IL NUOVO CIMENTO Vol. 26 C, N. 5 Settembre-Ottobre 2003
DOI 10.1393/ncc/i2003-10009-5

A simple model on the influence of the greenhouse effect on the efficiency of solar-to-wind energy conversion(*)

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(ricevuto il 2 Maggio 2003; approvato il 30 Settembre 2003)

Summary. — In the present paper we study the Gordon and Zarmi model (*Am. J. Phys.*, **57** (1989) 995) for dealing with the earth's wind energy as a solar-driven Carnot-like heat engine, incorporating the role of the greenhouse effect on the performance of this heat engine model, following the De Vos approach. We find that when the greenhouse effect is considered only at the low-temperature half part of the cycle, the efficiency of the conversion of solar energy into wind energy strongly depends on the greenhouse effect under both maximum-power and maximum-ecological-function conditions. We also analyze the De Vos-van der Wel model corresponding to the so-called two-reservoir case and find that the efficiency of conversion of solar energy into wind energy under maximum ecological function reaches a reasonable value within the interval of values reported in the literature.

PACS 92.70.Cp – Atmosphere.
PACS 44.40.+a – Thermal radiation.
PACS 44.90.+c – Other topics in heat transfer.

1. – Introduction

The problem of thermal balance between the planets of the solar system and the Sun under a finite-time thermodynamics (FTT) approach has been treated by several authors [1-7]. In some of these articles the question of the conversion of solar energy into wind energy is also treated. As is well known [4], cosmic radiation, starlight, and moonlight can be neglected for the thermal balance of any of the planets of the solar system and only the following quantities have an influence: The incident solar influx or solar constant S, the planet's albedo ρ, and the greenhouse effect of the planet's

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atmosphere crudely evaluated by means of a coefficient γ. This coefficient can be taken as the normalized greenhouse effect introduced by Raval and Ramanathan in ref. [8] which is defined as the infrared radiation energy trapped by atmospheric gases and clouds. When only the global thermal balance between the Sun and a planet is considered, one can roughly obtain the planet's surface temperature assumed as a uniform temperature T_P. If the conversion of solar energy into wind energy is to be modeled, it is necessary to involve at least two representative atmospheric temperatures for making the creation of work possible; that is, to take the planet's atmosphere as a working fluid that converts heat into mechanical work. This permits to introduce in a natural way the concept of atmospheric "heat engine". In this context process variables as work rate, heat fluxes and efficiency for instance, find a simple theoretical framework, where thermodynamical restrictions play a major role. This is in contrast with disciplines as non-equilibrium thermodynamics and hydrodynamics based on local differential equations where the transition from local to global variables is not a trivial task [1]. In 1989, Gordon and Zarmi (GZ) [2] introduced a FTT model taking the Sun-Earth-Wind system as a FTT-cyclic heat engine where the heat input is solar radiation, the working fluid is the Earth's atmosphere and the energy in the winds is the work produced, the cold reservoir to which the engine rejects heat is the 3 K surrounding universe. By means of this oversimplified model, Gordon and Zarmi were able to obtain reasonable values for the annual average power in the Earth's winds and for the average maximum and minimum temperatures of the atmosphere, without resorting to detailed dynamic models of the Earth's atmosphere, and without considering any other effect (such as Earth's rotation, Earth's traslation around the Sun, ocean currents, etc.). Later, De Vos and Flater [3] extended the GZ model to take into account the wind energy dissipation and obtained an upper bound for the conversion efficiency of solar energy into wind energy given by $w_{\max} \cong 8.3\%$, assuming that the atmospheric "heat engine" works at maximum-power regime. This model, in turn, was extended by De Vos and van der Wel [5] to obtain a new upper bound $w_{\max} \cong 10.23\%$. These same authors [6] considerably improved their numerical results for $w_{\max}$ by means of a model based on convective Hadley cells. All the models used in refs. [1-6] are endoreversible ones in the sense of FTT [9], that is, all irreversibilities are located in the exchanges between the engine and the external world and the engine model is internally reversible. In ref. [7], the GZ model was studied under a nonendoreversible approach using a criterion of merit called ecological optimization criterion. This criterion [10] consists of maximizing a function E that represents a good compromise between high-power output and low-entropy production. The function E is given by

$$E = P - T\Delta S_u, \tag{1}$$

where P is the power output of the cycle, ΔS_u the total entropy production (system plus surroundings) per cycle, and T is the temperature of the cold reservoir. This optimization criterion for the case of the so-called Curzon-Ahlborn cycle [11], for instance, leads to a cycle configuration such that for maximum E it produces around 75% of the maximum power and only about 25% of the entropy produced in the maximum-power regime [12]. By means of employing this criterion in a nonendoreversible GZ model, the authors of ref. [7] also found reasonable values for the annual average power of the winds and for the extreme temperatures of the Earth´s atmosphere. In the present paper we again study the GZ model but including the planet's greenhouse factor. It is convenient to remark that the GZ-type models are based on annual average quantities and thus they do not represent actual convective cells, but a kind of annual virtual cells that take into account

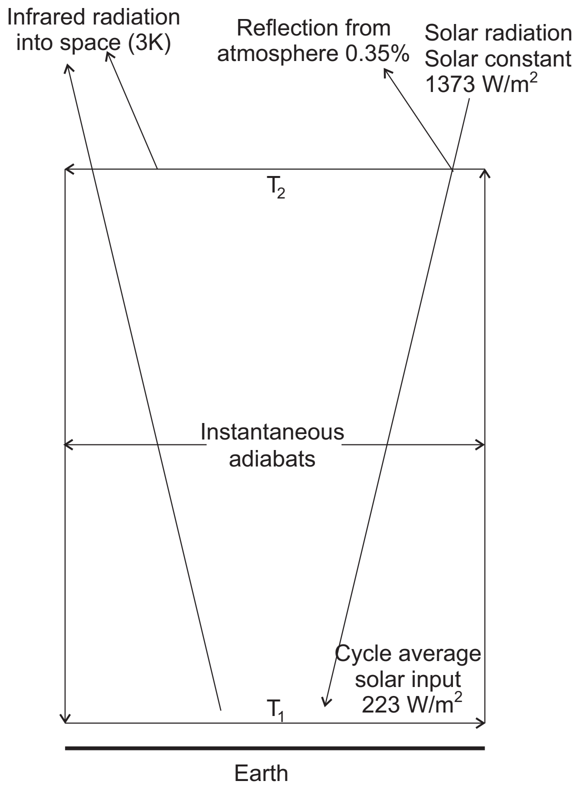

Fig. 1. – Scheme of a simplified solar-driven heat engine (taken from ref. [1]).

the global thermodynamic restrictions over the convection as dominant energy transfer phenomenon in the air (which has a big Rayleigh number). Besides, this kind of models must be only taken as the ones producing better upper bounds than those calculated by means of classical equilibrium thermodynamics (CET), which is the main purpose of FTT. As De Vos and Flater [3] assert, no mechanism guarantees that the atmosphere maximizes the wind's power; thus, we include also an analysis based on the ecological criterion. The paper is organized as follows: In sect. **2**, we discuss the GZ model under both maximum-power and maximum-E criteria including the planet's albedo ρ and the greenhouse factor γ. For the case of the maximum-power regime we recover the expression for the solar energy efficiency w, given by De Vos in refs. [3] and [4]. Nevertheless, we find that both the power output P and the solar energy efficiency w may depend on the greenhouse effect. In sect. **3**, we study the two-reservoir model of De Vos and van der Wel [5] under the ecological criterion and we find a value for the solar energy efficiency w within the reported values [13-18]. Finally, we present our conclusions in sect. **4**.

2. – GZ model including greenhouse effect

2·1. *The maximum-power regime.* – In fig. 1, a schematic view of a simplified Sun-Earth-Winds system as a heat engine cycle is depicted. This cycle consists of four branches:

a) two isothermal branches, one in which the atmosphere absorbs solar radiation at low altitudes and one in which the atmosphere rejects heat at high altitudes to the universe, and

b) two intermediate instantaneous adiabats with rising and falling air currents [9, 19].

According to GZ, this oversimplified Carnot-like engine corresponds very approximately to the global scale motion of wind in convective cells. In what follows, we use all of GZ-model's assumptions. For instance, the work performed by the working fluid in one cycle W, the internal energy of the working fluid U, and the yearly average solar radiation flux q_s are expressed per unit area of earth surface. The temperatures of the four-branches cycle are taken as follows: T_1 is the working fluid temperature in the isothermal branch at lowest altitude, where the working fluid absorbs solar radiation for half the cycle, during the second half of the cycle heat is rejected via black-body radiation from the working fluid at temperature T_2 (highest altitude of the cell) to the cold reservoir at temperature T_{ex} (the 3 K surrounding universe). In the GZ model the objective is to maximize the work per cycle (average power) subject to the endoreversibility constraint [9], that is

$$\Delta S_{\mathrm{int}} = \int_0^{t_0} \left(\frac{q_s(t) - \sigma\left[T^4(t) - T_{\mathrm{ex}}^4(t)\right]}{T(t)} \right) \mathrm{d}t = 0, \tag{2}$$

where ΔS_{int} is the change in entropy per unit area, t_0 is the time of one cycle, σ is the Stefan-Boltzmann constant ($5.67 \times 10^{-8}\mathrm{W/m^2K^4}$), and q_s, T and T_{ex} are functions of time t, taken as [2]

$$\begin{gathered} T(t) = \left\{ \begin{array}{cc} T_1, & 0 \le t \le t_0/2 \\ T_2, & t_0/2 \le t \le t_0 \end{array} \right\} \\ T_{\mathrm{ex}} = 3\ \mathrm{K}, \quad 0 \le t \le t_0 \\ q_s(t) = \left\{ \begin{array}{cc} 0 & t_0/2 \le t \le t_0 \\ \dfrac{I_{\mathrm{sc}}(1-\rho)}{2} & 0 \le t \le t_0 \end{array} \right\}, \end{gathered} \tag{3}$$

with I_{sc} the yearly average solar constant ($1373\ \mathrm{W/m^2}$) and $\rho \approx 0.35$ [2], the effective average albedo of the earth's atmosphere. The GZ model maximizes the work per cycle W, taken from the first law of thermodynamics for one cycle,

$$\Delta U = -W + \int_0^{t_0} \left(q_s(t) - \sigma\left[T^4(t) - T_{\mathrm{ex}}^4(t)\right]\right) \mathrm{d}t = 0, \tag{4}$$

by means of the Euler-Lagrange formalism and denoting average values as

$$\begin{aligned} \overline{T} &= \frac{T_1 + T_2}{2}, \\ \overline{T^n} &= \frac{T_1^n + T_2^n}{2}, \\ \overline{q_s} &= I_{\mathrm{sc}} \frac{(1-\rho)}{4}. \end{aligned} \tag{5}$$

From eqs. (4) and (5) and taking into account the constraint given by eq. (2), GZ construct the following Lagrangian L:

$$L = T^4(t) + \lambda \left[\frac{q_s(t)}{T(t)} - \sigma T^3(t) \right], \tag{6}$$

where λ is a Lagrange multiplier. By finding the extremum of L by means of $\partial L/\partial T(t) = 0$, GZ found for the Earth's atmosphere the following values: $T_1 = 277$ K, $T_2 = 192$ K and $P_{\max} = W_{\max}/t_0 = 17.1$ W/m^2. These numerical values are not so far from "actual" values, which are $P \approx 7$ W/m^2 [15], $T_1 \approx 290$ K (at ground level) and $T_2 \approx 195$ K (at an altitude of around 75–90 km). However, as GZ assert, their power calculation must be taken as an upper bound due to several idealizations in their model.

In ref. [7], the GZ model was used to maximize the ecological function given by eq. (1). This was made for both an endoreversible and a nonendoreversible case. For the endoreversible case (with a 3 K surrounding universe) the following values were obtained: $T_1 = 294$ K, $T_2 = 109.5$ K and $P_{\max} = 6.89$ W/m^2, which also are good values, remarkably for T_1 and P. Another endoreversible case was proposed, but using as cold reservoir the tropopause shell with $T_{\mathrm{ex}} \approx 200$ K. In this case the numerical results were: $T_1 = 293.3$ K, $T_2 = 239.2$ K and $P_{\max} = 10.75$ W/m^2, which also are reasonable values for the troposphere characteristics. The nonendoreversible version of the GZ model was accomplished by means of a lumped nonendoreversibility parameter R which may be considered as a measure of the departure from an endoreversible regime, due to internal losses. This parameter arises from Clausius' inequality [20-24]. With this approach, T_1, T_2 and P also reach reasonable values [7]. When the endoreversibility parameter R and the greenhouse factor γ [4] are considered, the integral constraint given by eq. (2) must be modified, becoming

$$\Delta S_R = \int_0^{t_0} \left(\frac{q_s(t) - \sigma(1-\gamma)R\left[T^4(t) - T^4_{\mathrm{ex}}(t)\right]}{T(t)} \right) \mathrm{d}t = 0, \tag{7}$$

while the first law of thermodynamics applied over one cycle becomes

$$\Delta U = -W + \int_0^{t_0} \left(q_s(t) - \sigma(1-\gamma)\left[T^4(t) - T^4_{\mathrm{ex}}(t)\right] \right) \mathrm{d}t = 0. \tag{8}$$

By using eqs. (3) into eq. (8), we obtain

$$P = \frac{W}{t_0} = \overline{q_s} - \frac{\sigma}{2}(1-\gamma)\left[T_1^4 + T_2^4\right] + \sigma(1-\gamma)T^4_{\mathrm{ex}}. \tag{9}$$

For $T_{\mathrm{ex}} = 3$ K, we take the appoximation $\overline{q_s} \gg \sigma T^4_{\mathrm{ex}}$ (223 W/m^2 $\gg 4.59 \times 10^{-6}$W/m^2) and eq. (9) becomes

$$P = \frac{W}{t_0} = \overline{q_s} - \frac{\sigma}{2}(1-\gamma)\left[T_1^4 + T_2^4\right]. \tag{10}$$

By using again eqs. (3) into eq. (7), we obtain

$$\frac{\overline{q_s}}{T_1} = \frac{\sigma R}{2}(1-\gamma)\left[T_1^3 + T_2^3\right]. \tag{11}$$

Since our objective is the maximization of the average power output per cycle, subject to the integral constraint, we construct the following Lagrangian:

$$L_R = \overline{q_s} - \frac{\sigma}{2}(1-\gamma)\left[T_1^4 + T_2^4\right] - \lambda\left\{\frac{\overline{q_s}}{T_1} - \frac{\sigma R}{2}(1-\gamma)\left[T_1^3 + T_2^3\right]\right\}, \tag{12}$$

λ being a Lagrange multiplier. By means of $\partial L_R/\partial T(t) = 0$, we obtain the following equations:

For $\partial L_R/\partial T_1 = 0$,

$$-2\sigma(1-\gamma)T_1^5 + \lambda\overline{q_s} + \frac{3\sigma R}{2}(1-\gamma)\lambda T_1^4 = 0; \tag{13}$$

for $\partial L_R/\partial T_2 = 0$,

$$\lambda = \frac{4}{3R}T_2; \tag{14}$$

and for $\partial L_R/\partial\lambda = 0$,

$$\frac{\overline{q_s}}{T_1} - \frac{\sigma R(1-\gamma)}{2}\left(T_1^3 + T_2^3\right) = 0. \tag{15}$$

By the elimination of λ, eqs. (13)-(15) reduce to

$$T_1^5 - T_2T_1^4 - \frac{2\overline{q_s}}{3}\frac{1}{\sigma(1-\gamma)R}T_2 = 0, \tag{16}$$

and

$$T_1^4 + T_1T_2^3 - \frac{2\overline{q_s}}{\sigma(1-\gamma)R} = 0. \tag{17}$$

We numerically solve these equations and obtain, for example, the following results: For the Earth with $\rho = 0.33$ [25], $\gamma = 0.3$ [8], $R = 1$ and $\overline{q_s} = 229.5$ W/m^2 [25] we get $T_1 = 305.2$ K, $T_2 = 211.4$ K [26] and $P_{\max} = 17.5$ W/m^2; and for Venus, with $\rho = 0.71$ [25], $\gamma = 0.983$, $R = 1$ and $\overline{q_s} = 188.5$ W/m^2 [25] we get $T_1 = 736.1$ K, $T_2 = 509.7$ K and $P_{\max} = 14.4$ W/m^2, which are good values for the surface temperatures of these planets [3].

From eq. (11) and taking the nonendoreversible Carnot efficiency [24] as

$$\eta = 1 - \frac{T_2}{RT_1}, \tag{18}$$

we obtain

$$T_1^4 = \frac{2\overline{q_s}}{\sigma}\left\{\frac{1}{R\left[1 + R^3(1-\eta)^3\right](1-\gamma)}\right\}, \tag{19}$$

and

$$T_2^4 = \frac{2\overline{q_s}}{\sigma}\left\{\frac{R^4(1-\eta)^4}{R\left[1+R^3(1-\eta)^3\right](1-\gamma)}\right\}. \tag{20}$$

By substituting eqs. (19) and (20) into eq. (10) we get

$$P = \overline{q_s}\left\{\frac{(R-1)+R^4(1-\eta)^3\eta}{R\left[1+R^3(1-\eta)^3\right]}\right\}. \tag{21}$$

Then, for the solar energy efficiency w defined as $w = P/\overline{q_s}$ [4], we obtain

$$w = \frac{(R-1)+R^4(1-\eta)^3\eta}{R\left[1+R^3(1-\eta)^3\right]}, \tag{22}$$

which for $R = 1$ (endoreversible case) reduces to

$$w = \frac{\eta(1-\eta)^3}{1+(1-\eta)^3}, \tag{23}$$

which is the same result reported by De Vos [4]. By solving $\mathrm{d}w/\mathrm{d}\eta = 0$, De Vos found that for a maximum-power regime, 7.67% of the solar energy $\overline{q_s}$ can be converted into wind energy. Besides, as De Vos remarks this value is independent of planet and yet of solar system, given that eq. (23) does not involve ρ, γ and the so-called dilution factor $f = R_s^2/r^2$ [4], where R_s is the radius of the Sun and r is the radius of Earth's orbit around the Sun. Our equation (22) suggests that w depends on the parameter R which in principle embraces the internal losses of the working fluid. We also can approach to the maximization of the power output of the GZ model by considering that the factor $(1-\gamma)$ corresponding to the greenhouse effect [4] only participates in the low half of the cycle depicted in fig. 1. Thus, by integrating eq. (8) with the approximation $\overline{q_s} \gg \sigma T_{\mathrm{ex}}^4$, the expression for the average power output becomes

$$P = \overline{q_s} - \frac{\sigma}{2}\left[(1-\gamma)T_1^4 + T_2^4\right]. \tag{24}$$

With the same previous assumptions for the greenhouse effect, the integral constraint (eq. (7)) turns out to be

$$\frac{\overline{q_s}}{T_1} = \frac{\sigma R}{2}\left[(1-\gamma)T_1^3 + T_2^3\right]. \tag{25}$$

For the maximization of the power output, we construct the following Lagrangian:

$$L_R' = \overline{q_s} - \frac{\sigma}{2}\left[(1-\gamma)T_1^4 + T_2^4\right] - \lambda\left\{\frac{\overline{q_s}}{T_1} - \frac{\sigma R}{2}\left[(1-\gamma)T_1^3 + T_2^3\right]\right\}, \tag{26}$$

λ being a Lagrange multiplier. From eq. (26), we obtain the following three equations:

$$T_1^5 - \frac{3}{4}R\lambda T_1^4 - \frac{\overline{q_s}}{2\sigma(1-\gamma)}\lambda = 0, \tag{27}$$

$$\lambda = \frac{4}{3R} T_2, \tag{28}$$

and

$$T_1^4 - \frac{1}{(1-\gamma)} T_1 T_2^3 - \frac{2\overline{q_s}}{\sigma R (1-\gamma)} = 0, \tag{29}$$

which by the elimination of λ reduce to

$$T_1^5 - T_2 T_1^4 - \frac{2\overline{q_s}}{3\sigma R (1-\gamma)} T_2 = 0, \tag{30}$$

and

$$T_1^4 - \frac{1}{(1-\gamma)} T_1 T_2^3 - \frac{2\overline{q_s}}{\sigma R (1-\gamma)} = 0. \tag{31}$$

By numerically solving eqs. (30) and (31) we find the following results: For the Earth with $\rho = 0.33$ [25], $R = 1$, $\overline{q_s} = 229.5$ W/m^2 [25] and $\gamma = 0.3$ [8] we get $T_1 = 299.3$ K, $T_2 = 202.2$ K [26] and $P_{\max} = 22.8$ W/m^2, and for Venus, $\rho = 0.71$ [25], $R = 1$, $\overline{q_s} = 188.5$ W/m^2 [25] and $\gamma = 0.997$ [22], we find $T_1 = 727.7$ K, $T_2 = 199.9$ K and $P_{\max} = 119.4$ W/m^2. Both results are reasonable values for the surface temperatures of these planets and improve the numerical results obtained in the previous calculations, which consider the greenhouse effect in both isothermal branches of the Carnot-like cycle. If in eq. (7) we do not use the appoximation $\overline{q_s} \gg \sigma T_{\rm ex}^4$, and we take $T_{\rm ex} = 200$ K (the tropopause temperature), eqs. (30) and (31) become

$$T_1^5 T_2^4 - T_1^4 T_2^5 + \frac{T_{\rm ex}^4}{3}\left(T_1^5 - T_2^5\right) - \frac{2\overline{q_s}}{3\sigma R (1-\gamma)} T_2^5 = 0, \tag{32}$$

and

$$T_1^4 T_2 + \frac{1}{(1-\gamma)} T_1 T_2^4 - \left[T_{\rm ex}^4 + \frac{2\overline{q_s}}{\sigma R(1-\gamma)}\right] T_2 - \frac{T_{\rm ex}^4}{(1-\gamma)} T_1 = 0. \tag{33}$$

The numerical solution of these equations provides the following results: for $\gamma = 0$ and $R = 1$, $T_1 = 293.4$ K and $T_2 = 239.3$ K (the same results obtained in [7]) and for $\gamma = 0.3$ [8] and $R = 1$, $T_1 = 310.4$ K and $T_2 = 244.2$ K, which are not so good values. If we take eq. (25) for the integral constraint and eq. (18) for the nonendoreversible Carnot efficiency, we obtain

$$T_1^4 = \frac{2\overline{q_s}}{\sigma}\left\{\frac{1}{R\left[(1-\gamma) + R^3(1-\eta)^3\right]}\right\}, \tag{34}$$

and

$$T_2^4 = \frac{2\overline{q_s}}{\sigma}\left\{\frac{R^3(1-\eta)^4}{\left[(1-\gamma) + R^3(1-\eta)^3\right]}\right\}. \tag{35}$$

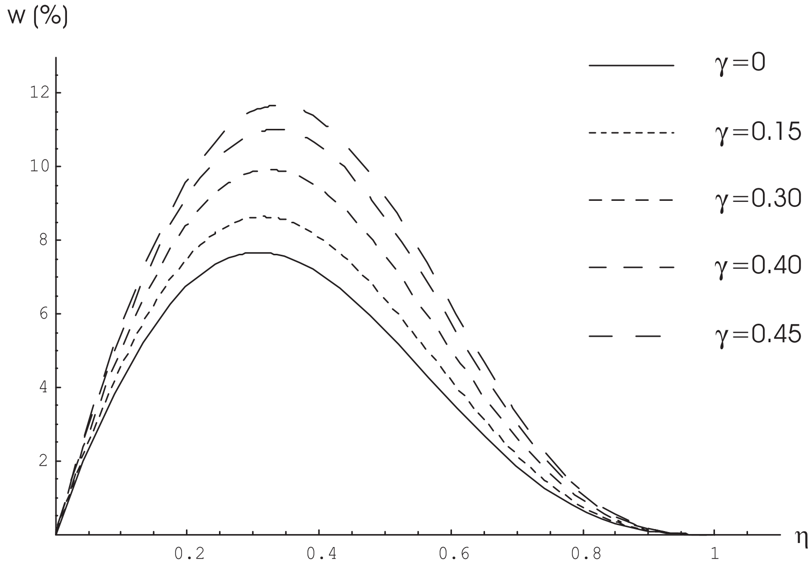

Fig. 2. – Solar-energy conversion efficiency w as a function of the thermal efficiency η for several values of the greenhouse coefficient γ, under maximum-power conditions with $R = 1$.

By substitution of these equations into eq. (24) for the average power output, we get

$$w(R,\gamma,\eta) = \frac{P}{\overline{q_s}} = \frac{(1-\gamma)(R-1) + R^3(1-\eta)^3\,[1-R(1-\eta)]}{R\,[(1-\gamma)+R^3(1-\eta)^3]}, \tag{36}$$

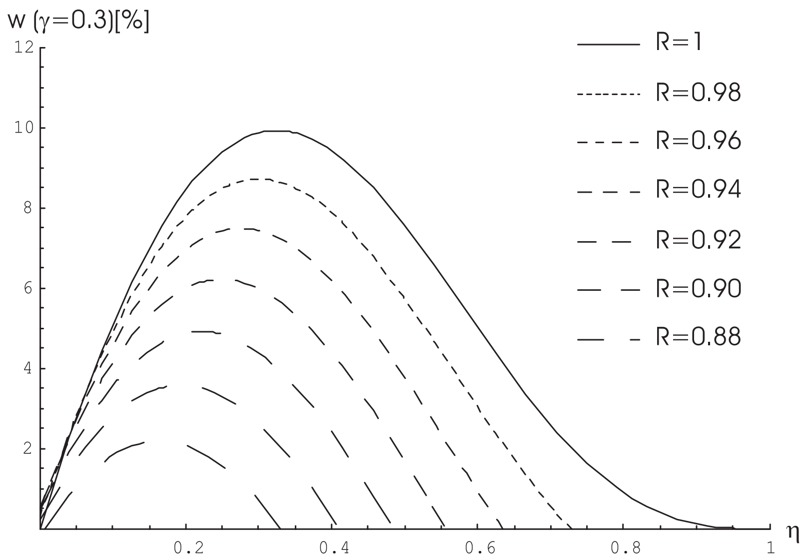

Fig. 3. – w behavior in terms of η for several values of the nonendoreversibility parameter R.

which, for the endoreversible case $R = 1$, becomes

$$w(\gamma,\eta) = \frac{\eta(1-\eta)^3}{(1-\gamma)+(1-\eta)^3}. \tag{37}$$

That is, even in the endoreversible case the solar energy efficiency depends on the greenhouse effect and therefore on the planet's atmosphere. Equation (37) reproduces De Vos' equation (23) only in the case $\gamma = 0$, but this situation does not correspond to a realistic model. In fig. 2, we show $w(\gamma, \eta)$ for several values of γ and $R = 1$. We can see that w increases as γ increases. That is, the efficiency of solar-to-wind energy conversion strongly depends on the greenhouse factor. In fig. 3, we can see how the parameter R affects the conversion efficiency w.

2˙2. *The maximum-ecological regime.* – Now, our objective is the maximization of the so-called ecological function given by eq. (1). First, we need to calculate the mean entropy production per cycle of the thermodynamic universe, $\Delta S_u/t_0$. Starting from fig. 1, we have

$$\Delta S_u = \int_0^{t_0} \left(\frac{-q_s(t) + \sigma(1-\gamma)R\left[T^4(t) - T_{\rm ex}^4(t)\right]}{T(t)} \right) \mathrm{d}t. \tag{38}$$

If we consider the greenhouse effect only in the first half of the cycle, by means of eqs. (3), we get

$$\Delta S_u = \int_0^{t_0/2} \left(\frac{\frac{I_{\rm sc}(1-\rho)}{2} + \sigma(1-\gamma)R\left[T_1^4 - T_{\rm ex}^4\right]}{T_1} \right) \mathrm{d}t + \int_{t_0/2}^{t_0} \left(\frac{\sigma R\left[T_2^4 - T_{\rm ex}^4\right]}{T_{\rm ex}} \right) \mathrm{d}t, \tag{39}$$

and

$$\frac{\Delta S_u}{t_0} = -\frac{\overline{q_s}}{T_1} + \frac{\sigma R}{2}\left[(1-\gamma)T_1^3 + \frac{T_2^4}{T_{\rm ex}}\right], \tag{40}$$

where we have used the approximation, $\overline{q_s} \gg \sigma T_{\rm ex}^4$.

Therefore, by substitution of eqs. (24) and (40) into eq. (1), the ecological function E becomes

$$E = \overline{q_s} - \frac{\sigma}{2}\left[(1-\gamma)T_1^4 + T_2^4\right] + \frac{T_{\rm ex}\overline{q_s}}{T_1} - \frac{\sigma R T_{\rm ex}}{2}\left[(1-\gamma)T_1^3 + \frac{T_2^4}{T_{\rm ex}}\right]. \tag{41}$$

Then, we propose the following Lagrangian:

$$L_E = \overline{q_s} - \frac{\sigma}{2}\left[(1-\gamma)T_1^4 + T_2^4\right] + \frac{T_{\rm ex}\overline{q_s}}{T_1} - \frac{\sigma R T_{\rm ex}}{2}\left[(1-\gamma)T_1^3 + \frac{T_2^4}{T_{ex}}\right] - \\ - \lambda\left\{\frac{\overline{q_s}}{T_1} - \frac{\sigma R}{2}\left[(1-\gamma)T_1^3 + T_2^3\right]\right\}, \tag{42}$$

λ being a Lagrange multiplier. By solving the Euler-Lagrange equations for this Lagrangian, we obtain

$$T_1^5 + \left[\frac{3T_{\rm ex}}{4}R - (1+R)T_2\right] T_1^4 - \\ - \frac{2\overline{q_s}}{3\sigma}\left(\frac{1+R}{R}\right)\left(\frac{1}{1-\gamma}\right)T_2 + \frac{T_{\rm ex}\overline{q_s}}{2\sigma}\left(\frac{1}{1-\gamma}\right) = 0, \tag{43}$$

and

$$T_1^4 + \frac{1}{(1-\gamma)}T_1T_2^3 - \frac{2\overline{q_s}}{\sigma R}\left(\frac{1}{1-\gamma}\right) = 0. \tag{44}$$

For $T_{\rm ex} = 3$ K and $\overline{q_s} = 223$ W/m^2 [25], the numerical solution of these equations gives us for $\gamma = 0$ and $R = 1$, $T_1 = 294.03$ K and $T_2 = 109.95$ K (same results obtained in [7]); and for $\gamma = 0.3$ and $R = 1$, $T_1 = 319.86$ K and $T_2 = 118.897$ K, which are not so good values. If in eq. (38) we do not use the appoximation $\overline{q_s} \gg \sigma T_{\rm ex}^4$, and we take $T_{\rm ex} = 200$ K (the tropopause temperature), we obtain the following three equations:

$$T_1^5 + \frac{3R}{4}(T_{\rm ex} - \lambda)T_1^4 - \frac{\overline{q_s}}{2\sigma(1-\gamma)}(T_{\rm ex} - \lambda) + \frac{RT_{\rm ex}^4}{4}(T_{\rm ex} - \lambda) = 0, \tag{45}$$

$$T_2^5 + RT_2^5 - \frac{3R}{4}\lambda T_2^4 - \frac{RT_{\rm ex}^4}{4}\lambda = 0 \tag{46}$$

and

$$(1-\gamma)T_1^4T_2 + T_1T_2^4 - \left(\frac{2\overline{q_s}}{\sigma R} + (1-\gamma)T_{\rm ex}^4\right)T_2 - T_{\rm ex}^4T_1 = 0, \tag{47}$$

with λ a Lagrange multiplier. The numerical solution of these equations gives us the following results: For $\gamma = 0$ and $R = 1$, $\lambda \approx 490.8$ m^{2}K^5/W, $T_1 \approx 303$ K and $T_2 \approx 219$ K (the same results obtained in [7]) and for $\gamma = 0.3$ and $R = 1$, $T_1 = 321.3$ K and $T_2 = 227.5$ K, which are not good values. Thus, we can conclude that the best numerical values are given by the maximum power regime with greenhouse effect only in the first half of the Carnot-like cycle depicted in fig. 1. However, we must have in mind that all of these numerical results are only rough semi-ideal limits of actual values.

3. – The De Vos-Flater-van der Wel model

In 1991, De Vos and Flater [3] interpreted the GZ model for the conversion of solar energy into wind energy as a Carnot-like endoreversible engine with the atmosphere as the working fluid operating between the sunny side and the dark side of a planet (see fig. 4a). By means of this simple model they found a solar energy efficiency given by eq. (23), which under maximum-power conditions leads to 7.67%, independently of f, ρ and γ. According to De Vos and Flater [3] this simple model has one important drawback; it is only correct if the mechanical power W is either continuously stored or continuously drained away from the planet. Thus, these authors proposed a correction to the GZ model, by means of the consideration that wind energy W is dissipated as heat in the

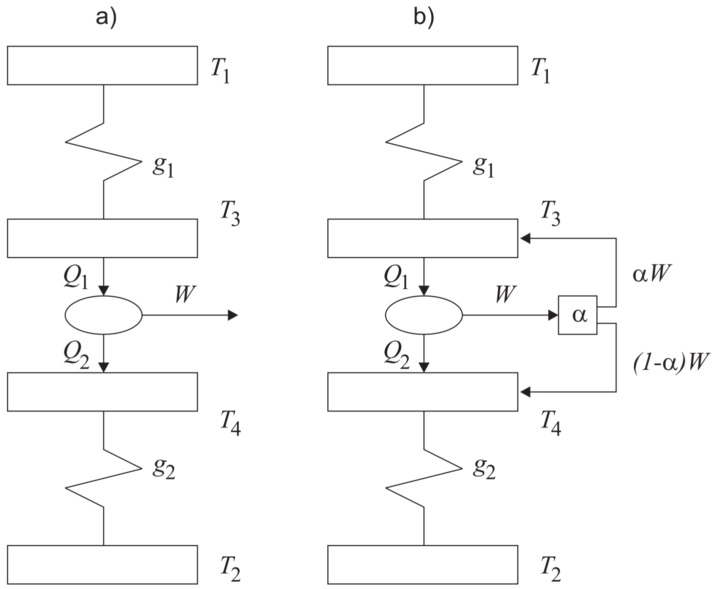

Fig. 4. – a) De Vos-Flater endoreversible model without wind energy dissipation. b) De Vos-Flater endoreversible model with energy dissipation.

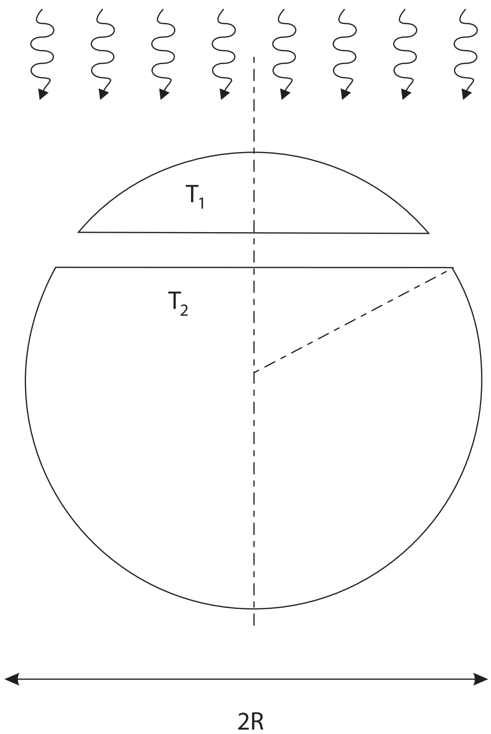

Fig. 5. – Two-reservoir model of De Vos-van der Wel.

same planet after a temporary storage in the atmosphere. This idea is accomplished by means of the model depicted in fig. 4b. They assume that a fraction α of W is deposited as heat on the illuminated side of the planet and the rest is deposited on the dark side. With this model De Vos and Flater show that a maximum solar efficiency $w_s = 8.3\%$ is reached when $\alpha = 1$ for an engine efficiency $\eta = 0.307$, by means of the maximization of the expression [3]

$$w(\gamma,\eta) = \frac{\eta(1-\eta)^3}{1+(1-\eta)^4}, \tag{48}$$

which is also independent of f, ρ and γ. In a later paper [5], De Vos and van der Wel extended the previous dissipative model to the one depicted in fig. 5. This model is an extension of the De Vos-Flater model for the case in which the angle θ is in the interval $0 < \theta \leq \pi/2$, in contrast with the former one, where $\theta = \pi/2$ only.

In the present section we take the two-reservoir model of De Vos and van der Wel (DVVW) depicted in fig. 5 and analyze its operation under the maximum-ecological-function regime. In the DVVW model the heat exchange between the planet and Sun consists of four contributions [5]:

a) the hot cap receives

$$\Pi_1(1-\rho)f\sigma T_s^4,$$

from the Sun with a surface temperature T_s;

b) the hot cap emits

$$\Sigma_1(1-\gamma)\sigma T_1^4,$$

c) the cold cap receives

$$\Pi_2(1-\rho)f\sigma T_s^4,$$

and

d) the cold cap emits

$$\Sigma_2(1-\gamma)\sigma T_2^4.$$

Thus the heat fluxes Q_1 and Q_2 (see fig. 4b) are

$$Q_1 = \Pi_1(1-\rho)f\sigma T_s^4 - \Sigma_1(1-\gamma)\sigma T_1^4, \tag{49}$$

and

$$Q_2 = \Pi_2(1-\rho)f\sigma T_s^4 - \Sigma_2(1-\gamma)\sigma T_2^4, \tag{50}$$

the meaning of f, ρ and γ are the same as in the previous section. The quantities Σ_1 and Σ_2 are the surface areas of the two spherical caps, whereas Π_1 and Π_2 are the cross-sections of the sunray beams incident on them. Therefore, $\Sigma_1 = 2\pi R^2(1-\cos\theta)$,

$\Sigma_2 = 2\pi R^2(1+\cos\theta)$, $\Pi_1 = \pi R^2 \sin^2\theta$ ($\theta \le \pi/2$) and $\Pi_2 = \pi R \cos^2\theta$ ($\theta \le \pi/2$). De Vos and van der Wel rewrite Q_1 and Q_2, given by eqs. (49) and (50), as

$$Q_1 = g_1(T_{s1}^4 - T_1^4), \tag{51}$$

and

$$Q_2 = g_2(T_{s2}^4 - T_2^4), \tag{52}$$

where T_{s1} and T_{s2} are some effective sky temperatures given by

$$T_{s1} = \left(\frac{\Pi_1}{\Sigma_1}\frac{1-\rho}{1-\gamma}f\right)^{1/4} T_s, \tag{53}$$

and

$$T_{s2} = \left(\frac{\Pi_2}{\Sigma_2}\frac{1-\rho}{1-\gamma}f\right)^{1/4} T_s, \tag{54}$$

and where g_1 and g_2 are some kind of heat conductances

$$g_1 = \Sigma_1(1-\gamma)\sigma,$$

and

$$g_2 = \Sigma_2(1-\gamma)\sigma.$$

With all these definitions and by means of the engine depicted in fig. 4b, these authors found for the power output W [5]

$$W = \frac{\eta}{1-\alpha\eta}\left[\Pi_1 - \frac{\pi R^2 \Sigma_1}{\Sigma_1 + \Sigma_2(1-\eta)^4}\right](1-\rho) f\sigma T_s^4, \tag{55}$$

which leads to a solar efficiency given by

$$w = \frac{W}{\pi R^2 (1-\rho) f \sigma T_s^4} = \frac{\eta}{1-\alpha\eta}\left[\Pi_1 - \frac{\pi R^2 \Sigma_1}{\Sigma_1 + \Sigma_2(1-\eta)^4}\right]. \tag{56}$$

The maximum work is produced for the case $\alpha = 1$ (see fig. 3 of [4]) and for the case $\theta = \pi/2$ assumed by De Vos and Flater in ref. [3], eq. (48) is immediately obtained. Because w in eq. (56) is a function of α, η and θ, for $\alpha = 1$, the maximum w was obtained by means of $\partial w/\partial\theta = 0$ and $\partial w/\partial\eta = 0$ resulting [5]: $w = 9.64\%$ for $\theta = 77.8°$ and $\eta = 0.291$. All the w's calculated by De Vos *et al.* are smaller than CET calculations as that reported by Peixoto and Oort, which is of the order of 10% [18]. As we said before, our objective is the maximization of the DVVW model under the so-called ecological criterion [10,12]. That is, the optimization of the function E given by eq. (1). For doing this, we first need to calculate the average entropy production per cycle, ΔS_u. From fig. 4b, we obtain

$$\Delta S_u = -\frac{Q_1}{T_{s1}} + \frac{Q_1 + \alpha W}{T_1} + \frac{(1-\alpha)W}{T_2} - \frac{Q_2}{T_2} + \frac{Q_2}{T_{s2}}. \tag{57}$$

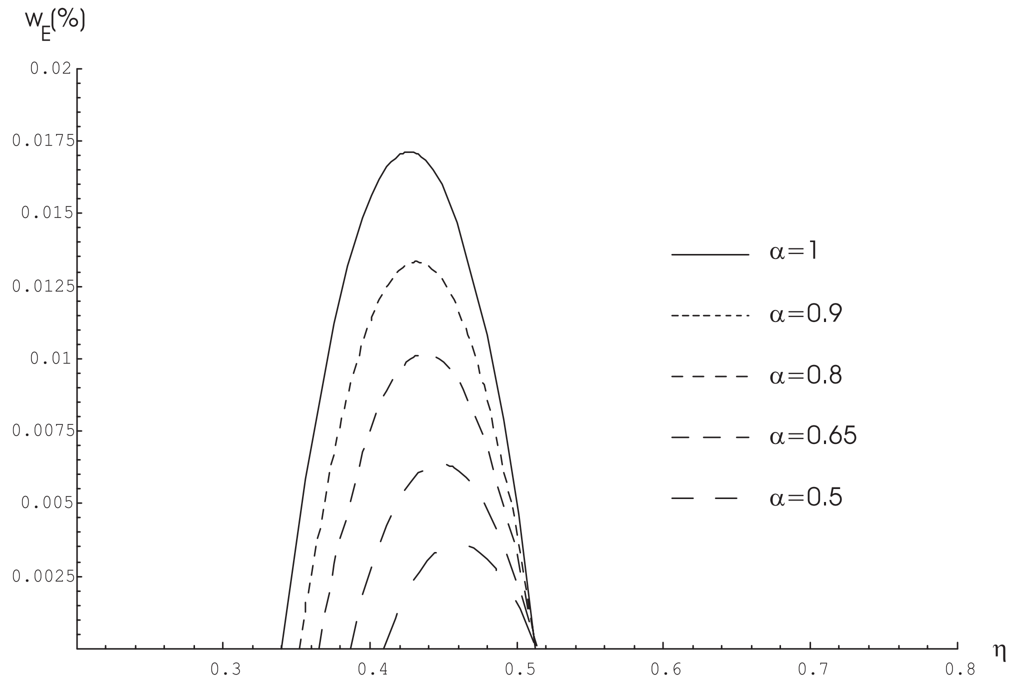

Fig. 6. – Illustration of the maximum w-curve ($\alpha = 1$) in terms of η under the maximum ecological regimen.

By the application of the endoreversibility hypothesis,

$$\Delta S_{\text{int}} = \frac{Q_1 + \alpha W}{T_1} + \frac{(1-\alpha)W - Q_2}{T_2} = 0, \tag{58}$$

into eq. (57), we get

$$\Delta S_u = \frac{Q_2}{T_{s2}} - \frac{Q_1}{T_{s1}} > 0. \tag{59}$$

Therefore, the ecological function $E = W - T_{s2}\Delta S_u$ becomes

$$E = \left[\frac{\eta}{1-\alpha\eta} + \left(\frac{\Pi_2\Sigma_1}{\Pi_1\Sigma_2}\right)^{1/4} - 1\right]\left[\Pi_1 - \frac{\pi R^2\Sigma_1}{\Sigma_1 + \Sigma_2(1-\eta)^4}\right](1-\rho)f\sigma T_s^4, \tag{60}$$

where we have used eqs. (49) and (50) for Q_1 and Q_2, respectively. Now, we define an ecological solar efficiency as

$$w_E \equiv \frac{E}{Z},$$

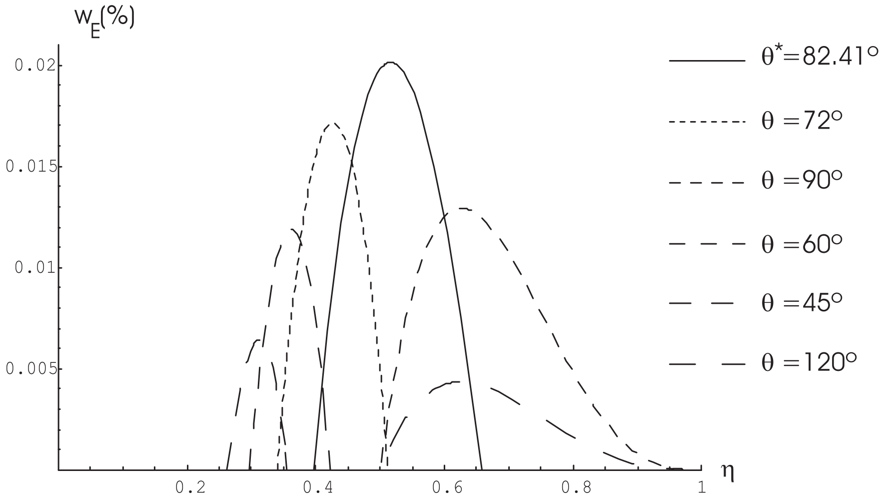

Fig. 7. – w_E behavior in terms of η for several values of the angle θ.

Z being the total amount of absorbed solar energy given by $Z = \pi R^2(1-\rho) f\sigma T_s^4$. Thus,

$$w_E(\eta,\theta) = \left[\frac{\eta}{1-\alpha\eta} + \left(ctg^2\theta\frac{1-\cos\theta}{1+\cos\theta}\right)^{1/4} - 1\right] \times \times \left[\sin^2\theta - \frac{1-\cos\theta}{1+\cos\theta+(1+\cos\theta)(1-\eta)^4}\right]. \tag{61}$$

Analogously to the case of the maximum-power regime [5], this function reaches its maximum for $\alpha = 1$ (see fig. 6), *i.e.* for the highest α physically meaningful, since $\partial w_E/\partial\alpha > 0$ for all α. Therefore, for $\alpha = 1$, we numerically calculate $\partial w_E/\partial\eta = 0$ and $\partial w_E/\partial\theta = 0$, obtaining $\eta^* = 0.516$ and $\theta^* = 82.41°$ (see fig. 7). After substituting these values into eq. (61), we get $w_E^* \approx 0.02012$ ($w_E^* \approx 2\%$), which is an excellent value for the solar efficiency accordingly to the values reported in the literature [13-18], which are in the interval [0.3%, 3%]. However, if we substitute the values of η ($\eta^* = 0.516$) and θ ($\theta^* = 82.41°$) which maximize $w_E(\text{or}E)$ into eq. (56), we obtain $w = 5.5\%$, yet larger than the reported values for w [0.3%, 3%], but smaller than CET calculations [18].

4. – Concluding remarks

FTT models for studying the problem of the conversion of solar energy into winds energy can be useful as a crude first approximation to this matter. In spite of its simplicity these models provide reasonable values for the average power of the winds and for the extreme temperatures of the atmosphere of a planet. In the FTT approach to this issue a maximization criterion is usually involved. Several authors have used the hypothesis that the atmospheric “heat engine” maximizes the power output, but as De Vos and Flater assert no mechanism guarantees that the atmosphere maximizes the wind’s power. Thus, we consider appropriate to essay with another optimization criterion, which is the so-called ecological one. In fact, some authors [25, 27, 28] have recognized that the Earth’s atmosphere operates at nearly its maximum efficiency, thus, for endoreversible models it is possible that some kind of ecological regime would be more plausible than a maximum-power regime. In the present paper, we have used the Gordon-Zarmi model for the conversion of solar energy into wind energy but involving De Vos' ideas about the role of the greenhouse effect on the thermodynamical behavior of the atmosphere. Our results suggest that if the greenhouse effect is included only in the first half of the Carnot-like cycle of the Gordon-Zarmi model, the solar-energy conversion efficiency, w, strongly depends on this effect (see fig. 3). When we take the parameter γ as acting in both branches of the cycle, we recover the first w-expression reported by De Vos and Flater (eq. (23)). In our version of the GZ model, we also include the so-called nonendoreversibility parameter, R, and see that this quantity may also influence the solar-energy efficiency w (see fig. 4). In sect. **3**, we have analyzed the two-reservoir De Vos-van der Wel model under an ecological optimization, and we found a w-value within the interval of values reported in the literature [13-18] referred by De Vos and van de Wel [6]. Despite we used the simplest DVVW model, we have obtained $w_E \approx 2\%$, which is a better number than that obtained by De Vos and van der Wel for their two-reservoir model under a maximum-power criterion. However, when we maximize the conversion efficiency w, by considering the power output W under maximum ecological regime, the resulting value is $w = 5.25\%$ which is not a very good value (although is smaller than the DVVW value). All these numerical values are smaller than the Carnot limits reported by Piexoto and Oort [18], $w_c \leq 10\%$, such as it must be expected from FTT models in comparison with CET models. In summary, by means of the models due to Gordon-Zarmi, De Vos-Flater and De Vos-van der Wel we have found some simple insights on the process of conversion of solar into wind energy.

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This work was supported in part by COFAA-IPN-México. We thank M. Santillán for fruitful discussions. MABJ thanks support by SUPERA and the doctoral program of UAEM-México.

REFERENCES

[1] Pollarolo G. and Sertorio L., *Nuovo Cimento C*, **2** (1979) 335.
[2] Gordon M. and Zarmi Y., *Am. J. Phys.*, **57** (1989) 995.
[3] De Vos A. and Flater G., *Am. J. Phys.*, **59** (1991) 751.
[4] De Vos A., *Endoreversible Thermodynamics of Solar Energy Conversion* (Oxford University Press, Oxford) 1992.
[5] De Vos A. and van del Wel P., *J. Non-Equilib. Thermodyn.*, **17** (1992) 77.
[6] De Vos A. and van del Wel P., *Theor. Appl. Climatol.*, **46** (1993) 193.
[7] Barranco-Jiménez M. A. and Angulo-Brown F., *J. Appl. Phys.*, **80** (1996) 4872.
[8] Raval A. and Ramanathan V., *Nature*, **342** (1989) 758.
[9] Rubin M., 1979 *Phys. Rev. A*, **19** 1272.
[10] Angulo-Brown F., *J. Appl. Phys.*, **69** (1991) 7465.
[11] Curzon F. and Ahlborn B., *Am. J. Phys.*, **43** (1975) 22.
[12] Arias-Hernández L. A. and Angulo-Brown F., *J. Appl. Phys.*, **81** (1997) 2973.
[13] Lettau H., *Arch. Met. Geoph. Biokl.*, **7** (1954) 133.
[14] Hubbert M., *Sci. Am.*, **225** (1971) 60.
[15] Gustavson M., *M Science*, **204** (1979) 13.
[16] Ingersoll A., *Sci. Am.*, **245** (1981) 66.
[17] Arpe K., *Beitr. Phys. Atmos.*, **59** (1986) 321.

[18] Peixoto J. and Oort A., *Physics of Climate* (American Institute of Physics: New York) 1992, p. 365.
[19] Gutkowicz-Krusin D., Procaccia I. and Ross J., *J. Chem. Phys.*, **69** (1978) 3898. These authors showed that the CA endoreversible limit (with instataneous adiabats) is reached for a large compression ratio. In the GZ virtual cells it is feasible to consider this condition is fullfilled.
[20] Oskaynak S., Goktan S. and Yavuz H., *J. Phys. D*, **27** (1994) 1139.
[21] Chen J., *J. Phys. D*, **27** (1994) 1144.
[22] Velasco S., Roco J. M. M., Medina A., White and Calvo-Hernandez A., *J. Phys. D*, **33** (2000) 355.
[23] Chen L., Wu C. and Sun F., *J. Non-Equilib. Thermodyn.*, **24** (1999) 327.
[24] Ares de Parga G., Angulo-Brown F. and Navarrete-González T. D., *Energy*, **24** (1999) 997.
[25] Curry J. A. and Wester P. J., *Thermodynamics of Atmospheres & Oceans* (International Geophysics Series: Academic Press) 1999.
[26] Zemansky M. and Dittman R., *Heat and Thermodynamics* (Mc Graw Hill, Boston) 1997. This numerical result is in reasonable agreement with the relation between altitude and temperature change given by $\mathrm{d}h = -(\frac{\gamma}{\gamma-1})(\frac{R}{mg})\mathrm{d}T$ for an atmosphere undergoing an ideal adiabatic expansion. $\gamma = \frac{c_p}{c_v}$ is the adiabatic exponent, R is the universal gas constant, m is the molar mass of air and g is the acceleration of gravity.
[27] Lorentz E., in *Dynamic of Climate* (Pergamon Press, New York) 1960, p. 86.
[28] Schulman L., *J. Atmos. Sci.*, **34** (1977) 559.