IL NUOVO CIMENTO Vol. 32 C, N. 3-4 Maggio-Agosto 2009
DOI 10.1393/ncc/i2009-10420-x

Colloquia: IFAE 2009

New techniques for one-loop scattering amplitudes

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(ricevuto il 19 Settembre 2009; pubblicato online il 9 Ottobre 2009)

Summary. — We review the main features of the Ossola, Papadopoulos and Pittau (OPP) method for the evaluation of one-loop amplitudes of arbitrary scattering processes. In this approach, the coefficients of the scalar integrals are extracted by means of simple algebraic equations constructed by numerically evaluating the numerator of the integrand for specific choices of the integration momentum. After a discussion of the first results obtained with this method, we will focus on the latest developments in the direction of a fully automatized approach to multi-loop NLO calculation.

PACS 11.10.-z – Field theory.
PACS 12.38.Bx – Perturbative calculations.
PACS 13.85.Hd – Inelastic scattering: many-particle final states.

1. – Introduction

The experimental programs at the LHC will require high-precision predictions for multi-particle processes. As a consequence, several different groups have been recently working on the construction of tools for the computation of one-loop amplitudes for processes involving more than four particles [1].

Any one-loop amplitude $\mathcal{M}$ can be expressed in terms of scalar integrals as

$$\mathcal{M} = \sum_i d_i \ \mathrm{Box}_i + \sum_i c_i \ \mathrm{Triangle}_i + \sum_i b_i \ \mathrm{Bubble}_i + \sum_i a_i \ \mathrm{Tadpole}_i + R, \quad (1)$$

where d_i, c_i, b_i and a_i are the coefficients of 4-, 3-, 2- and 1-point scalar integrals [2, 3] and R is a polynomial, function of ratios of the external kinematic invariants, known as rational term. Since the scalar functions can be calculated for all kinematic configurations, by means of codes such as QCDLoop [4, 5] or OneLOop [6], the knowledge of all coefficients and of the rational term formally completes the calculation of the scattering amplitude.

Besides standard techniques, where a tensor reduction is explicitly performed, new numerical and analytical developments have emerged recently. The common features of the so-called "unitarity-based" methods [7, 8] is a different approach to the reduction: they aim at the direct determination of the coefficients of the scalar one-loop functions.

Both approaches have been pursued and led to very important results (see ref. [9] for a review). In particular, in the last few months, very challenging calculations involving four particles in the final state have been successfully completed [10, 11].

2. – A walk through the OPP method

In two recent papers [12, 13], we proposed a reduction method (OPP) for arbitrary one-loop amplitudes at *the integrand level* [14]. The method is based on the idea of expressing the integrand of the one-loop amplitude in terms of the propagators that depend on the integration momentum.

In this approach, the coefficients in eq. (1) are numerically determined by solving a system of algebraic equations that are obtained by: i) the numerical evaluation of the numerator of the integrand at explicit values of the loop-variable; ii) and the knowledge of the most general polynomial structure of the integrand itself [14].

The solution of the system of equations becomes particularly simple if we exploit numerically the set of kinematical equations for the integration momentum corresponding to the so-called quadruple, triple and double cuts also used in the unitarity-cut method.

Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \tag{2}$$

with $\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$. The bar denotes objects living in $n = 4 + \epsilon$ dimensions and a tilde objects of dimension ϵ. Physical external momenta p_i are 4-dimensional objects, while the integration momentum q is in general n-dimensional. Following this notation, we have $\bar{q}^2 = q^2 + \tilde{q}^2$ and $\bar{D}_i = D_i + \tilde{q}^2$.

2˙1. *Reduction in dimension 4.* – Assuming for the moment that the numerator $N(q)$ is fully 4-dimensional, we can rewrite it at the integrand level in terms of $D_i = (q + p_i)^2 - m_i^2$ as

$$\begin{aligned} N(q) = {} & \sum_{i_0<i_1<i_2<i_3}^{m-1} \Big[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\Big] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\ & + \sum_{i_0<i_1<i_2}^{m-1} \Big[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\Big] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ & + \sum_{i_0<i_1}^{m-1} \Big[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\Big] \prod_{i \neq i_0, i_1}^{m-1} D_i \\ & + \sum_{i_0}^{m-1} \Big[a(i_0) + \tilde{a}(q; i_0)\Big] \prod_{i \neq i_0}^{m-1} D_i \,. \end{aligned} \tag{3}$$

The quantities $d(i_0 i_1 i_2 i_3)$ are the coefficients of 4-point scalar functions with denominators labeled by i_0, i_1, i_2, and i_3. In the same way, we call $c(i_0 i_1 i_2)$, $b(i_0 i_1)$ and $a(i_0)$

the coefficients of all possible 3-point, 2-point and 1-point scalar functions, respectively. The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are called "spurious" terms, *i.e.* terms that are present in the decomposition at the integrand level, but will vanish upon integration. These terms still depend on the integration momentum q.

Inserted back in eq. (2), this expression simply states the multi-pole nature of any m-point one-loop amplitude, that contains a pole for any propagator in the loop.

Once eq. (3) is established by fixing the process-independent form of all the spurious terms, the task of computing the one-loop amplitude is reduced to the algebraic problem of extracting all the coefficients d, c, b, a. This is achieved simply by evaluating the function $N(q)$ a sufficient number of times, at different values of q, and then solving the system with respect to d, c, b, a. This can be achieved quite efficiently by choosing particular values of q such that a subset of denominators D_i vanish. Operating in this manner, the system becomes "triangular" and it is solved iteratively.

2·2. *Rational terms*. – As mentioned in the previous section, the reduction has been performed assuming a purely four-dimensional numerator (this singles out the so-called cut-constructable part of the amplitude).

However, as is well known, even starting from a perfectly finite tensor integral, the tensor reduction may lead to integrals that need to be regularized (we use dimensional regularization). For this reason, we introduced in eq. (2) the d-dimensional denominators $\bar{D}_i = D_i + \tilde{q}^2$, that differ by an amount $\tilde{q}^2$ from their 4-dimensional counterparts D_i.

This leads to a mismatch in the cancellation of the d-dimensional denominators of eq. (2) with the 4-dimensional ones of eq. (3). The rational part of the amplitude, called R_1 [15], comes from such a lack of cancellation. A different source of rational terms, called R_2, can also be generated from the ϵ-dimensional part of $N(q)$ [15,16]. The sum of these two contributions reconstructs the rational term in eq. (1).

If we go back to the integrand $A(\bar{q})$ of eq. (2) and insert the expression for $N(q)$ of eq. (3), we observe the presence of ratios of denominators in different dimensions, namely $D_i/\bar{D}_i$. We can rewrite them using $D_i/\bar{D}_i = \bar{Z}_i$, with $\bar{Z}_i \equiv (1 - \tilde{q}^2/\bar{D}_i)$. The rational part is produced, after integrating over $d^n q$, by the $\tilde{q}^2$ dependence in $\bar{Z}_i$. The expressions for all relevant integrals are reported in the Appendix of ref. [13].

As an alternative approach to determine R_1, we can reconstruct the dependence of the various coefficients on $\tilde{q}^2$, assuming that the reduction of the numerator was performed directly in terms of n-dimensional denominators $\bar{D}_i$. This approach is described in refs. [12,15], and leads to identical results for R_1.

The rational part R_2, generated from the ϵ-dimensional part of the numerator $N(q)$, can be determined using a dedicated set of Feynman rules in a tree-level computation [15,17]. More information about the reduction algorithm has been given in previous talks [18]. For a complete and detailed discussion, please refer to the original papers.

3. – Applications and automatization

The rest of this presentation will be devoted to a summary of various developments within the OPP framework.

As a first example of application of the method, we calculated the 6-photon amplitude, via a fermionic massive loop [13]. Previous results were available in the massless limit [19] and we found complete agreement. This process provided a playground for testing our method and a good benchmark to compare different methods of calculation.

As a next step, we released a public code called `CutTools` [20] that allows to extract all the coefficients of the scalar integral and the rational term R_1 in a fully automatized way. The user only needs to provide a routine for the evaluation of the numerator $N(q)$.

More recently [21], we exploited the polynomial structure of the integrand and the freedom in choosing the solutions of the cuts, to improve the system-solving algorithm. By selecting the variables of each polynomial to be proportional to the roots of unity, the extraction of the polynomial's coefficients is carried out through projections, using the orthogonality relation among plane waves, rather than by system inversion. We are currently working on further improvements to this approach.

In ref. [22], the OPP reduction is applied to the calculation of the next-to-leading order QCD correction for the production of three vector bosons at the LHC. This includes the case of ZZZ production, as well as the W^+W^-Z, W^+ZZ, and $W^+W^-W^+$ production. The virtual part of the NLO calculation has been performed using `CutTools` [20]. Scalar integrals are computed using the package `OneLOop` [6]. The tree-level cross-section has been evaluated using the `HELAC` event generator [23]. The same code, appropriately adapted to implement the dipole subtraction method, in the form proposed by the authors in ref. [24], has also been used to calculate the real corrections. Concerning the finite parts, we agree with the results obtained by the authors of ref. [25], for the production of three Z bosons. We also compared the results for W^+ZZ and W^+W^-Z with Hankele and Zeppenfeld [26]. A complete discussion, including leptonic decays, was presented in [27].

This last calculation, aside from confirming once more the validity of our approach, was very instructive. In order to go beyond this level of complexity, we need further automatization, both in the generation of the numerator $N(q)$ and in combining the different contributions to the result.

In a very recent paper [28], van Hameren, Papadopoulos and Pittau presented a technique for the automatized evaluation of the numerator $N(q)$, based on a 1-loop extension of `HELAC` [23]. In fact, after fixing the integration momentum q, as required by the OPP reduction, any n-point one-loop amplitude is an $(n + 2)$-point tree level amplitude. As a powerful proof of concept for the validity of this approach, which combines HELAC-1loop with the OPP reduction, they calculated amplitudes for all 6-particle processes in the 2005 Les Houches wishlist.

Even more recently, Czakon, Papadopoulos and Worek, produced an automatized tool [29] to include dipole subtraction [24], that works within the framework of `HELAC`.

4. – Conclusions

Motivated by the upcoming LHC experiment, we observed tremendous progress in the calculation of one-loop multi-leg processes. This involves several new results for cross-sections of great importance for the LHC, but also the developments of many new techniques and codes.

The next frontier is a systematic automatization of the calculations. In this process, we should address important issues, such as stability, precision and versatility. Also, the computational efficiency of the codes should be good enough to make their use realistic within the experimental community for the LHC analysis.

Important results have been obtained with traditional methods [11,30]. On the other hand, unitarity-based approaches led to the development of advanced tools, such as `Rocket` [31] or `Blackhat` [32], that proved to be very effective for massless processes, such as multi-gluon production or $W + 3$ jets.

The OPP reduction was developed from the beginning in order to be suited for automatization. The recent progress in the automated generation of the numerators [28], or in the generation of dipole terms [29], is of crucial importance in this direction. It is likely that in the near future the combination of HELAC and OPP reduction will lead to a fully automatized tool for the calculation of any process, both in QCD and electroweak theory, without limitations on the masses of internal and external particles. At the same time, it will be important to continue to improve the efficiency and the stability of the method, to reach the same level of reliability of the traditional methods.

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Work developed in collaboration with C. Papadopoulos, R. Pittau, P. Mastrolia, T. Binoth and A. van Hameren. Many thanks to M. Czakon and M. Worek for fruitful discussions.

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