Violation of the Feynman scaling law as a manifestation of nonextensivity(*)

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Summary. — We demonstrate that the apparently *ad hoc* parametrization of the particle production spectra discussed in the literature and used in the description of cosmic ray data can be *derived* from the information theory approach to multiparticle production processes. In particular, the violation of the Feynman scaling law can be interpreted as a manifestation of nonextensivity of the production processes.

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1. – Introduction

The shape of the $x = E/E_0$ spectra of secondaries is of great importance in all investigations concerning developments of cosmic ray cascades [1] (cf. also [2]). The crucial problem of practical importance is the *existence* or *nonexistence* of the Feynman scalling, which says that x-spectra of secondaries are energy independent. Because in cosmic ray applications one is sensitive essentially to the large x region (of, say, $x > 0.1$),

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the commonly used formula [1]

$$(1) \qquad \frac{\mathrm{d}N}{\mathrm{d}x} = Da\frac{(1 - a'x)^4}{x}$$

stresses this fact by the power-like form with exponent obtained from the $x \to 1$ limit of fragmentation data [1]. Here a' is the parameter responsible for the Feynman scaling violation, whereas D and a are additional parameters obtained from the fit to data — all are energy dependent (cf. [1] for details).

We shall demonstrate that (1) emerges in a natural way from the information theory approach to hadronization processes presented in [3] and extended to allow for the possible nonextensivity of the hadronization process by applying Tsallis entropy [4]. (See [5] for details of the notion of nonextensivity and [6] for its application to high energy reactions.)

2. – Particle production spectra from information theory

In order to describe the hadronization process in information theory, one uses [3] the *least biased and most plausible* single particle distribution $f(y) = \frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}y}$ (where y is the rapidity) resulting from a hadronization process in which a mass M hadronizes into N secondaries of mean transverse mass $\mu_\mathrm{T} = \sqrt{\mu^2 + \langle p_\mathrm{T}\rangle^2}$ each (for simplicity we consider only one-dimensional hadronization with limited transverse momenta which can, however, depend on M and N). This is done maximizing the Shannon (or Boltzmann-Gibbs) information entropy, $S = -\int \mathrm{d}y\, f(y) \ln f(y)$, under constraints of normalization ($\int \mathrm{d}y f(y) = 1$) and energy conservation ($\int \mathrm{d}y\mu_\mathrm{T} \cosh y f(y) = M/N$), which leads to the following (extensive) distribution function (cf. [3] for details):

$$(2) \qquad f(y) = \frac{1}{Z(M,N)} \exp\left[-\beta(M,N)\cdot\mu_\mathrm{T}\cosh y\right].$$

Here $Z(M,N)$ comes from the normalization of $f(y)$, whereas the Lagrange multiplier $\beta(M,N)$ is to be calculated from the energy conservation constraint. Notice that *there is no free parameter here.* This should be contrasted with the popular use of eq. (2) as a "thermodynamical parametrization" with inverse "temperature" $1/\beta = T$ being a free, positively defined parameter. In [3] it was shown that, in a wide range of energies and multiplicities, $\beta(M,N) \sim \frac{N}{M} = \frac{1}{\sqrt{s}}\frac{N}{K}$.

However, comparison with data cannot be done in such model-independent way, because of the fluctuations of the inelasticity K (given by the inelasticity distribution $\chi(K)$ [7]) causing fluctuations of M and because of fluctuations in the number of particles $N = N(M)$ produced from a mass M given by multiparticle distribution $P(N; M)$ [8] (its actual shape accounts here for the possible multisource structure of the production process). One has therefore

$$(3)\ \frac{\mathrm{d}N}{\mathrm{d}y} = \int_0^1 \mathrm{d}K\chi(K) \sum_N P(N; K\sqrt{s})N \frac{1}{Z(K\sqrt{s}, N)} \exp\left[-\beta(K\sqrt{s}; N)\mu_\mathrm{T}\cosh y\right],$$

where $\beta(K\sqrt{s}, N)$ is still calculated from the energy conservation constraint, but now for a given event, *i.e.*, it is a fluctuating quantity with fluctuations resembling gamma-like distribution (and characterised by mean value $\bar{\beta}$ and normalised variance ω_β).
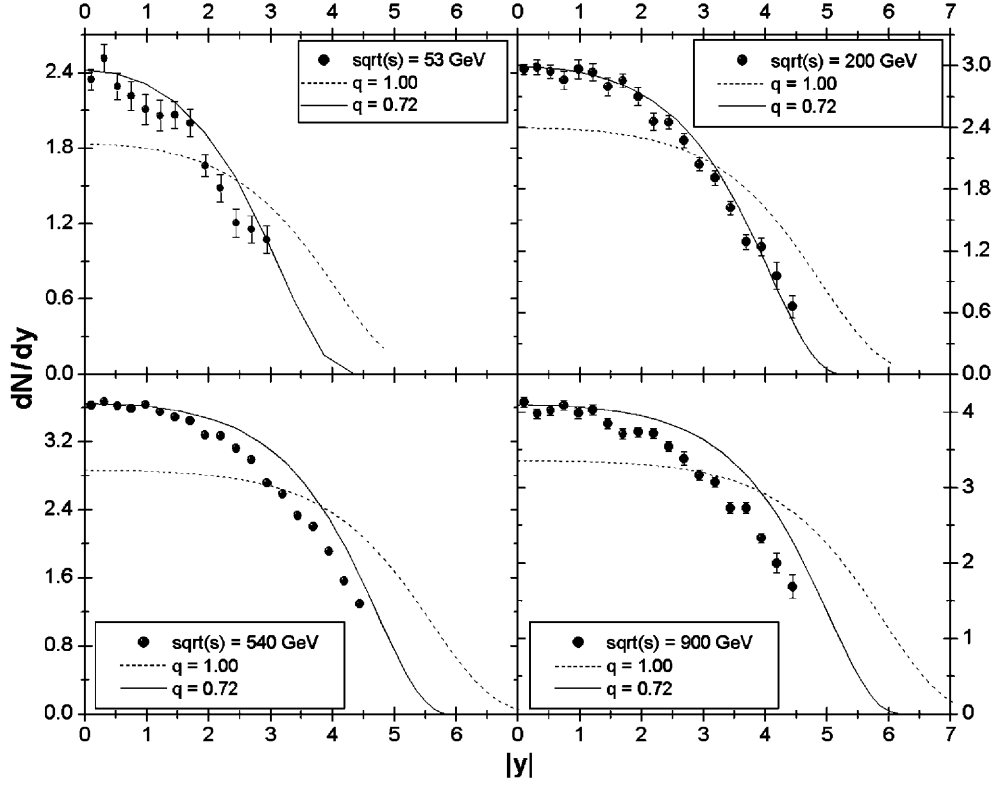
Fig. 1. – Comparison of eq. (5) with UA5 [10] data for $q = 1$ and $q = 072$.

It has been shown in [9, 6] that such fluctuations of the parameter of the exponential distribution convert it to a power-like distribution: $\exp[-\hat{x}/x_0] \to \exp_q[-\hat{x}/x_0] = (1 - \hat{x}/x_0/\alpha)^\alpha$ with $\alpha = \pm 1/\omega_\beta$ in our case, where $\hat{x} = \frac{2\mu_T}{\sqrt{s}}\cosh y$ and x_0 is a parameter connected with the mean value of $\beta \sim \frac{N}{K}$.

This is precisely the form corresponding to the distribution obtained using the non-extensive Tsallis q-entropy [4, 5], instead of the Shannon one (and reproducing it in the limit $q \to 1$), $S_q = -(1 - \int \mathrm{d}y\, [f(y)]^q)/(1 - q) \to S_{q=1} = -\int \mathrm{d}y f(y) \ln f(y)$, together with the modified constraint equation $\int \mathrm{d}y \mu_T \cosh y [f(y)]^q = M/N$. Using it, one gets instead of (2) its nonextensive version:

$$(4) \qquad f_q(y) = \frac{1}{Z_q(M, N)} \left[1 - (1 - q)\beta_q(M, N) \cdot \mu_T \cosh y\right]^{\frac{1}{1-q}}.$$

Nonextensivity means that the entropy of the composition $(A + B)$ of two independent systems A and B is equal to [5] $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1 - q) S_q^{(A)} \cdot S_q^{(B)}$ proceeding to its usual additive form only in the $q = 1$ limit. It arises, whenever in the system one encounters long-range correlations, memory effects or fractal structure of the corresponding space-time or phase space. Such situation is expected to occur also in the hadronization processes [6].
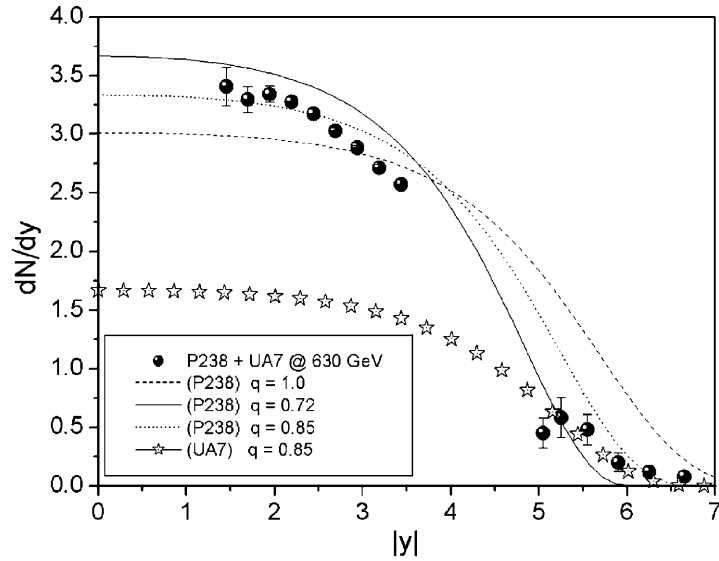
Fig. 2. – Comparison of eq. (5) with P238 [11] and UA7 [12] data for rapidity distributions.

3. – Comparison with experimental data

We make therefore the following conjecture: the convolution (3) can, in practice, be replaced by the following simple one-parameter formula of the type of eq. (1) which should be used to describe the existing data of [10-12]:

$$(5) \qquad \frac{\mathrm{d}N}{\mathrm{d}y} = \langle N(s)\rangle \frac{1}{Z_q} \left[1 - (1-q)\beta_q(\sqrt{s}, \frac{3}{2}\langle N(s)\rangle)\mu_{\mathrm{T}}\cosh y \right]^{\frac{1}{1-q}}.$$

The only free parameter (characterising the strenght of fluctuations in the system according to the previous discussion) is the nonextensivity parameter q. Here $Z_q = \int \mathrm{d}y \exp_q[-\beta_q\mu_{\mathrm{T}}\cosh y]$ and $\langle N(s)\rangle$ is the mean (single non-diffractive) charged multiplicity at a given energy $\sqrt{s}$. Notice that, since β_q is calculated from the energy conservation constraint which involves all produced particles (*i.e.*, total multiplicity), it is calculated here for $\frac{3}{2}\langle N(s)\rangle = \langle N_{\mathrm{total}}(s)\rangle$ particles. For the same reason care must be taken when one addresses data at $\sqrt{s} = 630$ GeV, as part of them is for charged and part for neutral particles only. In both cases the β_q must be the same (calculated for total multiplicity at a given energy) whereas the multiplicity in front of the formula has to be chosen according to the actual situation.

In fig. 1 we show our results both for $q = 1$ and our best fit with $q = 0.72$. In all calculations the experimentally observed variation of μ_{T} with energy has also been accounted for by using the following simple interpolation formula: $\mu_{\mathrm{T}} = 0.3 + 0.044\ln(\sqrt{s}/20)$ GeV. The results are reasonable, especially for 53 and 200 GeV. For higher energies our distributions start to be broader than data and this cannot be improved by changing q, as diminishing its value, in order to make distributions narrower, will spoil the agreement with data for small rapidities. It turns out that UA7 and P238 data cannot be fitted together with UA5 data, as they demand a slightly bigger value of $q = 0.85$, cf. fig. 2.
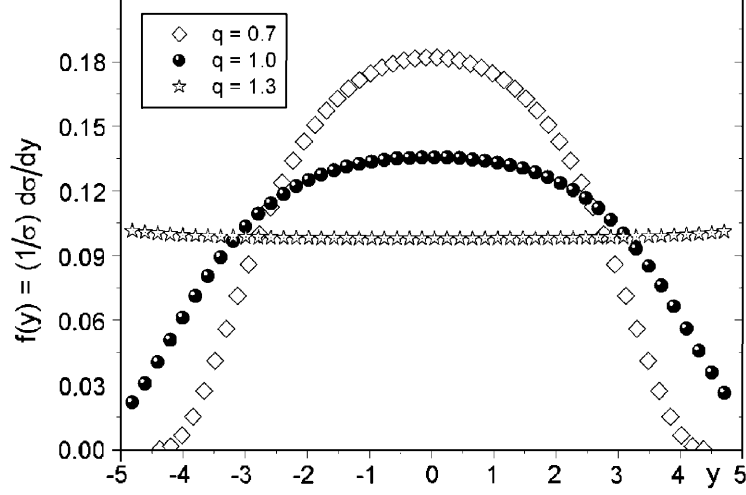
Fig. 3. – The examples of distribution (4) for mass $M = 100$ GeV hadronizing into $N = 20$ secondaries of (transverse) mass $\mu_\mathrm{T} = 0.4$ GeV each.

Notice that P238 data are for charged and UA7 data for neutral particles, therefore they must be described with, respectively, $\frac{2}{3}\langle N_\mathrm{total}\rangle$ and $\frac{1}{3}\langle N_\mathrm{total}\rangle$ in eq. (5), this leads to differences clearly seen in fig. 2.

4. – Summary and conclusions

Our approach consists in noticing the striking similarity of eq. (4) to eq. (1) of [1] ($\mathrm{d}y = \mathrm{d}x/x$). Figure 3 shows the characteristic features of $f_q(y)$ for the case of hadronization of fireball of mass $M = 100$ GeV into $N = 20$ particles of transverse mass $\mu_\mathrm{T} = 0.4$ GeV. Notice that for $q < 1$ one obtains an increase of particle densities in the central region connected with its decrease at the edges of rapidity range — clearly resembling the characteristic pattern of Feynman scaling violation. We have replaced therefore the "exact" formula (3) by a one-parameter fit represented by (5) where q summarizes the action of the averaging over the fluctuations caused by the reaction initial conditions which are represented by the inelasticity and multiplicity distributions. We find it very amazing that such simple approach coincides practically with the empirical formula (1) and with only one parameter q describes all data fairly well. Notice that $q = 0.72$ is not very far from $q = 0.75$, which gives power $1/(1-q) = 4$ in (1). Notice also that our general formula allows for the description of the small x region as well. It would certainly be interesting to connect q directly to the parameters describing inelasticity and multiplicity distributions or to descriptions of leading particles, for example of the type of that presented in [13]. Our results seem to indicate that most probably the parameter q should be x-dependent reflecting the different character of fluctuations of the quantity N/M [9] in the central (mostly pionization) and fragmentation regions. This problem will be addressed elsewhere.

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