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Comparison between an integral and algebraic formulation for the eddy diffusivity using the Copenhagen experimental dataset

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Summary. — In this work an algebraic formulation to evaluate the eddy diffusivities in the Convective Boundary Layer (CBL) is derived. The expression depends on the turbulence properties (z height dependence) and the distance from the source. It is based on the turbulent kinetic energy spectra and Taylor's statistical diffusion theory. It has been tested and compared through an experimental dataset, with another complex integral formulation taken from the literature. The agreement between the complex integral formulation and simple algebraic expression points out that this new parameterization is valid and can be used as a surrogate for eddy diffusivities in the inhomogeneous convective turbulence present in the CBL. The validation shows that the proposed algebraic vertical eddy diffusivity is suitable for application in advanced air quality regulatory models.

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1. – Introduction

In principle from the Eulerian dispersion model concept it is possible to construct a pratical model of dispersion from a continuous point source given adequate boundary and initial conditions plus a knowledge of the time and space fields of U (mean wind vector) and K_α (eddy diffusivities, with $\alpha = x, y, z$) [1].

The choice of an adequate parameterization of concentration turbulent fluxes plays a relevant role in air quality dispersion models which are based on the advection-diffusion equation. As a consequence, many of the turbulent dispersion researches are related to the specification of these fluxes.

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The principal scheme for closing the advection-diffusion equation is to relate turbulent concentration fluxes to the gradient of the mean concentration by K eddy diffusivities, which carry with them the physical structure of the transport. In this aspect, these eddy diffusivities, for a continuous point source, must consider the memory effect present in the near-source turbulent field. In fact, following Arya [2]: "To represent the near-source diffusion in weak winds the eddy diffusivities should be considered as function of not only turbulence (*e.g.*, large eddy length and velocity scales), but also of distance from the source". Following this idea, Degrazia *et al.* [3] proposed for the CBL a complex integral formulation for the eddy diffusivities having the form

$$\frac{K_\alpha}{w_* z_i} = \frac{0.09 c_i^{1/2} \psi^{1/3} (z/z_i)^{4/3}}{(f_m^*)_i^{4/3}} \int_0^\infty \frac{\sin\left[\frac{7.84 c_i^{1/2} \psi^{1/3} (f_m^*)_i^{2/3} n' X}{(z/z_i)^{2/3}}\right]}{(1+n')^{5/3}} \frac{\mathrm{d}n'}{n'}, \tag{1}$$

where $c_i = \alpha_i \alpha_u (2\pi k)^{-2/3}$, $\alpha_u = 0.5 \pm 0.05$ and $\alpha_i =$1, 4/3, 4/3 for u, v and w components, respectively (values of α_i are derived from the turbulence isotropy in the inertial subrange) [4, 5]; $(f_m^*)_i$ is the normalized frequency of the spectral peak regardless of stratification; z is the height above ground; z_i is the CBL height; $\psi = \varepsilon z_i / w_*^3$ is the nondimensional molecular dissipation rate function; w_* is the convective velocity scale; n' is a nondimensional frequency and finally $X = x w_* / U z_i$ can be thought as a nondimensional time, since it is the ratio of travel time x/U to the convective timescale z_i / w_*.

The present approach (eq. (1)) fundamentally hinges on Batchelor's [6] time-dependent equation for the evolution of eddy diffusivities K_α:

$$K_\alpha = \frac{1}{2} \frac{\mathrm{d}\sigma_\alpha^2}{\mathrm{d}t} \tag{2}$$

which says that the generalized eddy diffusivity is the time derivative of the spacial variance. It is important to point out the benefits of using the parameterization given by eq. (1). Taylor's theory is valid only for homogeneous turbulence, whereas eq. (1) that employs a velocity spectra dependent on z is more general and can be also applied in inhomogeneous turbulence [3,7]. More recently, Arya [2] suggested a simple interpolation formula for the eddy diffusivities given by

$$K_\alpha = \frac{\sigma_i^2 x}{U} \left(1 + \frac{x}{b L_\alpha}\right)^{-1}, \tag{3}$$

where σ_i^2 is the variance of the turbulent wind velocity (normally dependent on z), $L_\alpha = U T_{L_i}$ in which T_{L_i} is the Lagrangian decorrelation time scale of velocity fluctuations (also dependent on z) and b is a constant of order 1. We stress here that an expression as (3) to be applicable to real cases in the planetary boundary layer (PBL), must be dependent on z (σ_i^2, U and L_α functions of z). Only in this case we can construct an eddy diffusivity for inhomogeneous turbulence. If it is not a function of z, then this limits the approach to much fewer atmospheric conditions and locales.

In this work we derive initially the turbulent decorrelation time-scale T_{L_i} for the CBL based upon convective similarity and statistical diffusion theory [7]. This decorrelation time-scale will be used to obtain a simple algebraic expression for the eddy diffusivities in the CBL which depends on the turbulence properties (inhomogeneous turbulence) and the distance from the source. The hypothesis to be tested in this study is whether the complex integral formulation (1) for eddy diffusivities in inhomogeneous turbulence can be expressed by a simple algebraic equation of the same sort of eq. (3) but here valid for the CBL. Furthermore, the simple algebraic expression is evaluated against the complex integral formulation (eq. (1)) using an air pollution model and atmospheric dispersion experiments that were carried out in the Copenhagen area under moderately unstable conditions [8].

2. – Algebraic formulation for the eddy diffusivity depending on height and source distance

An analysis of estimates for σ_α [9] (the generalized-dispersion parameter) obtained from data collected during a field study suggests the following model for the dispersion parameters of elevated releases in the CBL:

$$\sigma_\alpha = \frac{\sigma_i t}{\left(1 + t/2T_{L_i}\right)^{1/2}}\,, \tag{4}$$

where σ_i^2 is given by [7]

$$\sigma_i^2 = \frac{1.06 c_i \Psi^{2/3} \left(z/z_i\right)^{2/3} w_*^2}{\left(f_m^*\right)^{2/3}} \tag{5}$$

and T_{L_i} is obtained for large diffusion travel times of the relation [10]

$$K_\alpha = \sigma_i^2 T_{L_i} = \frac{\sigma_i^2 \beta_i F_i(0)}{4} \tag{6}$$

and consequently

$$T_{L_i} = \frac{\beta_i F_i(0)}{4}\,, \tag{7}$$

where according to Degrazia and Anfossi [11], $\beta_i = 0.55\, U/\sigma_i$ is defined as the ratio of the Lagrangian to the Eulerian time-scales; $F_i(0)$ is an Eulerian one-dimensional turbulent velocity spectrum at the origin normalized by the turbulent velocity variance having the form

$$F_i(0) = \frac{z}{\left(f_m^*\right) U}\,. \tag{8}$$

The substitution of β_i and (8) in (7) results in the following equation for the turbulent decorrelation time-scale in the CBL:

$$T_{L_i} = \frac{0.13 z^{2/3} z_i^{1/3}}{c_i^{1/2} \Psi^{1/3} \left(f_m^*\right)_i^{2/3} w_*}\,. \tag{9}$$

A formulation for the time-dependent coefficients K_α has been derived by [6] and as a consequence from eqs. (2) and (4) we obtain

$$K_\alpha = \frac{\sigma_i^2 t}{(1+t/2T_{L_i})^2}\left(1+\frac{t}{4T_{L_i}}\right). \tag{10}$$

Equations (1), (3) and (10) have the same behaviour for small and large times. As a consequence, eq. (10) can be utilised to generate a simple algebraic expression that can be used as a surrogate of eq. (1). Thus, replacing eqs. (5) and (9) into algebraic expression (10) the following generalized eddy diffusivities depending on the source distance can be written as

$$\frac{K_\alpha}{w_* z_i} = \frac{0.583 c_i \Psi^{2/3}(z/z_i)^{4/3} X\left[0.55(z/z_i)^{2/3} + 1.03 c_i^{1/2}\Psi^{1/3}\left(f_m^*\right)_i^{2/3} X\right]}{\left[0.55(z/z_i)^{2/3}\left(f_m^*\right)_i^{1/3} + 2.06 c_i^{1/2}\Psi^{1/3}\left(f_m^*\right)_i X\right]^2}. \tag{11}$$

3. – Vertical eddy diffusivities from eqs. (1) and (11)

For horizontal homogeneity the CBL evolution is mainly driven by the vertical transport of heat. Therefore, the analysis will focus on the vertical eddy diffusivities. These eddy diffusivities can be derived from eqs. (1) and (11) by assuming

$$\left(f_m^*\right)_w = \frac{z}{(\lambda_m)_w} = 0.55\left(\frac{z}{z_i}\right)\left[1-\exp\left[-\frac{4z}{z_i}\right] - 0.0003\exp\left[\frac{8z}{z_i}\right]\right]^{-1}, \tag{12}$$

where $(\lambda_m)_w = 1.8 z_i\left[1-\exp\left[-\frac{4z}{z_i}\right] - 0.0003\exp\left[\frac{8z}{z_i}\right]\right]$ is the value of the vertical wavelength at the spectral peak, which was obtained from empirical data by Caughey and Palmer [12]. To proceed, the vertical eddy diffusivities described in terms of energy-containing eddies and function of the downwind distance X and the height z can be obtained from eqs. (1), (11) and (12) using $c_w = 0.36$ [7], as follows:

$$\frac{K_z}{w_* z_i} = 0.12\psi^{1/3} q^{4/3}\int_0^\infty \frac{\sin\left(3.17 q^{-2/3}\psi^{1/3} X n'\right)}{(1+n')^{5/3}}\frac{\mathrm{d}n'}{n'} \tag{13}$$

and

$$\frac{K_z}{w_* z_i} = \frac{0.38\psi^{2/3} X\left[1+0.75\psi^{1/3} q^{-2/3} X\right]}{\left[0.82 q^{-1/3} + 1.24\psi^{1/3} q^{-1} X\right]^2}, \tag{14}$$

where $q = \left[1-\exp\left[\frac{-4z}{z_i}\right] - 0.0003\exp\left[\frac{8z}{z_i}\right]\right]$.

The dissipation function ψ according to ref. [13] has the form

$$\psi^{1/3} = \left[\left(1-\frac{z}{z_i}\right)^2\left(\frac{z}{-L}\right)^{-2/3} + 0.75\right]^{1/2}, \tag{15}$$

where L is the Monin-Obukhov length in the surface layer.

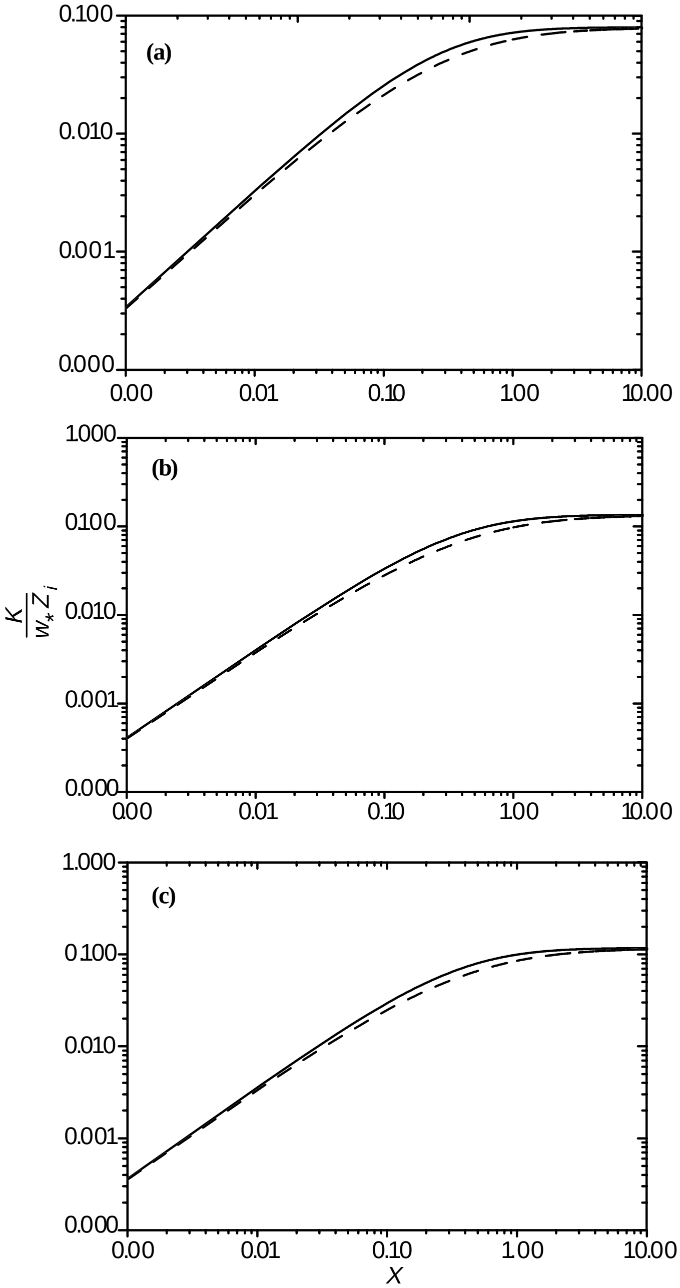

Fig. 1. – Vertical eddy diffusivities calculated from eqs. (13) (integral, dashed line) and (14) (algebraic, solid line) for three different values of $z/z_i = 0.2$ (a), 0.5 (b), 0.8 (c).

In fig. 1 are depicted the curves for $K_z/w_* z_i$ given by integral (13) and algebraic (14) formulas for three different heights: $z/z_i = 0.2$ (a), 0.5 (b), 0.8 (c) considering $z/L = -10$. The curves show a good agreement between the two expressions. For a given height, the $K_z/w_* z_i$ is initially zero, increases with X, at first linearly and then more slowly, and finally tends to a constant value.

As a test for the algebraic formula (14) we introduce the vertical eddy diffusivities (13) and (14) in an air pollution model to simulate the ground level cross-wind integrated concentrations of contaminants released from an elevated continuous source in an unstable PBL.

4. – Air pollution model

Following Vilhena [14,15], for a Cartesian coordinate system in which the x direction coincides with that one of the average wind, the steady-state advection-diffusion equation is written as [2]

$$U\frac{\partial \overline{c}}{\partial x} = \frac{\partial}{\partial x}\left(K_x \frac{\partial \overline{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \overline{c}}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{c}}{\partial z}\right), \tag{16}$$

where $\overline{c}$ denotes the average concentration, U the mean wind speed in x direction and K_x, K_y and K_z are the eddy diffusivities. The cross-wind integration of eq. (16) (neglecting the longitudinal diffusion) leads to

$$U\frac{\partial \overline{c^y}}{\partial x} = \frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{c^y}}{\partial z}\right) \tag{17}$$

subject to the boundary conditions of zero flux at the ground and CBL top, and a source with emission rate Q at height H_s

$$K_z \frac{\partial \overline{c^y}}{\partial z} = 0 \qquad \text{in} \quad z = 0, z_i\,, \tag{18}$$

$$U\overline{c^y}(0,z) = Q\delta(z - H_s) \qquad \text{in} \quad x = 0\,, \tag{19}$$

where now $\overline{c^y}$ represents the average cross-wind integrated concentration.

Bearing in mind the dependence of the K_z coefficient and wind speed profile U on the variable z, the height z_i of a CBL is discretized in N sub-intervals in such a manner that inside each interval $K_z(z)$ and $U(z)$ assume the average value

$$K_n = \frac{1}{z_{n+1} - z_n} \int_{z_n}^{z_{n+1}} K_z(z)\mathrm{d}z\,, \tag{20}$$

$$U_n = \frac{1}{z_{n+1} - z_n} \int_{z_n}^{z_{n+1}} U(z)\mathrm{d}z\,. \tag{21}$$

For the vertical eddy diffusivity depending on x and z (eqs. (13) and (14)), initially we take the average in z variable

$$K_n(x) = \frac{1}{z_{n+1} - z_n} \int\limits_{z_n}^{z_{n+1}} K_z(x, z) \mathrm{d}z \,. \tag{22}$$

The procedure is quite similar for x variable. Indeed the domain in x variable is discretized into variable sub-intervals of length Δx_i and in each sub-interval the following average value for the eddy diffusivity is considered:

$$K_{i,n} = \frac{1}{x_{i+1} - x_i} \int\limits_{x_i}^{x_{i+1}} K_n(x') \mathrm{d}x' \,. \tag{23}$$

Recall that $K_{i,n}$ assumes a constant value at $x_i \leqslant x \leqslant x_{i+1}$ and $z_n \leqslant z \leqslant z_{n+1}$. Therefore the solution of problem (17) is reduced to the solution of N problems of the type

$$U_n \frac{\partial \overline{c_n^y}}{\partial x} = K_{i,n} \frac{\partial^2 \overline{c_n^y}}{\partial z^2}, \qquad z_n \leqslant z \leqslant z_{n+1}, \qquad x_i \leqslant x \leqslant x_{i+1} \tag{24}$$

for $n = 1 : N$, where $\overline{c_n^y}$ denotes the concentration at the n-th sub-interval. To determine the $2N$ integration constants the additional $(2N - 2)$ conditions, namely continuity of concentration and flux at interface are considered

$$\overline{c_n^y} = \overline{c_{n+1}^y} \qquad n = 1, 2, ...(N - 1) \,, \tag{25}$$

$$K_{i,n} \frac{\partial \overline{c_n^y}}{\partial z} = K_{i,n+1} \frac{\partial \overline{c_{n+1}^y}}{\partial z} \qquad n = 1, 2, ...(N - 1) \,. \tag{26}$$

Applying the Laplace transform in eq. (24) there results

$$\frac{\partial^2}{\partial z^2} \overline{c_n^y}(s, z) - \frac{U_n s}{K_{i,n}} \overline{c_n^y}(s, z) = -\frac{U_n}{K_{i,n}} \overline{c_n^y}(0, z) \,, \tag{27}$$

where $\overline{c_n^y}(s, z) = L_p\left\{\overline{c_n^y}(x, z); x \to s\right\}$, which has the well-known solution

$$\overline{c_n^y}(s, z) = A_n e^{-R_n z} + B_n e^{R_n z} + \frac{Q}{2R_a} \left(e^{-R_n (z - H_s)} - e^{R_n (z - H_s)} \right) , \tag{28}$$

where

$$R_n = \pm \sqrt{\frac{U_n s}{K_{i,n}}} \qquad \text{and} \qquad R_a = \pm \sqrt{U_n K_{i,n} s} \,.$$

Finally, applying the interface and boundary conditions, we come out with a linear system for the integration constants. Henceforth the concentration is obtained inverting numerically the transformed concentration $\overline{c^y}$ by the Gaussian quadrature scheme [16]

$$\overline{c_n^y}(x,z) = \sum_{j=1}^{8} A_j \frac{P_j}{x} \left(A_n e^{-\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right) z} + B_n e^{\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right) z} \right), \tag{29}$$

$$\overline{c_n^y}(x,z) = \sum_{j=1}^{8} A_j \frac{P_j}{x} \Bigg[A_n e^{-\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right) z} + B_n e^{\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right) z} + + \frac{1}{2} \frac{Q}{\sqrt{\frac{P_j K_n U_n}{x}}} \left(e^{-(z-H_s)\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right)} - e^{(z-H_s)\left(\sqrt{\frac{P_j U_n}{x K_{i,n}}}\right)} \right) \Bigg]. \tag{30}$$

The solution (29) is valid for layers that no contain the contaminant source. On the other hand, the solution (30) can be used to evaluate the concentration field in the layer that contains the continuous source. These solutions are only valid for $x > 0$, once the quadrature scheme of Laplace inversion does not work for $x = 0$. A_j and P_j are the weights and roots of the Gaussian quadrature scheme and are tabulated in the book by Stroud and Secrest [17].

At this point it is important to mention that this procedure leads to a solution for the concentration with a continuous dependence on z and sectionally continuous on x because there is imposed the condition of continuity of concentration and flux concentration at interface z_n. To get a solution with continuous dependence on x and z variables, we must apply, besides the boundary conditions, the interface conditions of continuity of concentration at x_i and z_n. The justificative for the adopted approach stems from the simplicity resulting from the straight application of the formulation for concentration encountered by Moreira [18], when the eddy diffusivity coefficient varies only the z variable. Furthermore, no additional computational effort is required to evaluate the concentration when the eddy diffusivity depends on x and z. We are awared that this procedure is an approximation because of the discontinuity of concentration at interface x_i, but it improves the results. It is also relevant to recall that for both approaches, the number of integration constants are equal to the number of boundary and interface conditions, consequently the integration constants are uniquely determined.

5. – Model evaluation

The performance of the present model (eqs. (29), (30), (13) and (14)) has been evaluated against experimental ground-level concentration using tracer SF_6 data from dispersion experiments carried out in the northern part of Copenhagen, described in Gryning *et al.* [19]. The tracer was released without buoyancy from a tower at a height of 115 m, and collected at the ground-level positions at a maximum of three crosswind arcs of tracer sampling units. The sampling units were positioned 2–6 km from the point of release. Tracer releases typically started 1 h before the start of tracer sampling and stopped at the end of the sampling period; the average sampling time was 1 h. The site was mainly residential with a roughness length of 0.6 m. Table I shows the meteorological data from Gryning and Lick [8] and Gryning *et al.* [19] utilized during the experiments that were

TABLE I. – *Summary of meteorological conditions during the Copenhagen experiments.*

Exp.	U (ms^{-1})	u_* (ms^{-1})	L (m)	w_* (ms^{-1})	z_i (m)
1	3.4	0.36	−37	1.8	1980
2	10.6	0.73	−292	1.8	1920
3	5.0	0.38	−71	1.3	1120
4	4.6	0.38	−133	0.7	390
5	6.7	0.45	−444	0.7	820
6	13.2	1.05	−432	2.0	1300
7	7.6	0.64	−104	2.2	1850
8	9.4	0.69	−56	2.2	810
9	10.5	0.75	−289	1.9	2090

used for the validation of the proposed approach. The mean wind speed measured at the released heigth and presented in table I was used to calculate the vertical eddy diffusivity (eqs. (13) and (14)) for each dispersion experiment. To calculate w_*, the relation $w_*/u_* = (-z_i/kL)^{1/3}$ was used. The Copenhagen data set was chosen since most of the experiments were performed during unstable moderately atmospheric conditions, and without strong buoyancy, so that ground-level cross-wind integrated concentration can be simulated by an advection-diffusion equation. The stability parameter z_i/L indicates cases in which the unstable PBL presents weak to moderate convection.

The wind speed profile used in eqs. (29) and (30) has been parameterized following the similarity theory of Monin-Obukhov and OML model [20]:

$$U = \frac{u_*}{k}[\ln(z/z_0) - \Psi_m(z/L) + \Psi_m(z_0/L)] \qquad \text{if} \quad z \leqslant z_b\,, \tag{31}$$

$$U = U(z_b) \qquad \text{if} \quad z > z_b\,, \tag{32}$$

where $z_b = \min\,[|L|, 0.1z_i]$, and Ψ_m is a stability function given by [21]

$$\Psi_m = 2\ln\left[\frac{1+A}{2}\right] + \ln\left[\frac{1+A^2}{2}\right] - 2\tan^{-1}(A) + \frac{\pi}{2} \tag{33}$$

with

$$A = (1 - 16z/L)^{1/4}\,, \tag{34}$$

$k = 0.4$ is the Von Karman constant, u_* is the friction velocity and z_0 the roughness length.

In table II the measured and computed ground-level crosswind concentrations of the both approaches (eqs. (29), (30), (13) and eqs. (29), (30), (14)) are presented. A good agreement with the observed results of ground-level crosswind concentrations was obtained by the use of the algebraic vertical eddy diffusivity in eqs. (29) and (30). Figure 2 shows the scatter diagram between the observed and predicted cross-wind integrated concentations. The results given by the simulations are quite satisfactory for both parameterizations (eqs. (13) and (14)). Subsequently, aiming to confirm this analysis, an

TABLE II. – *Observed and modeled ground-level crosswind integrated concentration* $\overline{c^y}(x,0)/Q$ *at different distances from the source.*

Exp.	Distance (m)	Data (10^{-4}s m^{-2})	Model equations (29), (30), (13) (10^{-4}s m^{-2})	Model equations (29), (30), (14) (10^{-4}s m^{-2})
1	1900	6.48	8.33	7.58
	3700	2.31	4.53	4.07
2	2100	5.38	4.28	4.62
	4200	2.95	2.56	2.60
3	1900	8.20	8.67	8.92
	3700	6.22	5.32	5.06
	5400	4.30	3.99	3.86
4	4000	11.66	8.97	8.58
5	2100	6.72	7.55	8.88
	4200	5.84	5.64	5.79
	6100	4.97	4.43	4.47
6	2000	3.96	3.22	3.63
	4200	2.22	2.02	2.10
	5900	1.83	1.58	1.61
7	2000	6.70	4.91	4.87
	4100	3.25	2.73	2.59
	5300	2.23	2.21	2.10
8	1900	4.16	5.30	5.39
	3600	2.02	3.35	3.22
	5300	1.52	2.60	2.57
9	2100	4.58	4.19	4.31
	4200	3.11	2.48	2.47
	6000	2.59	1.80	1.82

evaluation between the observed and predicted results was made by applying the following statistical indices [22]:

- Nmse (normalized mean square error) $= \overline{(C_\mathrm{o} - C_\mathrm{p})^2}/\overline{C_\mathrm{o} C_\mathrm{p}}$,
- Fa2 = fraction of data (%) for $0.5 \leqslant (C_\mathrm{p}/C_\mathrm{o}) \leqslant 2$
- Cor (correlation coefficient) $= \overline{(C_\mathrm{o} - \overline{C_\mathrm{o}})(C_\mathrm{p} - \overline{C_\mathrm{p}}}/\sigma_\mathrm{o}\sigma_\mathrm{p}$,
- Fb (fractional bias) $= \overline{C_\mathrm{o}} - \overline{C_\mathrm{p}}/0.5(\overline{C_\mathrm{o}} + \overline{C_\mathrm{p}})$,
- Fs (fractional standart deviations) $= (\sigma_\mathrm{o} - \sigma_\mathrm{p})/0.5(\sigma_\mathrm{o} + \sigma_\mathrm{p})$,

TABLE III. – *Statistical indices evaluating the model performance.*

Model	Nmse	Fa2	Cor	Fb	Fs
Eqs. (29), (30), (13)	0.06	1.00	0.89	0.025	0.095
Eqs. (29), (30), (14)	0.07	1.00	0.88	0.020	0.078

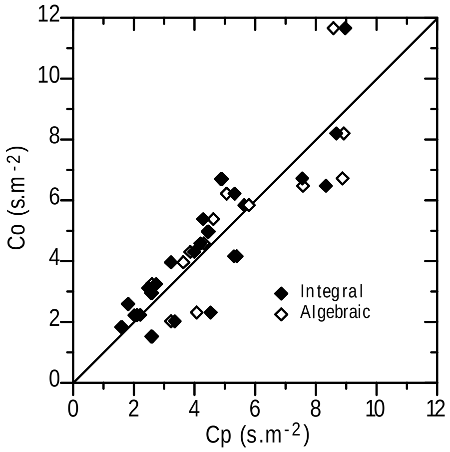

Fig. 2. – Observed (C_{o}) and predicted (C_{p}) scatter diagram of ground-level crosswind concentrations, normalised with emission ($\overline{c^y}/Q$), using the approach (29) and (30) with vertical eddy diffusivities given by eqs. (13) and (14).

where subscripts "o" and "p" refer to observed and predicted quantities, and an overbar indicates an average. These results are shown in table III.

The statistical evaluation highlights a quite satisfactory performance of both vertical eddy diffusivities for all the experiments considered: all the values for the numerical indices are within ranges that are characteristics of those found for other state-of-the-art models applied to other field datasets. The results point out that the parameterization of the turbulent transport given by a simple algebraic interpolation formula can accuratelly represent the near-source diffusion in weak winds. Therefore, the hypothesis that the eddy diffusivities as functions of distance from the source can be estimated from a simple algebraic equation is valid.

6. – Conclusions

This paper describes the development and testing of an algebraic formulation for the eddy diffusivity depending on the source distance and turbulence in the CBL. The approach is based on spectral properties and Taylor's statistical diffusion theory. By considering a model for the dispersion parameters that incorporates a decorrelation time scale and a variance of the turbulent wind velocity dependent on z, an algebraic vertical eddy diffusivity for inhomogeneous turbulence in a CBL is derived. This simple algebraic formulation (eq. (14)), for different heights was compared with a vertical eddy diffusivity calculated from a complex integral. The comparison exhibits a good degree of numerical agreement between algebraic and integral formulations. Furthermore, both expressions (eqs. (13), (14)) were introduced in an air pollution model and validated with a dataset from Copenhagen experiments. The statistical analysis of the results shows a good agreement between the observed ground-level crosswind concentrations and those simulated employing both formulations. These validations point out that the algebraic vertical eddy diffusivity can accurately represent the turbulent transport of contaminants released from a continuous point source in a CBL. The great advantage of using the algebraic expression for K_z is the fact that this formula, under the computational point of view, is roughly forty times faster than the numerical integration of eq. (13) and, consequently, it is useful in the solution of large and complex atmospheric diffusion models.

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