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HPC in global geodynamics: Advances in normal-mode analytical modeling

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Summary. — Analytical models based on normal-mode theory have been successfully employed for decades in the modeling of global response of the Earth to seismic dislocations, postglacial rebound and wave propagation. Despite their limited capabilities with respect to fully numerical approaches, they are yet a valuable modeling tool, for instance in benchmarking applications or when automated procedures have to be implemented, as in massive inversion problems when a large number of forward models have to be solved. The availability of high-performance computer systems ignited new applications for analytical modeling, allowing to remove limiting approximations and to carry out extensive simulations on large global datasets.

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1. – Introduction

While fully analytical models have been for decades a valuable tool in geodynamical modeling, in more recent years their importance has been reduced by the wide employment of fully numerical approaches, such as the finite-element method, which allows to set up realistic models including, for instance, detailed topography or 3D heterogeneities of the Earth's interior. Nevertheless, analytical modeling has not lost its relevance, since it allows a better understanding of the solution process with more control over numerical artifacts and it is therefore commonly used as a benchmarking tool for numerical models.

In this paper, I focus on the normal-mode framework (hereafter NM), which was originally introduced by Peltier [1] in the realm of viscoelastic Earth models, and it is employed to obtain the response of a spherical layered Earth to various excitations, such as surface loads or body force distributions. Its main shortcoming is the numerical instability connected with the solution of the so-called "secular equation", which may imply a loss of accuracy in the numerical solution. Since the polynomial degree of the secular equation scales with rheological model complexity, only coarse models can be safely employed.

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Several workarounds have been proposed in the literature [2-4] which allow to overcome these shortcomings either by introducing a purely numerical stage in the solution scheme or by assuming *a priori* some functional characteristics of the solution. Recently, a new solution scheme based on the application of the "Post-Widder formula" [5, 6] has been proposed in the realm of postglacial rebound [7] and postseismic relaxation [8]. With this method, the structure of the NM formalism is preserved but the explicit solution of the secular equation is not needed; at the same time, the resulting numerical codes are much simplified [9]. The cost of the Post-Widder method is a dramatic increase of computation times; for this reason, its application has become viable only with the wide availability of high-preformance computer systems.

2. – The viscoelastic normal mode framework

The theoretical details of the application of NM framework to postglacial and postseismic rebound models are widely discussed in the literature [10,11]; only the key aspects are outlined here. The equilibrium equations and the Poisson equation for a spherical, incompressible, self-gravitating viscoelastic body with spherical symmetry can be reduced to a system of algebraic equations. This is accomplished by i) expressing the physical observables on a spherical harmonic basis and ii) Laplace-transforming the governing equations. As a result, for each harmonic degree and order, the Laplace-transformed harmonic terms of the observables assume the following form:

$$\mathbf{x}(s) = \mathbf{Q}(s)\mathbf{R}(s)^{-1}\mathbf{b} + \mathbf{p}, \tag{1}$$

where s is the Laplace variable, the arrays $\mathbf{Q}(s)$ and $\mathbf{R}(s)$ are obtained by propagating the fundamental matrix of the system through the mantle and the vectors $\mathbf{b}$ and $\mathbf{q}$ account for boundary conditions at the free surface and at the core-mantle boundary. The time-domain solution has to be recovered by a Laplace inversion of eq. (1), which can be accomplished by an explicit integration over a Bromwich path integral, obtaining

$$\mathbf{x}(t) = \mathbf{x}_e\delta(t) + \sum_{k=1}^{N}\left[\frac{\mathbf{Q}(s)\mathbf{R}^{\dagger}(s) + |\mathbf{R}(s)|\mathbf{p} - |\mathbf{R}(s)|\mathbf{x}_e}{\frac{\mathrm{d}}{\mathrm{d}s}|\mathbf{R}(s)|}\right]_{s=s_k}, \tag{2}$$

where $\mathbf{x}_e = \lim_{s\to\infty}\mathbf{x}(s)$ is the elastic response and s_k are the isolated roots of the secular equation

$$|\mathbf{R}(s)| = 0. \tag{3}$$

When the solution scheme outlined above is implemented in a numerical code, several difficulties arise. The most remarkable ones are:

1) Assuming a Maxwell rheology, the number of roots of eq. (3) is $N = 4L$, where L is the number of mechanically distinct rheological layers. Since for high polynomial degrees the root-finding algorithms become unstable due to numerical noise and roots coalescence [9], the complexity of practically solvable models is actually limited to small values of L.

2) In order to compute the elastic limit $\mathbf{x}_e$ and the derivative of $\mathbf{R}(s)$ in eq. (2), every single polynomial coefficient in $\mathbf{Q}$ and $\mathbf{R}$ must be tracked through the products of

propagation arrays. This implies a rapidly increasing complexity of the code as L increases; moreover, if a rheological law more realistic than a Maxwell rheology is introduced, the algebraic complexity may be eventually tackled only through the use of symbolic manipulators [12].

3) If the excitation is represented by a pointlike seismic source, which contains virtually all harmonics, a stable convergence of the harmonic expansion is obtained only at degrees of the order 10^3–10^4 [8]. If a recursive expression is employed in evaluation of Legendre functions, error propagation may lead to numerical degeneration before the convergence of the harmonic series.

Despite these limitations, the NM framework can be conveniently implemented on a parallel architecture. Indeed, for each harmonic degree the solution scheme outlined above is completely independent of other degrees, so that a considerable speed-up of the solution can be obtained with little effort by distributing harmonic degrees over CPUs. This approach requires few communications between threads so that the performance of parallelized codes scales well up to hundreds of CPUs. This is possible as long as a spherically symmetric Earth is modeled, since the introduction of lateral heterogeneities leads to mode coupling effects [13] that may require a more complicated (and possibly less efficient) parallelization approach. The speed-up obtained with a parallelized NM code enables to compute a large number of forward solutions; this can be employed, for instance, in i) running global-scale simulations to compute the cumulative effect of a large number of sources [14], ii) modeling a high-resolution discretization of a 3D source structure [15], or iii) when solving large inverse problems.

3. – A new class of models from the application of the Post-Widder Laplace inversion

To overcome the numerical difficulties outlined in the previous section, a new solution scheme has been proposed both in the realm of postglacial [7] and postseismic [8] rebound models. This scheme is based on the application of the so-called "Post-Widder formula" [5, 6], which provides a convenient approximation of the Laplace inverse of a function through a sampling of the anti-transform on the real positive axis. For practical applications, a discretized version [16] of the Post-Widder formula can be employed:

$$f(t, M) = \frac{\ln 2}{t} \sum_{k=1}^{M} \zeta_k \tilde{f}\left(\frac{k \ln 2}{t}\right), \tag{4}$$

where $\tilde{f}(s)$ is the Laplace transform of $f(t)$, M is the order of the approximation, ζ_k are weights that depend only on k and M, and $f(t, M) \to f(t)$ for $M \to \infty$. This algorithm allows to recover the time-domain solutions without invoking the residue theorem, as in eq. (2). In this way, there is no need to find the roots of eq. (3) and the matricial polynomials $\mathbf{Q}(s)$, $\mathbf{R}(s)$ can be sampled at $s_k = k \ln 2/t$ without tracking their polynomial coefficients. As a result, i) rheological profiles of arbitrary complexity can be safely modeled and ii) generalized linear rheologies can be easily accounted for. These modelistic improvements come at a cost: the sum in eq. (4) contains oscillating terms that can lead to catastrophic cancellations with loss of precision or, at worse, numerical degeneration. For this reason, a safe implementation of the PW algorithm requires the use of high-precision arithmetic, through one of the publicly available multi-precision libraries. This results in a substantial computational overhead with respect to standard hardware-supported numerical formats, which leads to a consistent performance degradation. For this reason, the PW Laplace inversion has not found application until the wide availability of high-performance computer systems.

The most attractive feature of the application of PW Laplace inversion in postglacial and postseismic rebound models is that it allows to expand their modelistic capabilities while retaining the same formal structure of normal modes. By allowing to account for rheological models of arbitrary layering resolution, it has been possible, for instance, to obtain the minimum resolution required to fit the postglacial rebound obtained with reference models within a specified threshold [7] or to investigate the rheological layering resolution needed to model postseismic deformation within typical experimental errors of geodetic techniques [8].

4. – Conclusions

The rapidly increasing availability of high-performance computing systems can enable new applications for analytical models based on the normal-mode framework. Analytical models, opposed to fully numerical approaches such as the finite-element method, offer better control over numerical artifacts and the advantage of being easier to employ in automated procedures. With the computational speed-up that can be obtained with a parallel NM code, large-scale global simulations, inverse problems, or automated "near-real time" applications can be conveniently implemented. While the standard normal-mode approaches suffer from intrinsic limitations that may affect their actual modeling capabilities, it has been shown that a suitable reformulation of the solution scheme by taking advantage of the Post-Widder Laplace inversion allows to bypass many shortcomings. The extended modelistic capabilities come at the cost of a large increase in computational requirements, that can be handled with the wide availability of processing power.

* * *

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