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Statistical analysis of three series of daily rainfall in North-Western Italy

D. Anfossi(1), C. Cappa(1), J. von Hardenberg(2) and A. Provenzale(1)(2)

(1) *Istituto di Scienze dell'Atmosfera e del Clima, CNR - C.so Fiume 4, I-10133 Torino, Italy*
(2) *ISI Foundation - V.le S. Severo 65, I-10133 Torino, Italy*

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Summary. — In this work we study three long series of daily rainfall measured in North-Western Italy. We analyze the global statistical properties of the three data sets and we discuss both the seasonal distribution of rainfall intensity and the long-term variation in rainfall properties. We show that the three series display a vanishingly small autocorrelation for periods longer than one or two days, consistent with the absence of multifractality in these records. These time series are largely consistent with the output of a simple chain-dependent stochastic process.

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1. – Introduction

The statistical properties of rainfall intensity play a major role in determining the climatology of several mid-latitude areas, with important effects on land management and use. More dramatically, major rainfall episodes can create serious environmental problems, generating, for example, severe floods and landslides.

These issues are particularly important in the Alpine and sub-Alpine areas of North-Western Italy, where events of extreme rainfall, aggravated by the properties of the local geomorphology and by the rapid response of small catchement areas, have repeatedly induced disastrous floods in several different river basins.

The first step towards the prevention of natural disasters and a sensible management of the environment is necessarily based on the understanding of the processes at work. Among these, the spatio-temporal properties of rainfall play a crucial role. For this reason, historical rainfall records on both global and regional scales have often been analyzed, with the aim of determining the statistical properties of rainfall and the possible presence of climatic change.

For North-Western Italy, in 1911 Anfossi [1] discussed rainfall climatology in Piedmont and the Ligurian area. More recent studies of precipitation records in Italy include the analyses of individual precipitation series recorded in Napoli [2], Padova [3], Roma [4],

TABLE I. – *Characteristics of the measurement sites.*

Location	Latitude N	Longitude E (h:m)	Elevation (m a.s.l.)
Torino	45°3′	00:31	275
Cuneo	44°24′	00:30	536
Milano	45°28′	00:36	122

Milano [5] and Moncalieri [6], the analysis of an ensemble of 45y-long series of annual, seasonal and monthly rainfall data in the Central-Western Mediterranean [7], and the analysis of an ensemble of calibrated and homogeneized long precipitation records in the whole Italian area, see [8, 9] and [10]. The climatological analysis of a large set of daily, monthly and yearly rainfall data in Piedmont has also led to the classification of the overall geographical properties of cumulated yearly rainfall in the last century [11, 12].

In this work, we add another piece to the puzzle and conduct a statistical study of three long time series of daily rainfall, recorded at three representative sites in North-Western Italy. A preliminary analysis of one of these series is discussed in [13]. The present work is intended to provide specific information on the local climatology of the geographical area under study, and some general considerations on the analysis of daily rainfall time series. A forthcoming work will consider the statistical properties of a large dataset of three-year long rainfall records with high temporal resolution [14].

2. – The data set

The time series considered in this work are daily rainfall intensities, $p(t)$, where t indicates the day, measured at three representative sites in North-Western Italy, namely the series available from the Hydrographic Office of Torino, with branches in Torino and Cuneo, and the series measured at the Brera Observatory in Milano. The Milano series is discussed in [5]. Table I reports the characteristics of the measurement sites. Further information on the Torino and Cuneo records is given in [12], where the basic statistical properties of the monthly and yearly rainfall averages are analyzed.

The three rainfall records measured in Torino, Cuneo and Milano span different time intervals. In order to have a consistent set of statistical results, here we analyze only a 90y-long portion of data that is common to all three time series, namely, from the 1st of January 1900 to the 31st of December 1990. This allows to estimate a basic statistical sample against which one may contrast recent extreme rainfall episodes. Figure 1 shows the three records of daily rainfall intensity during this time span. The minimum daily rainfall below which a day is considered to be “dry” is set at 1 mm/day.

At this point, a word of caution is in order. Although we believe that these three data sets are representative of the overall statistical behavior of daily rainfall at the measurement sites, one must always keep in mind that this type of historical data can contain significant errors. Since the data span such a long period of time, in particular, one must be aware that there are differences in the precision and reliability of different portions of the data sets. Particularly during war periods, the time series are partially incomplete. The uncertain and/or missing values have been completed by interpolation from other available rainfall records in the vicinity of the measurement site. We also removed five isolated spikes with daily rainfall larger than 150 mm/day present in the

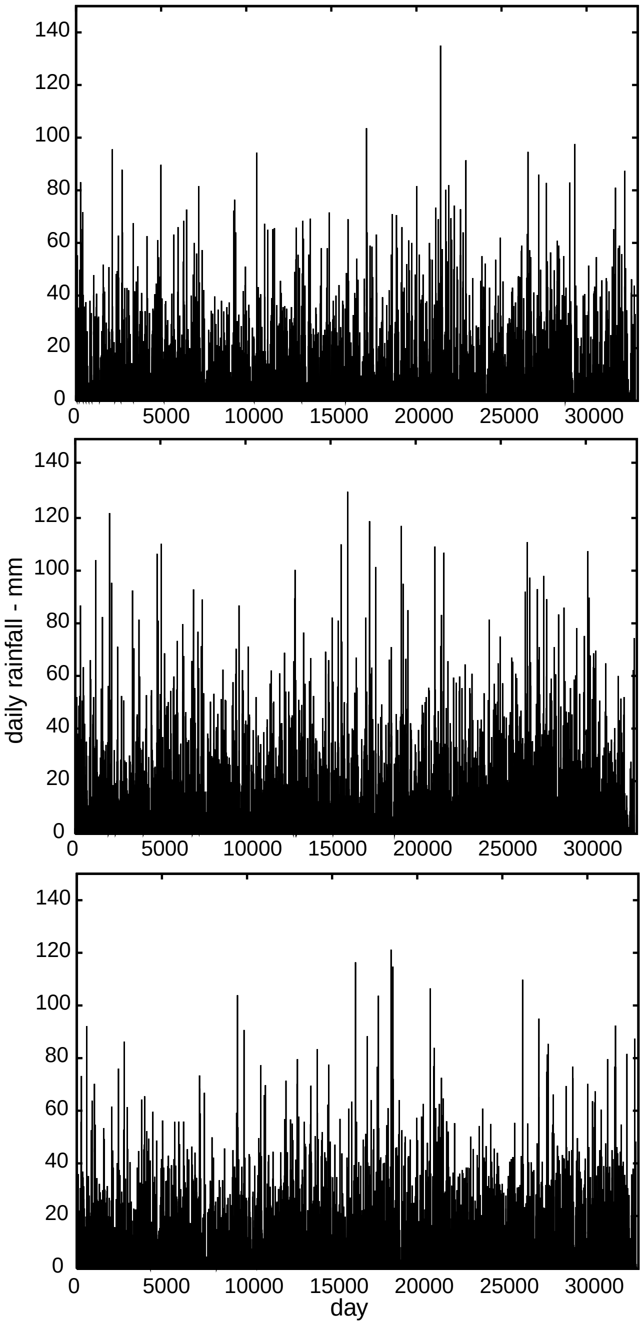

Fig. 1. – Time series of daily rainfall intensity in Torino (upper panel), Cuneo (middle panel) and Milano (lower panel).

TABLE II. – *Long-term linear trends in the rainfall records.* $\alpha_{\rm tot}$ *is the trend of the total rainfall and* $\alpha_{\rm day}$ *is the trend of the yearly number of rainy days. Error bars indicate the* $1\,\sigma$ *confidence limits.*

Site	$\alpha_{\rm tot}$	$\alpha_{\rm day}$
Torino	0.44 ± 0.89	0.05 ± 0.07
Cuneo	0.53 ± 1.05	-0.56 ± 0.06
Milano	-0.10 ± 0.94	-0.56 ± 0.08

Milano series. These five points were substituted by a linear interpolation between the previous and the subsequent point.

Note, also, that the geographical coordinates and the elevation of the measurement sites, and the sensitivity of the measuring gauges have been changed during the period considered. The maximum horizontal displacement has been less than 4 km, and the maximum change in elevation has been less than 60 meters. A discussion of the history of the Cuneo and Torino records, together with that of several other rainfall and temperature time series in Piedmont, is given in [12].

3. – Cumulated yearly rainfall and long-term behavior

First we concentrate on the cumulated yearly rainfall, in order to obtain a general qualitative picture of the evolution of rainfall intensity during the period 1900-1990. In this framework, an important question is whether the rainfall records indicate the presence of long-term changes in the rainfall climatology of the area under study. Detailed studies of long-term trends in an ensemble of precipitation records in regions of the Mediterranean area are reported in [7-9] and [10].

Figure 2 shows the cumulated yearly rainfall for the three sites considered in this work. This figure suggests that no long-term trend is present in the total rainfall data during the study period. A least-square linear fit to the time series confirms this inference. Table II reports the slopes of a linear fit to the rainfall data for the three series of total yearly rainfall shown in fig. 2. The error bars are the $1\,\sigma$ confidence limits. None of the trends in total yearly rainfall is significantly different from zero. Note, also, the similarity between the three series. This is consistent with the high level of correlation found for rainfall averaged over periods longer than 20 days, see fig. 5 below. For the yearly data, the Torino and Cuneo series have a cross-correlation larger than 0.8, while the cross-correlation with the series recorded in Milano is of the order of 0.6.

On the other hand, some interesting behavior is present in the number of rainy days. Figure 3 shows the yearly number of rainy days for the three sites considered. While the Torino series does not display any trend in the number of rainy days, both the Cuneo and Milano series display a significant decrease of the number of rainy days during the last 90 years, as indicated by table II. Since the total rainfall did not change significantly, such a result would suggest a tendency toward "tropicalization", characterized by sparser and more intense precipitation events. However, a closer look to the series reveals that all three series provide similar estimates of the number of rainy days in the period 1925-1963. The Milano series reveals a sudden change in the number of rainy days in the period 1910-1917, that is the main responsible for the trend of this series and could have a spurious origin. The Cuneo series, on the other hand, shows a marked decrease in the number

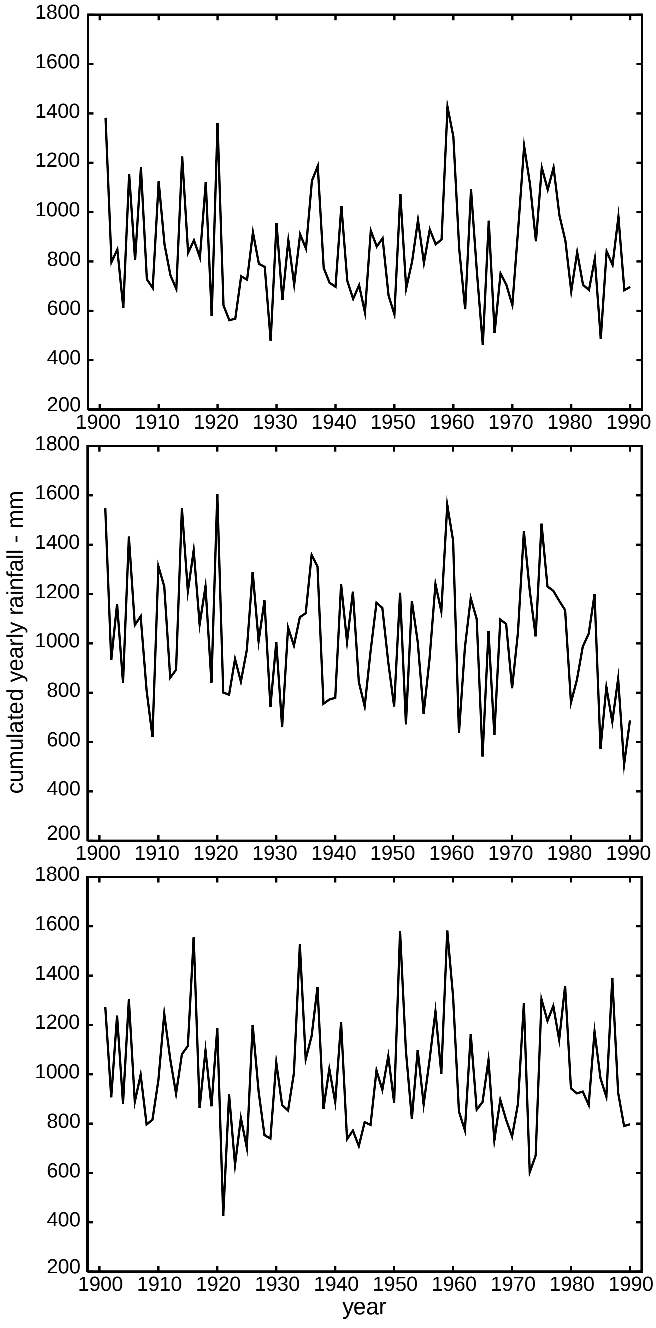

Fig. 2. – Cumulated yearly rainfall in mm for Torino (upper panel), Cuneo (middle panel) and Milano (lower panel).

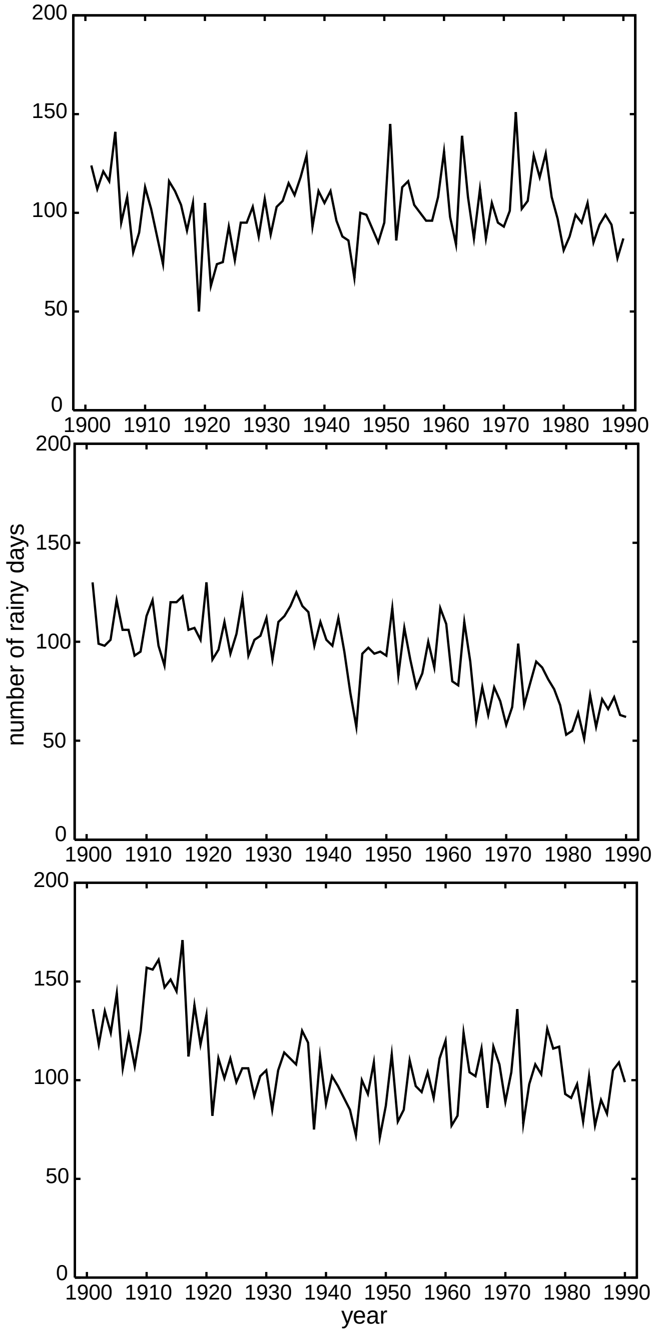

Fig. 3. – Yearly number of rainy days for Torino (upper panel), Cuneo (middle panel) and Milano (lower panel).

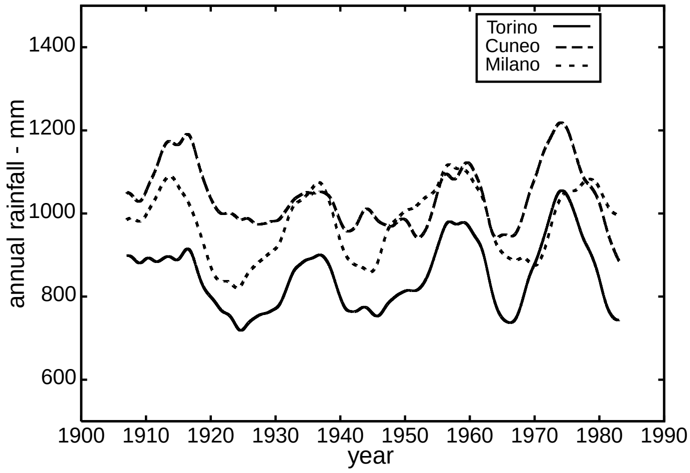

Fig. 4. – Running average of the three rainfall time series. The total rainfall is expressed as annual rainfall in mm.

of rainy days starting approximately in 1960. Although probably real, this trend can be a local effect. Thus, no clear indications of regional long-term changes emerge from the data considered here. A more complete exploration of long-term trends in precipitation records requires a larger number of series, as discussed, for example, in [7-9] and [10].

Before closing this section, we want to delve further into the possible presence of decadal periodicities in the rainfall record. Figure 4 shows a running average of the total rainfall record for the three sites considered. These signals have been obtained by convolving the daily rainfall signal with the window function $W(t-t') = W_0 \tanh(|t-t'|-\tau_{\mathrm{w}})$, where W_0 is a normalization factor and $\tau_{\mathrm{w}} = 4$ years is the width of the window. This choice leads to a unit weight for values of the time difference $|t-t'|$ up to three years, and to a rapidly decreasing weight for larger time differences. The weight is practically zero for $|t-t'|$ larger than seven years. Other choices of the window width in the range of a few years, or other choices of the window function (including running averages with constant weight) provide similar results. Note, also, that we have expressed the low-pass filtered rainfall series in terms of total annual rainfall.

Figure 4 indicates the presence of a well-defined oscillation in all three signals, with periodicity between 20 and 25 years. This oscillation is more regular in the Torino and Milano series, consistent with the indication of spectral analysis. It is interesting to note that this decadal oscillation has approximately the same phase in the three records considered. The limited number of oscillations present in the series analyzed here, and the limited number of series considered, suggest to defer further exploration of this oscillation to studies with a larger number and/or longer rainfall time series.

4. – Global statistical properties of daily rainfall

Table III reports the main global statistical properties of the records considered, namely, the average annual rainfall, $\bar{p}_{\mathrm{y}}$, the average daily rainfall, $\bar{p}_{\mathrm{d}}$, the average fraction of rainy days, ϕ_{w}, the average rainfall intensity, $\bar{p}_{\mathrm{w}} = \bar{p}_{\mathrm{d}}/\phi_{\mathrm{w}}$, and the average durations of uninterrupted sequences of dry or rainy days, T_{dry} and T_{wet}. Here, we fix at 1 mm per day the minimum rainfall required to consider "rainy" a given day. These quantities

TABLE III. – *Global statistical properties of the rainfall records considered.* $\bar{p}_\mathrm{y}$ *is the average annual rainfall,* $\bar{p}_\mathrm{d}$ *is the average daily rainfall,* ϕ_w *the average fraction of rainy days,* $\bar{p}_\mathrm{w}$ *is the average rainfall intensity,* T_dry *is the average duration of an uninterrupted sequence of dry days,* T_wet *is the average duration of an uninterrupted sequence of rainy days.*

Site	$\bar{p}_\mathrm{y}$ (mm)	$\bar{p}_\mathrm{d}$ (mm)	ϕ_w	$\bar{p}_\mathrm{w}$ (mm)	T_dry (days)	T_wet (days)
Torino	849.9	2.3	0.27	8.4	5.5	2.1
Cuneo	1025.8	2.8	0.25	11.1	5.5	1.9
Milano	995.9	2.7	0.29	9.2	5.5	2.3

agree with the general pattern of rainfall distribution and climatology in the area under study.

In the following, we explore in detail the statistical properties of these time series.

4˙1. *Spatial correlation.* – It is well known that rainfall has a strong spatio-temporal variability. Thus, rainfall recorded at a point may provide little information on the history of rainfall at a nearby site. For this reason, it is interesting to study the spatial correlation between the three series under study. Figure 5 shows the cross-correlation $r_{a,b}(T)$ between any pair a, b of time series in the data set under study. This is computed for time series of cumulated rain over an integration period T, with T between 1 day (the sampling time) and more than four months, as

$$r_{a,b}(T) = \frac{\langle p_a(t;T)\, p_b(t;T)\rangle}{\langle p_a^2(t;T)\rangle^{1/2}\, \langle p_b^2(t;T)\rangle^{1/2}}\,, \tag{1}$$

where $p_a(t;T)$ is the precipitation recorded at site a during an interval of duration T starting at time t, and the symbol $\langle ...\rangle$ indicates average over all the time intervals of duration T. Clearly, $p(t,1) = p(t)$.

The results of fig. 5 indicate that the correlation between the series is relatively low when the data are averaged over periods shorter than three or four days. At larger cumulating times, however, the series become more correlated. For periods longer than about 20 days, the cross-correlation saturates to a constant value. This latter is different for the different pairs considered. The saturation value is quite large, $r(T) \approx 0.8$, for the series of Torino and Cuneo. Conversely, the correlation with the data recorded in Milano is smaller, being $r \approx 0.6$, as can be expected from the larger distance between these sites and the different climatological conditions.

4˙2. *Rainfall intensity.* – First, we consider the amplitude distribution of rainfall intensity. Figure 6 shows the probability density function (pdf) of daily rainfall, $\pi(p)$, as obtained from the three data sets under study. All three distributions have a similar shape, close to an exponential at large rainfall intensities. At smaller rainfall intensity, the distributions deviate from an exponential, becoming closer to a power law.

The exact shape of the rainfall intensity distribution is a debated issue. One option, that reproduces the combination of power law behavior for small rainfall and exponential distributions for large rainfall, is the Γ-function, that has been widely used in the past as an empirical representation of rainfall intensity distributions [15]. The best-fit estimate

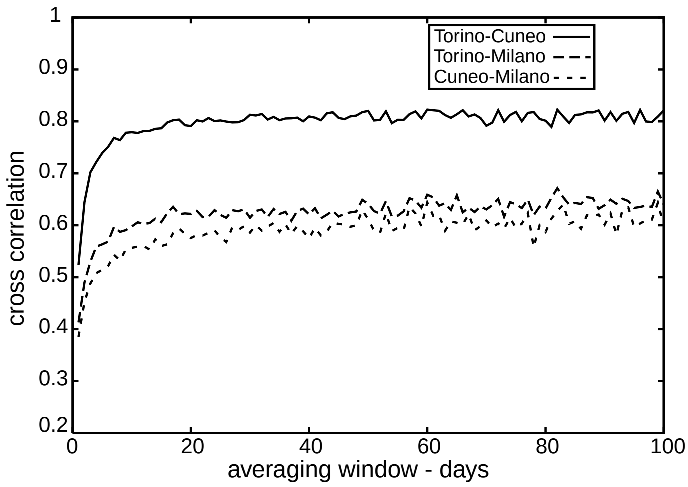

Fig. 5. – Cross-correlation $r(T)$ between any two pair of the three time series considered *vs.* the time interval T for which rain is cumulated.

to a Γ-function, as obtained following Clarke [15], is also plotted in fig. 6. As can be seen from this figure, the Γ distribution provides a reasonable fit to the data.

From the rainfall intensity distribution, it is also possible to estimate the return period for a daily rainfall of given intensity. This is given by $T_{\rm ret}(p) = \left[1 - \int_0^p \pi(p')\mathrm{d}p'\right]^{-1}$, and it is plotted in fig. 7. From this, we see that Cuneo has slightly shorter return periods, while Torino has the longest return periods for extreme rainfall events.

4˙3. *Distribution of dry and rainy periods.* – The distribution of the duration of uninterrupted sequences of dry or rainy days is an important piece of information for properly addressing issues such as flood and drought forecasting. Figure 8 shows the pdfs of dry and rainy periods for the three data sets. These distributions turn out to be very close to an exponential. In general, dry periods can be significantly longer than rainy intervals.

We recall that an exponential distribution of the time intervals between two events is typical of a Poisson process generating uncorrelated events on the time axis (*e.g.*, [15]). This suggests a mild correlation between rainfall on different days, an issue that is explored further below.

4˙4. *Autocorrelation.* – Figure 9 shows the autocorrelation functions, $R(\tau)$, for the three time series considered. The autocorrelation is defined as

$$R(\tau) = \frac{\langle p(t)p(t+\tau)\rangle}{\langle p^2(t)\rangle}\,, \qquad (2)$$

where τ is the time delay.

This figure shows that the autocorrelation is already quite low, $R(\tau) \approx 0.3$, for $\tau =$ 1 day. If we use the rough definition of decorrelation time as the delay for which $R = 1/e$, the above result implies that the decorrelation time for rainfall is less than one day. We also note that the autocorrelations have an approximate power law shape for delays up to about 10 days, even though, given the very small level of correlation, it is not clear how to exploit this property for prediction and/or modelling purposes.

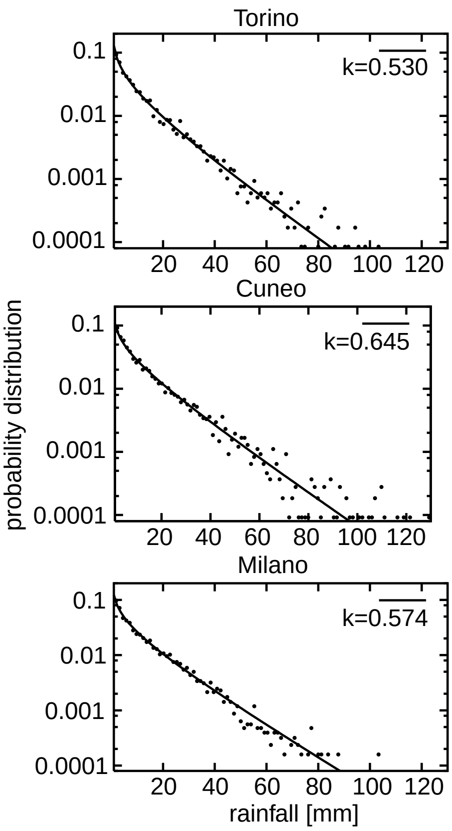

Fig. 6. – Probability distributions of daily rainfall intensity for the three time series considered. The solid lines represent best-fit Γ distributions.

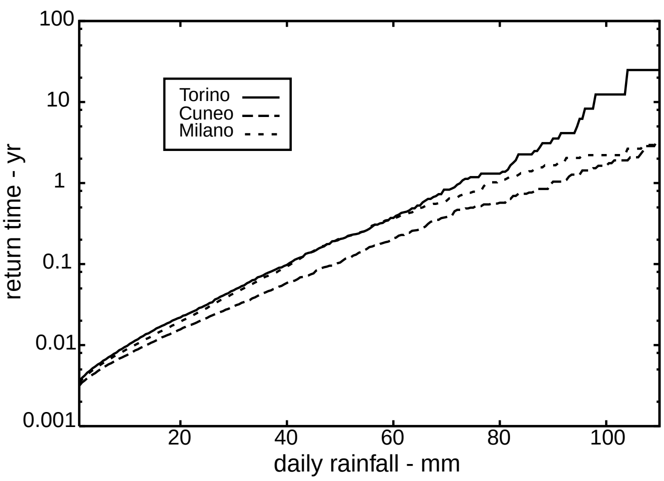

Fig. 7. – Return periods (in years) for daily rainfall, for the three data sets under study.

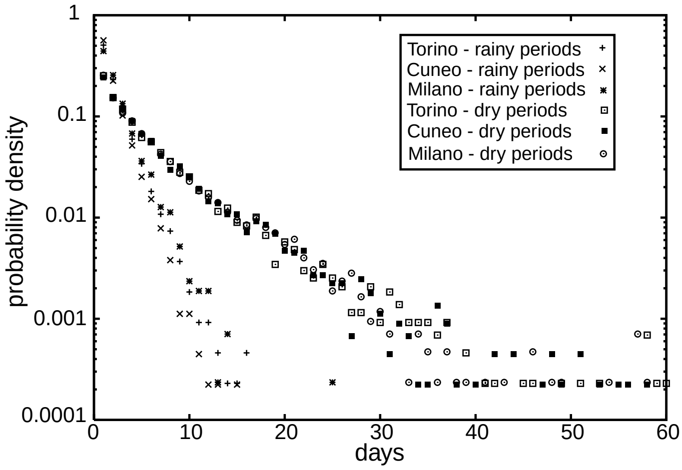

Fig. 8. – Probability density distributions for the length of uninterrupted sequences of dry and rainy periods.

4˙5. *Power spectra.* – The power spectrum of a time series is defined as the Fourier transform of the autocorrelation, and it does not add new information with respect to the latter. However, the information is displayed in a different format, and this can be helpful for detecting periodicities and understanding how the variance is distributed among the different components.

Figure 10 shows the power spectra for the three time series considered as a function of the periodicity T of the harmonic components. For all three data sets, the spectra are characterized by (a) the presence of a strong six-month periodicity and (for the Torino record) of an annual periodicity; (b) a white-noise appearance for periodicities longer than about ten days; and (c) a modest decrease of the spectral power for periodicities

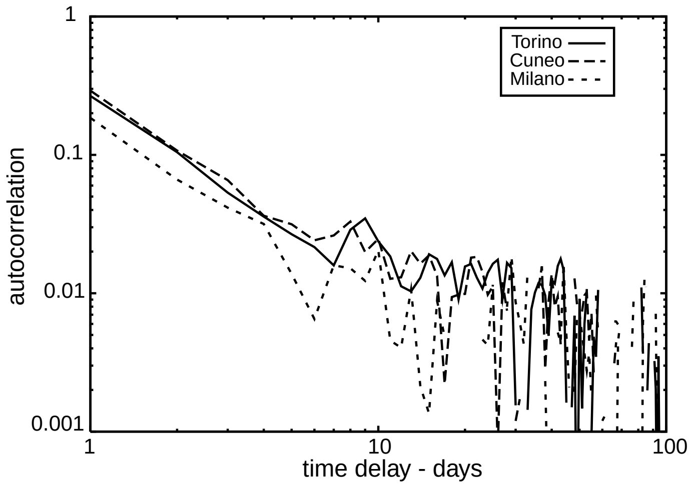

Fig. 9. – Autocorrelation functions for the three time series considered.

shorter than about ten days. These two latter properties are consistent with the weak correlations detected for short delays (a few days), and with the lack of correlation at longer delays.

For the Torino and Milano records, in addition, one can see increased spectral power at periodicities of about 20–30 y, suggesting the presence of a decadal oscillation with this approximate periodicity in the rainfall record. However, as discussed in sect. **3**, the limited length of the time series and the fact that this oscillation is not seen in the Cuneo record require a cautionary attitude.

The presence of the peaks at six months and one year indicate a strong seasonal nature of the rainfall record, a well-known fact for the area under study, which is characterized by rainfall maxima in spring and fall. It is interesting to note that the presence of the annual peak in the Torino record indicate that the rainfall regime in this latter site must have a significant asymmetry between spring and fall. In sect. **5**, we explore in detail the seasonality of the precipitation regimes at the three sites considered.

5. – Rainfall seasonality

The rainfall climatology of the study area is characterized by the presence of two rainfall maxima in spring and fall, and two minima during winter and summer. Figure 11 shows the monthly rainfall averages for the three sites under study, confirming this general picture.

Some differences are visible between the different records. In particular, rainfall in Torino is characterized by dominance of the spring rainfall maximum over the fall one, and by a deeper rainfall minimum in winter than in summer. This generates the two peaks at 6 and 12 months in the power spectrum of the Torino series. The peak at 12 months is absent in the Cuneo and Milano records. These latters display an almost pure 6-month periodicity, with comparable rainfall maxima in spring and fall and minima in summer and winter.

Another important information comes from the monthly averages of the number of rainy and dry days, and from the average daily rainfall intensity (clearly, this quantity can be derived from the total rainfall and the number of rainy days). The two panels of fig. 12 show these quantities for the three sites considered.

Finally, we check whether the different seasons are characterized by different levels of correlation between subsequent days. Figure 13 shows the four autocorrelation functions as obtained by considering the rainfall record in winter (December-February), spring (March-May), summer (June-August) and fall (September-November), for the three sites considered. This figure shows that the autocorrelation falls very rapidly, as already shown by the total averages. The only interesting evidence is a slightly faster decay in summer, for all three stations, presumably due to the presence of localized (in space and time) thunderstorm activity.

6. – Scaling and intermittency

An important characteristic of rainfall records is their intermittent nature, with prolonged periods of quiescence (dry periods) intermingled with sudden bursts of activity (the rainy intervals). This behavior has stimulated several attempts to quantify the nature of rainfall intermittency, often by using the notion of multifractality and anomalous scaling.

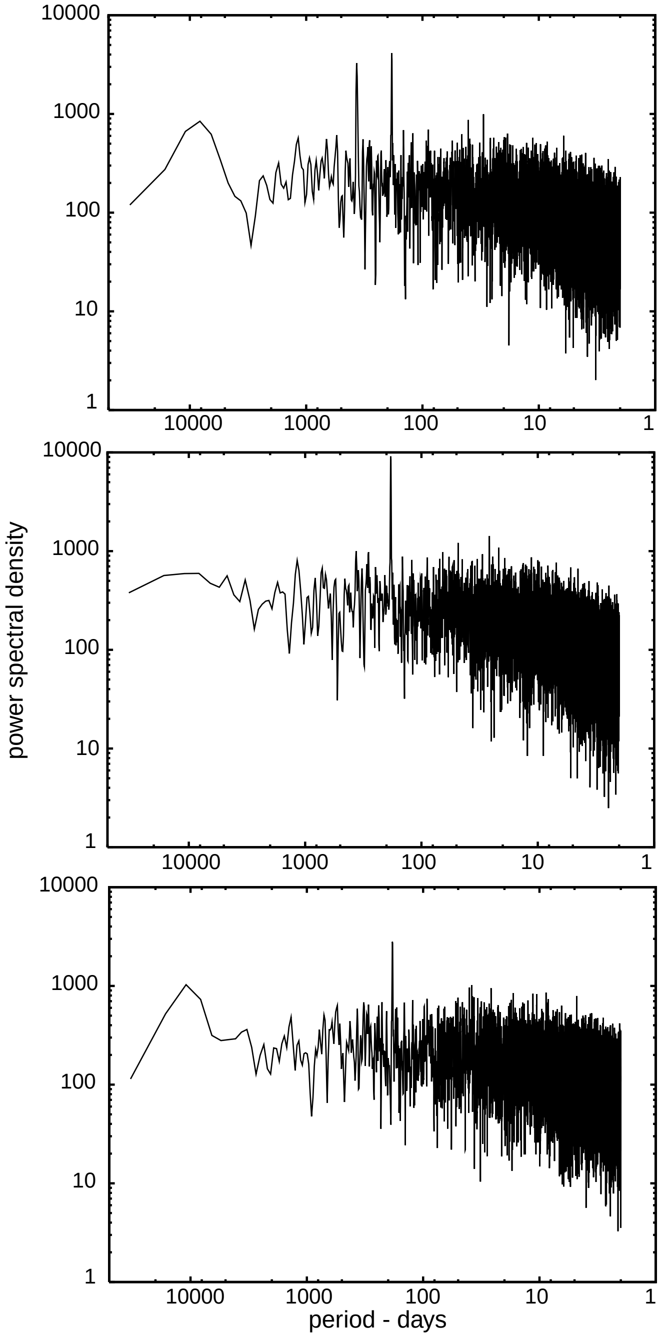

Fig. 10. – Power spectra for the three time series considered. Torino: upper panel; Cuneo: middle panel; Milano: lower panel.

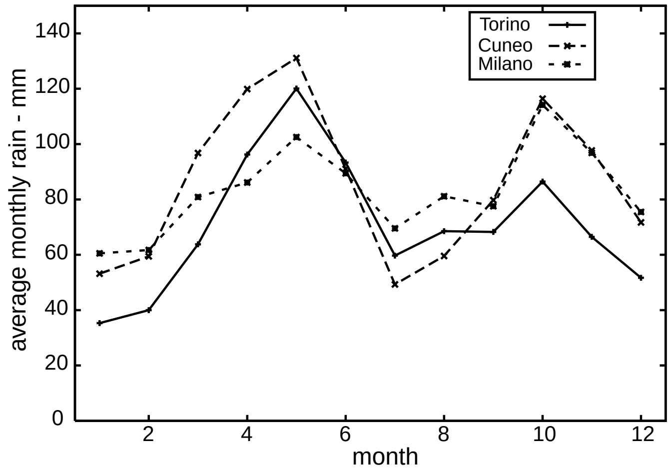

Fig. 11. – Average monthly rain for the three sites considered.

A way to quantitatively assess the scaling properties of a given signal is based on the evaluation of the generalized structure functions (see, *e.g.*, [16] for a more detailed discussion). The structure functions can be defined as

$$S_q(\tau) = \left\langle \left| \frac{p(t+\tau) - p(t)}{\sigma} \right|^q \right\rangle , \tag{3}$$

where σ is the r.m.s. of the record $p(t)$ and $\langle \ldots \rangle$ indicates a time average over the time series.

The second-order structure function, $S_2(\tau)$, is obtained from the autocorrelation function $R(\tau)$ defined above by a simple linear transformation. In general, a signal is considered to have *scaling properties* if the second-order structure function has a power law dependence on its argument, *i.e.* $S_2(\tau) \propto \tau^{2H_2}$, where H_2 is the scaling exponent. Simple examples of scaling signals are uncorrelated white noise, characterized by $H_2 = 0$, and Brownian motion, characterized by $H_2 = 0.5$.

For scaling signals, also the higher-order structure functions are characterized by power law dependence on their argument, *i.e.* $S_q(\tau) \propto \tau^{qH_q}$, where the H_q are called the generalized scaling exponents. Here, an important distinction occurs. Signals characterized by equality of all the generalized scaling exponents, $H_q = H_p$ for $q \neq p$, are called self-affine or *monofractal*, and display a simple scaling behavior. Both uncorrelated white noise and Brownian motion are examples of this type.

By contrast, signals characterized by an entire set of differing scaling exponents, $H_q < H_p$ for $q > p$, are called *multifractal* and they are told to possess *anomalous* scaling properties. Multifractality is closely tied with intermittency, and it is indeed one of the characterizing properties of intermittent systems.

In order to assess the possible presence of intermittency and multifractality in daily rainfall records, we evaluated the generalized structure functions for the three time series considered here. The three panels of fig. 14 show $\log S_q(\tau)/q$ *vs.* $\log \tau$ for $q = 2, 4, 6, 8, 10, 12$ for the three sites considered. In this representation, a monofractal signal is characterized by a set of structure functions with the same slope, while a multifractal

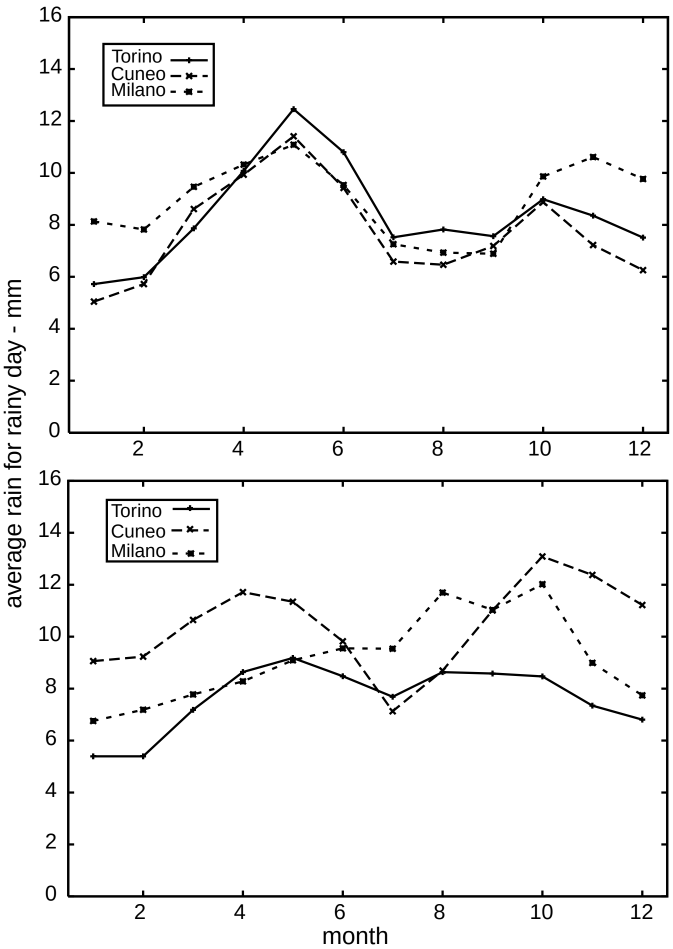

Fig. 12. – Average monthly number of rainy days (upper panel) and average rain intensity (lower panel) for the three sites considered.

signal would lead to structure functions with a slope that varies with the order q of the moment.

Two facts emerge from fig. 14:

i) The structure functions for different values of q are simply shifted with respect to each other (due to their definition), but display the same dependence on τ for any q. This indicates the absence of multifractality in the daily rainfall records considered here. The small oscillations visible in the higher-order structure functions are related to the amplification of small fluctuations due to the large moment order.

ii) Except than for a mild non-zero slope up to three or four days, consistent with the weak correlation on this time scale already detected by the autocorrelation function, the structure functions are flat, indicating the absence of correlation (memory) on time scales larger than a few days.

Overall, these results indicate that the daily rainfall records considered here are *not*

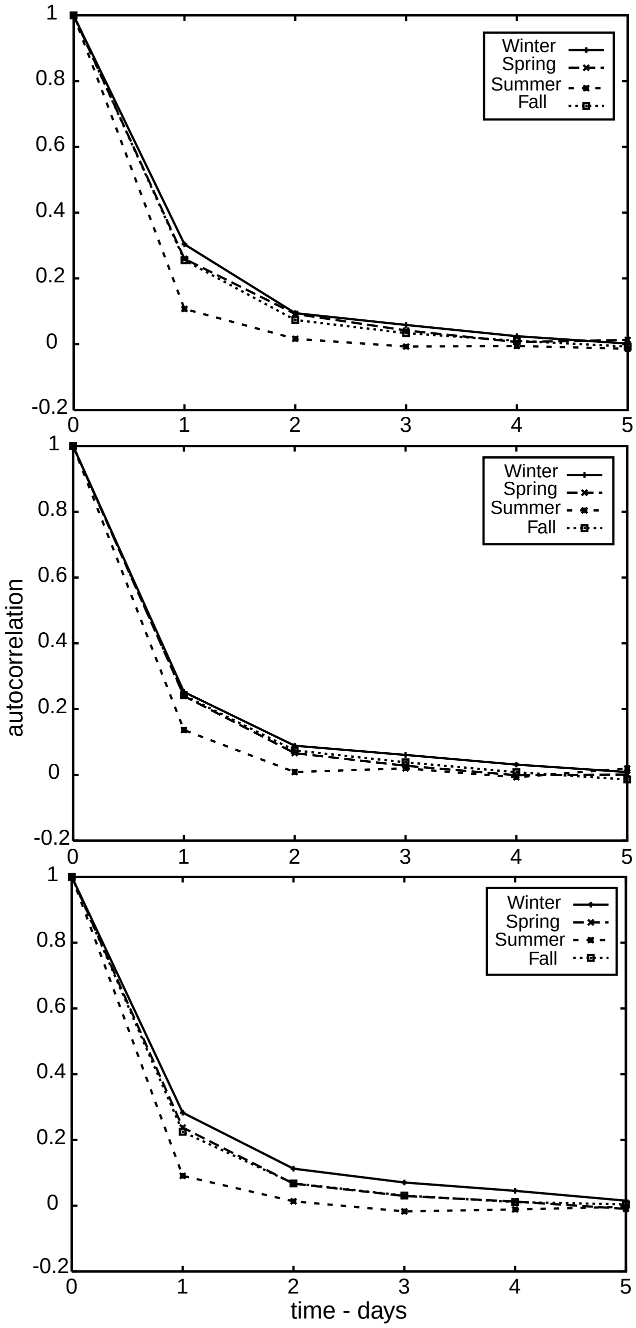

Fig. 13. – Seasonally averaged autocorrelation functions for Torino (upper panel), Cuneo (midle panel) and Milano (lower panel).

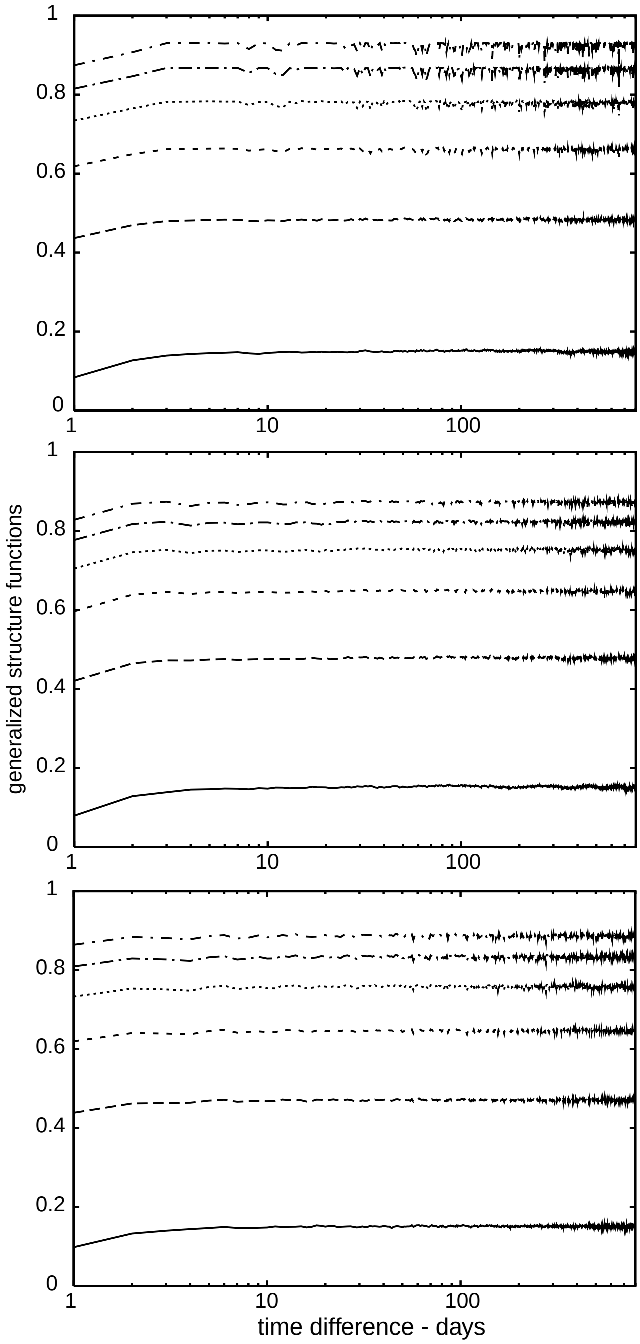

Fig. 14. – Logarithm of the generalized structure functions $S_q(\tau)/q$ *vs.* the logarithm of the time difference τ for Torino (upper panel), Cuneo (middle panel) and Milano (lower panel).

multifractal, and are basically uncorrelated on time scales larger than a few days.

At this point, a comment is in order. Sometimes, the search for multifractality is performed using another approach, based on the so-called box-counting method. This method, however, has been shown to be strongly biassed in the case of signals with exponential or hyper-exponential amplitude distributions, providing false detections of multifractality even for uncorrelated signals with no multifractality [16]. In the same work, it was shown that simple surrogate-data tests, based for example on shuffling the data points, thus destroying the temporal correlations, and re-evaluating the multifractal properties, can allow for distinguishing truly multifractality from spurious one.

The rainfall records seem to be a classic example of this problem. By evaluating the multifractal properties with the box-counting method, we detected an apparent multifractality, in contrast with the results provided by the structure function analysis. However, the use of surrogate-data tests confirmed that the results of the box-counting were spurious, being generated by the quasi-exponential one-point amplitude pdf of the rainfall intensity. Claims of rainfall multifractality, based on a box-counting analysis, must thus be considered with caution until appropriate surrogate-data tests are applied and the analysis is repeated.

7. – Comparison with a simple stochastic model

The short autocorrelation time of the rainfall series, and the absence of multifractal behavior, suggest that an approximate description based on a low-order Markov chain could capture most of the properties of the daily rainfall data under study.

Here we consider the example of a first-order, two-state Markov chain where the system can be either "dry" or "wet", with a preselected probability of transition from one state to another. Such a model is called a chain-dependent process in [17]. Given the significant seasonality of the rainfall records, we need to define seasonal, or monthly, transition and persistence probabilities. Figure 15 shows the persistence probabilities for subsequent days, $dry \rightarrow dry$ and $wet \rightarrow wet$, as a function of the month for the three signals considered, as obtained by an average over the whole time series. The transition probabilities are easily obtained from these.

The persistence (and transition) probabilities display a seasonal dependence, in accordance with the analysis already discussed in sect. **4**. On the other hand, the rainfall intensity distributions for the different months display a smaller variability. Thus, in a first approximation we build a two-state Markov chain where only the transition from one state to the other (or the persistence in a state) does depend on the specific month. In "wet" days, we extract the value of the rainfall intensity from a preselected distribution which does not change with the season. Also, the extraction is completely random, and it does not depend on the previous state. Clearly, such an approach is an oversimplification, and it is used here to assess whether such a minimal model can capture some aspects of measured rainfall.

In the following, we consider only the Torino record. By construction, the simulated and the original rainfall records have the same amplitude distribution. Figure 16 shows the distribution of the durations of the dry periods for the orginal rainfall record (diamonds) and for the stochastic model based on the above assumptions. No clear difference between the two is visible.

Figure 17 shows the power spectrum for the series generated by one realization of the stochastic model. The overall appearance is the same as for the original record (compare with fig. 10), with the presence of the peaks at 6 and 12 months. However, in

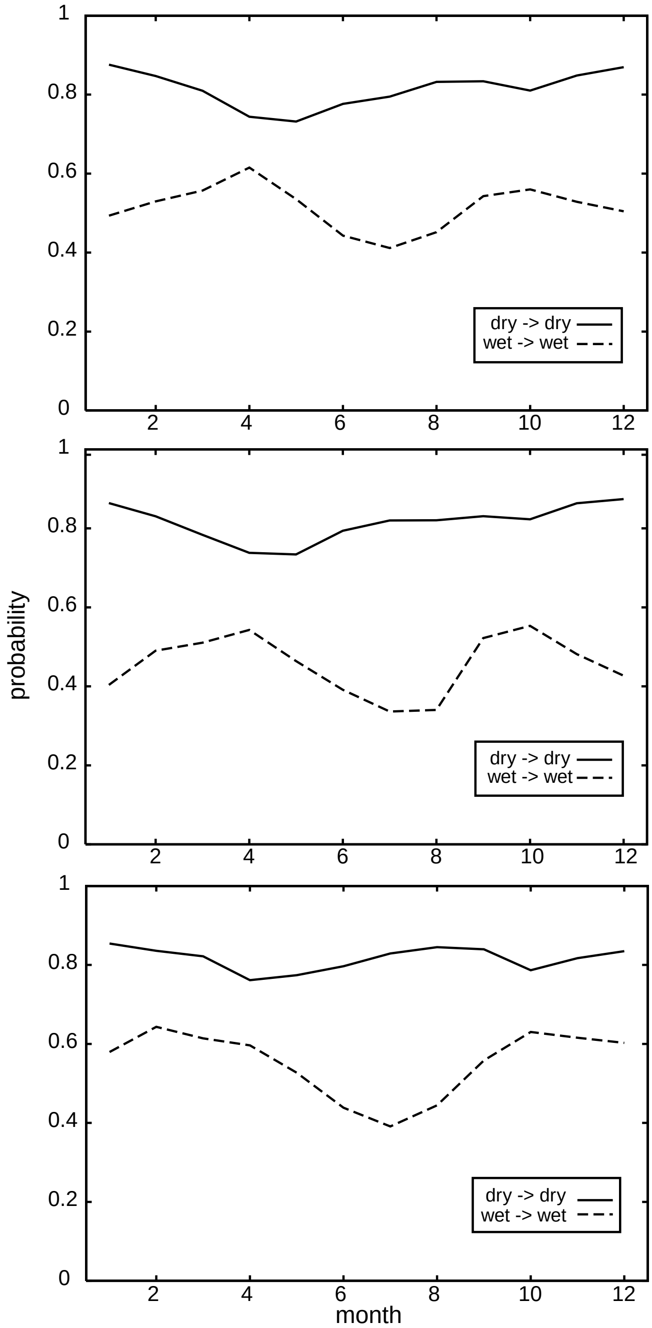

Fig. 15. – Persistence probabilities $dry \to dry$ and $wet \to wet$ for the three sites considered (Torino, upper panel; Cuneo, middle panel; Milano, lower panel).

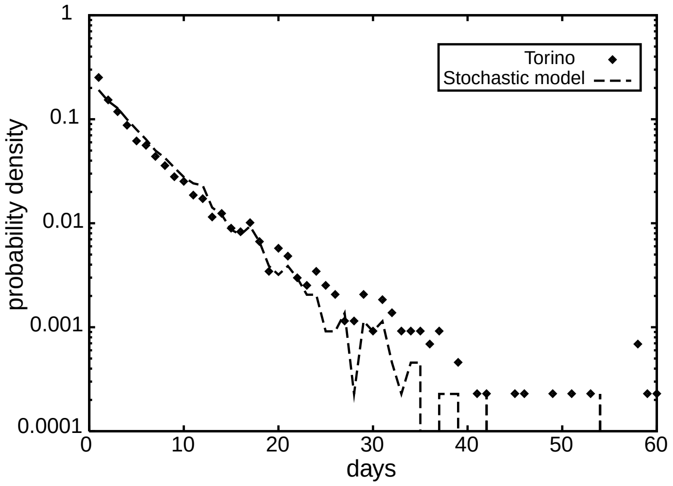

Fig. 16. – Distribution of the durations of the dry periods for the Torino record (diamonds) and for the stochastic model discussed in the text.

the stochastic model the increased spectral power at periods between 20 and 25 years is not present. This latter is absent *ab initio* in the construction of the stochastic model.

An interesting discrepancy between the original data and the stochastic model emerges in the behavior of the autocorrelation function. Figure 18 shows the autocorrelation function for the Torino series and for one realization of the stochastic process. Although both are quite small already for a delay of one day, there is an apparent difference between the two. At any time lag below about ten days, the stochastic process introduced above has a lower correlation with respect to real rainfall data.

A slightly more refined version of the above model is based on considering two different

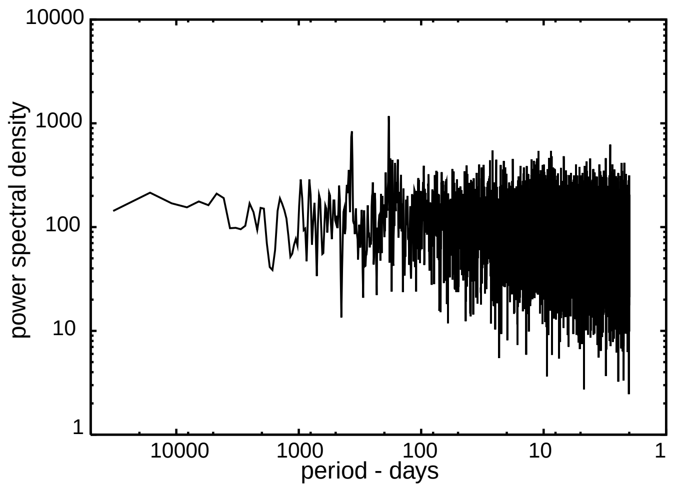

Fig. 17. – Power spectral density for the synthetic series generated by the stochastic model discussed in the text.

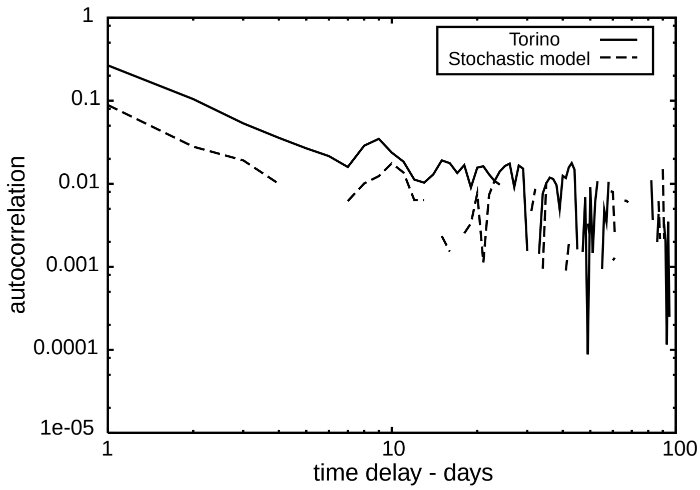

Fig. 18. – Autocorrelation function for the original Torino rainfall record (upper curve) and for one realization of the stochastic process discussed in the text.

distributions for the extraction of the random precipitation value on a given day, conditioned, respectively, by the fact that the previous day was rainy or dry (see, *e.g.*, [17]). Figure 19 shows these two distributions, and it suggests that some difference may indeed exist between them. However, a two-state chain-dependent model using this additional information does not provide a better agreement with the autocorrelation function.

The difference between the model and the data is due to the correlation existing between the rainfall intensity on two subsequent rainy days, that is absent in the stochastic model. An estimate of the correlation C_r between the rainfall intensity on day i and

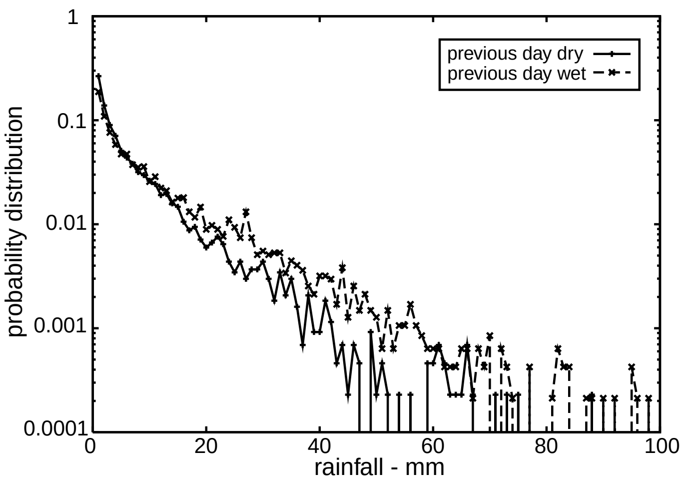

Fig. 19. – Probability distributions of daily rainfall, conditioned by the fact that the previous day was dry (lower curve) or rainy (upper curve).

that on day $i+1$, conditioned by requiring that both days are rainy, provides the value $C_r = 0.22$, very close to the rainfall autocorrelation at the delay of one day (we recall that this was computed by considering both rainy and dry days). Thus, the value of the autocorrelation function seems to be completely determined by the small correlation between the rainfall intensity on two subsequent rainy days. A better stochastic model should exploit this property.

8. – Conclusions

The three rainfall records considered here do not indicate the presence of a significant trend in rainfall intensity. The presence of oscillations with approximate period between 20 and 25 years is consistent with the results obtained from the analysis of other similar records of rainfall intensity.

The global properties of the time series have indicated that the power spectra are approximately white, with well-defined periodicities at 6 months for the Cuneo and Milano records, and at both 6 and 12 months for the Torino series. The amplitude distribution is well fitted by a Γ-function, and the distribution of dry and rainy periods is approximately exponential, consistent with what is expected for a Poisson process.

The study of the seasonal distribution of these rainfall records has confirmed the presence of rainfall maxima in spring and fall.

The scaling analysis of the rainfall series has revealed that these data are not multifractal, contrary to what is suggested by an analysis based on the box-counting method. The low autocorrelation of the rainfall record, already detected for a delay of one day, and the absence of multifractality strongly limit the predictability properties of daily rainfall at a single site. The similarity of the measured series with the synthetic data generated by a simple linear stochastic process confirms this view.

* * *

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