IL NUOVO CIMENTO Vol. 32 C, N. 2 Marzo-Aprile 2009
DOI 10.1393/ncc/i2009-10394-7

Colloquia: CSFI 2008

A Vlasov solver for collective effects in particle accelerators

M. Migliorati([1])(*), G. Dattoli([2]), A. Schiavi([1]) and M. Venturini([3])

([1]) *Sapienza Università di Roma - Rome, Italy*
([2]) *ENEA Centro Ricerche Frascati - Rome, Italy*
([3]) *LBNL - Berkeley CA 94720-8211, USA*

(ricevuto il 22 Giugno 2009; pubblicato online il 7 Settembre 2009)

Summary. — Integration techniques based on Lie algebraic methods have been successfully used in beam transport codes for particle accelerators. Generally these methods have been applied to problems of single-particle beam dynamics. Here we present an application of Lie algebraic techniques to the development of a Vlasov solver suitable for problems of beam transport in the presence of non-negligible particle self-fields. The solver we discuss is suitable for modelling a variety of collective effects that may arise at high current. In particular, we consider the case of coherent synchrotron radiation effects in magnetic bunch compressors which can cause instabilities limiting performance of high current accelerators.

PACS 41.85.Ja – Particle beam transport.
PACS 29.27.Bd – Beam dynamics; collective effects and instabilities.

1. – Introduction

The dynamics of charged particle beams through circular and linear accelerators or, more in general, through any kind of beam transport line, has been successfully studied using integration techniques based on Lie algebraic methods when collective effects due to electromagnetic self-fields (or wake fields) are neglected [1,2]. On the other hand, when self-fields are important, one generally relies on multi-particle tracking codes which allow for a generally straightforward inclusion of collective effects both in the longitudinal and transverse degrees of freedom (*e.g.*, see ref. [3]). These codes are of common use in the design of the accelerator devices to monitor possible beam instabilities excited by the wake fields. However, tracking codes of this kind suffer from spurious sampling noise inherent in employing a number of macroparticles smaller than the actual beam population and, as result, may introduce artificial instabilities. In this contribution we extend Lie algebraic techniques to develop a method of simulation of beam dynamics that fully accounts

(*) E-mail: mauro.migliorati@uniroma1.it

for wake-field effects. The method uses a representation of the beam density in phase space on a grid and, in contrast to macroparticle tracking codes, yields a relatively noise-free time evolution. The technique, which uses exponential operators similar to those commonly adopted in quantum mechanics, is flexible enough to allow for the account of a variety of effects in the beam transport and caused by different kinds of wake fields.

In the next sections we will briefly review the concept of wake potential that provide a general formalism for treating many collective effects. We introduce the Vlasov solver and finally show a specific application involving the effects of the coherent synchrotron radiation (CSR) in a magnetic bunch compressor, which is a contributing factor to the so-called "microbunching instability".

2. – Collective effects in particle accelerators

Direct inter-particle forces or inter-particle forces mediated by the interaction of a beam with its surroundings [4] are of fundamental importance in beam dynamics. Both forces can often be described in terms of a "wake potential". To introduce the concept of wake potential, consider the situation in which a charge q_1 travels at the velocity of light through an accelerator device. A particle of charge q, that follows at a distance Δz, experiences a Lorentz force $\mathbf{F}_L$ which, in first approximation, will depend only on Δz and the position s inside the structure. The longitudinal wake field is then defined in terms of the energy variation of q due to the longitudinal component of the Lorentz force generated by the source particle with charge q_1 along the whole structure:

$$w(\Delta z) = \frac{1}{qq_1} \int_{\text{Structure}} F_{L,z}(\Delta z; s)\mathrm{d}s. \tag{1}$$

The convolution integral of the wake function with the longitudinal charge density $\lambda(z)$ of the bunch represents the energy variation of the test particle due to the wake fields of all particles in the bunch:

$$U(z) = q \int_{-\infty}^{\infty} \lambda(z')w(z'-z)\mathrm{d}z'. \tag{2}$$

The main effects of longitudinal wake fields are a net energy loss and correlated energy spread along the bunch that typically worsen the beam quality and may cause instability.

3. – The Vlasov solver

Analytical methods can be useful for studying the beam dynamics under the effects of wake fields, but are usually limited to the domain of validity of linear theory and often over simplified models of beam dynamics. For more accurate modelling beyond this domain one has to resort to numerical calculations. Macroparticle simulation codes are generally used. A particle orbit is described by six differential first-order equations (two equations for each plane), typically coupled to each other, that are solved step by step. Present limitations to computational power make simulations using more than a few million macroparticles generally impractical on a single processor.

Simulations of this kind are time consuming, in part because of the convolution of the wake fields with the bunch distribution that must be carried out at each step of integration. In many cases they reproduce quite well the experimental observations of beam behavior, but present a noticeable disadvantage: due to the limited number of macroparticles, the numerical noise introduced in the simulations may produce artificial instabilities, especially in those cases where the beam dynamics is highly sensitive to small initial perturbations (caused for example by shot noise). Indeed, a simulation employing N_{mp} randomly deposed macroparticles to represent a bunch containing N physical electrons will overestimate the amplitude of the shot noise by a factor $\sqrt{N/N_{\mathrm{mp}}}$.

An effective alternative approach consists in describing the bunch by a continuous distribution function in the phase-space beam density and in following its evolution along the accelerator machine [5, 6]. The differential equation governing this function is the Vlasov equation

$$\frac{\partial}{\partial s}\rho = H\rho \qquad \text{with} \qquad \rho|_{s=0} = \rho_0, \tag{3}$$

that describes the evolution along the coordinate s of a charged beam density distribution ρ to be represented on a discrete grid by interpolation polynomials; s is the independent or "time-like" variable. The operator H, containing the physical properties of the system, may be specified by simple differential operators, when describing the bunch evolution through magnetic systems, or by integral operators, when accounting for the effects of wake fields. This problem is nonlinear in that the operator H in eq. (3) acting on the beam density ρ also depends on ρ through expressions of the form eq. (2).

Vlasov solvers are not sensitive to sampling noise and may therefore be used as an alternative effective tool to characterize beam instabilities. Most solvers developed so far, however, treat 2D systems and are therefore suited to model only the longitudinal phase space of beams with negligible transverse effects. These methods have proved useful for both beam dynamics studies and lattice optimization when some heuristic and approximate account of the effect of the transverse dynamics on the longitudinal motion is included [6]. However, a more accurate characterization of certain instabilities requires a complete description of the coupling between the longitudinal and transverse motion. In higher dimension, however, Vlasov methods become less competitive in comparison to tracking codes as the number of grid points needed for representing the solution of the Vlasov equation quickly escalates with the dimension of the system. The ensuing computer memory problem is the main disadvantage limiting the use of Vlasov solvers.

Equation (3) is an initial value problem whose formal solution, for s-independent H, can be written as

$$\rho = \exp[Hs]\rho_0. \tag{4}$$

In the following section we discuss the evolution of eq. (4) for a beam passing through a magnetic compressor system and subject to wake fields effects due to CSR.

4. – Simulations of a magnetic compressor

A magnetic compressor is a system of bending magnets and drifts that can be used for shortening a bunch. Once the operator H is known for each device of the compressor, we divide the beam evolution into small "time" (*i.e.* s) steps and use the operator splitting technique to obtain the distribution function at each step [5] under the assumption that H is locally constant. For each grid point in the phase space, the advanced distribution ρ is obtained by numerical interpolation of the distribution at the beginning of the step

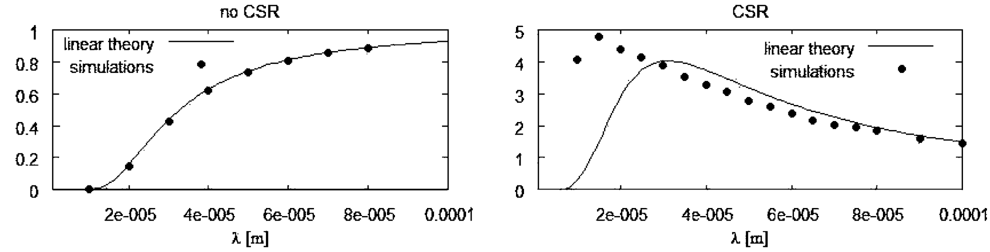

Fig. 1. – Modulation amplitude gain as a function of the wavelength for a bunch entering in a magnetic compressor.

using the method of local characteristics, see eq. (4). Lagrange polynomials of the 5th order are used for the off-grid interpolation of the distribution.

If a bunch enters the compressor with an initial sinusoidal modulation (*e.g.*, it could be a component of the Fourier spectrum of shot noise), the modulation amplitude can be enhanced as a consequence of the concurring effects of the coherent synchrotron radiation wake field and the compressor optics. The ratio between the amplitudes of the modulation at exit and entrance of the compressor (or instability "gain") is plotted in fig. 1 as a function of the wavelength of the initial modulation and compared with linear theory [7] for the cases without and with the CSR wake field, respectively. The simulation was performed using a 2D Vlasov solver for a beam with vanishing transverse emittance. In order to take into account the transverse beam dynamics, a 4D parallel code is now being developed to work on multi-processors. The work is still in progress; preliminary results can be found in ref. [8].

5. – Conclusions

Macroparticle codes used for simulations of beam dynamics in the presence of wake fields suffer from intrinsic numerical noise problems, that may seed artificial instabilities. Vlasov solvers, on the contrary, are immune from sampling noise and may therefore be used as a more effective tool to characterize the instability, particularly in low dimension. Their main limitation comes from the required number of grid points, which may become prohibitively large for a full 6D simulation. The Vlasov solver method that we have presented, based on the use of the exponential operator, includes a full account of collective effects due to wake fields and represents an alternative complementary approach to macroparticle simulations.

REFERENCES

[1] Douglas D. R. *et al.*, *9th PAC Proceedings, Washington DC, USA* (1981) p. 2522.
[2] Grote H. and Iselin F. C., CERN/SL/90-13 (AP) CERN (1993).
[3] Borland M., *ELEGANT*, LS-287, ANL, Argonne, IL 60439, USA.
[4] Palumbo L. *et al.*, CERN 95-06 (1995).
[5] Dattoli G. *et al.*, *Nucl. Instrum. Methods A*, **574** (2007) 244.
[6] Venturini M. *et al.*, *Phys. Rev. ST Accel. Beams*, **10** (2007) 054403.
[7] See, *e.g.*, Heifets S. *et al.*, *Phys. Rev. ST Accel. Beams*, **5** (2002) 064401.
[8] Migliorati M. *et al.*, *EPAC 08 Proceedings, Genova, Italy* (2008) p. 1764.