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Simulations of a turbulent channel flow with rodlike polymers

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Summary. — The modification of turbulence due to the introduction of dilute fibres in a channel flow is analysed by means of Direct Numerical Simulation (DNS). Using a simplified rheological model it is possible to assess that the modification of turbulent structure is far simpler than in the flexible case. In fact, at a given fibre concentration some relevant statistics show a substantial insensitivity on Reynolds number.

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PACS `47.27.nd` – Channel flow.
PACS `47.57.Ng` – Polymers and polymer solutions.

1. – Introduction

It is a relatively long time since the first simulation of drag reduction by polymers has been reproduced numerically [1, 2]. The velocity field obtained by the use of very simple models, that still maintain the viscoelastic feature of polymers solutions, has successfully reproduced the well-known drag reducing characteristics of such flows. Despite the incertanties that come from the use of such a basic model, numerical studies have the advantage to give access to the whole fluid dynamic field allowing for a thorough analysis of the spatial and temporal structure of turbulence. With respect to the flexible case the simulations started only recently due to the higher complexity of the model.

We present here some results from a DNS of a turbulent channel flow with rodlike polymers. In particular some single-point statistics for two different Reynolds numbers at two different concentrations will we discussed.

2. – Mathematical model

The present data have been obtained using an appropriate model for the rod-like polymers. As is customary in dilute solutions, the total stress tensor is obtained from the superposition of the Newtonian contribution, with dynamic viscosity μ, and a part due to the polymer molecules, T^p_{ij}. Specifically, in this approach each rod is represented by a neutrally boyant axisymmetric particle whose configuration is given in terms of the vector n_i which accounts for its orientation. In the model, at each point the evolution of an ensemble of polymers, forced by the fluid dynamic field and by a Brownian noise, is considered and the moments of n_i, $\mathcal{R}_{ij} = \langle n_i n_j \rangle$, $\mathcal{R}_{ijkl} = \langle n_i n_j n_k n_l \rangle$, are introduced. Hence the field equation for the covariance matrix reads according to [3]

$$\frac{\partial \mathcal{R}_{ij}}{\partial t} + u_k \frac{\partial \mathcal{R}_{ij}}{\partial x_k} = K_{ir}\mathcal{R}_{rj} + \mathcal{R}_{ir}K_{jr} - 2E_{kl}\mathcal{R}_{ijkl} - 6\gamma_B \left(\mathcal{R}_{ij} - \frac{\delta_{ij}}{3} \right), \tag{1}$$

where K_{ij} and E_{ij} are the velocity gradient and its symmetric part, respectively, and γ_B is the Brownian rotational diffusion. For the stress tensor, again following [3] we have

$$T^p_{ij} = \mu_p \left[E_{kl}\mathcal{R}_{ijkl} + 6\gamma_B \left(\mathcal{R}_{ij} - \frac{\delta_{ij}}{3} \right) \right], \tag{2}$$

μ_p is the rodlike polymers contribution to viscosity depending on the number density, which is proportional to the zero shear viscosity. In the present work we will consider a case where the Brownian diffusion term can be neglected, *i.e.* $\gamma_B \to 0$. Let us note that in this limit the polymer model depends only on the parameter μ_p. In the simulations the simple closure hypothesis will be used, *i.e.* $\mathcal{R}_{ijkl} = \mathcal{R}_{ij}\mathcal{R}_{kl}$ [4]. Considering the closure and the hypotesis of the absence of Brownian effects, the dynamics of the solution is described by the continuity equation, a slightly modified form of the Navier-Stokes equation and an equation for the conformation tensor, whose nondimensional forms read

$$\begin{aligned}
&\frac{\partial u_i}{\partial x_i} = 0, \\
&\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\eta_p}{Re}\frac{\partial}{\partial x_j}\left(E_{kl}\mathcal{R}_{ij}\mathcal{R}_{kl} \right), \\
&\frac{\partial \mathcal{R}_{ij}}{\partial t} + u_k \frac{\partial \mathcal{R}_{ij}}{\partial x_k} = K_{ir}\mathcal{R}_{rj} + \mathcal{R}_{ir}K_{jr} - 2E_{kl}\mathcal{R}_{ijkl}\,.
\end{aligned} \tag{3}$$

Only two nondimensional numbers appear in the previous system, namely the Reynolds number $Re = U_0 L_0/\nu$ and the relative viscosity $\eta_p = \mu_p/\mu$.

3. – Results

For the present DNS, the dimensions of the domain are $2\pi h \times 2h \times \pi h$, where h is half the channel height. The numerical formulation is a pseudo-spectral method with Fourier expansion in the directions parallel to the wall and Chebyshev in the wall-normal direction. The simulations have been performed at a nominal Reynolds number of 10000, for all cases, Newtonian and rodlike polymers. For the polymer simulations the value of η_p were set to 25 and 50. The flow has been forced on average with a fixed pressure drop, so the resulting Reynolds number based in the friction velocity was pre-established and equal to $Re_\tau = 300$ and $Re_\tau = 400$. The four runs for the rodlike polymers are summarised in table I.

In this framework drag reduction corresponds to an increased flow rate, and, when the mean velocity profile is plotted in the usual viscous units an increased value of the

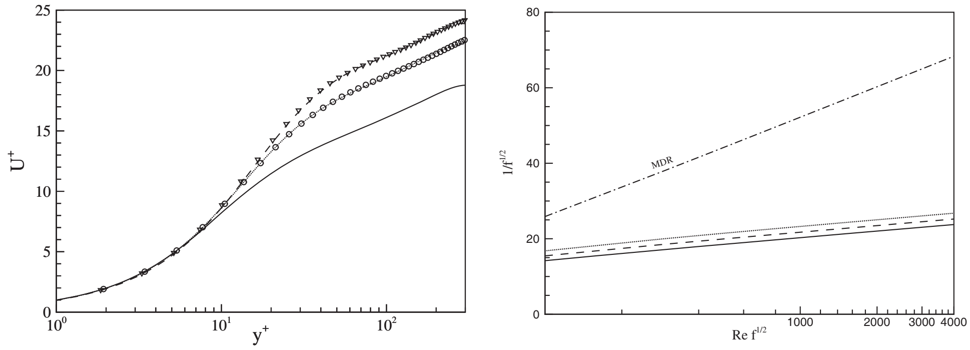

Fig. 1. – Left panel: Mean-velocity profiles for the simulations of table I. $Re_\tau = 300$ at $\eta_p = 25$ (dotted line) and $\eta_p = 50$ (dashed line), $Re_\tau = 400$ at $\eta_p = 25$ (circles) and $\eta_p = 50$ (triangles). For comparison the mean profile from a Newtonian simulation at $Re_\tau = 300$ is reported (solid line). Right panel: Prandtl-Karman plot. Newtonian (solid line), rodlike at $\eta_p = 25$ (dashed line), rodlike at $\eta_p = 50$ (dotted line) and MDR (dash-dotted line) high-Reynolds-number extrapolations are shown.

constant on the logarithmic region results (see fig. 1). Quantitatively drag reduction can be measured as the relative decrease of the friction factor

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho\overline{U}^2}\,, \tag{4}$$

which equals twice the value of the Fanning factor f. Introducing the viscous velocity $u_\tau = \sqrt{\tau_w/\rho}$, the decrease of f is described as

$$\frac{1}{\sqrt{f}} = \frac{\overline{U}}{u_\tau} = \overline{U}^+ = \frac{1}{Re_\tau}\int_0^{Re_\tau} \overline{u}^+ \mathrm{d}y^+. \tag{5}$$

The previous result establishes the relationship between the drag reduction evaluated in the mean-velocity profile and in the Prandtl-Karman plot. In particular, we observe that in the mean profile the shifting of the logarithmic region is independent of Reynolds number while it depends on the rodlike polymers concentration, see [5]. The consequences of this observation can be extrapolated in the P-K plot. Assuming the same behaviour at all Reynolds numbers it is possible to obtain the curve corresponding to a given concentration η_p. In the large Re_τ limit the function has a logarithmic dependence

Table I. – *Summary of the simulations.*

Re_τ	η_p	Domain	Collocation points
300	25	$2\pi \times 2 \times \pi$	$128 \times 193 \times 64$
300	50	$2\pi \times 2 \times \pi$	$128 \times 193 \times 64$
400	25	$2\pi \times 2 \times \pi$	$192 \times 193 \times 96$
400	50	$2\pi \times 2 \times \pi$	$192 \times 193 \times 96$

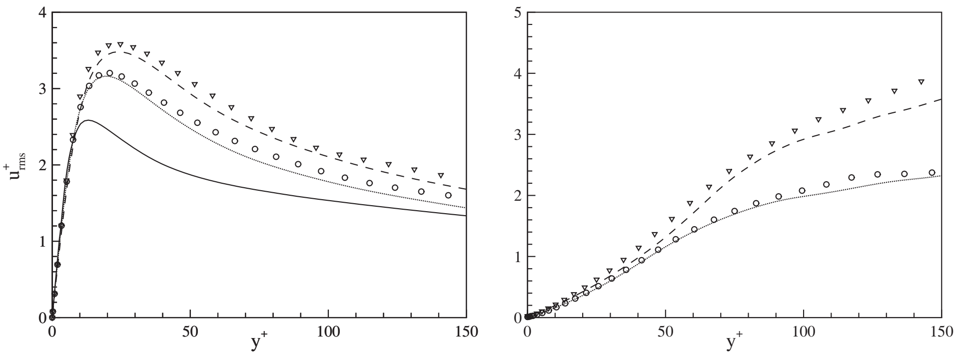

Fig. 2. – Left panel: Root-mean square velocities, $u'_{\mathrm{r.m.s.}}$. Right panel: effective viscosity evaluated as T^p_{xy}/S, where S is the mean shear. The symbols legend is the same as fig. 1.

$1/\kappa \log Re_\tau + C$, as for the Newtonian case, where the slope is the same as the mean-velocity profile while the intercept depends on the one in fig. 1. The ladder behaviour expected for rodlike polymers [6] is then reproduced. It is worth noting the difference with a flexible case where in the scaling of the mean velocity the value of the intercept would increase with Reynolds number. This behaviour is consistent with a line in the P-K plot whose slope is larger than the Newtonian case (a fan-like behaviour).

As is customary in drag reducing flows the velocity fluctuations are altered with respect to a corresponding Newtonian flow. Comparing the four simulations we observe an increase in the streamwise fluctuations more pronounced at the higher concentration but almost insensitive to Reynolds number, see left panel of fig. 2. This last evidence is consistent with the idea that the increase of the streamwise turbulence intensities in a drag reducing flow can be simply explained with a general increase of the kinetic energy of the flow at a given pressure drop. This means that most of the changes in the structure of the boundary layer can be considered proportional to the intercept shift in the logarithmic profile of the mean velocity. In order to strengthen the scenario that turbulent structure in rodlike polymers depends strongly only on one parameter, *i.e.* concentration, the effective viscosity, evaluated as the ratio between the mean polymeric shear stress T^p_{xy} and the mean shear $S(y) = \mathrm{d}U/\mathrm{d}y$, is shown in fig. 2 right panel. Even this observable shows a negligeable dependence on Reynolds number.

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REFERENCES

[1] Sureshkumar R., Beris A. N. and Handler A. H., *Phys. Fluids*, **9** (1997) 3.
[2] De Angelis E., Casciola C. M. and Piva R., *Comput. Fluids*, **31** (2002) 495.
[3] Doi M. and Edwards S. F., *The Theory of Polymer Dynamics* (Oxford, New York) 1988.
[4] Benzi R., Ching E. S. C., De Angelis E. and Procaccia I., *Phys. Rev. E*, **77** (2008) 046309.
[5] Gillissen J. J. J., Boersma B. J., Mortensen P. H. and Andersson H. I., *J. Fluid Mech.*, **602** (2008) 209.
[6] Sasaki S., *J. Phys. Soc. Jpn.*, **60** (1991) 868.