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Neutrino magnetic moment and neutrino energy quantization in rotating media

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Summary. — After a brief discussion on neutrino electromagnetic properties, we consider the problem of neutrino energy spectra in different media. It is shown that in two particular cases (*i.e.*, neutrino propagation in a) transversally moving with increasing speed medium and b) rotating medium) neutrino energies are quantized. These phenomena can be important for astrophysical applications, for instance, for physics of rotating neutron stars.

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1. – Introduction—neutrino electromagnetic properties

Initially the problem considered in this paper has originated from the studies of neutrino electromagnetic properties and related items. There is no doubt that the recent experimental and theoretical studies of flavour conversion in solar, atmospheric, reactor and accelerator neutrino fluxes give strong evidence of nonzero neutrino mass. A massive neutrino can have nontrivial electromagnetic properties [1]. A recent review on neutrino electromagnetic properties can be found in [2]. The present situation in the domain is characterized by the fact that in spite of reasonable efforts in studies of neutrino electromagnetic properties, there is no any experimental confirmation in favour of neutrino electromagnetic characteristics being nonvanishing. However, it is very plausible to assume that a neutrino may have nonzero electromagnetic properties. In particular, it seems very reasonable that a neutrino has a nonvanishing magnetic moment [2, 3].

Neutrino magnetic moment interaction effects. If a neutrino has nontrivial electromagnetic properties, notably nonvanishing magnetic (or electric) transition dipole moments or nonzero millicharge and charge radius, then a direct neutrino coupling to photons becomes possible and several processes exist important for applications [4]. A set of typical and most important neutrino electromagnetic processes involving the direct neutrino couplings with photons is: 1) a neutrino radiative decay $\nu_1 \to \nu_2 + \gamma$, neutrino

Cherenkov radiation in external environment (plasma and/or electromagnetic fields), 2) photon (plasmon) decay to a neutrino-antineutrino pair in plasma $\gamma \to \nu\bar{\nu}$, 3) neutrino scattering off electrons (or nuclei), 4) neutrino spin (spin-flavor) precession in magnetic field. Another very important phenomenon is the resonant amplification of the neutrino spin-flavour oscillations in matter that was first considered in [5].

Note a new mechanism of electromagnetic radiation produced by a neutrino moving in matter and originated due to neutrino magnetic moment [6-8]. It was termed the spin light of neutrino in matter ($SL\nu$) [6]. Although the $SL\nu$ was considered first within quasiclassical approach, it was clear that this is a quantum phenomenon by its nature. The quantum theory of this radiation has been elaborated [7] (see also [8]) within development [9, 10] of a quite powerful method that implies the use of the exact solutions of the modified Dirac equation for the neutrino wave function in matter. For elaboration of the quantum theory of the $SL\nu$ one has to find the solution of the quantum equation for the neutrino wave function and for the neutrino energy spectrum in medium.

2. – Quantum equation for neutrino in medium

The modified Dirac equation for the neutrino wave function exactly accounting for the neutrino interaction with matter [7]:

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0. \tag{1}$$

This is the most general form of the equation for the neutrino wave function in which the effective potential $V_\mu = \frac{1}{2}(1+\gamma_5) f_\mu$ includes both the neutral and charged current interactions of neutrino with the background particles and which can also account for effects of matter motion and polarization. It should be mentioned that other modifications of the Dirac equation were previously used in [11] for studies of the neutrino dispersion relations, neutrino mass generation and neutrino oscillations in the presence of matter.

In the case of matter composed of electrons, neutrons, and protons and for the neutrino interaction with background particles given by the standard model supplied with the singlet right-handed neutrino one has

$$f^\mu = \sqrt{2} G_F \sum_{f=e,p,n} j_f^\mu q_f^{(1)} + \lambda_f^\mu q_f^{(2)}, \tag{2}$$

where

$$q_f^{(1)} = \left(I_{3L}^{(f)} - 2Q^{(f)} \sin^2\theta_W + \delta_{ef} \right), \qquad q_f^{(2)} = -\left(I_{3L}^{(f)} + \delta_{ef} \right), \tag{3}$$
$$\delta_{ef} = \begin{cases} 1 & \text{for } f = e, \\ 0 & \text{for } f = n, p. \end{cases}$$

Here $I_{3L}^{(f)}$ and $Q^{(f)}$ are, respectively, the values of the isospin third components and the electric charges of matter particles ($f = e, n, p$). The corresponding currents j_f^μ and polarization vectors λ_f^μ are

$$j_f^\mu = (n_f, n_f \mathbf{v}_f), \qquad \lambda_f^\mu = \left(n_f(\boldsymbol{\zeta}_f \mathbf{v}_f), n_f \boldsymbol{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \mathbf{v}_f (\boldsymbol{\zeta}_f \mathbf{v}_f)}{1 + \sqrt{1 - v_f^2}} \right), \tag{4}$$

where θ_W is the Weinberg angle. In the above formulas (4), n_f, $\mathbf{v}_f$ and $\boldsymbol{\zeta}_f$ $(0 \le |\boldsymbol{\zeta}_f|^2 \le 1)$ stand, respectively, for the invariant number densities, average speeds and polarization vectors of the matter components.

In the case of matter at rest it is possible to solve the modified Dirac equation for different types of neutrinos moving in matter of different composition, as is shown in [7]. The energy spectrum of different neutrinos moving in matter is given by

$$E_\varepsilon = \varepsilon \eta \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2} + \alpha m. \tag{5}$$

In the general case of matter composed of electrons, neutrons and protons the matter density parameter α for different neutrino species is

$$\alpha_{\nu_e, \nu_\mu, \nu_\tau} = \frac{1}{2\sqrt{2}} \frac{G_F}{m} (n_e (4 \sin^2 \theta_W + \varrho) + n_p (1 - 4 \sin^2 \theta_W) - n_n), \tag{6}$$

where $\varrho = 1$ for the electron neutrino and $\varrho = -1$ for the muon and tau neutrinos.

The value $\eta = \mathrm{sign}(1 - s\alpha \frac{m}{p})$ in (5) provides a proper behavior of the wave function in the hypothetical massless case. The values $s = \pm 1$ specify the two neutrino helicity states, ν_+ and ν_-. The quantity $\varepsilon = \pm 1$ splits the solutions into the two branches that in the limit of the vanishing matter density, $\alpha \to 0$, reproduce the positive- and negative-frequency solutions, respectively.

In the next two sections we apply the developed method of exact solutions to two particular cases when neutrino is propagating a medium transversally moving with increasing speed [12, 9] and in a rotating medium of constant density. In both cases the obtained energy spectrum of the neutrino is quantized like the energy spectrum of an electron is quantized in a constant magnetic field.

3. – Neutrino quantum states in a medium transversally moving with increasing speed

First we consider a neutrino propagating in a medium composed of neutrons that move perpendicular to the neutrino path with linearly increasing speed. This can be regarded as the first approach to modelling of neutrino propagation inside a rotating neutron star [12, 9]. The corresponding modified Dirac equation for the neutrino wave function is given by (1) with the matter potential accounting for rotation,

$$f^\mu = -G(n, n\mathbf{v}), \qquad \mathbf{v} = (\omega y, 0, 0), \tag{7}$$

where $G = \frac{G_F}{\sqrt{2}}$. Here ω is the angular frequency of matter rotation around the OZ axis, it also is supposed that the neutrino propagates along the OY axis. For the neutrino wave function components $\psi(x)$ we get from the modified Dirac equation (1), a set of equations(1),

$$
\begin{aligned}
[i(\partial_0-\partial_3)+Gn]\psi_1+[-(i\partial_1+\partial_2)+Gn\omega y]\psi_2&=m\psi_3,\\
[(-i\partial_1+\partial_2)+Gn\omega y]\,\psi_1+[i(\partial_0+\partial_3)+Gn]\psi_2&=m\psi_4,\\
i(\partial_0+\partial_3)\psi_3+(i\partial_1+\partial_2)\psi_4&=m\psi_1,\\
(i\partial_1-\partial_2)\psi_3+i(\partial_0-\partial_3)\psi_4&=m\psi_2.
\end{aligned}
\tag{8}
$$

In a general case, it is not a trivial task to find solutions of this set of equations.

The problem is reasonably simplified in the limit of a very small neutrino mass, *i.e.* when the neutrino mass can be ignored in the left-hand side of (8) with respect to the kinetic and interaction terms in the right-hand sides of these equations. In this case two pairs of the neutrino wave function components decouple one from each other and four equations (8) split into the two independent sets of two equations, that couple together the neutrino wave function components in pairs, (ψ_1,ψ_2) and (ψ_3,ψ_4).

The second pair of eqs. (8) does not contain a matter term and is attributed to the sterile right-handed chiral neutrino state, ψ_R. The corresponding solution can be taken in the plain-wave form

$$
\psi_R\sim L^{-\frac{3}{2}}\exp[i(-p_0t+p_1x+p_2y+p_3z)]\psi,
\tag{9}
$$

where p_μ is the neutrino momentum. Then for the components ψ_3 and ψ_4 we obtain from (8) the following equations:

$$
\begin{aligned}
(p_0-p_3)\psi_3-(p_1-ip_2)\psi_4&=0,\\
-(p_1+ip_2)\psi_3+(p_0+p_3)\psi_4&=0.
\end{aligned}
\tag{10}
$$

Finally, from (10) for the sterile right-handed neutrino we get

$$
\psi_R=\frac{\mathrm{e}^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0-p_3)}}\begin{pmatrix}0\\0\\-p_1+ip_2\\p_3-p_0\end{pmatrix},
\tag{11}
$$

where $px=p_\mu x^\mu$, $p_\mu=(p_0,p_1,p_2,p_3)$ and $x_\mu=(t,x,y,z)$. This solution, as it should do, has the vacuum dispersion relation.

In the neutrino mass vanishing limit the first pair of eqs. (8) corresponds to the active left-handed neutrino. The form of these equations is similar to the corresponding equations for a charged particle (*e.g.*, an electron) moving in a constant magnetic field B given by the potential $\boldsymbol{A}=(By,0,0)$ (see, for instance, [13]). To display the analogy, we note that in our case the matter current component $n\boldsymbol{v}$ plays the role of the vector potential $\boldsymbol{A}$. The existed analogy between an electron dynamics in an external electromagnetic field and a neutrino dynamics in background matter is discussed in [9].

(1) The chiral representation for Dirac matrices is used.

The solution of the first pair of eqs. (8) can be taken in the form

$$\psi_L \sim \frac{1}{L}\exp[i(-p_0 t + p_1 x + p_3 z)]\psi(y), \tag{12}$$

and for the components ψ_1 and ψ_2 of the neutrino wave function we obtain from (8) the following equations:

$$\begin{aligned} &(p_0 + p_3 + Gn)\psi_1 - \sqrt{\rho}\left(\frac{\partial}{\partial\eta} - \eta\right)\psi_2 = 0, \\ &\sqrt{\rho}\left(\frac{\partial}{\partial\eta} + \eta\right)\psi_1 + (p_0 - p_3 + Gn)\psi_2 = 0, \end{aligned} \tag{13}$$

where

$$\eta = \sqrt{\rho}\left(x_2 + \frac{p_1}{\rho}\right), \quad \rho = Gn\omega. \tag{14}$$

For the wave function we finally get

$$\psi_L = \frac{\rho^{\frac{1}{4}} \mathrm{e}^{-ip_0 t + ip_1 x + ip_3 z}}{L\sqrt{(p_0 - p_3 + Gn)^2 + 2\rho N}} \begin{pmatrix} (p_0 - p_3 + Gn)u_N(\eta) \\ -\sqrt{2\rho N}u_{N-1}(\eta) \\ 0 \\ 0 \end{pmatrix}, \tag{15}$$

where $u_N(\eta)$ are Hermite functions of order N. For the energy of the active left-handed neutrino we get

$$p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \quad N = 0, 1, 2, \ldots . \tag{16}$$

The energy depends on the neutrino momentum component p_3 along the rotation axis of matter and the quantum number N that determines the magnitude of the neutrino momentum in the orthogonal plane. For description of antineutrinos one has to consider the "negative sign" energy eigeinvalues (see similar discussion in sect. **2**). Thus, the energy of an electron antineutrino in the rotating matter composed of neutrons is given by

$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \quad N = 0, 1, 2, \ldots . \tag{17}$$

Obviously, generalization for different other neutrino flavours and matter composition is just straightforward (see (4) and (6)).

Thus, it is shown [12] that the transversal motion of an active neutrino and antineutrino is quantized in moving matter very much like an electron energy is quantized in a constant magnetic field that corresponds to the relativistic form of the Landau energy levels (see [13]).

4. – Neutrino energy in a rotating medium

Now we consider a more consistent model of a neutrino motion in a rotating matter. For this case we choose the effective matter potential in (1) in the following form:

$$f^{\mu} = -G(n, n\mathbf{v}), \qquad \mathbf{v} = (-\omega y, \omega x, 0). \tag{18}$$

Contrary to the case considered in the previous section, eq. (1) with the potential (18) describes the case when a neutrino is moving in a rotating medium. It is shown below how eq. (1) with (18) can be solved and the corresponding neutrino energy spectrum is obtained.

The solution of eq. (1) with (18) can be sought in the form

$$\psi(t,x,y,z) = \mathrm{e}^{-ip_0 t + ip_3 z}\begin{pmatrix}\psi_1(x,y)\\ \psi_2(x,y)\\ \psi_3(x,y)\\ \psi_4(x,y)\end{pmatrix}. \tag{19}$$

Substituting (19) into (1) with (18) and using the explicit form of the Dirac matrices in the chiral representation, we arrive at a system of linear equations for the neutrino wave function components:

$$\begin{aligned}
-(p_0+p_3+Gn)\psi_1 + i\left\{\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) + Gn\omega(x-iy)\right\}\psi_2 &= -m\psi_3,\\
i\left\{\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) - Gn\omega(x+iy)\right\}\psi_1 + (p_3-p_0-Gn)\psi_2 &= -m\psi_4,\\
(p_0-p_3)\psi_3 + i\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_4 &= m\psi_1,\\
+i\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 + (p_0+p_3)\psi_4 &= m\psi_2.
\end{aligned} \tag{20}$$

In the polar coordinates $x+iy = r\mathrm{e}^{i\phi}$, $x-iy = r\mathrm{e}^{-i\phi}$ one has

$$\frac{\partial}{\partial x} + i\frac{\partial}{\partial y} = \mathrm{e}^{i\phi}\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi}\right), \qquad \frac{\partial}{\partial x} - i\frac{\partial}{\partial y} = \mathrm{e}^{-i\phi}\left(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi}\right), \tag{21}$$

and the system of eqs. (20) transforms to

$$\begin{aligned}
-(p_0+p_3+Gn)\psi_1 + i\mathrm{e}^{-i\phi}\left\{\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi} + \rho r\right\}\psi_2 &= -m\psi_3,\\
i\mathrm{e}^{i\phi}\left\{\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi} - \rho r\right\}\psi_1 + (p_3-p_0-Gn)\psi_2 &= -m\psi_4,\\
(p_0-p_3)\psi_3 + i\mathrm{e}^{-i\phi}\left(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi}\right)\psi_4 &= m\psi_1,\\
+i\mathrm{e}^{i\phi}\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi}\right)\psi_3 + (p_0+p_3)\psi_4 &= m\psi_2.
\end{aligned} \tag{22}$$

It is possible to show that the operator of the total momentum $J_z = L_z + S_z$, where $L_z = -i\frac{\partial}{\partial\phi}$, $S_z = \frac{1}{2}\sigma_3$, commutes with the corresponding Hamiltonian of the considered system. Therefore the solutions can be taken in the form

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} = \begin{pmatrix} i\chi_1(r)e^{i(l-1)\phi}\\ \chi_2(r)e^{il\phi}\\ i\chi_3(r)e^{i(l-1)\phi}\\ \chi_4(r)e^{il\phi}\end{pmatrix}, \tag{23}$$

that are the eigenvectors for the total momentum operator J_z with the corresponding eigenvalues $l - \frac{1}{2}$. After substitution of (23) the system (22) can be rewritten in the following form:

$$\begin{aligned} -(p_0 + p_3 + Gn)\chi_1 + \left\{\frac{\mathrm{d}}{\mathrm{d}r} + \frac{l}{r} + \rho r\right\}\chi_2 &= -m\chi_3, \\ \left\{\frac{\mathrm{d}}{\mathrm{d}r} - \frac{l-1}{r} - \rho r\right\}\chi_1 + (p_0 - p_3 + Gn)\chi_2 &= m\chi_4, \\ (p_0 - p_3)\chi_3 + \left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{l}{r}\right)\chi_4 &= m\chi_1, \\ \left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{l-1}{r}\right)\chi_3 - (p_0 + p_3)\chi_4 &= -m\chi_2. \end{aligned} \tag{24}$$

For further consideration it is convenient to introduce the rising and decreasing operators

$$R^+ = \frac{\mathrm{d}}{\mathrm{d}r} - \frac{l-1}{r} - \rho r, \qquad R^- = \frac{\mathrm{d}}{\mathrm{d}r} + \frac{l}{r} + \rho r. \tag{25}$$

After application of the decreasing R^- and increasing R^+ operators to the second and first equations of (22) correspondingly, one gets

$$\begin{aligned} R^-R^+\chi_1 + ((p_0 + Gn)^2 - p_3^2 - m^2)\chi_1 &= m(Gn\chi_3 + \rho r\chi_4), \\ R^+R^-\chi_2 + ((p_0 + Gn)^2 - p_3^2 - m^2)\chi_2 &= m(Gn\chi_4 + \rho r\chi_3). \end{aligned} \tag{26}$$

Note that the system (24), as well as the system (8), can be solved exactly in the limit of vanishing neutrino mass $m \to 0$. In order to find the nonzero-mass correction to the energy spectrum of a neutrino in a bound state in matter, the neutrino square integrable wave function should be found. Therefore, in analogy with the zero-mass case we take $\chi_3 = \chi_4 = 0$ in the lowest order of perturbation series expansion. Thus we arrive at the system

$$\begin{aligned} R^-R^+\chi_1 + ((p_0 + Gn)^2 - p_3^2 - m^2)\chi_1 &= 0, \\ R^+R^-\chi_2 + ((p_0 + Gn)^2 - p_3^2 - m^2)\chi_2 &= 0. \end{aligned} \tag{27}$$

The solution of (27) can be written in the form

$$\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = \begin{pmatrix} C_1\mathcal{L}_s^{l-1}(\rho r^2)\\ C_2\mathcal{L}_s^{l}(\rho r^2)\end{pmatrix}, \tag{28}$$

where $\mathcal{L}_s^l$ are the Laguerre functions [13]. After substitution of (28) into (27) and taking into account the properties of the increasing and decreasing rising operators,

$$
\begin{aligned}
R^+\mathcal{L}_s^{l-1}(\rho r^2) &= -2\sqrt{\rho(s+l)}\,\mathcal{L}_s^l(\rho r^2),\\
R^-\mathcal{L}_s^{l}(\rho r^2) &= 2\sqrt{\rho(s+l)}\,\mathcal{L}_s^{l-1}(\rho r^2),
\end{aligned}
\tag{29}
$$

we get from (27) the equation for the neutrino energy spectrum in matter:

$$
m^2 + p_3^2 + 4(s+l)\rho - (p_0+Gn)^2 = 0. \tag{30}
$$

Solving this equation we get for the neutrino energies

$$
p_0 = \pm\sqrt{m^2+p_3^2+4N\rho} - Gn, \quad N = 0,1,2,\ldots, \tag{31}
$$

where the quantum number $N = s+l$ is introduced. As usually, two signs in the solution correspond to the neutrino and antineutrino energies, correspondingly,

$$
p_0 = \sqrt{m^2+p_3^2+4N\rho} - Gn, \qquad \tilde{p}_0 = \sqrt{m^2+p_3^2+4N\rho} + Gn. \tag{32}
$$

From the obtained energy spectrum it is just straightforward that the transversal motion momentum of an antineutrino is given by

$$
\tilde{p}_\perp = 2\sqrt{NG\omega}. \tag{33}
$$

The quantum number N determines also the radius of the quasiclassical orbit in matter (it is supposed that $N \gg 1$ and $p_3 = 0$),

$$
R = \sqrt{\frac{N}{Gn\omega}}. \tag{34}
$$

It follows that antineutrinos can have bound orbits inside a rotating star. To make an estimation of magnitudes, let us consider a model of a rotating neutron star with radius $R_{NS} = 10\,\mathrm{km}$, matter density $n = 10^{37}\,\mathrm{cm}^{-3}$ and angular frequency $\omega = 2\pi \times 10^3\,\mathrm{s}^{-1}$. For this set of parameters, the radius of an orbit is less than the typical star radius R_{NS} if the quantum number $N \le N_{\max} = 10^{10}$. Therefore, antineutrinos that occupy orbits with $N \le 10^{10}$ can be bounded inside the star. The scale of the bounded antineutrinos energy estimated by (32) is of the order $\tilde{p}_0 \sim 1\,\mathrm{eV}$. It should be underlined that within the quasiclassical approach the neutrino binding on circular orbits is due to an effective force that is orthogonal to the particle speed. Note that there is another mechanism of neutrinos binding inside a neutron star when the effect is produced by a gradient of the matter density (see the last paper in [11]). A discussion on the "matter-induced Lorentz force" that can be introduced in order to explain a neutrino motion on quasiclassical circular orbits can be found in [9].

We argue that the effect of a neutrino energy quantization can have important consequences for physics of rotating neutron stars.

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