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Waves and granulation(*)

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Summary. — Propagation of hydrodynamic waves is considered in frames of the one-dimensional model of granulation. An exact analytical solution of the problem is obtained. The theory is applied only to high-frequency p-modes, because the effect of gravity is neglected. It appears that acoustic waves in a structured atmosphere are not plane ones. Besides, there are hydrodynamic phonon and wave guide wave modes. The results of calculations of the wave functions for high-frequency waves with periods 100–200 s are presented. Upgoing waves are captured in integranular lanes, while downgoing ones are trapped in granules. The effect of wave capture is of greater efficiency for phonon and wave guide modes. The phase velocities of waves differ from the mean sound speed in the photosphere.

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1. – Introduction

The observations clear show the interrelation between granulation and p-modes [1-5]. Without question p-modes oscillations in granules and lanes are different. The limited space resolution of current observations gives no way for a detailed exploration of the effect. For example, the width of lanes is about 100–200 km, that is beyond the resolution of current observation. Thus, to understand the results of observations and predict the effects, which can be observed with better resolution, the development of an adequate theory is needed. There is the naive opinion that the problem has been solved in the frames of the theory of acoustic waves in a uniform atmosphere. Before the results are presented the difficulties encountered in the theory of waves in granulation are outlined.

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The solar granulation makes the photosphere inhomogeneous. A treatment of p-modes propagation in the photosphere faces fundamental problems, because wavelength is about the size of granules. The granulation has its greatest impact on waves, when both temperature fluctuations and flows are taken into account. The temperature in granules and lanes is about 6000 K and 4000 K, respectively. Consequently, local sound velocity in lanes is less than in granules by 20%, approximately. But this is not the whole story. The upflows and downflows affect hydrodynamic waves propagation due to acoustic Doppler effect. For example, upgoing waves slow down in the lanes due to downflows. If velocity of downflows is about 3 $\mathrm{km\,s^{-1}}$, while velocities of upflows in the granules are about 1 $\mathrm{km\,s^{-1}}$, the local phase velocities of upgoing waves in laboratory frame are two times less in lanes in comparison with granules. This is true for high-frequency p-modes in the photosphere. But, a similar problem exists for 5 min oscillations in the subphotospheric layers. It is interesting that downgoing waves do not meet so strong inhomogeneity, because in this case the effects of the temperature fluctuations and flows on local phase velocity of sound waves are opposite and compensate each other. Thus, there is no question that only a treatment of wave equation can solve the problem of hydrodynamic wave propagation in the photospere. In some respects this is similar to the treatment of p-modes in a stratified atmosphere, when the eigenvalue problem has to be considered and the wave function has to be obtained. The application of perturbation or asymptotic methods to the case when phase velocities vary two times in granulation is under question. The method of the treatment of hydrodynamic waves in a structured atmosphere has been developed by [6-8]. Wave propagation in one-dimensional compressible periodical shear flow with temperature variations has been considered. Originally, the method has been developed to explore the effect of temperature and flow fluctuations on acoustic waves. But it was revealed that new wave modes appear alongside with acoustic waves. This is the hydrodynamic phonon mode, which has nothing to do with a heat transfer, and wave guide mode. The shortcoming of the treatment is the neglecting of the gravity. Consequently, the results of the theory can be applied only to the case of the high-frequency p-modes, because the neglecting of gravity is acceptable. The first applications of the model of periodical shear flow to granulation [9, 10] were restricted by the case of p-modes with $l = 0$, that corresponds to the propagation of waves along the direction of flows. Besides, the sinusoidal dependence of the vertical velocity of equilibrium flow and temperature variations on the horizontal coordinate was considered. Thus, the sizes of granules and lanes are the same in the models considered by [9, 10]. This is in conflict with real granulation. In the current paper the model with narrow lanes is considered. Besides, all possible modes for the case $l \neq 0$ are explored.

2. – Basic equation and solution

An equilibrium atmosphere with vertical hot upflows and cold downflows is considered. The equilibrium pressure p_0 is constant in the entire atmosphere, while the equilibrium values of temperature $T_0(x)$, density $\rho_0(x)$ and vertical velocity $\vec{V} = (0, 0, V(x))$ are arbitrary functions of x. All equilibrium variables are independent of the vertical coordinate z. The problem of linear waves in the atmosphere with periodical shear flow is reduced to the generalized Rayleich equation, which reads

$$\frac{\mathrm{d}^2 p}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}\ln c_0^2(x)}{\mathrm{d}x} + \frac{2}{V_{\mathrm{ph}} - V(x)}\frac{\mathrm{d}V(x)}{\mathrm{d}x}\right)\frac{\mathrm{d}p}{\mathrm{d}x} + \left[\frac{k_z^2(V_{\mathrm{ph}} - V(x))^2}{c_0^2(x)} - k_z^2\right] p = 0, \tag{1}$$

where p is the amplitude of pressure fluctuations $p' = p\exp[\imath(\omega t - k_z z)]$, $c_0(x)$ is the sound speed, $V_{\mathrm{ph}} = \omega/k_z$ is the phase velocity. In the limit of a uniform atmosphere, where $V(x) = \mathrm{const}$ and $c_0(x) = \mathrm{const}$, the solution of (1) is $p = C\exp[\imath k_\perp x]$ and the equation is reduced to the dispersion relation for acoustic waves $\omega^2 = (k_z^2 + k_\perp^2)c_0^2$. In the case of periodical dependence of the temperature and equilibrium velocity on the x-coordinate the problem is reduced to the classical problem of periodical shear flow, the stability of which was explored in details for the incompressible flows. The stability exploration was performed by the usage of classical energy principle. Zhugzhda was the first who pointed out that the problem could exactly be solved by the usage of methods, which were developed for the differential equations with periodical coefficients.

In the case of periodical shear flow, when $T_0(x) = T_0(x + 2d)$ and $V(x) = V(x + 2d)$, the general solution of eq. (1) is

$$p = Ae^{\imath(\omega t - k_z z - k_\perp x)} \sum_{m=\infty}^{-\infty} C_m e^{imk_g x}, \tag{2}$$

where A is an arbitrary constant and $k_g = \pi/d$ is the space wave number. The coefficients C_m in the solution are not arbitrary constants. The relative values of the coefficients C_m are defined by the equilibrium profiles of the velocity of shear flow and temperature. The solution of (1) is obtained as follows. After substitution of the general solution (2) in (1) the terms of the same space wave numbers mk_g are collected. The equation is satisfied only if the coefficients at all space harmonic obtained by the collection are equal to zero. This provides an infinite set of coupled uniform linear equations with respect to coefficients C_m of the general solution (2). The set of equations has a nontrivial solution, when the infinite determinant of the equation set, which is known in the theory of equations with periodical coefficients as a Hill determinant, equals zero. Setting Hill determinant equal to zero provides a dispersion equation of the problem. The main unexpected result of the solution is that the dispersion equation has an infinite number of eigenvalues, which correspond to the infinite number of wave modes. Moreover, there are few different kinds of the wave modes. For the first time new wave modes in the periodical shear flow have been revealed in [6]. But it has took a considerable time before the physical meaning of new modes was understood.

3. – Wave modes

The main features of different modes are outlined here, while the complete discription will be published in [8]. There is only one mode among the infinite number of wave modes in the compressible periodic shear flow, which in the limit of uniform atmosphere, when $c_0(x) = \mathrm{const}$ and $V(x) = \mathrm{const}$, reduces to the acoustic wave. All other modes appear only in the periodical shear flow. In the case of acoustic waves the main term in the general solution (2) corresponds to $m = 0$, namely, the coefficient C_0 has the largest modulus among all others coefficients. In the limit of uniform atmosphere all coefficients in (2) for $m \neq 0$ tend to zero. The solutions (2) for upgoing and downgoing waves are different. Their phase velocities are different and differ from the mean sound speed in the structured atmosphere.

Besides acoustic waves there is the hydrodynamic phonon mode, which got his name due to the analogy with classical phonon mode in one-dimensional lattice. Their dispersion relations are the same in the case of propagation across shear flows. The distinctive

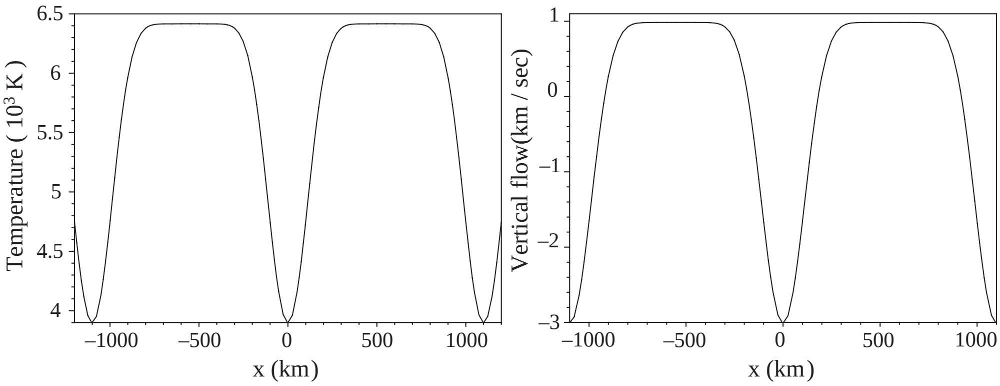

Fig. 1. – 1D model of solar granulation. The dependence of the temperature (left plot) and vertical velocity of flows (right plot) on the horizontal coordinate.

feature of the phonon mode is a finite frequency $\omega \neq 0$ for $\lambda = \infty$ and opposite signs of phase and group velocities. However, the exact analogy is violated in the case of an oblique propagation of hydrodynamic phonon waves in the periodical shear flow. Different phonon modes differ in frequencies $\omega(\lambda = \infty)$. Hydrodynamic phonon modes are running along shear flows only if their frequencies are high enough. Cut-off frequency increases with the number of phonon modes.

The third kind of modes is the wave guide mode, which is similar to waves in the multilayer wave guide. These modes also have cut-off frequency, which increases with the number of modes. There is also the vortex mode, which is not discussed for reasons of space.

4. – Model of granulation

The model of granulation, which has been used in [9, 10], is in conflict with observations, because it assumes that granules and lanes are of the same size. The current model removes this shortcoming. The problem was solved for a more general profile of periodical shear flow, when the temperature and flow velocity depends on x in the following manner:

$$T_0 = T_{00} + \delta(\cos k_g x)^n, \quad V = V_0 + Ma(\cos k_g x)^n. \tag{3}$$

Figure 1 shows the plots for $T_0(x)$ and $V(x)$ for a special choice of parameters, which from my standpoint fits satisfactorily the observations and simulations. Shortcomings of the model are neglecting of gravity and stratification. That is the reason why the model is applied to the consideration of high-frequency p-modes, whose properties are weakly dependent on gravity. The size of granules in the model is 1100 km, while lanes are one order narrower ($n = 10$).

5. – High-frequency phonon and p-modes in the photosphere

Figure 2 (upper row) shows the dependence of the velocity amplitudes of upgoing and downgoing acoustic waves of period $P = 180, 200$ s and $l = 500, 1000$, which is connected

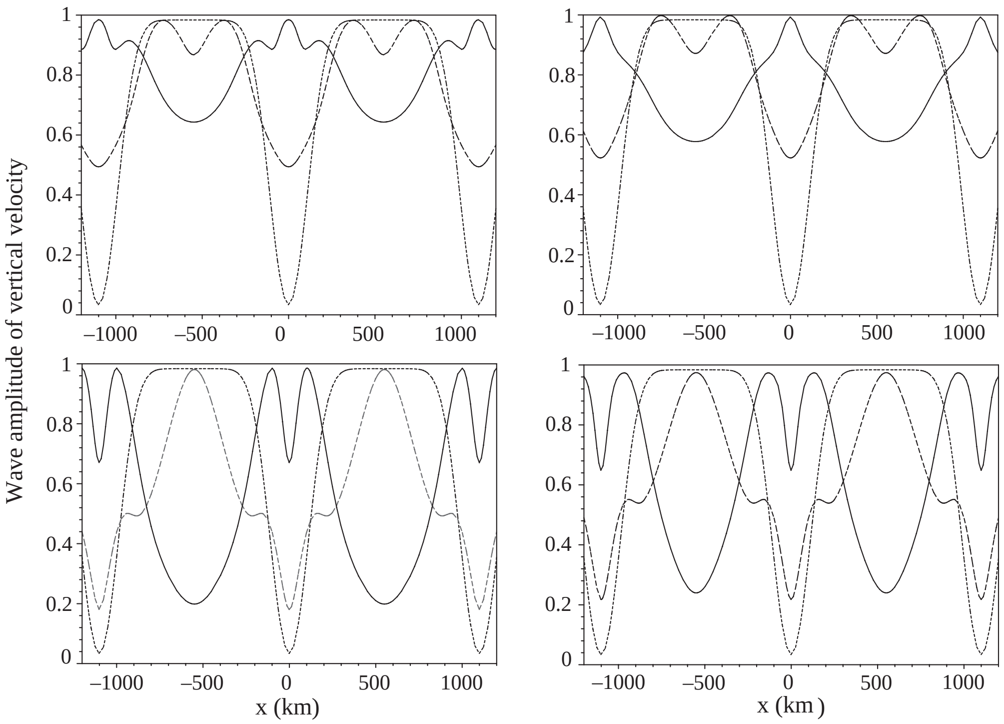

Fig. 2. – The dependence of the amplitude of vertical velocity in arbitrary units of upgoing (solid line) and downgoing (dashed line) acoustic (upper row) and phonon (bottom row) waves on the horizontal coordinate. The temperature profile of the granulation model in arbitrary units is shown by the dotted line. (Left plots) period $P = 200$, $l = 1000$; (right plots) period $P = 180$, $l = 500$.

with the horizontal wave number by the relation $\sqrt{l(l+0.5)} = k_{\perp}R_{\odot}$. The effect of the capture of upgoing waves by intergranular lanes and downgoing ones by granules is clear pronounced. The amplitude of upgoing wave in the intergranular lanes is about two times larger than in granules. If high-frequency p-modes do not undergo reflection in the photosphere, because their frequency is above cut-off frequency in the photosphere, the energy flux in the intergranular lanes has to be few times larger than in granules. The essential enhancement of energy flux of high-frequency p-modes is bound to be under intergranular lanes in the chromosphere, because the distance between lanes is about 1000 km, that compares to the thickness of the chromosphere. The current model does not take into account that downflows in some of the lanes are rather fast. The effect of the capture of upgoing waves has to be reinforced in the lanes with fast downflows. This effect deserves more exploration in connection with bright points in the chromosphere. The phase velocities of upgoing and downgoing waves in terms of mean sound speed in the photosphere are $V_{\mathrm{ph}}/\overline{c_0} = 1.04,\ -1.02$ (for $P = 200$ s) $1.02,\ -0.99$ (for $P = 180$ s). Thus, phase velocities of high-frequency p-modes just slightly differ from mean sound speed and corrections to eigenfrequencies due to granulation are rather small. Figure 2 (bottom row) shows the dependence of the velocity amplitudes of upgoing and downgoing phonon waves for the same frequencies and l, as for acoustic waves. The effect of capture of waves is markedly enhanced in comparison with the acoustic waves. The amplitudes

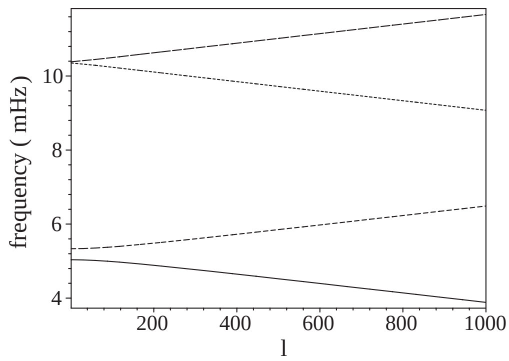

Fig. 3. – The cut-off frequencies of first phonon (solid line), first wave guide (short-dashed line), second phonon (dotted line) and second wave guide (long-dashed line) modes as a function of l.

of upgoing phonone mode in the intergranular lanes are about four times larger than in granules, while the energy flux has to be 10–15 times more. The phase velocity of phonon modes is well below sound speed, namely, the phase velocities of upgoing and downgoing phonon waves are $V_{\mathrm{ph}}/\overline{c_0} = 1.61,\ -1.67$ for $P = 200$ s and $1.69,\ -1.81$ for $P = 180$ s. The crucial distinction of phonon waves from the acoustic ones is the opposite signs of their horizontal phase and group velocities. If phonon modes exist in the solar photospheric and subphotospheric layers, they have to manifest themselves on the time-distance diagram in a way, which is very different from p-modes. For reasons of space exploration phase relations for different modes are not discussed here. However, it can help to distinguish different wave modes. The different wave modes are outlined shortly. An ensemble of all possible wave modes has to be produced by compressible turbulence. Roughly speaking monopole generation produces mainly acoustic mode, dipole, quadrupole, etc. and generation produces main phonon and wave guide modes. An important point is that phonon and wave guide modes have cut-off frequency. Figure 3 shows cut-off frequencies of few modes calculated for the current model of granulation. Waves with frequencies below lines shown on the plot do not propagate in the photosphere. Thus, rather low-frequency phonon and wave guide modes generated under the photosphere by convection are reflected from the photosphere and can be captured in subphotospheric layers. This effect is the valid one for farther exploration, which is beyond the scope of this short contribution. The current model offers a means of complete exploration of all wave modes in the photosphere. Amplitudes and phase relations of all components of velocities, density and temperature fluctuations can be found, that makes the comparison with any observations possible.

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