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Magnetic interactions: A multimedia interactive tutorial

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Summary. — Magnetic interactions, besides their intrinsic interest, constitute a didactical topic suitable to contextualize the fundamental physical concept of field. Moreover, the magnetic properties of matter, in particular ferromagnetism, are usually poorly dealt in italian high schools. By employing the Java programming language, we have developed an interactive tutorial allowing learners to explore the ways in which two Geomag$^{\mathrm{TM}}$ magnets interact with a ferromagnetic sphere. The simulation (whose quantitative aspects are based on a magnetostatics transposition of the electrostatic's method of images) interactively shows the sphere's magnetization pattern as a magnetic dipole approaches to it. The interaction force between magnet and ferromagnetic object is also shown. The proposed tutorial is usefully employed to explain the so-called "Geomag paradoxes" (Defrancesco *et al.* 2007).

PACS `01.50.-i` – Educational aids.
PACS `01.50.F-` – Audio and visual aids.

1. – Introduction

The use of toys in physics teaching is a common practice (Güémenez *et al.* 2009, Aref *et al.* 2007, Featonby 2005), since a lot of physics may come out of them if properly employed. In particular, there is a well known toy suitable to get many insights on magnetic field properties: the GEOMAG$^{\mathrm{TM}}$ magnetic building kit. Such a kit consists of a number of strong cylindrical bar magnets (6 mm diameter and length 25 mm, completely covered in hard plastic except at their two ends) and of a number of ferromagnetic steel balls (diameter 12.7 mm). Geomags have been used in the past for educational purposes either to qualitatively elucidate some issues about magnetic force/torque (Allasia *et al.* 2006) and field (Defrancesco *et al.* 2007), or as a tool to practically illustrate the well known "method of images" in electrostatic by establishing analogies between electric and magnetic phenomena (Poon 2003). In a recent paper (Defrancesco *et al.* 2007) an apparent paradox involving Geomags interactions has been proposed: two like-poles may attract if properly faced through a ferromagnetic sphere. This phenomenon has been quantitatively investigated in a more recent paper (Bonanno *et al.* 2009a).

In the present work we propose a multimedia tool allowing to interactively visualize the behavior of the surface magnetization of a ferromagnetic sphere in presence of one or more little magnets, and the interaction force among them. The computational part of the simulation is based on a magnetostatics transposition of the electrostatic's method of images, following previous literature results (Lindell 1993, Poon 2003, Redžić 2006).

2. – The physical system

As is well known, the problem of an insulated conducting sphere in front of a static point electric charge Q can be solved for the potential outside the sphere by replacing it by two image point charges (Jackson 1999). These are:

$$Q' = -\eta Q \tag{1}$$

located at a distance from the center of the sphere

$$d' = \eta R; \tag{2}$$

and a charge $Q'' = -Q'$ located just in the center (here d is the distance of Q from the sphere's center, and the three charges are aligned).

In the previous relations the adimensional scale factor η is given by:

$$\eta = \frac{R}{d}\,. \tag{3}$$

Looking now at magnetic phenomena, we point out that in a region of space devoid of *free currents* (as opposite to *magnetization currents*) the magnetic field is irrotational. In this way a magnetic scalar potential may be introduced, so achieving a complete formal mapping of magnetostatics onto electrostatics (Poon 2003). In other words, the equations satisfied by the magnetic field are formally equivalent to those satisfied by the electric field, provided that the electric charge is replaced by a fictitious "magnetic charge" (Kip 1969). In this analogical picture, conducting bodies are replaced by high magnetic permeability ones. Once the formal analogy has been established, one can obtain the surface magnetization on the sphere as the normal derivative of the magnetic scalar potential at the surface of the sphere (Jackson 1999, pag. 29; Poon 2003 pag. 945). Similarly, the interaction forces between the sphere and the real magnets may be calculated considering the interaction of these last with the "image" ones

The application of this formalism to a magnetic dipole put in front of a ferromagnetic sphere, leads to the correspondence illustrated in fig. 1: the effects of the sphere outside it (and so the interaction force experienced by the magnetic dipole) may be simulated by an "image" dipole m':

$$m' = \eta^2 m \tag{4}$$

placed at a distance d' from the sphere's center given by the expression (2).

In this regard, we need to make some comments. As illustrated above, In the monopolar case (*i.e.*, single electric charge in front of a conductive sphere) there are two image charges supposed inside the sphere, one of which is always located in its center. Instead, when a dipole (either electric or magnetic) is considered, there are no image

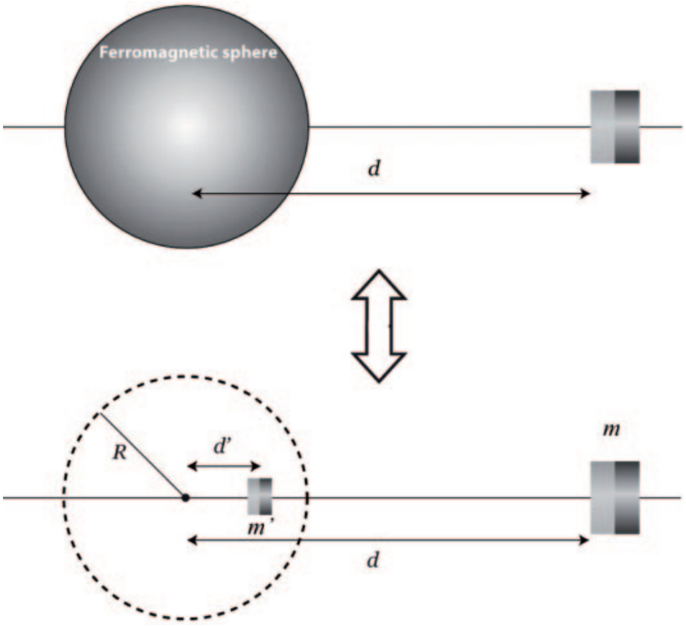

Fig. 1. – An high permeability sphere in front of a magnetic dipole m may be replaced, as long as one is concerned with the region outside the sphere, by another dipole m': the image dipole.

charges/dipoles placed in the sphere center: in this case, indeed, two "magnetic charges" having equal strength and opposite sign should be placed in that point, so giving rise to a null magnetic (or electric) moment. Moreover, we should observe that, while image charge in eq. (1) is linearly dependent on the scale factor η, image moment shows a square dependence, as pointed out elsewhere (Bonanno *et al.* 2009b)

In the slightly more complex case of two like-poles faced through a ferromagnetic sphere, which recently has been experimentally treated (Bonanno *et al.* 2009a), the system can be modeled by replacing the real sphere with two fictitious image dipoles (fig. 2). In this case, the strengths and positions of images are calculated according to formulae (2) and (4), where the values of η for the two dipoles are determined by the respective distances from the sphere center.

In any case, the force acting on the right-side dipole may be determined by calculating the force acted upon it by either one image magnet (fig. 1) or the other real magnet and two image ones (fig. 2). Correspondingly, the surface magnetization on the sphere may be obtained as the radial component of the magnetic field, by properly using spherical polar coordinates.

3. – The Java simulation

The interactive multimedia tutorial(1), aimed to support the understanding of the "Geomag paradoxes" (Defrancesco *et al.* 2007, Bonanno *et al.* 2009a) consists of two Java applets: one modeling the interaction of a single dipole with a high permeability

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(1) Available on the web:
`http://www.fis.unical.it/didattica/mptl14/magneticInteractionsTutorial.html`

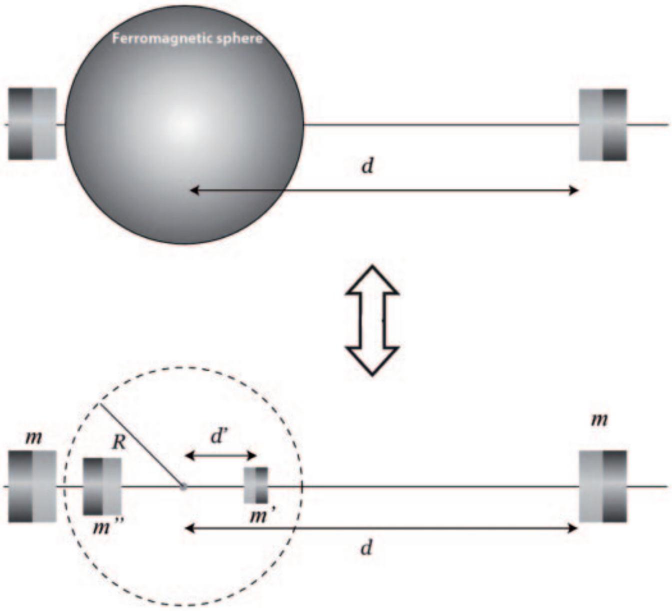

Fig. 2. – Thanks to the superposition principle, an high permeability sphere in front of two magnetic dipoles m —one on each side— may be replaced by two image dipoles m' and m'', as long as one is concerned with the region outside the sphere.

ferromagnetic sphere (fig. 3), the other modeling the case in which two dipoles are axially placed on opposite sides of the sphere, facing like-poles through it (fig. 4). In this last case,

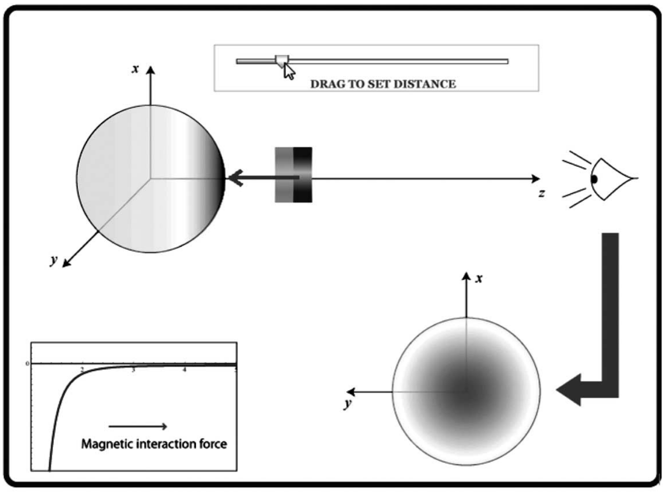

Fig. 3. – GUI of the applet modeling the interaction of a magnetic dipole with an high permeability sphere. The upper left part of the window shows the side view of the magnetization, while in the left right is shown the polar view (along the z-axis).In the insert on the bottom left, the interaction force between the magnet and the sphere is shown as a function of the distance.

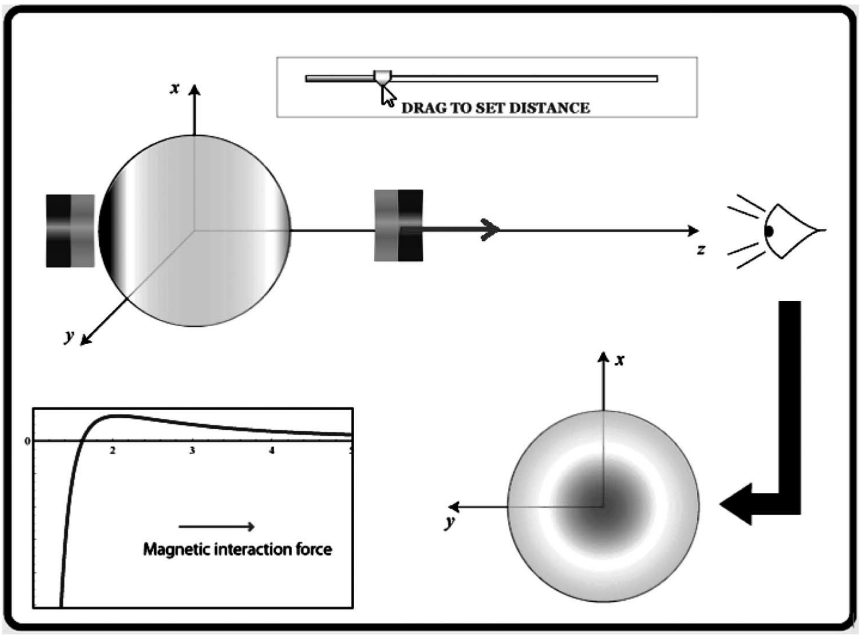

Fig. 4. – GUI of the applet modeling the interaction of two magnetic dipoles facing their like-poles through a high permeability sphere. The left magnet is fixed very close to the sphere, while the right one's position may be varied by using the slider in the upper part of the GUI. Other details are described in the caption of fig. 3.

one of the dipoles (on the left in the figure) is fixed very close to the sphere, while the other one (on the right in the figure) is positioned on the opposite side of the sphere at a distance that may be varied by acting on a slider In both cases, the structure of the Graphical User Interface (GUI) of the applet is the following. The upper left part of the window shows the side view of the magnetization, while in the left right the magnetization distribution is shown in polar view (along the z-axis).In the insert on the bottom left, the interaction force between the mobile magnet and the sphere is shown as a function of the distance, while the same force is displayed as a variable length vector applied to the magnet.

Due to the symmetry of this analyzed system around the z-axis, the surface magnetization density σ shows the same axial symmetry. So, in a spherical polar system of coordinates having its origin in the sphere center and its polar axis along the z-axis (see figs. 3 and 4), the magnetization in a given point on the surface will depend only on the angle θ between the position vector of this point and the polar axis:

$$\sigma \equiv \sigma(\theta). \tag{5}$$

To visually display the magnetization, this function has been mapped on the *Hue-Saturation-Brightness-Alpha (HSBA) color space* (Jewett 2009) by using a suitable coding algorithm. In particular, the sign of the function has been coded in the Hue HSBA-space coordinate (assigning Red-Hue to the south magnetization and Blue-Hue to the north one). Instead, the modulus of σ has been coded in the S- and A- color coordinates, by using a linear combination of them so to obtain a color whose saturation and opacity are increasing with σ value. An appropriate projection has been employed to show the polar distribution of magnetization as a function of *plane* polar coordinates in the *equatorial plane* (*x-y plane*).

4. – Conclusions

In this paper we present an interactive multimedia tutorial, realized by using the Java programming language, aimed to clarify some peculiarity of magnetic interactions, either between magnets and ferromagnetic materials or among magnets when a ferromagnetic material is interposed. In particular the multimedia permits to elucidate why and how two magnetic like-poles may attract when faced through a ferromagnetic sphere: what has been referred to as the "Geomag paradox". The simulation is carried out by modeling the interaction between magnets and a ferromagnetic sphere by means of a magnetostatic transposition of the method of images, well known in the electrostatic context. The novelty of the simulation with respect to other applets on similar themes (NTNUJAVA 2009) consists mainly in the interactive visualization of both the surface distribution of the magnetization on the sphere and the interaction force between involved bodies.

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