IL NUOVO CIMENTO Vol. 29 C, N. 6 Novembre-Dicembre 2006
DOI 10.1393/ncc/i2005-10195-0

Dry and wet spells at Itabaina (NE Brazil)(*)

K. Karuna Kumar and T. V. Ramana Rao

Department of Atmospheric Sciences, Federal University of Campina Grande
58109-970 Campina Grande-PB, Brazil

(ricevuto il 10 Ottobre 2005; revisionato l'8 Agosto 2006; approvato il 2 Settembre 2006; pubblicato online il 29 Novembre 2006)

Summary. — Results of a study of dry and wet spells at Itabaina, a semi-arid station in northeast Brazil, are presented in this paper. Frequency distributions of dry spells of different durations at the station are compared with those derived from three statistical models. It is found that the Eggenberger-Polya and the geometric model provide better results than the logarithmic model. Frequency distribution of wet spells during the three rainiest months of the year shows that rainfall persistency reaches a maximum value on the fifth day of a rainy spell.

PACS 92.60.-e – Properties and dynamics of the atmosphere; meteorology.
PACS 92.60.Jq – Water in the atmosphere.

1. – Introduction

The semi-arid zone of northeast Brazil is 860000 km^2 in area and contains nearly 10% of the country's population. The main climatic characteristics of this region are: annual rainfall of 400–800 mm with a coefficient of variability of up to 60%, high air temperature and high potential evapotranspiration values. In the semi-arid region the main constraint to crop production is the rainfall and its extreme variability.

Information on the occurrence of dry and wet spells during the crop-growing season is a matter of much importance in regions with limited water resources. Results of a study of dry and wet spells at Itabaina (35° 20′ W, 7° 20′ S), a semi-arid station in northeast Brazil, are presented in this paper. Daily rainfall data at the station during the period 1911-1976 is used in the study. Due to missing information the year 1916 is not considered.

Various studies have been carried out in the past to estimate the frequency distribution of sequences of dry days. The most often used model is the simple Markov chain which assumes that the probability of any particular day being dry or wet depends only on

(*) The authors of this paper have agreed to not receive the proofs for correction.

the nature of the previous day [1-3]. Higher-order Markov chain models have been used by Feyerherm and Bark [4]. The logarithmic series has been suggested by Williams [5] as a fit to dry- and wet-spell distribution. The Eggenberger-Polya model [6] has been employed by Berger and Goossens [7] to estimate dry- and wet-spell frequencies.

2. – Methodology

In this paper a dry day is defined as a day with no rainfall. A dry spell is a sequence of dry days bracketed by wet days on both sides. Daily rainfall data at Itabaina for the months April-June during the period 1911-1976 is used to obtain the frequency distribution of dry spells of different durations. These frequencies are compared with those computed using a) the logarithmic model, b) the Eggenberger-Polya model and c) the geometric model.

The logarithmic series is mathematically written as [5]

$$\alpha x, \alpha\frac{x^2}{2}, \alpha\frac{x^3}{3}, \alpha\frac{x^4}{4}\dots,$$

where each term, in the case of dry-spell series, represents the number of spells of dry days lasting over 1, 2, 3, etc. days.

The two constants x (which is always less than unity) and α can be computed from the two simultaneous equations:

$$S = \alpha \ln(1 + N/\alpha),$$
$$N = \frac{\alpha x}{(1-x)},$$

where S is the total number of dry spells and N is the total number of dry days involved in the study period. For the estimation of α and x, the procedure given by Williams [8] is used.

In the Eggenberger-Polya [6] model the frequencies (ϕ) of dry spells of 1, 2, 3 etc. days are given by

$$\phi_1 = \frac{S}{(1+d)^{m/d}}, \quad \phi_2 = \phi_1\frac{m}{1+d}, \quad \phi_3 = \phi_2\frac{m+d}{2(1+d)},$$

where $d = (\sigma^2/m) - 1$, $(m+1)$ is the mean length of a dry spell and S is the total number of dry spells.

The geometric series is written as $Ay, Ay^2, Ay^3, Ay^4, \dots$ where the terms represent frequencies of dry spells of duration 1, 2, 3 etc. days. The values of A and Y are computed using the procedure given in Cooke [9].

The dry-spell frequencies derived from these models are compared with observed frequencies at Itabaina. The three models are evaluated in terms of Mean Bias Error

(MBE), Root Mean Square Error (RMSE) and the Mean Percentage Error (MPE).

$$\text{MBE} = \frac{\sum_{i=1}^{n}(x_{ie} - x_{im})}{n},$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(x_{ie} - x_{im})^2}{n}},$$

$$\text{MPE} = \frac{\sum_{i=1}^{n}\left[\frac{(x_{ie} - x_{im})}{x_{im}} \cdot 100\right]}{n}.$$

In these expressions X_{ie} and X_{im} are the i-th estimated and measured values and n is the number of observations. A positive value of MBE suggests an overestimation and a negative value an underestimation by the model. The values of MBE represent a systematic error or bias while the RMSE represents a non-systematic error. In the case of MPE the signs of the errors are neglected and the percentage errors are added up to derive the mean value. For a model to be a good estimator MPE, MBE and the RMSE should be small.

Daily rainfall data for the months April-June during the period 1911-1976 is used for the study of rainfall persistency. These months represent the wettest part of the year at this station. Rainfall during these months is attributed to easterly disturbances which enter the region from the Atlantic ocean. Their frequency varies on interannual and interseasonal time scales but there are from three to five events per month. The influence of any particular event on local rainfall can last from one to five or six days.

A wet spell is defined as a sequence of rainy days bracketed by dry days on both sides. Rainfall persistency is defined as the observed percentage frequency in excess of chance that a period of M rainy days is followed by N or more consecutive days with rain beginning L days later [10].

From the daily rainfall data frequencies of wet spells of different durations are obtained. Frequencies of occurrence of rain on the first, second and third etc. days following wet spells of various durations are also derived.

3. – Results

The total number of dry days is 4015 or approximately 68% of the number of days considered in the study. The number of dry spells of all durations is 875. The number of dry spells of more than eight days duration is less than 10% of this total. Dry spells which occur rarely are not of much importance and observed frequencies of spells of up to eight days duration are compared with frequencies derived from the three models described above (table I). Parameters of the three models are included in the table. From table I it can be seen that the geometric and Eggenberger-Polya models yield better results than the logarithmic model. Frequencies obtained from the Eggenberger-Polya model are closer to the observed frequencies than those from the geometric series. According to the EP model the total number of dry spells is 870 and 66% of the first dry days will be followed by at least one more dry day. Similarly 73% and 75% of the second and third

TABLE I. – *Observed and theoretical frequencies of dry spells at Itabaina.*

Duration of dry spells in days	Observed frequencies	Theoretical frequencies		
		EP	Log	Geometric
1	289	300	317	275
2	163	156	146	188
3	114	104	90	129
4	73	74	62	89
5	59	55	46	61
6	43	41	35	42
7	29	31	28	29
8	21	24	22	20
MBE		−0.8	−5.6	5.3
RMSE		6.2	15.8	12.8
MPE		6.4	13.1	8.2

$A = 400.7574$, $\alpha = 344.9109$, $d = 4.3054$, $Y = 0.68587$, $X = 0.92089$, $m = 2.7611$, $S = 875$.

dry days will be followed by at least one more dry day. The chances of a four-day dry spell lasting for one more day is 76%.

In many studies on rainfall climatology a first-order Markov chain model is used. If the probability of a dry day being followed by a wet day $P(\text{w/d})$ is denoted by α and the probability of a wet day being followed by a dry day $P(\text{d/w})$ is denoted by β the mean length of a dry spell is $1/\alpha$ and the mean length of a wet spell is $1/\beta$.

In the present case $\alpha = 0.22$ and $\beta = 0.46$ and the mean lengths of dry and wet spells at the station are about 5 and 2 days, respectively. The period of study is the rainiest part of the year at the station and is hence the central part of the crop-growing period. The mean dry spell length of 5 days during this period suggests that the rainfall regime is not very favourable for crop growth.

It may be mentioned here that the mean rainfall during the period April-June at the station for two 30 year periods 1917-1946 and 1947-1976 is 357 mm and 320 mm, respectively. The corresponding change in the number of rainy days is from 38 to 33.

The total number of days considered in the study is 5915 and the number of wet days is 1900. The probability on chance of rain on any day (P) is 0.32 and on this basis the probability of a wet spell of one, two or three days is 0.32, 0.10, 0.03 and so on.

Frequencies of occurrence of wet spells of different durations expected on chance are computed using the expression [11]

$$F_r = P^r Q(2 + Q(m - r - 1))n,$$

where F_r is the frequency of a wet spell of duration r days, P is the probability of a given day being wet, $Q = 1 - P$, m is the mean number of days in a season and n is the number of years considered. The frequencies obtained agree with those computed from the equation NP^rQ^2 [7], where N is the total number of days considered in the study.

TABLE II. – *Wet spells of different durations at Itabaina.*

	1	2	3	4	5	6	7	8	9	10	11	12
Observed frequencies	477	197	76	43	20	23	16	10	3	2	2	5
Frequencies expected on chance	884	280	89	28	9	3	1	0	0	0	0	0
Cumulative totals of observed frequencies	878	401	204	128	85	65	42	26	16	13	11	9

The frequencies obtained are given in table II together with the observed frequencies of wet spells. Also included in the table are cumulative totals of observed wet spells from the longest to the shortest. Spells of duration 14, 16, 19 and 21days—each of which occurred once—are not included in the table.

The occurrence of wet spells lasting one, two and three days is less than that expected on chance and wet spells lasting longer than three days are more frequent than expected on chance basis.

From table II it is seen that out of 878 spells of at least one day duration 401 or 46% are of at least two days duration, 204 cases or 23% are of at least three days duration, etc. Of the 401 cases of wet spells of at least two days length 51% are of at least three days length, and 32% are of at least four days length. Such information is presented in table III together with the probabilities of a wet spell continuing for 1, 2, 3, etc. days on a chance basis.

The highest value of probability of a wet spell continuing for at least one more day occurs after a wet spell of five days with a value of 76% as compared with 32% expected on chance. Probability of a wet spell continuing for at least two more days is the highest after a sequence of four rainy days... According to the definition adopted in this study,

TABLE III. – *Percentage frequencies of occurrence of rain on N days following a wet spell of M days. Probabilities of N consecutive rainy days expected on chance are given at the top.*

	32%	10%	3%	1%	< 1%	< 1%	< 1%
M	61	50	42	35			
8	62	38	31	26	21		
7	65	40	25	20	17	14	
6	76	49	31	19	15	13	10
5	66	51	33	20	13	10	8
4	63	42	32	21	13	8	6
3	51	32	21	16	10	6	
2	46	23	15	10	7	5	
1							
N	1	2	3	4	5	6	7

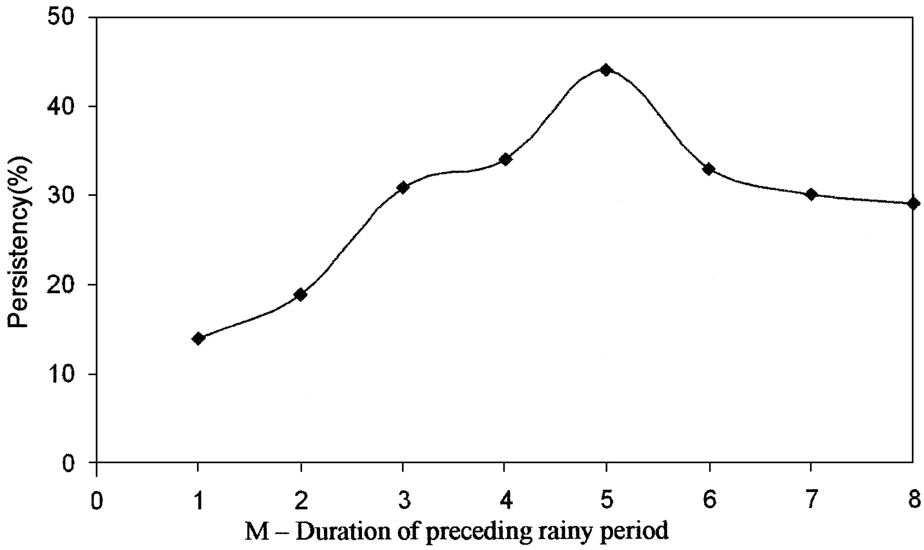

Fig. 1. – Persistency of rainfall at Itabaiana with $L = 1$, $N = 1$.

rainfall persistency is given by the difference between the observed probabilities of rain continuing for one, two, three, etc., days and the corresponding probabilities based on chance. Persistency values for $L = 1$ and $N = 1$ are shown in fig. 1. It can be seen that persistency reaches a maximum value of 44% on the fifth day of a wet spell. On the other hand, it has a low value of 14% on the first day of a wet spell.

4. – Conclusions

Frequencies of dry spells of different durations at Itabaina (PB) are compared with those derived from three statistical models. It is found that the Eggenberger-Polya model and the geometric models provide good estimates of dry-spell frequencies at the station. The incidence of wet spells lasting one, two and three days is less than expected on chance and spells lasting longer than three days are more frequent than expected on chance basis. The rainfall persistency reaches a maximum value on the fifth day of a wet spell.

REFERENCES

[1] Gabriel K. R. and Neumann, *J. Q. J. R. Meteorol. Soc., London*, **88** (1962) 90.
[2] Caskey J. E., *Mon. Weather Rev.*, **91** (1963) 298.
[3] Weiss L. L., *Mon. Weather Rev.*, **92** (1964) 169.
[4] Feyerherm A. M. and Bark L. D., *J. Appl. Meteorol.*, **6** (1967) 770.
[5] Williams C. B., *Q. J. R. Meteorol. Soc.*, **78** (1952) 91.
[6] Eggenberger F. and Polya G., *Z. Angew. Math. Mech. (Berlin)*, **3** (1923) 279.
[7] Berger A and Goossens C. H. R., *J. Climatol*, **3** (1983) 21.
[8] Williams C. B., *J. Ecol.*, **34** (1947) 253.
[9] Cooke D. S., *Q. J. R. Meteorol. Soc., London*, **79** (1953) 536.
[10] Jorgensen D. L., *Mon. Weather Rev.*, **77** (1949) 303.
[11] Cochran W. G., *Q. J. R. Meteorol. Soc. London*, **64** (1938) 631.