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On classical meteor light curves and utilitarian model atmospheres

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Summary. — We present a series of classical meteor light curve profiles based upon a set of simplified analytic atmospheric models. The model atmospheres specifically express the density variation as a power law in atmospheric height, and are derived under a variety of assumptions relating to the atmospheric temperature profile and the variation of the acceleration due to gravity. We find that the light curve profiles show only small differences with respect to any variation in the temperature profile and the geometry imposed upon the atmospheres.

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1. – Introduction

Useful pedagogical and physical insight may be gained through the construction of simplified models that yield straightforward analytical expressions for otherwise complex variable quantities. Here we look at a series of simplified atmospheric models that yield analytic expressions for the variation of a classical meteor's brightness with height. We consider a range of atmospheric models, each allowing for varying degrees of physical sophistication, and we consider the meteoroid to interact with the atmosphere under the idealized, classical ablation condition that imposes constant meteoroid velocity.

The classical meteor light curve model was initially developed under the constraint of numerous physical simplifications (see, *e.g.*, [1]). It is assumed, for example, that a meteoroid is a monolithic spherical grain that ablates in the Earth's upper atmosphere without deceleration. It is also implicitly assumed in the classical approximation that the meteoroid adopts a constant shape factor (*i.e.*, typically that of a sphere), and that the meteoroid has a uniform composition. Likewise parameters such as the heat and momentum transfer coefficients and the luminous efficiency are assumed to be constant

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during the ablation process. While all of the above assumptions may be questioned and modified according to various levels of physical sophistication the classical model is still useful since it provides a standard light curve against which more complex numerical morphologies can be compared.

The classic meteor light curve may be described as being late peaked in the sense that the rise time to maximum brightness is always greater than the fall time from maximum brightness. Indeed, it can be shown that this characteristic of classical light curves must hold true irrespective of the assumed mass, composition and initial velocity of the meteoroid when an isothermal, exponential atmosphere model is employed [2]. In addition, it has often been said and, indeed, it is often written that the shape of a classical light curve is derived according to the multiplication of two competing exponential terms. The two competing terms are related to the atmospheric density and the meteoroid mass, with the latter term decreasing and the former term increasing exponentially with decreasing atmospheric height. In this communication we show that the classic meteor light curve can also be effectively described according to atmospheric models in which the atmospheric density varies as a power law in height. In sect. **2** below we briefly describe the ablation equations for the classical model, while in sect. **3** we turn our attention to a very brief discussion of simplified model atmospheres. The classic light curve is then described in sect. **4** and sect. **5** provides a final discussion.

2. – The classical ablation equations

The ablation mass loss and meteoroid deceleration equations may be derived according to conservation of energy and conservation of momentum arguments [1]. Here we present the equations in the parameterized form introduced by Beech and Murray [3]. According we define the variables $\mu = \eta m^{1/3}$ and $U = \varepsilon V^2$, where m is the meteoroid mass, and V is the meteoroid velocity. The variable μ has units of mass per unit area, while U is dimensionless, and η and ε are constants relating to the physical characteristics of the meteoroid (see Appendix A of Beech and Murray [3]). Under the constant velocity condition the two equations for meteoroid ablation reduce to a differential equation of the form $\mathrm{d}\mu/\mathrm{d}\psi = U$, where $\mathrm{d}\psi = \rho(h)\mathrm{d}h$, and where $\rho(h)$ is the atmospheric density at height h. In the classical situation, the equation describing the variation of the meteoroids mass per unit area is accordingly

$$\mu(h) = \mu_0 + U \int \rho(h)\mathrm{d}h \,, \tag{1}$$

where μ_0 is the value of μ at h_0 and where the integral on the r.h.s. of (1) is taken over the height interval $h < h_0$ to h_0.

3. – Model atmospheres

By far the most commonly employed analytic atmosphere model is that corresponding to an isothermal ideal gas with a constant pressure scale height. This is the model atmosphere outlined in all introductory astronomy and planetary science texts, and indeed it provides an analytically useful variation of the atmospheric density with height [4, 5]. Accordingly, the isothermal atmosphere has a density variation of the form $\rho(h) = \rho_0' \exp[-h/H]$, where h is the atmospheric height, ρ_0' is a constant reference density, and H is a constant atmospheric scale height. Various levels of sophistication may be imposed upon the isothermal approximation. If, for example, one allows for a constant gradient scale height such that $H = H_0 + \theta h$, where H_0 is the pressure scale height at $h = 0$, and $\theta = \mathrm{d}H/\mathrm{d}h$, then the density variation may be written as a power law (see, *e.g.*, [4]),

$$\rho(h) = \rho_0 \left(1 + \frac{\theta\, h}{H_0}\right)^{-(1+\theta)/\theta} . \tag{2}$$

In the limit that $\theta \to 0$, it can be seen that eq. (2) reduces to the "standard" exponential form of density variation. In terms of limiting approximation we note that if $\theta \sim 0$, and $H_0 \gg h$, then eq. (2) (and also for that matter the standard exponential representation) reduces to a linear density approximation: $\rho(h) = \rho_0(1 - h/H_0)$. This being said, the condition that $H_0 \gg h$ does not apply in the region in which meteors typically ablate.

As an alternative to the formalism introduced above we may alternative produce power law representations of $\rho(h)$ by allowing the atmospheric temperature and gravitational acceleration to vary with height in the following form:

$$g/g_0 = (h_0/h)^{\alpha} , \tag{3}$$

$$T/T_0 = (h_0/h)^{\beta} , \tag{4}$$

where T_0 and g_0 are the temperature and gravitational acceleration at some reference height h_0 [6]. For an atmosphere in hydrostatic equilibrium the downward gravitational attraction term is counterbalanced by the upward pressure gradient so that $\partial P/\partial h + \rho(h)g(h) = 0$. When variations in gravitational acceleration are assumed negligible (*i.e.*, $g(h) \approx g = \mathrm{const}$) then, for an isothermal ideal gas, the height dependency of the atmospheric pressure will be described by the familiar exponential variation: $P = P_0 \exp[-h/H]$, where $H = (R/n)T/g$ is the pressure scale height, and where R is the gas constant and n is the mean molecular weight of the atmospheric gas (assumed in this analysis to be a single atomic species). In the discussion presented below, however, we investigate some of the general analytic solutions to the equation of hydrostatic equilibrium when both the acceleration due to gravity and temperature are allowed to vary with height according to eqs. (3) and (4) above.

Formally, for a perfect gas in local thermal and hydrostatic equilibrium, the atmospheric pressure P will vary with height according to the equations

$$\mathrm{d}P/\mathrm{d}h + \rho(h)g(h) = 0 , \tag{5}$$

$$P = (R/n)\, \rho(h)\, T(h) . \tag{6}$$

Combining eqs. (3) and (4) with eqs. (5) and (6), we find the following expression for the atmospheric density variation:

$$\frac{\mathrm{d}\rho}{\rho} = \left[\frac{\beta}{x} - \varphi\, x^{(\beta-\alpha)}\right] \mathrm{d}x , \tag{7}$$

where $x = h/h_0$ and $\varphi = h_0/H_0$ and where $H_0 = (R/n)T_0/g_0$ is the atmospheric scale height at h_0. In principle any number of atmospheric models may be generated according

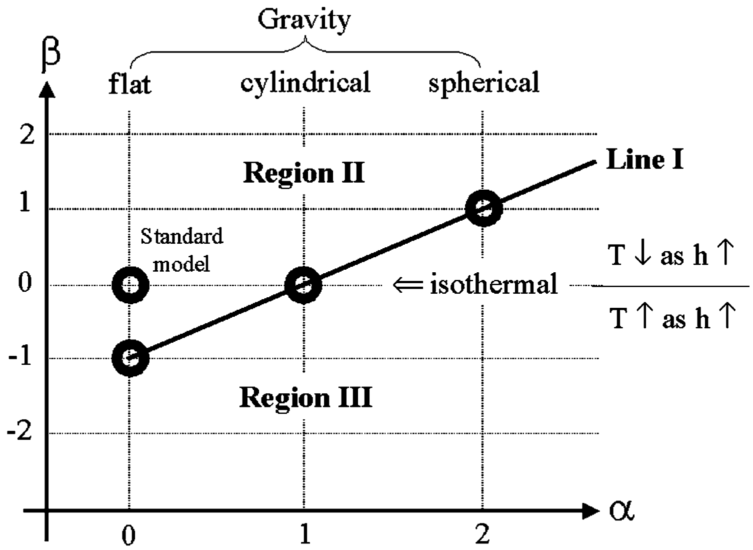

Fig. 1. – The schematic solution space to eq. (7). The "standard model" corresponds to the exponential, isothermal model atmosphere. All models with $\beta = 0$ are isothermal, whereas models with $\beta > 0$ have a temperature profile that decreases with increasing height. Models with $\beta < 0$ have a temperature profile that increases with increasing height. Diagram based upon Eshleman and Gurrola [6].

to the assumed parings of (α, β). In practice, however, only a restricted range of (α, β) pairs are worthy of investigation.

The solution space to eq. (7) that we wish to study is shown schematically in fig. 1. As the index α varies from 0, to 1 to 2, the gravitational acceleration term, as given by eq. (3), corresponds, respectively, to flat, cylindrical and spherical geometry. When $\beta = 0$ the atmosphere is isothermal. If $\beta < 0$ the atmospheric temperature increases with increasing height—as indicated by eq. (4). In contrast, if $\beta > 0$ the atmospheric temperature decreases with increasing height. The "standard" exponential, isothermal atmosphere model corresponds to the combination (α, β) = (0, 0). It might, at first thought, seem that a good choice for an atmospheric model would be the one corresponding to (α, β) = (2, 0), to allow for "spherical" geometry, but the resultant analytic form for the density variation with height is not well suited to analytic manipulation (see below).

In fig. 1 we identify two main regions (Regions II and III) and the boundary line between them (Line I). The boundary line corresponds to those models for which $\alpha - \beta = 1$. On this boundary line the solutions to eq. (7) are power laws in the normalized atmospheric height $x = h/h_0$, and the atmospheric density varies as $\rho \sim x^{(\beta-\varphi)}$. In Region II the solutions to eq. (7) are all of the form $\rho \sim x^{\beta} \exp[-\varphi x]$. While in Region III the solutions to eq. (7) are of the form $\rho \sim x^{\beta} \exp[\varphi/x]$. It is the form of the solutions to eq. (7) in Region III that makes the (α, β) = (2, 0) model unwieldy to analytic manipulation. The pressure scale height H is defined according to the equation $H\mathrm{d}P/\mathrm{d}h + P = 0$, and consequently substituting from eqs. (3) through (6), we find that the scale height varies as $H = H_0 x^{(\alpha-\beta)}$ for the models under consideration. Since, however, we are primarily interested in those atmospheres for which $\alpha - \beta = 1$, it is revealed that we are dealing with model atmospheres in which the pressure scale height varies linearly with height.

TABLE I. – *Model atmosphere characteristics. Note in column 5 that the reference density, ρ_0', for the "standard", exponential isothermal atmosphere is not numerically equal to ρ_0 applicable in the other models. This follows since for the exponential atmosphere $\rho_0' = \rho(h_0)\exp[\varphi]$, while for the power law atmosphere models $\rho_0 = \rho(h_0)$. The vertical arrows indicate whether a quantity is increasing or decreasing.*

Model	$(\alpha,\ \beta)$	Geometry	Temperature	Density	Power
1	(0, 0)	Flat	Isothermal	$\rho_0' e^A$	$A = -x\varphi$
2	(0, −1)	Flat	$T \uparrow$ as $h \uparrow$	$\rho_0 x^B$	$B = -(1+\varphi)$
3	(1, 0)	Cylindrical	Isothermal	$\rho_0 x^C$	$C = -\varphi$
4	(2, 1)	Spherical	$T \downarrow$ as $h \uparrow$	$\rho_0 x^D$	$D = 1-\varphi$

There are approximate physical analogues to some of the $(\alpha,\ \beta)$ combinations falling on Line I. Specifically, and with reference to table I, model 2 corresponds approximately to the thermosphere, whereas models 1 and 3 approximate to the mesopause; model 4, in turn, approximates the mesosphere. None of the model atmospheres described in table I have a formal truncation where the density goes to zero and the atmosphere, as such, physically ends. In addition, the "Line I" power law models have surface densities $\rho(h = 0)$ that are either singular if $\beta < \varphi$ (which will, in fact, be the most likely situation since in the typical small mass meteoroid ablation region $\varphi \approx 10$), or zero if $\beta \geqslant \varphi$. These boundary condition issues, however, are not our direct concern since we are only interested in the behaviour of the atmosphere in the region where meteoroid ablation actually takes place.

4. – Classical light curve models

If one assumes that the energy radiated at optical wavelengths is proportional to the kinetic energy of the ablated meteoroid material [1], then the intensity $I(h)$ of a classical meteor will vary according to the meteoroid mass and the atmospheric density, such that

$$I(h) = I_0 U^{5/2} \rho(h) \mu^2 \,, \tag{8}$$

where I_0 is a constant (see Appendix I of Beech and Murray [3] for an evaluation of this term). It is the last two terms of eq. (8) that determine the classical light curve profile as a function of height. Indeed, as the meteor descends through the atmosphere $\rho(h)$ will increase and since ablation is assumed to be taking place μ will decrease. It is these latter two terms that are the competing expressions that produce the classical meteor light curve profile.

Perhaps the simplest, although certainly not the most physically realistic, first example application of eq. (8) is that when the atmospheric density is taken to be a constant. In this case, we write $\rho(h) = \rho_0 \mathbf{H}(1-x)$, where $\mathbf{H}(1-x)$ is the Heaviside step function. We introduce the step function, where $\mathbf{H}(1-x) = 0$ for $x > 1$, and $\mathbf{H}(1-x) = 1$ for $x < 1$, to provide a "formal" reference height at which the atmosphere is deemed to "begin". For the constant density atmosphere we find from eq. (1) that

$$\left(\frac{\mu}{\mu_0}\right) = 1 - G_{\text{const}}[1-x] \,, \tag{9}$$

where $G_{\text{const}} = U\rho_0 h_0/\mu_0$. Equation (9) indicates an end height where $\mu = 0$ (that is, where the meteoroid mass goes to zero) at $x_{\text{end}} = 1 - 1/G_{\text{const}}$, and this also sets the condition $G_{\text{const}} \geqslant 1$. In this model case the light curves have a maximum intensity at $x = 1$, and there is no competition, as such, between the mass term and the atmospheric density. One might anticipate such light curve profiles to result when very small mass grains (or fragments) are ejected from a meteoroid at heights below that which they would normally have started ablative mass loss. During such "flare" events, vigorous grain ablation will proceed over a very short atmospheric path, and the constant atmospheric density model will approximately apply. A constant density model might also be applicable in the rare cases of earth-grazing fireballs (such as the August 10, 1972 event described by Jacchia [7] and Ceplecha [8].

The term G_{const} is related to both the mass per unit area of the meteoroid (through μ_0), and the mass per unit area of the atmospheric column at the reference height h_0 (through the $\rho_0 h_0$ term). And, as an "idealized" aside pertaining to this special model atmosphere, we note that the condition to produce a meteorite on the ground can be expressed as $x_{\text{end}} = 0$ or $G_{\text{const}} = 1$, which requires that $\mu_0 = U\rho_0 h_0$. Hence to produce a meteorite in the constant density atmosphere approximation the initial mass per unit area of the meteoroid must be U times greater than the mass per unit area of the atmospheric column through which it has to pass in order to reach the ground. This approximation provides a lower bound on the minimum mass per unit area for a meteoroid to produce a meteorite on the ground in the Earth's actual atmosphere. An upper bound on μ_0 may be placed according to the argument that a meteoroid will be appreciably decelerated if during its flight it has to displace more than its own body mass of atmospheric gas. Hence, under the constant velocity condition imposed in this analysis, we find that $(\rho_0 h_0)/4 > \mu_0$ must hold true. This limit sets, in turn, an upper bound on the value of $U < 1/4$. We find, therefore, that to produce a meteorite on the ground under the constant density atmosphere, constant velocity approximation a meteorite must have a diameter $D_{\text{met}} > 3(\rho_0 h_0)/2\delta$, where δ is the meteoroid density.

By inserting the density expressions for model atmospheres 2, 3, and 4, as described in table I, into eq. (1) and eq. (8) the general expression for the meteor intensity as a function of normalized height becomes

$$I(x) = I_{00} x^{(\beta-\varphi)} \left[1 - G_\alpha x^{(\alpha-\varphi)}\right]^2, \tag{10}$$

where $I_{00} = I_0 \rho_0 \mu_0^2 U^{5/2}$ and G_α is a constant such that

$$G_\alpha = G_{\text{const}}/(\varphi - \alpha) \tag{11}$$

and, as before, $x = h/h_0$ and $\varphi = h_0/H_0$. The height of maximum brightness and the end height at which $\mu = 0$ are easily found from eqs. (1) and (10), and they are

$$x_{\text{end}} = [G_\alpha]^{1/(\varphi-\alpha)} \tag{12}$$

and

$$x_{\max} = \left[\frac{3(\varphi - \alpha) + 1}{\varphi - \beta}\right]^{1/(\varphi-\alpha)} x_{\text{end}}\,. \tag{13}$$

TABLE II. – *End height and height of maximum brightness comparisons. In each calculation the meteoroid has a mass of* 10^{-6} *kg and a velocity of 70 km/s. Columns four and five give the end and maximum brightness heights in kilometers. The last column shows the* F*-value for each of the light curves shown in fig. 2. The value of* G_{exp} *in model 1 (exponential, isothermal atmosphere) is five orders of magnitude greater than the* G_α *values applicable in the other models due to the difference between* ρ_0' *and* ρ_0 *(see text for details).*

Model	(α, β)	G constant	h_{end} (km)	h_{max} (km)	F-value
1	(0, 0)	6.724×10^4	90.0	98.9	0.56
2	(0, −1)	0.0851	91.8	99.2	0.60
3	(1, 0)	0.0919	91.0	98.9	0.58
4	(2, 1)	0.0998	90.1	98.7	0.57

The equivalent expressions for the height of maximum brightness and the end height in the case of the exponential, isothermal atmosphere are

$$x_{\mathrm{end}} = (1/\varphi)\ln(G_{\mathrm{exp}}) \tag{14}$$

and

$$x_{\mathrm{max}} = (1/\varphi)\ln(3G_{\mathrm{exp}})\,, \tag{15}$$

where $G_{\mathrm{exp}} = (U\rho_0' h_0/\mu_0)/\varphi = (\rho_0'/\rho_0)\ (G_{\mathrm{const}}/\varphi)$, and recall that $\rho_0' \neq \rho_0$. The equation for the intensity is accordingly

$$I(x) = I_{00'}\left(\frac{\mu}{\mu_0}\right)^2 \exp[-x\varphi] \tag{16}$$

with, upon substitution in eq. (1),

$$\left(\frac{\mu}{\mu_0}\right) = 1 - G_{\mathrm{exp}}\exp[-x\varphi]\,; \tag{17}$$

eqs. (16) and (17) express the statement that the profile of the classical meteor light curve is the result of two competing exponential terms.

5. – Discussion

It has not been our intention in this paper to suggest that the power law approximations for the atmospheric density are necessarily superior to the standard isothermal model. Nor are we advocating them as alternatives to rival the full numerical integration of the ablation equations with a detailed atmospheric model. The latter calculations will always be superior with respect to the "quality" of their results. This being said we do advocate the use of such analytic models as useful guides to understanding what physical processes are at play in the determination of meteor light curve morphology.

The classical light curve analysis will hold true to good approximation when the meteoroid entry velocity is very high. For illustrative purposes only, therefore, we show in fig. 2 a comparison of classical meteor light curves constructed for a 10^{-6} kg monolithic

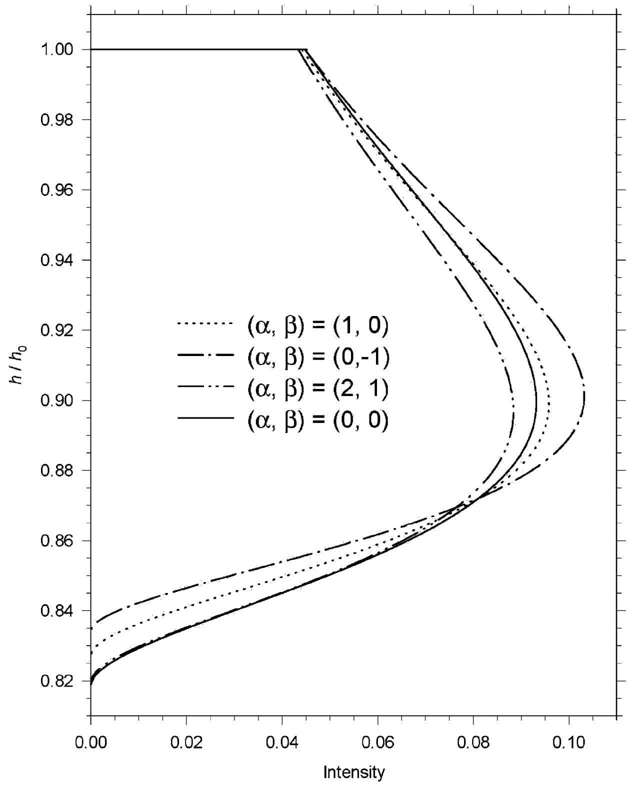

Fig. 2. – Comparison of classic light curves. The "standard model", isothermal atmosphere $[(\alpha,\ \beta) = (0,\ 0)]$, classic light curve is shown by the solid line. The light curves have been constructed with the following atmospheric parameters: $h_0 = 110$ km, $\rho_0 = 10^{-7}$ kg/m^3, $H = 8.1$ km (from [9]), which yields $G_{\mathrm{const}} = 1.156$, $\varphi = 13.58$, and $\rho_0' = 0.079$ kg/m^3. The terms in G_{const} assume a stone-like composition with a density of 3300 kg/m^3, a specific heat of 1200 J/kg/K, and a specific latent heat of vaporization of 6×10^6 J/kg. The momentum and heat transfer coefficients are assumed to be unity and the initial velocity is taken to be 70 km/s.

meteoroid entering the Earth's atmosphere at 70 km/s for each of the model atmospheres 1 through 4 (from table I). A comparison of end heights and heights of maximum brightness values are further given in table II. The end height values differ by less than 2 km, while the heights of maximum brightness differ by 0.5 km. The magnitudes at maximum brightness differ by less than 0.5 magnitude, and each light curve is late peaked (*i.e.*, the fall time from maximum is less than the rise time to maximum). Meteor light curve profiles are often described in terms of the F-parameter which is defined such $F = (1 - x_{\mathrm{max}})/(1 - x_{\mathrm{end}})$. Accordingly, $0 < F < 1$, and a perfectly symmetric light curve would have an F-value of 0.5. An F-value greater than 0.5 indicates that the light curve is late peaked. The F-value for each atmospheric model is given in the last column of table II. The classic light curves for models 2, 3 and 4 are all slightly, but not significantly, later peaked than the light curve for the standard isothermal atmosphere (model 1).

In general descriptive terms, for the isothermal models, going from flat to cylindrical geometry results in a slight brighter meteor at maximum but a slightly shorter trail length over all. Both light curves follow a similar rise to maximum brightness, but the brightness of the cylindrical geometry model falls off more rapidly than that of the flat geometry model. After maximum brightness, the spherical geometry, temperature decreasing with increasing height model "converges" with the flat geometry, isothermal model. The temperature increasing with increasing height, flat geometry model produces the shortest trail but the brightest meteor.

In conclusion, it would appear in general that variations in the atmospheric temperature profile (*i.e.*, whether it is constant, increasing or decreasing) and variations in the imposed atmospheric geometry (*i.e.*, flat, cylindrical or spherical) do not significantly effect the inherent shape of the classical meteor light curve. And, in addition, the classical meteor light curve need not be thought of as resulting specifically or exclusively from the competition of two competing exponential terms. Rather, as eq. (8) indicates, it is the variation of the atmospheric density with height and the variation of the meteoroids mass per unit area that determine the shape of the light curve.

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