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Neutrino physics theory

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Summary. — Neutrino physics covers a wide range in theoretical physics. I briefly review the state of the art of neutrino theory, with particular regard to the measure of masses and mixings. Some issues for the future are outlined.

PACS `14.60.Lm` – Ordinary neutrinos.
PACS `14.60.Pq` – Neutrino mass and mixing.

1. – Introduction

It is of course impossible to summarize in few pages the role of the neutrino in theoretical physics. Neutrinos play a role in almost all the corners of fundamental physics. Let us remind for example that the discovery of the neutrino mass was probably the first convincing evidence of physics beyond the Standard Model. The (Dirac or Majorana) nature of the neutrino mass, the precise measure of the masses (and mass hierarchy) and the mixings at the future long baseline experiments and neutrino factories could give us valuable pieces of information on the mass generation mechanism. Also studying non-standard neutrino interactions can help us to understand the underlying grand unified theory beyond the Standard Model. The phenomenon of neutrino oscillations could also help us to investigate on the fundamental aspects of space-time and quantum mechanics.

Moreover, there is a strong interplay among neutrino physics, astrophysics and cosmology. About the 99% of the energy of a core-collapse supernova is radiated into neutrinos of all flavors. Therefore the observation of neutrinos from the next galactic supernova(e) will be an exceptional laboratory not only for astrophysics (*e.g.*, the study of the explosion mechanism, the subsequent formation of a neutron star/black hole) but also for a plethora of neutrino properties. In this regard stochastic turbulence and non-linear effects such as neutrino self-interactions in the stellar core are new and interesting features that are under study.

Historically, our Sun was the first astrophysical object observed in the light of neutrinos. Solar neutrinos gave the first indication for neutrino oscillations. Today, the

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(*) E-mail: `daniele.montanino@le.infn.it` observation of solar neutrinos yield valuable information on our star and a new discrepancy between solar neutrino fluxes and helioseismology open new interesting questions on how our star works.

Neutrinos have a role in the evolution of the Universe, in particular in stellar and primordial nucleosynthesis and in the cosmic structure formation. Furthermore, neutrinos could play an important role in the matter-antimatter asymmetry (the "leptogenesys"). Neutrinos could also be in interaction with the dark-energy field (Mass Varying neutrinos "MaVaNs") or massive sterile neutrinos could be an important (or dominant) dark matter component. Conversely, dark matter particles could annihilate (or decay) into neutrinos that can be detected in current or future experiments. Finally, the future observation of very high energy neutrinos will shed light on the most violent phenomena in our universe [1].

Last but not least, very recently neutrinos from the decay of the U-Th in the crust and in the mantel (the so-called "Geo-neutrinos") were observed, opening a new window for the study the of energetic of the Earth interior.

For this reason I will not attempt here to give a complete overview of the state-of-the-art of the neutrino theory. I just focus on some recent issues in neutrino physics. For recent reviews I address to [2-4].

2. – Neutrino mixing and oscillations

As well known, neutrino flavor eigenstates ν_α with $\alpha = e, \mu, \tau$ are related to the mass eigenstates $\tilde{\nu}_i$ through a unitary mixing matrix $\nu_\alpha = \sum_i U_{\alpha i} \tilde{\nu}_i$, where the matrix $\mathbf{U}$ can be written in the MNS parameterization [5]:

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\delta} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\delta} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\delta} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\delta} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix} \cdot \mathbf{R}, \tag{1}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $\mathbf{R} = \mathrm{diag}\{1, e^{i\phi_2}, e^{i\phi_3}\}$ if neutrinos are Majorana particles ($\mathbf{R} = \mathbf{1}$ for Dirac neutrinos). Conventionally, the masses of the neutrinos are labeled in such a way that $m_1 < m_2 \ll m_3$ for "normal" hierarchy (NH) and $m_3 \ll m_1 < m_2$ for "inverted" hierarchy (IH).

When a neutrino propagates in space undergoes to a phenomenon of oscillatory flavor conversion caused by the interference between the different de Broglie waves of the mass eigenstates. In neutrino oscillations the Majorana phases $\phi_{1,2}$ are unobservable. Essentially, neutrino oscillations are a combination of a "short" wave with wave number $k_H = \Delta m^2_{13}/2E_\nu$ and a "long" wave with wave number $k_L = \Delta m^2_{12}/2E_\nu$, where $\Delta m^2_{ij} = m^2_j - m^2_i$ and E_ν is the neutrino energy. On a fixed pathlength, probing one or the other wave means tune on different neutrino energies. To this regard, on a typical baseline of $\sim O(100\,\mathrm{km})$ experiments sensitive to $\sim$ GeV neutrinos (*e.g.*, to neutrinos produced in atmosphere by cosmic rays and by accelerators) are typically sensitive to Δm^2_{13} and experiments sensitive to $\sim$ MeV neutrinos (*e.g.*, from neutrinos produced in far nuclear reactors as for the KamLand detector) are typically sensitive to Δm^2_{12}.

A different argument applies for solar ($\sim$ MeV) neutrinos: ν_e's experience a different refraction index in matter due to charge current interactions and this affects neutrino propagation. This effect is known as Mikheyev-Smirnov-Wolfenstein (MSW) effect [6]. For solar neutrinos, the dominant effect is the MSW effect in solar matter while the vacuum oscillations are averaged.

TABLE I. – *Current best-fit values with* 1σ *errors, best-fit values in degrees (angles only), and* 2σ *and* 3σ *intervals (1 d.o.f.) for the three-flavor neutrino oscillation parameters from global data.*

Parameter	Best-fit	Degrees	2σ	3σ
Δm^2_{12} ($10^{-5}\,\mathrm{eV}^2$)	$7.65^{+0.23}_{-0.20}$		7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{13}\rvert$ ($10^{-3}\,\mathrm{eV}^2$)	$2.40^{+0.12}_{-0.11}$		2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	33°	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	45°	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	6°	≤ 0.040	≤ 0.056
δ, sign(Δm^2_{13}) (+1: NH, −1: IH)	Currently no information			

To make short the long story, a plethora of experiment was performed to detect flavor oscillations in order to reconstruct the mixing matrix and measure the Δm^2's. Combining all experimental data we obtain the limits in table I [7].

At the moment there is only an upper limit on the mixing angle θ_{13} at more than 1σ, although data push toward a non-zero value. However this hint has been recently weakened by the non-observation of $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions in the long baseline ($\sim 700\,\mathrm{km}$) MINOS experiment [8]. The measure of θ_{13} as well the determination of the CP violations in the leptonic sector and the determination of the mass hierarchy will be the goal of the next reactor and long-baseline (LBL) experiments [9].

Other questions have to be solved. One of these is if there are new (light) sterile states mixed with active neutrinos. These sterile neutrinos were invoked in the past in order to reconciling the LSND evidence of $\nu_\mu \to \nu_e$ on a short baseline with the other evidence of oscillations. However, the experiment MiniBOONE (conceived to test directly the LSND evidence) has not confirmed the LSND anomaly [10]. Moreover, no deficit of events has been observed in the neutral current sample of MINOS [11]. Therefore, at the moment there is no convincing indication for further sterile neutrinos. There are also several limits on possible non-standard features in neutrino oscillations. Just to mention one of them, the possibility of a non-Hamiltonian dynamics for the neutrino propagation that would lead to the decoherence of oscillations has been proposed in literature. However, no evidence for decoherence has been found yet and only limits on the decoherence parameter can be set [12, 13].

3. – Absolute masses and the nature of the neutrino

Neutrino oscillations cannot give information on absolute neutrino masses. In particular, we do not know whether the lightest neutrino state is (almost) massless or the three neutrinos are almost degenerate. There are in practice three possibilities to measure absolute neutrino masses: 1) the measure of the distortion of the electron spectrum in tritium beta decay, 2) the influence of massive neutrinos in the evolution of some cosmological observables (in particular, the Cosmic Microvave Radiation (CMB) anisotropy and the matter power spectrum), 3) the neutrinoless double beta decay ($0\nu2\beta$). The last possibility is effective only if neutrinos are Majorana particles (*i.e.*, coincide with their own antiparticles): in this case processes violating the leptonic number by two unity such as the double beta decay $(Z, A) \to (Z \pm 2, A) + 2e^{\mp}$ are possible. The first observable is

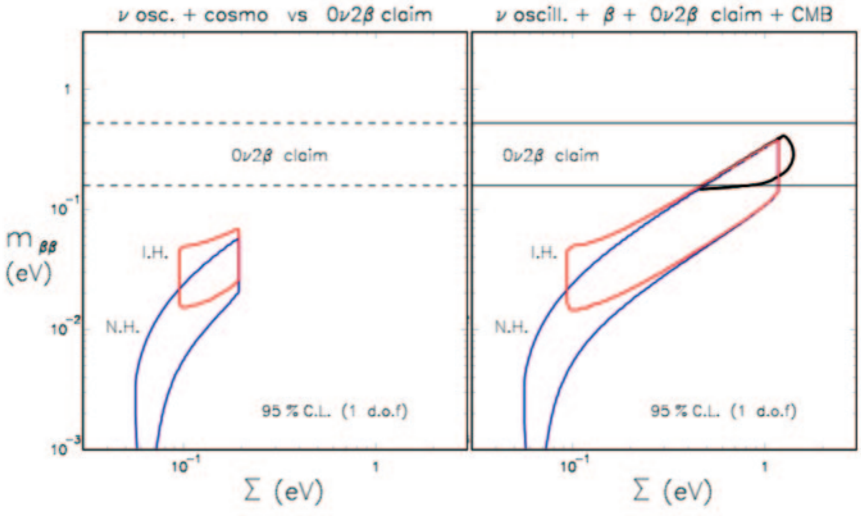

Fig. 1. – Left: bounds from oscillation plus all cosmological data, contrasted with the $0\nu 2\beta$ decay claim. Right: same as left but with relaxed cosmological data [15].

sensitive to the quantity

$$m_\beta = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}, \tag{2}$$

while the second is sensitive to the sum of the three masses $\Sigma = m_1 + m_2 + m_3$. The third quantity is instead sensitive to

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right| . \tag{3}$$

At the present none of the above-mentioned observables have a positive evidence for a non-zero neutrino mass, except for the Heidelberg-Moscow (HM) experiment [14] which reports a $0\nu 2\beta$ signal in ^{76}Ge with half-life $T_{1/2}^{0\nu} = 2.23^{+0.44}_{-0.31} \times 10^{25}\,$y ($1\sigma$ errors) at $> 6\sigma$. The exact determination of $m_{\beta\beta}$ is difficult since the decay rate is affected by the uncertainties on the nuclear matrix element. From reasonable nuclear models we obtain a range for $m_{\beta\beta}$ at 2σ : $0.16 < m_{\beta\beta}/\text{eV} < 0.52$ [15]. This range is still compatible with other direct searches of mass, but not with an "aggressive" cosmology. In fact, combining all the cosmological data, the limit on Σ is very stringent: $\Sigma < 0.19\,$eV at 2σ [15]. The situation is illustrated in the left panel of fig. 1 were the allowed zones for all the cosmological data in the plane $(m_{\beta\beta}, \Sigma)$ is shown in contrast with the strip allowed by the Heidelberg-Moscow experiment for both the mass hierarchies. As is clear, no compatibility is possible and something should be wrong. However, if more relaxed cosmological data are used (*i.e.*, only the CMB data) a compatibility appears in the zone $\Sigma \sim 1\,$eV ($m_i \sim 0.3\,$eV). This mass is in the reach of the future beta decay KATRIN experiment. In any case future double-beta decay experiments can reach a sensitivity up to $m_{\beta\beta} \sim 0.01\,$eV [16], and together with more precise cosmological measurements will be able to (dis)prove the HM experiment and solve the puzzle.

Beyond mass determination, other valuable pieces of information can be extracted by cosmological data. One of these is the mass hierarchy [17]. In fact, non-degenerate massive neutrinos become non-relativistic at different times. This leaves an imprint on the CMB anisotropy spectrum if Σ is at least $\sim 0.1\,\mathrm{eV}$. Another important issue is the determination of the (Dirac or Majorana) nature of neutrinos [18]. In fact, if Σ is less than $0.07\,\mathrm{eV}$, from fig. 1 we argue that the mass spectrum must be normal but the nature the of the neutrinos remains undetermined also with the next generation of $0\nu 2\beta$ decay experiments ($m_{\beta\beta}$ can vanish for an unlike combination of parameters). In the range $0.07 < \Sigma/\mathrm{eV} < 0.1$ we have instead two possibilities: if the mass hierarchy (as measured by LBL experiments) is normal the neutrinos nature must be determined by $0\nu 2\beta$ experiments; conversely, if the mass hierarchy is inverted, neutrinos are surely Dirac particles. For $\Sigma > 0.1\,\mathrm{eV}$ the mass hierarchy can be determined by cosmology, and the nature of neutrinos will be in the reach of next generation $0\nu 2\beta$ experiments.

4. – Final remarks

Neutrino physics has experienced its golden age in the last years. The fact that neutrinos are massive has been established beyond any doubt. The square mass differences and two of the three mixing angles have been measured with great accuracy, opening a new era of precision measurements in neutrino physics. But still many questions wait for an answer: 1) if there are CP violations in the leptonic sector, 2) the absolute neutrino masses and the mass hierarchy, 3) whether neutrinos are Dirac or Majorana particles, 4) the mechanism of the generation of mass in the neutrino sector (and, more in general, in the fermionic sector), 5) the discovery of possible non-standard neutrino interactions, 6) the role of neutrinos in the creation of matter-antimatter asymmetry. The answer to these questions will be the challenges for the future of the theoretical and experimental neutrino physics.

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