic flux not varying in time $d\Phi/dt = 0$. The persistent current I_p flows against the total electric field $E = -\nabla V$ in the wide half in contrast to the conventional current I_{sw} the direction of which corresponds to the direction of the total electric field $E = -\nabla V - dA/dt$ thanks to the Faraday electric field $-dA/dt = -l^{-1} d\Phi/dt$, see in the center. The photos of a real aluminum ring with the radius $r \approx 1\ \mu m$ is shown. Such ring was used for the observation of the $V_{dc}(\Phi)$ oscillations.

radiation in order to explain an energy dissipation V_{dc}^2/R which they observed. But they did not take into account that no rectification process can explain the dc power $V_{dc} I_p$, observed in loop's segment in which the circular current I_p flows against the voltage V_{dc}.

2. ANALOGY WITH THE SECONDARY WINDING OF THE ELECTRIC TRANSFORMER

The voltage (1) is observed when the circulating current I_{sw} is induced by the Faraday electromotive force $-d\Phi/dt$ in accordance with the well known relation $(R_{gr} + R_{sm}) I_{sw} = -d\Phi/dt$. The current I_{sw} flows against the potential difference V in the half of an asymmetric ring with a smaller resistance $R_{sm} < R_{gr}$ as well as in the secondary winding of the electric transformer, Fig.1. Therefore this half may be considered as the secondary winding whereas the half with the greater resistance $R_{gr} > R_{sm}$ may be considered as the load. The current I_{sw} in the secondary winding of the electric transformer is induced by the current in the primary winding I_{pw} in order to obtain the power $W_{load} = R_{load} I_{sw}^2$ on a load with the resistance R_{load}, Fig.1 at the left. The current in the primary winding I_{pw} may induce the current I_{sw} also in the asymmetric ring, Fig.1 at the center.

2.1. Increase the voltage and the power with the number of the asymmetric rings

A useful power can be obtained in this case at a load connected in parallel to the half with the smaller resistance R_{sm}. Such a transformer is not the most efficient. The power on the load cannot exceed $V I_{sw}$. But the voltage and the power may be increased with the help of a system of identical asymmetric rings connected in series, Fig.2. A homogeneous magnetic field B should induce the same magnetic flux $\Phi = BS = \pi r^2$ in all rings with the same radius r. Therefore the current $I_{sw} = (R_{gr} + R_{sm})^{-1}(-d\Phi/dt) = (R_{gr} + R_{sm})^{-1} S(-dB/dt)$ has the same direction and value in all identical rings when the magnetic field changes in time $dB/dt \neq 0$. The voltage (1) should also has the same sign and value in all identical rings and therefore $V_N = N V_1$. For example, if a current I induces the voltage (1) $V_1 \approx 1.5\ \mu V$ on each ring then the voltage $V_5 \approx 7.5\ \mu V$ will be observed on five rings connected in series, Fig.2.

2.2. The secondary winding without the primary winding

The asymmetric rings cannot be used as the secondary winding without the primary winding, if the current I_{sw} circulating in the rings with a non-zero resistance $R_{gr} + R_{sm} = R_l > 0$ will rapidly decay during a short relaxation

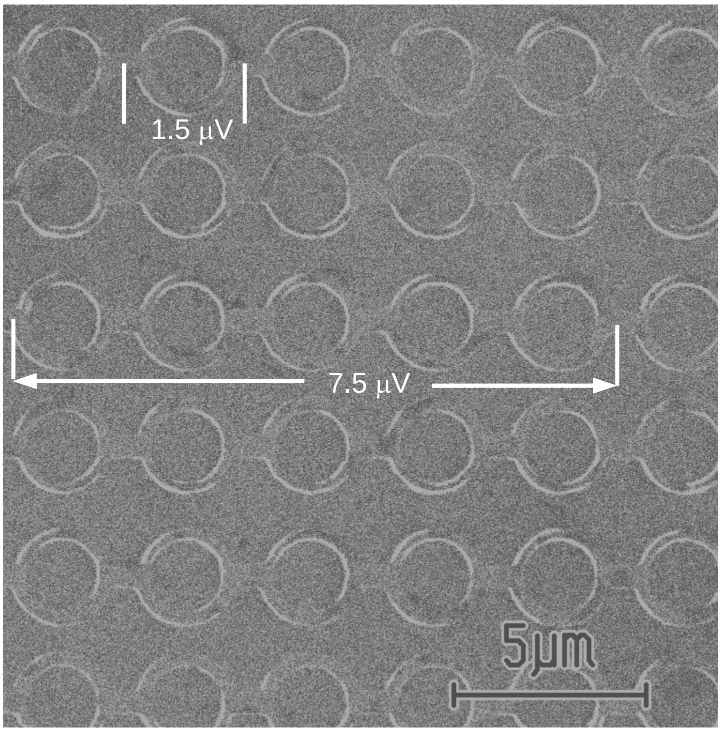

FIG. 2: A fragment of the system of 1080 identical asymmetric aluminum rings with $r \approx 1$ μm connected in series. The semi-ring widths are $w_w \approx 400$ nm and $w_n \approx 200$ nm. The voltage $V \approx 7.5$ μV is observed of the five rings connected in series when the voltage $V \approx 1.5$ μV is observed on each ring.

time $\tau_{RL} = L/R_l$ in the absence of the Faraday electromotive force, i.e in a magnetic field constant in time $dB/dt = 0$, here L is the total inductance of the ring. The persistent current does not decay in magnetic field constant in time $dB/dt = 0$ both in superconducting state where the resistance equal zero $R_l = 0$ and in the fluctuation region at $T \approx T_c$ where $0 < R_l < R_n$ [5, 7]. The proportionality $V_{dc}(\Phi) \propto I_p(\Phi)$ of the voltage (observed in [8, 9]) to the persistent current [7] allows to consider asymmetric superconducting ring (or loop) as the secondary winding which can provide a power without the primary winding. The current cannot induce a voltage when the resistance of the ring equals zero $R_{gr} = R_{sm} = 0$, see (1). Therefore the authors [8] did not observed a visible dc voltage when all parts of their equipment were shielded carefully. The voltage $V_{dc}(\Phi) \propto I_p(\Phi)$ is observed only if the ring [9] (or loop [8]) or its segments are switched between superconducting and normal state [10]. In this case both the persistent current $\overline{I_p} = \Theta^{-1} \int_{\Theta} dt I(t)$ and the resistance $\overline{R_l} = \Theta^{-1} \int_{\Theta} dt R_l(t)$ average in time $\Theta \gg 1/f_{sw}$ can be non-zero, here f_{sw} is a frequency of the switching. In the region of superconducting transition the ring is switched by thermal fluctuations [6]. Therefore both the persistent current $\overline{I_p} \neq 0$ and the resistance $\overline{R_l} > 0$ are measured in this temperature region [5, 7]. Thermal fluctuations cannot switch rings segments in the normal state below superconducting transition $T < T_c$. In this temperature region non-equilibrium noise, for example electromagnetic radiation [8], can are switching the ring [9] (or loop [8]) in the normal state.

According to the analogy with the secondary winding, the persistent current rather than non-equilibrium noise is responsible for the dc voltage observed in [8, 9]. Non-equilibrium noise, for example electromagnetic radiation [8], is responsible for the non-zero resistance $\overline{R_l} > 0$ average in time. The analogy with the secondary winding is more true than the analogy with the rectification process used in [8] since it allows to explain why the periodicity in the voltage repeats the periodicity in the persistent current $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$. We present in this work additional arguments in favor of this analogy. We demonstrate that the voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ increases with the number of identical asymmetric superconducting rings connected in series, similar to the rings with the usual current I_{sw}. We draw readers attention on the paradoxicality of the effect discovered in [8, 9] which cannot be described with the rectification process.

3. EXPERIMENTAL DETAILS

We use the system of 1080 asymmetric aluminum rings with the same radius $r \approx 1\ \mu m$, Fig.2, and the system of 667 aluminum rings with the same radius $r \approx 0.5\ \mu m$. The systems were fabricated by e-beam lithography and lift-off process of $d \approx 20\ nm$ (the 1080 rings) and $d \approx 30\ nm$ (the 667 rings) thick aluminum film. The 1080 rings were more asymmetric than the 667 one: the arm widths of all 1080 rings were $w_n \approx 200\ nm$ and $w_w \approx 400\ nm$ for narrow and wide parts, respectively and $w_n \approx 100\ nm$ and $w_w \approx 125\ nm$ of all 667 rings. The resistance in the normal state was $R_n \approx 8000\ \Omega$ of the 1080 rings and $R_n \approx 5400\ \Omega$ of of the 667 rings; the resistance ratio $R(300K)/R(4.2K) \approx 1.7$ and $R(300K)/R(4.2K) \approx 2$; superconducting transition temperature $T_c \approx 1.360\ K$ and $T_c \approx 1.320\ K$; the width of the resistive transition $\Delta T_c \approx 0.02\ K$ and $\Delta T_c \approx 0.01\ K$. The temperature dependence of the critical current is described by the relation $I_c = I_c(T=0)(1-T/T_c)^{3/2}$, Fig.3, where $I_c(T=0) \approx 700\ \mu A$ for the 1080 rings $I_c(T=0) \approx 520\ \mu A$ for the 667 rings. The critical current density $j_c(T=0) \approx 10^7 A/cm^2$ equals approximately the depairing current density [11].

4. THE INCREASE THE AMPLITUDE OF THE OSCILLATIONS OF THE DC VOLTAGE WITH THE NUMBER OF THE IDENTICAL ASYMMETRIC RINGS CONNECTED IN SERIES

The authors [8] did not observed a visible dc voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ when all parts of their equipment were shielded carefully since the $\overline{I_p}(\Phi)$ amplitude is very low in the fluctuations region at $T \approx T_c$ where the persistent current is observed [7] at a non-zero resistance $\overline{R_l} > 0$ without a non-equilibrium noise, for example electromagnetic radiation. Therefore it ought be accentuated that the amplitude of the persistent current is much higher in superconducting state $T < T_c$ and increases with the temperature decrease, Fig.3.

4.1. Temperature dependence of the amplitude of the persistent current and the critical current

The persistent current is observed due to the Bohr quantization and the Aharonov - Bohm effect. The wave function $\Psi = |\Psi|e^{i\varphi}$ describing a superconducting pairs is defined along the whole circle $l = 2\pi r$ when all ring's segments are in

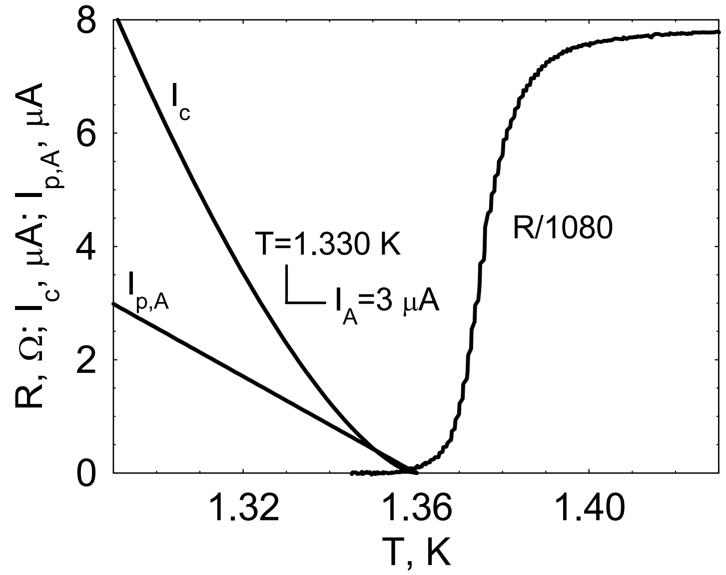

FIG. 3: Temperature dependence of the resistance R (superconducting resistive transition), of the critical current $I_c = I_c(T=0)(1-T/T_c)^{3/2}$ and the amplitude of the persistent current $I_{p,A} = I_{p,A}(T=0)(1-T/T_c)$ of the system of 1080 aluminium rings. $I_c(T=0) \approx 700\ \mu A$, $I_{p,A}(T=0) \approx 58\ \mu A$, $T_c \approx 1.36\ K$. The temperature $T \approx 1.330\ K$ and the current amplitude $I_A = 3\ \mu A$ at which the dc voltage oscillations are observed, Fig.4, are indicated.

superconducting state with a non-zero density of Cooper pairs $|\Psi|^2 = n_s > 0$. According to the canonical definition, the gradient operator $\hat{p} = -i\hbar\nabla$ corresponds to the canonical momentum $p = mv + qA$ of a particle with a mass m and a charge q both with $A \neq 0$ and without $A = 0$ magnetic field. Whereas the operator of the velocity $\hat{v} = (\hat{p} - qA)/m = (-i\hbar\nabla - qA)/m$ [12] depends on the magnetic vector potential A. The angular momentum of each Cooper pair in the superconducting state has discrete values $m_p = \oint_l dl\Psi^*\hat{p}\Psi/2\pi \oint_l dl\Psi^*(-i\hbar\nabla)\Psi/2\pi = \hbar n$ due to the Bohr quantization $m_p = rp = rmv = n\hbar$ or the requirement $\oint_l dl\nabla\varphi = n2\pi$ of uniqueness of the wave function at any point of the circle $\Psi = |\Psi|e^{i\varphi} = |\Psi|e^{i(\varphi+n2\pi)}$. The velocity

$$\oint_l dlv = \oint_l dl\frac{\hbar\nabla\varphi - qA}{m} = \frac{\hbar}{mr}(n - \frac{\Phi}{\Phi_0}) \quad (2)$$

cannot be equal zero when the magnetic flux $\Phi = \oint_l dlA$ inside the ring is not divisible $\Phi \neq n\Phi_0$ by the flux quantum Φ_0 due to the dependence of the operator of the velocity on the magnetic vector potential A. The effects connected with this dependence were first predicted by Aharonov and Bohm [13]. Therefore, they are referred as the Aharonov - Bohm effects.

The persistent current

$$I_p = sn_sqv = \frac{q\hbar}{mr\overline{(sn_s)^{-1}}}(n - \frac{\Phi}{\Phi_0}) = \frac{\Phi_0}{L_k}(n - \frac{\Phi}{\Phi_0}) \quad (3)$$

is proportional to the density of Cooper pairs since all $N_s = \oint_l dlsn_s$ pairs, being bosons, have the same quantum number n in superconducting ring with the macroscopic volume $V = \oint_l dls$. Here $\overline{(sn_s)^{-1}} = l^{-1}\oint_l dl(sn_s)^{-1}$ and $L_k = ml\overline{(sn_s)^{-1}}/q^2$ is the kinetic inductance of Cooper pairs in the ring with the section area s and the density of Cooper pairs n_s varying along the circumference $l = 2\pi r$. The value of the persistent current (3) and the discreteness of the permitted state spectrum depend on the density of Cooper pairs n_s in each ring's segment [14]. The difference the kinetic energy [6]

$$E_n = \frac{L_kI_p^2}{2} = \frac{\Phi_0^2}{2L_k}(n - \frac{\Phi}{\Phi_0})^2 \quad (4)$$

between the permitted states is large $E_{n+1} - E_n \approx \Phi_0^2/2L_k \gg k_BT$ [15] when $n_s > 0$ in all ring's segments. The quantum number n corresponding to the minimal kinetic energy should change at $\Phi = (n' + 0.5)\Phi_0$ according to (4). Therefore the persistent current corresponding to the thermodynamic equilibrium changes periodically in magnetic field B with the period $B_0 = \Phi_0/S$ [11]: the current is diamagnetic at $n'\Phi_0 < \Phi < (n' + 0.5)\Phi_0$, paramagnetic at $(n' + 0.5)\Phi_0 < \Phi < (n' + 1)\Phi_0$ and equal zero at $\Phi = n'\Phi_0$ and $\Phi = (n' + 0.5)\Phi_0$. The amplitude of the oscillations increases with the temperature decrease $I_{p,A} = I_{p,A}(T = 0)(1 - T/T_c)$, Fig.3, due to the increase of the density of Cooper pairs $n_s = n_s(T = 0)(1 - T/T_c)$ [11]. The amplitude of the oscillations of the dc voltage $V_{dc}(\Phi) \propto I_p(\Phi)$ should increase with the $I_{p,A}$ increase. But the critical current $I_c = I_c(T = 0)(1 - T/T_c)^{3/2}$ increases also with the temperature T decrease, Fig.3.

4.2. The dependence of the dc voltage amplitude on the amplitude of a non-equilibrium noise and temperature.

A non-equilibrium noise $I_{noise} = \int_{fmin}^{fmax} dfI_f\sin(2\pi ft+\phi_f)$ can make the resistance average in time non-zero $\overline{R_l} > 0$ when its amplitude $\overline{2I_{noise}^2}^{1/2} = (\int_{fmin}^{fmax} dfI_f^2)^{1/2}$ exceeds the value of the critical current $\overline{2I_{noise}^2}^{1/2} > I_c = I_c(T = 0)(1-T/T_c)^{3/2}$. The experimental investigation [11, 16] have corroborate that the oscillations $V_{dc}(\Phi)$ appear when the amplitude of the noises or a sinusoidal current $I_{sin} = I_A\sin(2\pi ft)$ reaches the critical current at the temperature of measurement $T < T_c$. Their amplitude quickly reaches a maximum and decreases with further increase in the current amplitude [11, 16]. The temperature dependence of the $V_{dc}(\Phi)$ amplitude $V_A(T)$ is also non-monotonic for a given value of the amplitude $\overline{2I_{noise}^2}^{1/2}$ [17–19]. The oscillations $V_{dc}(\Phi)$ appear when the critical current $I_c(T)$ decreases down to $\overline{2I_{noise}^2}^{1/2}$, the amplitude $V_A(T)$ increases with temperature T, reaches a maximum $V_{A,max}$ at $T = T_{max}$, and then decreases [17–19]. The maximum voltage $V_{A,max}$ increases and is observed at a lower temperature T_{max} with the increase of the amplitude $\overline{2I_{noise}^2}^{1/2}$ because of the temperature dependence of both the critical current and the persistent current, see Fig.3.

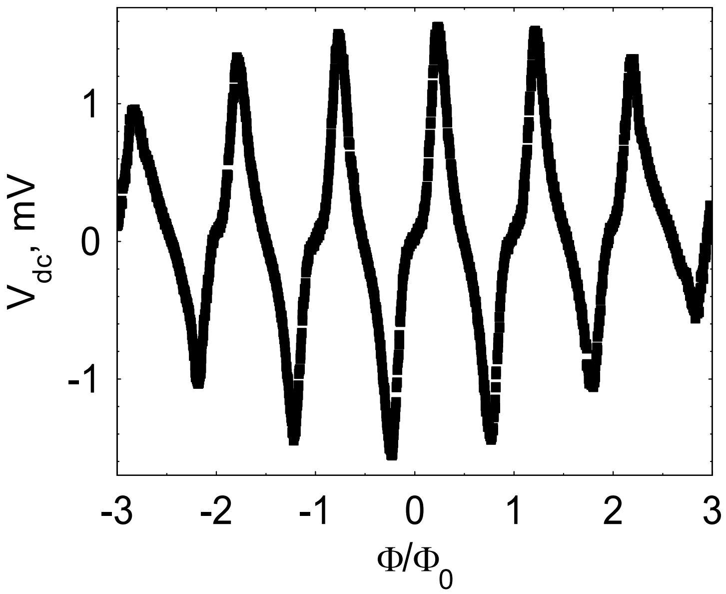

FIG. 4: Quantum oscillations of the dc voltage in magnetic field observed on the system of 1080 aluminium rings at the temperature $T \approx 1.330\ K \approx 0.978 T_c$ when the rings are switched between superconducting and normal states by the ac current with the amplitude $I_A \approx 3\ \mu A$. The period $B_0 = \Phi_0/S \approx 5.2\ Oe$ corresponds to the area $S = \pi r^2 \approx 20.7/5.2 \approx 4\ \mu m^2$ of the rings with $r \approx 1.1\ \mu m$ shown on Fig.2.

4.3. The amplitude of the dc voltage which was observed on the single asymmetric ring and on the system 1080 asymmetric rings connected in series.

According to the results presented in [9] the oscillations $V_{dc}(\Phi) \propto I_p(\Phi)$ with the amplitude $V_A \approx 1\ \mu V = 10^{-6}\ V$ are observed on a single asymmetric aluminum ring at the temperature $T \approx 0.98 T_c$ and non-equilibrium noises with the current amplitude $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$. In order to demonstrate that the amplitude of the oscillations increases with the number of the ring connected in series we have measured the oscillations $V_{dc}(\Phi) \propto I_p(\Phi)$ under similar conditions: $T \approx 0.98 T_c$ and $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$. The observation of the oscillations on the 1080 rings, Fig.4, with the amplitude $V_A \approx 1.6\ mV = 0.0016\ V$ about a thousand times more than on the single ring [9] leaves no doubt that the voltage induced by the persistent current increases with the number of rings connected in series, just as the voltage (1) induced in asymmetric rings by the conventional current.

5. THE PARADOX OF THE PHENOMENON.

The increase of the voltage $V_N = N V_1$ with the number N of the rings is a trivial effect when the voltage V_1 is induced by the conventional current I since the conventional current is induced by the Faraday electromotive force $-d\Phi/dt = -l dA/dt$. Our result is not trivial and is even paradoxical since we observed the dc voltage $V_{dc} \approx 1.6\ mV$ in magnetic field, for example $B \approx \Phi_0/4S$, constant in time $dB/dt = 0$, Fig.4.

5.1. The persistent current flows against the total electric field

The conventional electric current, induced by the Faraday electromotive force $-d\Phi/dt = -l dA/dt$, flows against the potential electric field $E = -\nabla V$ in the wide half (1), but its direction corresponds to the direction of the total electric field $E = -\nabla V - dA/dt$ in both halves, Fig.1 at the center. The Ohm law $j\rho = E = -\nabla V - dA/dt$ and the forces balance are valid in this case: $F_E + F_{dis} = 0$ at $dI/dt = 0$. The force $F_E = qE$ of the electric field $E = -\nabla V - dA/dt$ acting on the electrons is balanced the dissipation force F_{dis}. These laws are not valid for our observation, Fig.4. The observation of the dc voltage V_{dc} at $-d\Phi/dt = -l dA/dt = 0$ means that the persistent current flows against the total electric field $E = -\nabla V$ in the wide half, Fig.1 at the right.

5.2. The description of the paradox

The paradox can be described. The potential difference $V_A(t) = R_A I(t) = R_A I_p \exp -(t - t_i)/\tau_{RL}$ should appear on a segment l_A after each its transition at $t = t_{i,n}$ in the normal state with the resistance R_A. The sign of the voltage V_A will correspond to the direction of the persistent current I_p each time $t_{i,s}$. The voltage $V_A(t)$ and the current $I(t)$ will decay during a short relaxation time $\tau_{RL} = L/R_A$ after $t_{i,n}$. But the persistent current I_p (3) must appear again after the return of the segment in the superconducting state at $t_{i,s}$ due to the quantization (2). The direction of the persistent current (3) should be the same at each return in the superconducting state due to the predominate probability [15] of the permitted state n with the minimal kinetic energy (4). Therefore the voltage average in time

$$V_{dc} = \overline{V_A} = \Theta^{-1} \int_{\Theta} dt V_A(t) \approx R_A \Theta^{-1} \Sigma_{i=1}^{i=N_{sw}} I_{p,i-1} \int_{t_{i,n}}^{t_{i,s}} dt \exp - \frac{t}{\tau_{RL}} \quad (5)$$

should not equal zero when the same segment l_A of the ring is switched between superconducting and normal states [10]. Here N_{sw} is the number of the transitions between superconducting and normal states during the time Θ; $t_{i,n}$ and $t_{i,s}$ are the time of i - transition of the segment l_A in the normal state and in the superconducting state; $I_{p,i}$ is the persistent current after i - transition in the superconducting state. The transition from the continuous spectrum to the discrete spectrum of the permitted states (4) occurs when the segment $l_A$returns in the superconducting state [14]. The dc voltage $V_{dc} = \overline{V}$ can be observed in a magnetic field $B \neq n'\Phi_0/S$ and $B \neq (n' + 0.5)\Phi_0/S$ constant in time, Fig.4, due to the appearance of the velocity (2) of Cooper pairs during this transition. The mobile charge carriers accelerate at this transition in direction opposite to the electromagnetic force, similar the case of the Meissner effect [4]. We cannot answer on the Hirsch question [4] what is the force propelling the mobile charge carriers to move in direction opposite to the electromagnetic force. But we cannot doubt that Cooper pairs must accelerate since the state with zero velocity is forbidden at $B \neq n'\Phi_0/S$ according to the quantization (2) which is valid when the whole ring is in the superconducting state.

5.3. Quantum force

The paradoxical acceleration without a known force allows to explain also why the persistent current $\overline{I_p} \neq 0$ does not decay at non-zero resistance $\overline{R_l} > 0$ [5, 7] in spite of a non-zero dissipation $\overline{R_l I_p^2} > 0$. The angular momentum of each from N_s pairs changes from the value $m_p = \hbar n$ corresponding to the quantization to the value $m_p = q\Phi/2\pi = \hbar\Phi/\Phi_0$ corresponding to the zero velocity $v = 0$ when the circular electric current $I(t)$ changes from $I(t) = I_p$ to $I(t) = 0$. This change occurs under the influence of the dissipation force F_{dis}. The opposite change from the $m_p = \hbar\Phi/\Phi_0$ to $m_p = \hbar n$ should occur due to the quantization (2) when the entire ring returns in the superconducting state. The change

$$F_q = \hbar(\overline{n} - \frac{\Phi}{\Phi_0})\frac{f_{sw}}{r} \quad (6)$$

of the momentum p per an unit time due to the quantization when the ring is switched with a frequency $f_{sw} = N_{sw}/\Theta$ was called "quantum force" in [3]. The quantum force replaces the Faraday electromotive force in the balance of forces $-qd\Phi/dt + \oint_l dl F_{dis} = 0$ and compensates for the dissipation force

$$2\pi r F_q + \oint_l dl F_{dis} = 0 \quad (7)$$

5.4. The relation between values of the dc voltage and the persistent current

According to the relation (1) the voltage can be observed only at the measurement of asymmetric ring the halves of which have different resistance $R_{gr} > R_{sm}$. The dc voltage $V_{dc} = \overline{V} = \Theta^{-1} \int_{\Theta} dt V(t) \neq 0$ can be observed also only in an asymmetric ring with dissimilar segments. The dc voltage will not be observed if all ring's segments are switched between the superconducting and normal state identically. The persistent current $\overline{I_p} \neq 0$ will be observed in this case at non-zero resistance $\overline{R_l} > 0$, but the dc voltage $V_{dc} = \overline{V}$ will not be observed. The dc voltage will be observed only if the segments of the ring is switched in the normal state with different probability. For example, the dc voltage should be observed when only one segment l_A is switched between superconducting and normal states. According to (5) the relation between the dc voltage and the persistent current average in time should be described in this case by the relation $V_{dc} \approx f_{sw} L \overline{I_p}$ at the low frequency of the switching $f_{sw} \ll 1/\tau_{RL}$ and by the relation $V_{dc} \approx R_A \overline{I_p}$ at the high frequency $f_{sw} \gg 1/\tau_{RL}$ [10]. This relation cannot be deduced in the general case. Nevertheless we may be sure that the maximum value of the dc voltage V_{dc} should increase with the value of the persistent current $\overline{I_p}$.

5.5. Weak noise can induce high dc voltage

The dependence of the maximum dc voltage V_{dc} rather on the persistent current $\overline{I_p}$ than on the noise amplitude $\overline{2I_{noise}^2}^{1/2}$ testifies to the analogy with the secondary winding of the electric transformer and against the rectification effect. The measurements [11, 16] testify that the voltage V_{dc} is observed when the amplitude $\overline{2I_{noise}^2}^{1/2}$ exceeds the critical current. The persistent current reduces the critical current of both symmetric and asymmetric rings [21]. According to the predictions of the theory, confirmed experimentally [21], the critical current of the symmetric ring is described by the formula

$$I_c(\Phi) = I_{c0} - 2|I_p| = I_{c0} - 2I_{p,A}2|n - \frac{\Phi}{\Phi_0}| \qquad (8)$$

The ratio $I_{p,A}/I_{c0} = \sqrt{3}\xi(T)/4r$ of the critical currents $I_{c0} = I_{c0}(T = 0)(1 - T/T_c)^{3/2}$ at $I_p = 0$ to the amplitude $I_{p,A} = I_{p,A}(T = 0)(1 - T/T_c)$ of the persistent current is determined by the ratio of the correlation length of the superconductor $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$ to the radius of the ring r [6]. Therefore the critical current at $\Phi \approx (n + 0.5)\Phi_0$ of the ring with a small radius $r \approx \sqrt{3}\xi(T)/2$ may be equal zero or be much smaller than the persistent current $I_c \approx I_{c0} - 2I_{p,A} \ll I_{p,A}$. Measurements of the system of 667 of aluminum rings with the radius $r \approx 500$ nm, Fig.5, corroborate this possibility. The magnetic dependence of the critical current measured in the opposite directions are almost identical, Fig.5, because the rings are almost symmetric $w_w \approx w_n$. The dependence are described by the relation (8) at $I_{c0} \approx 2.9$ μA and $I_{p,A} \approx 1.25$ μA. The relation $I_{p,A}/I_{c0} = \sqrt{3}\xi(T)/4r \approx 0.43$

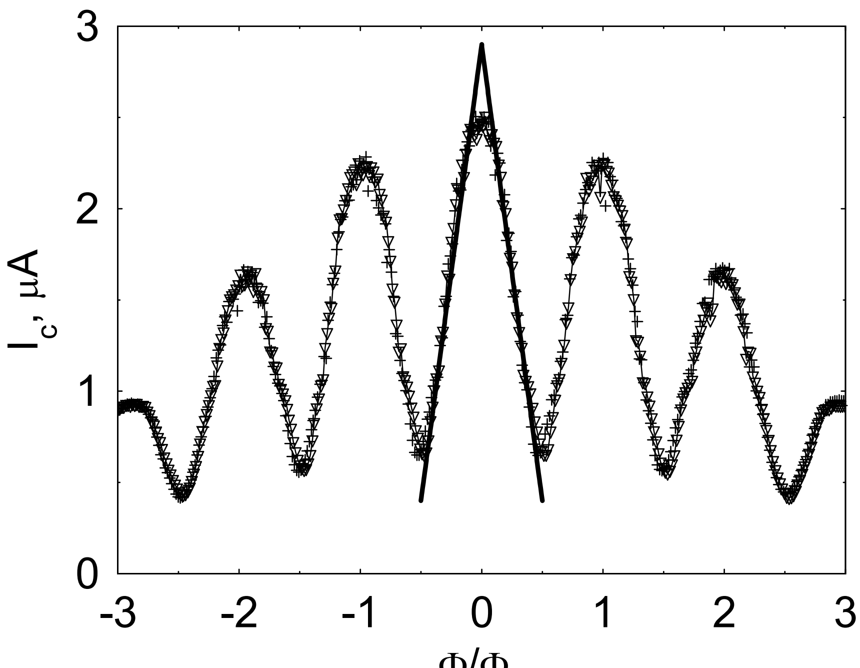

FIG. 5: Quantum oscillations in magnetic field of the critical current of 667 rings measured at the temperature $T \approx 1.283$ $K \approx 0.972T_c$ in the opposite directions $I_{c+}(\Phi/\Phi_0)$ (triangles) and $I_{c-}(\Phi/\Phi_0)$ (crosses). The experimental dependence correspond to the theoretical one for the symmetric rings (5) with $I_{c0} \approx 2.9$ μA and $I_{p,A} \approx 1.25$ μA (lines). The discrepancy between theoretical and experimental values near $\Phi = 0$ may be explained the influence of the contacts between rings the width of which is smaller than the total width of two ring-halves. The period $B_0 = \Phi_0/S \approx 22.6$ Oe corresponds to the area $S = \pi r^2 \approx 20.7/22.6 \approx 0.9$ μm^2 of the rings with $r \approx 0.54$ μm used for the measurements.

corresponds to $\xi(T = 0.97T_c) \approx 500\ nm$ and the value $\xi(0) \approx 100\ nm$ typical for aluminium film with small free path of electrons. The theoretical $I_c \approx I_{c0} - 2I_{p,A} \approx 0.4\ \mu A$ and measured $I_c \approx 0.6\ \mu A$ at $\Phi = \pm 0.5\Phi_0$ values of the critical current is smaller than the persistent current $|I_p| = I_{p,A} \approx 1.25\ \mu A$.

The dc voltage cannot be observed when the persistent current reduces the critical current down to zero $I_c = I_{c0} - 2|I_p| \leq 0$. The ring should be in the normal state all time in this case. A weak noise $\overline{2I_{noise}^2}^{1/2} > I_c$ can induce a high voltage V_{dc} and power $V_{dc}I_p \propto I_p^2$ when the persistent current reduces the critical current almost to zero. One can get more power $V_{dc}I_p$ at low noise when the critical current is much less than the persistent current $I_c(T,B) \ll I_p$.

6. PRACTICAL AND FUNDAMENTAL IMPORTANCE

The paradoxical phenomenon discovered by the authors [8, 9] has both practical and fundamental importance due to the increase of the dc voltage with the number of the identical asymmetric rings connected in series and due to the possibility to obtain a high dc voltage at weak noise considered above.

6.1. Detector of weak noise

The authors [8] interpreted the observation of the dc voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ as a consequence of a rectification process because of their false conclusion that this voltage disappears when all parts of the equipment are shielded carefully. The measurements [9, 17, 18] testify that the dc voltage decreases rather than disappears when the amplitude $\overline{2I_{noise}^2}^{1/2}$ of a non-equilibrium noise decreases. According to the results of measurements the relation $Eff_{Re} = V_{A,max}/R_n\overline{2I_{noise}^2}^{1/2}$ of the maximum amplitude $V_{A,max}$ of the dc voltage oscillations to the ring resistance in the normal state R_n and to the amplitude $\overline{2I_{noise}^2}^{1/2}$ of the noise current or the sine current $I_{sine} = I_A \sin(2\pi ft)$ does not exceed 0.25 [11]. The authors [8] observed the oscillation $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ with the amplitude $V_{A,max} \approx 15\ \mu V$ in the asymmetric superconducting quantum interference device with $R_n \approx 2\ \Omega$. Therefore we may conclude that the amplitude of the noise current created by the equipment used by the authors [8] should exceed $\overline{2I_{noise}^2}^{1/2} \approx 30\mu A$. The dc voltage $V_A \approx 1.2\ \mu V$ could have been induced on the single aluminum ring with $R_n \approx 6\ \Omega$ by a noise on order of magnitude less $\overline{2I_{noise}^2}^{1/2} \geq 1\ \mu A$. The oscillation $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ ceased to be visible on one ring when the noise amplitude was reduced with the help of screening and filtration. But the oscillations with the amplitude $V_{A,max} \approx 0.6\ \mu V$ were observed on the system of 110 asymmetric aluminum rings [17]. Additional filtration with low-temperature π-filters and coaxial resistive twisted pairs has allowed to reduce the amplitude of the uncontrollable noise down to $\overline{2I_{noise}^2}^{1/2} \approx 20\ nA$. The visible oscillation $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ were not observed on the system of 110 rings at this very low level of the noise. But they were observed on the system of 1080 rings [18]. Each time, the system with a greater number of asymmetric rings allowed detection of the reduced non-equilibrium noise.

The detection of very weak noise is possible due to the temperature dependence of the critical current and the persistent current, Fig.3, and the proportionality of the dc voltage $V_{dc}(\Phi)$ to the number of identical asymmetric rings connected in series. An arbitrarily weak noise can switch the rings to the normal state (inducing a non-zero resistance $\overline{R_l} > 0$) near the superconducting transition $T \approx T_c$ as the critical current is reduced to zero $I_c \to 0$ at $T \to T_c$, Fig.3. The dc voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ decreases at $T \to T_c$ because of the decrease of the persistent current. Therefore it is important that the amplitude of the oscillations $V_{dc}(\Phi)$ increases with the number N of the asymmetric rings. The proportionality $V_{A,N} \approx NV_{A,1}$ ensures detection of any weak noise down to equilibrium one with the help of the system of a sufficiently large number of identical asymmetric rings. The one asymmetric ring is enough in order to detect the non-equilibrium noise with the amplitude $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$ typical for the low-temperature part of a conventional measuring system used in [9]. But 110 rings [17] and even 1080 rings [18] are needed in order to detect the noise reduced with the help of the additional filtration. The reduction of the noise down to $\overline{2I_{noise}^2}^{1/2} \approx 20\ nA$ allowed to demonstrate the possibility of using asymmetric superconducting rings connected in series as a detector of very weak noise [19]. The amplitude of this noise $\overline{2I_{noise}^2}^{1/2} \approx 50\ nA$ [19] is about three orders of magnitude less than the amplitude $\overline{2I_{noise}^2}^{1/2} > 30\mu A$ of electromagnetic radiation responsible for the dc voltage oscillations $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ observed in [8].

6.2. The dc power source

It is enough easy to reduce the influence of the electromagnetic radiation emitted of broadcasting stations. It is more difficult to reduce the flow of thermal noise through the wires from room temperature $T \approx 300\ K$ to the low-temperature part $T \approx 1\ K$ of the measuring system. The thermal electric noise observed under equilibrium condition has been described first time theoretically by Nyquist [22] and observed by Johnson [23] as long ago as 1928. Therefore this phenomenon is called as the Nyquist noise or the Johnson noise. It is known that the power of the Nyquist noise $W_{Nyk,f} = 4k_BTdf$ is distributed evenly across all frequencies from zero $f_{min} = 0$ to the quantum limit $f_{max} \approx k_BT/h$ [20]. The total power of the Nyquist noise is equal $W_{Nyk,t} = 4(k_BT)^2/h$. Here $k_B \approx 1.4\ 10^{-23}\ J/K$ is the Boltzmann constant, $h \approx 6.6\ 10^{-34}\ Js$ is the Planck constant. The amplitude of the total Nyquist current in a closed circuit $\overline{2I_{noise}^2}^{1/2} \approx (W_{Nyk,t}/R_{hf})^{1/2} \approx (4(k_BT)^2/hR_{hf})^{1/2}$ depends on the effective resistance R_{hf} (first of all at high frequencies) and highest temperature of this circuit. The different parts of the closed circuit used for the observation of the dc voltage in [9, 17, 18] have different temperatures from $T \approx 1\ K$ (asymmetric rings) to $T \approx 300\ K$ (equipments at the room temperature). The total power of the Nyquist noise reaches $W_{Nyk,t} \approx 10^{-7}\ W$ at the room temperature $T \approx 300\ K$. This power equilibrium at $T \approx 300\ K$ can induce the noise non-equilibrium at $T \approx 1\ K$ with the amplitude $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$ when the effective resistance $R_{hf} \approx W_{Nyk,t}/\overline{2I_{noise}^2}$ between the room-temperature part and the low-temperature part of the circuit equals approximately $R_{hf} \approx 10\ k\Omega$. Therefore we may assume that the effective resistance of the measuring circuit used in [9] was equal $\approx 10\ k\Omega$. In order to decrease the noise non-equilibrium at $T \approx 1\ K$ down to $\overline{2I_{noise}^2}^{1/2} \approx 20\ nA$ the effective resistance should be increase in [18] up to $R_{hf} \approx 100\ M\Omega$.

The effective resistance was increased in [18] in order to use the system of asymmetric superconducting rings as the detector of a weak noise [19]. The resistance R_{hf} should be decrease rather than increase in order to use this system as the dc power source since the power $V_{dc}I_p$ increases with the increase of the amplitude $\overline{2I_{noise}^2}^{1/2}$. We observe the dc voltage $V_{dc} \approx 1.5\ \mu V$ on each ring and $V_{dc} \approx 1.6\ mV$ on 1080 rings at $B \approx \Phi_0/4S$ and $T \approx 0.98T_c$, Fig.4, when the persistent current equals $I_p \approx I_{p,A}/2 \approx 0.6\ \mu A$ (since $I_{p,A} \approx 1.2\ \mu A$ at $T \approx 0.98T_c$, Fig.3). Consequently the thermal noise $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$ induces the dc power $V_{dc}I_p \approx 2\ 10^{-12}\ W$ on each ring and $V_{dc}I_p \approx 2\ 10^{-9}\ W = 2\ nW$ on 1080 rings. A greater noise will induce greater power $V_{dc}I_p$ inducing the voltage V_{dc} at a lower temperature. The power $V_{dc}I_p$ will increase with the increase of the number of the rings, the amplitude of the thermal noise $\overline{2I_{noise}^2}^{1/2}$, and the ratio $I_{p,A}/I_c(\Phi)$.

6.3. Could the persistent current induce the voltage (1) above superconducting transition and in normal metal asymmetric rings?

The authors [8] claimed that they observe the dc voltage V because of the rectification process in order to explain the energy dissipation V_{dc}^2/R which they observed. But the rectification process cannot explain the energy dissipation $\overline{RI_p^2}$ observed in the phenomenon of the persistent current $\overline{I_p} \neq 0$ which does not decay in the rings with a non-zero resistance $\overline{R} > 0$. This paradoxical phenomenon is observed at thermodynamic equilibrium not only in the fluctuation region of the normal state $T \geq T_c$ [5, 7, 17, 24] but also in normal metal rings [25, 26]. The question "Could the persistent current induce the voltage (1)?" is provoked by this paradoxical phenomenon. The 1080 rings were needed in order to observe the voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ in the temperature region $T \geq T_c$ where the resistance is not zero $\overline{R_l} > 0$ at thermodynamic equilibrium [18]. The oscillations $V_{dc}(\Phi)$ with the visible amplitude $V_A > 0.02\ \mu V$ could be observed only in the lower part of the resistive transition $\overline{R_l} < 0.3R_n$ [18]. The persistent current $\overline{I_p}(\Phi)$ has been predicted predicted [27] and observed [28] in normal state of superconductor where $R_l \approx R_n$. But the amplitude of $\overline{I_p}(\Phi)$ is very small in this temperature region and therefore the 1080 rings are not enough in order to observe the visible voltage with the amplitude $V_A > 0.02\ \mu V$.

According to the experimental result [18], a system of more than tens of thousands of rings should be used to observe the visible dc voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ at the temperature corresponding to the upper part of the resistive transition where $R_l \approx R_n$. Even more rings will be needed to observe the visible oscillations $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ in a normal metal, since the persistent current of the electrons [25, 26] is much less than the persistent current of Cooper pairs [5, 7, 17, 24, 28]. The $\overline{I_p}(\Phi)$ amplitude of of Cooper pairs is $I_{p,A} \approx 100\ nA$ even in the fluctuation region [7, 17] where $\overline{R} > 0$ whereas the amplitude of the persistent current of electrons observed in [25] does not exceed $I_{p,A} \approx 1\ nA$. In addition, the amplitude of the oscillations $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ should increase proportionally to $\sqrt{N}$ rather than the number N of rings of normal metal, since the persistent current of electrons has random direction from ring to ring [25, 29] in contrast to the current of Cooper pairs (3).

Nevertheless modern experimental techniques can allow the observation of the visible dc voltage $V_{dc}(\Phi) \propto \overline{I_p}(\Phi)$ not only in normal state of superconductor rings but even of normal metal. The authors [30] have fabricated a system of approximately ten million ($N \approx 10000000!$) of rings with the radius $r \approx 300\ nm$ in order to observe the persistent current of electrons as far back as 1990. The 10^7 rings occupy a $7\ mm^2$ area on the substrate [30]. The experimental investigations of the system with big number of asymmetric rings connected in series may have fundamental importance. The authors [25] note fairly: "*An electrical current induced in a resistive circuit will rapidly decay in the absence of an applied voltage. This decay reflects the tendency of the circuits electrons to dissipate energy and relax to their ground state*" and claim that the persistent current is dissipationless in spite of the non-zero resistance of the rings. The author [31] agrees with the authors [25] although he recognizes: "*The idea that a normal, nonsuperconducting metal ring can sustain a persistent current - one that flows forever without dissipating energy - seems preposterous. Metal wires have an electrical resistance, and currents passing through resistors dissipate energy*".

The authors [25] don't even try to explain the contradiction with mathematics of their claim that the persistent current is dissipationless: they measure a non-zero resistance $R_l > 0$ and the persistent current $I_p \neq 0$ and claim that the product of the resistance R_l to the current squared I_p^2, i.e. the dissipation power $R_l I_p^2$, equals zero $R_l I_p^2 = 0$. The opinion of the author [32] (who has predicted in the first time the persistent current in normal metal) about the paradoxical possibility $I_p \neq 0$ at $R_l > 0$ does not contradict mathematics: "*The current state corresponds in this case to the minimum of free energy, so the account of dissipation does not lead to its disintegration*". It is argued in [24] that the author [32] rather than the authors [25, 31] is right. According to his opinion the $I_p \neq 0$ observed at $R_l > 0$ is a type of the Brownian motion [20] likewise the Nyquist noise. Nobody claims that the Brownian motion and its type - the Nyquist current are dissipationless. The kinetic energy of Brownians particles dissipates into the thermal energy $k_B T$ and is taken from the thermal energy [20]. The authors [25, 31] claim that the power of the persistent current equals zero $W_p = R_l I_p^2 = 0$ since it is the power of the direct current in contrast to the power of the Nyquist noise which equals zero at the zero frequency $f = 0$. This claim is not only preposterous but also useless: it cannot explain how the persistent current can flow against the electric field $E = -\nabla V$, see Fig.1 at the right. The description of this paradoxical phenomenon together with the phenomenon of the persistent current observed at non-zero resistance, given above, means that the observations $I_p \neq 0$ at $R_l > 0$ in normal state [5, 7, 17, 24] and normal metal [25, 26] testify to a dc power source $W_p = R_l I_p^2 \neq 0$ at thermodynamic equilibrium. These observations testify also to the possibility to observe $V_p(\Phi) \propto \overline{I_p}(\Phi)$ on asymmetric rings at the same condition in accordance with the relation (1).

7. CONCLUSION

We draw reader's attention (first of all experimentalists) to the fundamental and practical importance of experimental investigations of systems with big number of identical asymmetric superconducting rings. The proportionality $V_{A,N} \approx N V_{A,1}$ of the voltage $V_{A,N}$ with the number N of asymmetric rings connected in series means that the oscillation $V_{dc}(\Phi)$ with the amplitude up to $3\ V$ may be observed on the system of million rings when the rings are switched between superconducting and normal states by the noises with the amplitude $\overline{2I_{noise}^2}^{1/2} \approx 3\ \mu A$. Such system may allow also to observe the persistent voltage $V_p(\Phi) \propto \overline{I_p}(\Phi)$ in the temperature region above the superconducting transition in accordance with the relation (1). The observation of the persistent power $W_p = V_p^2/R \neq 0$ in the normal state of superconductor rings will have fundamental importance.

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