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Spectral transfer for refractivity in inhomogeneous turbulence in atmospheric surface layer

Sukaran Ram Patel

Departamento de Ciências Atmosféricas, Universidade Federal de Campina Grande-UFCG Campina Grande-PB, 58109-970, Brazil

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Summary. — The two-point spectral equation for refractivity is constructed, neglecting the pressure fluctuations and terms that are second order in the fluctuations. It is shown that for this case, certain terms can be interpreted as transfer terms, even for a general inhomogeneous turbulence.

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1. – Introduction

The Monin-Obukhov similarity theory is one of the most powerful tools in describing the physical properties of the atmospheric boundary layer. Similarity relationship does not only apply to the mean profiles of the meteorological parameters, but also to the statistical quantities and spectral behavior of turbulence. The Monin-Obukhov similarity empirical functions are now used routinely in many practical applications [1]. Very recently a number of applications of this theory for refractive index spectrum has been reported in [1-4]. Using refractive turbulence structure parameters and optical-scintillation measurements of inner scale, one may obtain complete micrometeorological characteristics of the surface layer; and also the refractive-index spectrum in the dissipation range is important to atmospheric optical propagation studies of both weak and very strong scintillation [5]. There are several studies of similarity principles for horizontally homogeneous boundary layers but very few for horizontally inhomogeneous surfaces because of their obvious difficulties. These motivate the study of the spectral transfer of refractivity in inhomogeneous turbulent motion.

The terms associated with both turbulence self-interaction [6] and mean gradients [7] in the two-point spectral equation for homogeneous turbulence can be interpreted as transfer terms. However, a similar interpretation does not seem obvious for inhomogeneous turbulence, because the condition of homogeneity is generally used in making the

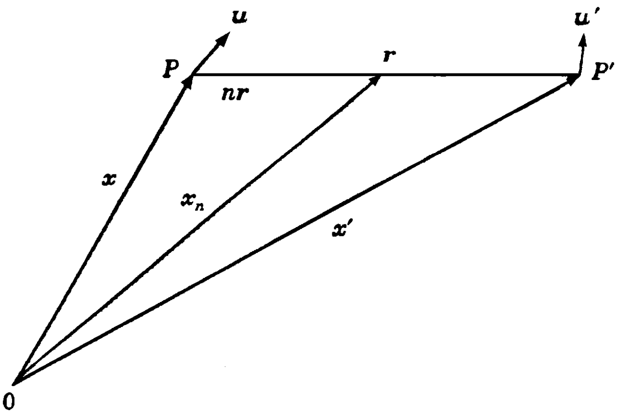

Fig. 1. – Vector configuration.

interpretation [7]. In this note it is shown, using the method of Deissler [7], that certain terms in the two-point spectral equation for refractivity in inhomogeneous turbulence in atmospheric surface layer can be interpreted as spectral-transfer terms, even for a general inhomogeneous turbulence.

2. – Methodology and discussion

Following Hill [8] the refractivity variance of air, $\overline{m^2}$, when the pressure fluctuations and terms that are second order in the fluctuations are neglected, may be written in the form

$$\overline{m^2} = a^2\overline{\theta^2} + b^2\overline{q^2} + 2ab\overline{q\theta}\,, \qquad a = (AP + CQ)/T^2\,, \qquad b = B - C/T\,, \tag{1}$$

where m is the fluctuation in the refractivity, T is the mean temperature, θ is the fluctuation of temperature, Q is the mean specific humidity, q is the fluctuation of humidity, P is the mean pressure; A, B, and C are functions of the wavelength. Now following Deissler [9, 10], the two-point correlation equations for temperature, humidity and cross-correlation between temperature and humidity can be constructed as

$$\begin{aligned}\frac{\partial\overline{\theta\theta'}}{\partial t} &= -\overline{u'_k\theta'}\frac{\partial T}{\partial x_k} - \overline{u'_k\theta}\frac{\partial T'}{\partial x'_k} - U_K\frac{\partial\overline{\theta\theta'}}{\partial x_k} - \\ &\quad -U'_k\frac{\partial\overline{\theta\theta'}}{\partial x'_k} - \frac{\partial\overline{u_k\theta\theta'}}{\partial x_k} - \frac{\partial\overline{u'_k\theta\theta'}}{\partial x'_k} + K\left(\frac{\partial^2\overline{\theta\theta'}}{\partial x_k^2} + \frac{\partial^2\overline{\theta\theta'}}{\partial x'^2_k}\right),\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial\overline{qq'}}{\partial t} &= -\overline{u'_k q'}\frac{\partial Q}{\partial x_k} - \overline{u'_k q}\frac{\partial Q'}{\partial x'_k} - U_K\frac{\partial\overline{qq'}}{\partial x_k} - \\ &\quad -U'_k\frac{\overline{qq'}}{\partial x'_k} - \frac{\partial\overline{u_k qq'}}{\partial x_k} - \frac{\partial\overline{u'_k qq'}}{\partial x'_k} + \nu_q\left(\frac{\partial^2\overline{qq'}}{\partial x_k^2} + \frac{\partial^2\overline{qq'}}{\partial x'^2_k}\right),\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial\overline{q'\theta}}{\partial t} &= -\overline{u'_k\theta}\frac{\partial Q'}{\partial x'_k} - \overline{u_k q'}\frac{\partial T}{\partial x_k} - U_k\frac{\partial\overline{q'\theta}}{\partial x_k} - \\ &\quad -U'_k\frac{\partial\overline{q'\theta}}{\partial x'_k} - \frac{\partial\overline{u_k q'\theta}}{\partial x_k} - \frac{\partial\overline{u'_k q'\theta}}{\partial x'_k} + \left(\nu_q\frac{\partial^2\overline{q'\theta}}{\partial x'^2_k} + K\frac{\partial^2\overline{q'\theta}}{\partial x_k^2}\right),\end{aligned} \tag{4}$$

where the usual tensor notation is used, the two points P and P' are separated by the vector r, the unprimed quantities are measured at point P and the primed quantities at P' (see fig. 1), the over bar indicates averaged quantities, U_k and u_k denote the mean and fluctuating components of velocity, respectively. K is the thermal diffusivity and ν_q is the molecular diffusivity of specific humidity. Now referring to fig. 1, one may have $\vec{x}+\vec{r}=\vec{x}', \vec{x}+n\vec{r}=\vec{x}_n$, from which one may obtain $\vec{x}_n = nx' + (1-n)x$ and

$$r_k = {x'}_k - x_k \qquad \text{and} \qquad (x_k)_n = n{x'}_k + (1-n)x_k\,, \tag{5}$$

where n is a number between 0 and 1. Using eqs. (1)-(4) and (5), one can write the two-point correlation equation for refractivity in the form

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{a^2\overline{\theta\theta'} + b^2\overline{qq'} + 2ab\overline{q'\theta}\right\} = \\
&-\frac{\partial}{\partial r_k}\left(a^2\left\{\overline{u'_k\theta\theta'} - \overline{u_k\theta\theta'}\right\} + b^2\left(\overline{u'_kqq'} - \overline{u_kqq'}\right) + 2ab\left\{\overline{u'_kq'\theta} - \overline{u_kq'\theta}\right\}\right) - \\
&-(U'_k - U_k)\frac{\partial}{\partial r_k}\left(a^2\overline{\theta\theta'} + b^2\overline{qq'} + 2ab\overline{q'\theta}\right) - \frac{\partial}{\partial(x_k)_n}\Big[a^2\{(1-n)\overline{u_k\theta\theta'} + \\
&+n\overline{u'_k\theta\theta'}\} + b^2\{(1-n)\overline{u_kqq'} + n\overline{u'_kqq'}\} + 2ab\{(1-n)\overline{u_kq'\theta} + n\overline{u'_kq'\theta}\}\Big] - \\
&-[(1-n)U_k + nU'_k]\cdot\frac{\partial}{\partial(x_k)_n}\left[a^2\overline{\theta\theta'} + b^2\overline{qq'} + 2ab\overline{q'\theta}\right] + F\,,
\end{aligned} \tag{6}$$

where the right-hand sides are, respectively, the turbulent self-interaction, interaction of turbulence with mean flow, diffusion, convection and F (sum of production, molecular diffusion of heat and specific humidity) terms. Now for self-interaction term consider

$$\begin{aligned}
&-\frac{\partial}{\partial r_k}\left[a^2\left(\overline{u'_k\theta\theta'} - \overline{u_k\theta\theta'}\right) + b^2\left(\overline{u'_kqq'} - \overline{u_kqq'}\right) + 2ab\left(\overline{u'_kq'\theta} - \overline{u_kq'\theta}\right)\right] = \\
&= \int_{-\infty}^{\infty}\left[a^2\Phi(\vec{\kappa},\vec{x}_n) + b^2\Psi(\vec{\kappa},\vec{x}_n) + 2ab\Omega(\vec{\kappa},\vec{x}_n)\right]\exp[i\vec{\kappa}\cdot\vec{r}]\,\mathrm{d}\vec{\kappa}\,,
\end{aligned} \tag{7}$$

where $(a^2\Phi + b^2\Psi + 2ab\Omega)$ is the three-dimensional Fourier transform of the self-interaction term, $\vec{\kappa}$ is the wave number vector and $\mathrm{d}\vec{\kappa} = \mathrm{d}\kappa_1\mathrm{d}\kappa_2\mathrm{d}\kappa_3$. As in the homogeneous turbulent field, $\partial\Big[a^2\left(\overline{u'_k\theta\theta'} - \overline{u_k\theta\theta'}\right) + b^2\left(\overline{u'_kqq'} - \overline{u_kqq'}\right) + 2ab\left(\overline{u'_kq'\theta} - \overline{u_kq\theta}\right)\Big]/\partial r_k$ for an inhomogeneous turbulence should be absolutely integrable over r in order for its Fourier transform to exist. Now our aim is to determine whether $\left[a^2\Phi + b^2\Psi + 2ab\Omega\right]$ can be interpreted as a spectral transfer term, as in the case of homogeneous turbulence. Normally this is true for the case of homogeneous turbulence [6], but here it is shown that this is true even for the case of a general inhomogeneous turbulence. To prove this, using eq. (5), eq. (7) is solved and can be written after some simplification as

$$\begin{aligned}
&\int_{-\infty}^{\infty}\left[a^2\Phi(\vec{\kappa},\vec{x}_n) + b^2\Psi(\vec{\kappa},\vec{x}_n) + 2ab\Omega(\vec{\kappa},\vec{x}_n)\right]\exp[i\vec{\kappa}\cdot\vec{r}]\,\mathrm{d}\vec{r} = \\
&= a^2\left[\overline{u'_k\theta\frac{\partial\theta'}{\partial {x'}_k}} - \overline{\theta'\frac{\partial u_k\theta}{\partial {x'}_k}} + \frac{\partial}{\partial x_k}\left(n\overline{u'_k\theta\theta'} + (1-n)\overline{u_k\theta\theta'}\right)\right] +
\end{aligned} \tag{8}$$

$$+b^2\left[\overline{qu'_k\frac{\partial q'}{\partial x'_k}}-\overline{q'\frac{\partial u'_k q}{\partial x'_k}}+\frac{\partial}{\partial x_k}\left(n\overline{qq'u'_k}+(1-n)\overline{qu_kq'}\right)\right]+$$
$$+2ab\left[\overline{q'u'_k\frac{\partial\theta}{\partial x'_k}}-\overline{\theta\frac{\partial q'u'_k}{\partial x'_k}}+\frac{\partial}{\partial x_k}\left(n\overline{q'u'_k\theta}-(1-n)\overline{q'u_k\theta}\right)\right],$$

where the continuity equation at P', and the fact that quantities at one point are independent of the position of the other point, are used. Further, it can be shown, after some simplification, that eq. (8) for $r=0$, reduces to zero; since for $r=0$, $x_k=x'_k$ in this case all the terms within the capital brackets on the right-hand side become zero. Thus $a^2\Phi+b^2\Psi+2ab\Omega$ when integrated over all wave numbers gives zero contribution to the rate of change of the refractivity variance even for a general inhomogeneous turbulence. Thus, $a^2\Phi+b^2\Psi+2ab\Omega$, can only transfer Fourier components of the correlation from one part of the wave number space to another, as in the case of homogeneous turbulence.

Now consider the second term of the right-hand side of eq. (6), which is associated with the interaction of the refractivity variance with mean flow. This is different from the first term, which represents the interaction of the refractivity variance with turbulence. However both terms are related to the transfer term. So, if $[a^2\Phi^*+b^2\Psi^*+2ab\Omega^*]$ is the Fourier transform of second term of the right-hand side of eq. (6), then one may have

$$-\left(U'_k-U_k\right)\frac{\partial}{\partial r_k}\left(a^2\overline{\theta\theta'}+b^2\overline{qq'}+2ab\overline{q'\theta}\right)= \tag{9}$$
$$=\int_{-\infty}^{\infty}\left[a^2\Phi^*\left(\vec{\kappa},\vec{x}_n\right)+b^2\Psi^*\left(\vec{\kappa},\vec{x}_n\right)+2ab\Omega^*\left(\vec{\kappa}.\vec{x}_n\right)\right]\exp\left[i\vec{\kappa}\cdot\vec{r}\right]\mathrm{d}\vec{\kappa}\,.$$

Considering $r=0$, one may have $U_k=U'_k$, in this case eq. (9) reduces to the form

$$\int_{-\infty}^{\infty}\left[a^2\Phi^*\left(\vec{\kappa},\vec{x}_n\right)+b^2\Psi^*\left(\vec{\kappa},\vec{x}_n\right)+2ab\Omega^*\left(\vec{\kappa},x_n\right)\right]\mathrm{d}\vec{\kappa}=0\,. \tag{10}$$

Thus, as in the case of $\left[a^2\Phi+b^2\Psi+2ab\Omega\right]$, $\left[a^2\Phi^*+b^2\Psi^*+2ab\Omega^*\right]$ also gives zero total contribution to the rate of change of refractivity $\left(a^2\overline{\theta^2}+b^2\overline{q^2}+2ab\overline{q\theta}\right)$ and can only change the distribution in the wave number space of contributions to refractivity $\left(a^2\overline{\theta^2}+b^2\overline{q^2}+2ab\overline{q\theta}\right)$. Thus the interaction of refractivity variance with the turbulent and the mean flow (the first two terms of the right-hand side of eq. (6)) appear to be the only ones associated with spectral transfer terms, while other terms may be interpreted as production, convection and diffusion terms. It is hoped that this study will help better understanding of the modeling the refractive turbulence parameters in the atmospheric surface layer.

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