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Global approaches and local strategies for phase unwrapping (*)(**)

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Summary. — Phase unwrapping, *i.e.* the retrieval of absolute phases from wrapped, noisy measures, is a tough problem because of the presence of rotational inconsistencies (residues), randomly generated by noise and undersampling on the principal phase gradient field. These inconsistencies prevent the recovery of the absolute phase field by direct integration of the wrapped gradients. In this paper we examine the relative merit of known global approaches and then we present evidence that our approach based on "stochastic annealing" can recover the true phase field also in noisy areas with severe undersampling, where other methods fail. Then, some experiments with local approaches are presented. A fast neural filter has been trained to eliminate close residue couples by joining them in a way which takes into account the local phase information. Performances are about 60–70% of the residues. Finally, other experiments have been aimed at designing an automated method for the determination of weight matrices to use in conjunction with local phase unwrapping algorithms. The method, tested with the minimum cost flow algorithm, gives good performances over both simulated and real data.

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1. – Introduction

Phase unwrapping is a problem which arises in various realms of signal processing, in particular Synthetic Aperture Radar Interferometry (INSAR) [1]. It consists of retrieving

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an absolute phase field, starting from the knowledge of the so-called wrapped phase, *i.e.* its principal value in $(-\pi, \pi]$, by adding to every sample the proper number of 2π-cycles. Since several solutions correspond to the same wrapped phase field, phase unwrapping is an ill-posed problem.

The most peculiar aspect of phase unwrapping in the two-dimensional case is the occurrence of point-like inconsistencies in the wrapped gradient field, called residues. Such singularities arise as a consequence of incorrect sampling of the phase field (aliasing), which can be due to excessive noise, critical surface slopes, or both.

In the absence of residues, phase unwrapping can be performed without problems, and any method gives the same, unique absolute phase surface. The real challenge in phase unwrapping is thus to eliminate the effect of the residues. Approaches to this problem are usually subdivided into "global" or "local".

Global phase unwrapping methods attempt to retrieve an absolute phase field which satisfies some *a priori* criteria, such as smoothness, while retaining its closeness to the measured phase data as much as possible.

Local algorithms aim at correcting locally the phase gradient field, so as to eliminate the residues. The choice of the locations where to apply the corrections must of course rely again on *a priori* assumptions about the solution.

In the first part of this paper, after introducing the mathematical problem of phase unwrapping, we present a brief review on known global methods and comparison of their performances, then the results obtained using an approach based on stochastic annealing with a Bayesian inference model, that proves to operate properly also with severely undersampled noisy phase fields. In this case, some global *a priori* information is used, by introducing a probabilistic model for the absolute phase surface.

We then introduce local methodologies, and the way in which local phase unwrapping methods try to eliminate residues by joining them in pairs. We illustrate an experiment aimed at training a feed-forward neural network to recognize and eliminate close residue couples by taking into account the local phase information. The Multi-Layer Perceptron (MLP), trained over simulated phase fields of various levels of coherence and slope levels, to recognize and eliminate in the best way adjacent residue couples, has been then tested over other simulated data. The measured performances are of the order of 60–70% of residues eliminated. This renders the approach a valid pre-processing step to drastically reduce the number of residues on an interferogram, which could increase the time performances of other, more powerful residue-joining algorithms.

Another issue of phase unwrapping which has been investigated concerns the consideration of any piece of information other than the wrapped phase alone, to set up weight maps to be used in conjunction with weighted phase unwrapping algorithms. In the last part of this paper, we present some recent results about a method to automatically arrange any number of information layers into a weight map having a given number of weight levels. The method uses a Self-Organizing Map (SOM) neural network to automatically cluster a set of feature vectors, obtained by applying an *a priori* rough ordering relationship to the information layer. The methodology, applied to simulated phase data, has been used in conjunction with the local phase unwrapping algorithm known as minimum cost flow. Results show significant improvements in the performances of the algorithm by using multi-valued, automatically-generated weight maps, with respect to double-valued, fixed-threshold ones.

2. – Phase unwrapping problem

2˙1. *Notation and definitions.* – In this section, we introduce the phase unwrapping problem. We denote the absolute phase field by $f(x, y)$, and the measured one by $g(x, y)$. Between the two fields there is the following relation:

$$g(x,y) = \mathcal{W}\left[f(x,y)\right] = \operatorname{mod}\left\{f(x,y) + \pi, 2\pi\right\} - \pi, \tag{1}$$

where $\mathcal{W}$ is the wrapping operator, defined here in terms of the modulo-2π operation.

Phase unwrapping means recovering the f field from the observed wrapped field g. An important quantity is the vector field $\mathbf{A}(x, y)$, given by

$$\mathbf{A}(x,y) = \mathcal{W}[\boldsymbol{\nabla} g(x,y)], \tag{2}$$

where $\boldsymbol{\nabla} = (\nabla_x\,, \nabla_y\,)$ is the discrete gradient operator, defined as

$$\begin{aligned} \nabla_x\, g(x,y) &= \left[g(x+1,y) - g(x,y)\right], \\ \nabla_y\, g(x,y) &= \left[g(x,y+1) - g(x,y)\right]. \end{aligned} \tag{3}$$

If the Nyquist condition

$$|\boldsymbol{\nabla} f(x,y)| < \pi \tag{4}$$

is verified everywhere on the grid, then phase unwrapping is exactly soluble and the solution is $\boldsymbol{\nabla} f = \mathbf{A}$; the absolute phase surface is obtained by integrating $\boldsymbol{\nabla} f$ over the whole grid along any path encompassing all the field sites once.

In real interferograms, however, condition (4) is often violated because of aliasing and noise; the detectable effect of this is the presence of pointlike inconsistencies for field $\mathbf{A}$. Quantitatively, one can calculate the integral of $\mathbf{A}$ over any 2×2 square having (x, y) as a corner, *i.e.*

$$\begin{aligned} I(x,y) &= \frac{1}{2\pi} \oint \mathbf{A} \cdot \mathrm{d}\mathbf{l} \\ &= \frac{1}{2\pi}\big[A_x(x,y) + A_y(x+1,y) - A_x(x,y+1) - A_y(x,y)\big]. \end{aligned} \tag{5}$$

It has been shown [2] that $I(x, y)$ will always be either -1, 0, or 1. Locations with $I \neq 0$ are called residues; the presence of residues implies the violation of the correct sampling condition (4). In the presence of inconsistencies, the relation $\boldsymbol{\nabla} f = \mathbf{A}$ is not valid, and the following one has to be used instead:

$$\boldsymbol{\nabla} f(x,y) = \mathbf{A}(x,y) + 2\pi \mathbf{k}(x,y), \tag{6}$$

where $\mathbf{k}(x, y)$ is a certain vector field of integers. Solving phase unwrapping is equivalent to finding a field $\mathbf{k}$ which restores the field $\boldsymbol{\nabla} f$. Such field must satisfy the consistency condition

$$\boldsymbol{\nabla} \times [\mathbf{A}(x,y) + 2\pi \mathbf{k}(x,y)] = 0. \tag{7}$$

Many methods have been proposed to solve this problem. However, finding a solution consistent with the real terrain features is very difficult and problem-dependent, so that no efficient method exists which is agreed upon by all researchers.

2˙2. *Residue topology.* – Normally, interference fringes behave as closed contour lines, so that, in good sampling conditions, every minimal circulation, defined in (5), involves an even number of crossings of these discontinuities, and no residue is generated. Noise, on the contrary, modifies locally some of the principal phase derivatives (PPDs), which can generate either single spurious fringe segments or local "breaks" into the existing topographical fringes. In the same way, a region of critical terrain slopes can also generate a local violation of the Nyquist condition, and so cause isolated absolute phase derivative (APD) values to exceed π in magnitude, so that one or more 2π-cycles are locally lost in the wrapping operation, and fringes result broken.

In the case of SAR interferometry, critical terrain slopes are those close to the look-angle value. Due to the particular geometric configuration of remotely-sensed SAR data, the mentioned condition generates phase gradients of more than π in magnitude between adjacent pixels, thus causing phase aliasing. Another critical condition is layover, which occurs when terrain slopes exceed locally the look angle value, thus causing a local inversion of the phase gradient sign.

Since every open fringeline brings two opposite-sign residues at its ends, it is clear that in both noisy and poor sampling conditions the residue number increases dramatically.

From a quantitative viewpoint, it has been shown by Monte Carlo simulations [3, 4] that, under conditions normally encountered on real interferograms, the residues located in adjacent circulations are the majority (more than 60%–70%). Nevertheless, only a small fraction of them is made of easily removable isolated dipoles, while the majority form clusters of inconsistencies, which cannot be easily paired due to possible ambiguities in their coupling order.

An isolated dipole can be easily eliminated by applying corrections taken among a finite set. For the easiest configurations, *i.e.* the two residues located on adjacent pixels, a single correction is needed to "solve" the dipole.

It is always possible to eliminate all inconsistencies, deterministically applying a predefined set of rules to all the dipoles present on the phase field [5]. However, different solutions are obtained for different scanning orders on the image. Therefore, to find acceptable residue pairings and the relative cutlines, it seems necessary to analyze the local phase field that possibly contains all relevant information.

3. – Global methods

As mentioned, global methods attempt to recover the absolute phase field by utilizing the wrapped values together with some additional assumptions, such as the absence of undersampled areas, the noise probability distribution, and the regularity of the phase field to be recovered. The prototype for all the global approaches is the least-squares method, where only the information on the probability distribution of the noise is added. In the absence of gradient field inconsistencies, this method gives results identical to those obtained using direct 2-D integration of the wrapped phase gradient field. If the phase noise distribution is actually similar to a Gaussian, the recovered phase field gives a good approximation of the true one, with errors localized near the inconsistencies. In case of severe undersampling, however, with noise distributions significantly different from the Gaussian case, this method badly fails, and additional information must be added.

Other methods are based on the ideas introduced by Tikhonov for solving ill-posed problems, by introducing as *a priori* knowledge some constraints expressed as a regularization condition.

The success of these methods applied to the unwrapping of the phase field depends strongly on the choice of the regularization operators. In the following we briefly review the proposed global methods comparing their performances on simulated noisy and undersampled fields. We show that our procedure based on stochastic annealing appears capable of recovering the true phase field also in heavily undersampled noisy areas, where the other known methods fail, provided that a proper regularizing constraint can be established.

3˙1. *Least-squares method.* – This method [6] is based on the following assumptions: i) a Gaussian model for noise and ii) the true phase gradient's being less than π everywhere; it follows that all the residues are assumed to be noise-induced. From (6), field $\mathbf{A}$ is evaluated as an estimate for the true phase gradient; the solution is searched as the scalar field f whose discrete gradient is closest to $\mathbf{A}$ in the least-squares sense, *i.e.* the minimizer of the following functional:

$$L[f] = \sum_{(x,y)} [\boldsymbol{\nabla} f(x,y) - \mathbf{A}(x,y)]^2. \tag{8}$$

The least-squares approach has been shown to be equivalent to solve a set of Poisson equations on a rectangular grid with Neumann boundary conditions [7]. This has suggested the use of fast, direct algorithms specialized in solving elliptic partial differential equations, such as Fast Fourier Transform (FFT) methods [8].

The LMS algorithm fails to retrieve the original absolute phase when the assumptions stated above are not met, *i.e.* in aliasing conditions. In the case of SAR interferometry, it has been shown [9, 10] that treating noise as Gaussian leads to a bias in the least-square estimation of the gradient of the true phase field.

This method gives identical results as the direct integration in the absence of gradient field inconsistencies. When Gaussian noise is the only source of residues, the recovered phase field gives a good approximation of the true one, with errors localized near the inconsistencies. In case of severe undersampling, this method badly fails.

3˙2. *Tikhonov regularization theory.* – Marroquin and Rivera [11] generalized the least-squares algorithm, by introducing a regularization term which performs noise reduction (even if this noise does not generate integration-path inconsistencies). Following Tikhonov regularization theory [12], the solution of phase unwrapping is assumed to be the minimizer of the quadratic functional:

$$H[f] = R[f] + \lambda L[f], \tag{9}$$

where R is a smoothing functional based on second-order differences:

$$R = \sum \left[\nabla_x f(x,y) - \nabla_x f(x-1,y)\right]^2 + \\ + \sum \left[\nabla_y f(x,y) - \nabla_y f(x,y-1)\right]^2 + \\ + \sum \left[\nabla_x f(x,y) - \nabla_x f(x,y-1)\right]^2 + \\ + \sum \left[\nabla_y f(x,y) - \nabla_y f(x-1,y)\right]^2, \tag{10}$$

and L is that defined in (8).

The parameter λ controls the compromise between smoothness (minimization of R) and fidelity to the data (minimization of L). If one has prior knowledge about the noise amplitude or the roughness of the solution, then the correct value of λ may be obtained as described by Marroquin and Rivera [11]. In most cases, however, λ must be adjusted by hand. Various algorithms for the minimization of H have been studied [11]. Although this algorithm is effective in noise reduction, it works under the same basic assumptions i) and ii) as LMS, reported in subsect. **3**˙1; hence it fails to reconstruct the correct input surface in aliasing conditions.

3˙3. *Parallel algorithm based on Bayesian theory.* – An interesting approach to PU in a Bayesian framework has been proposed by Marroquin and coworkers [13]. The true phase field is written as

$$f(x,y) = g(x,y) + 2\pi k(x,y), \tag{11}$$

where $k(x,y)$ counts the 2π-cycles to be added to the observed wrapped phase([1]). Instead of constraining $\{k\}$ to be integer-valued, the following constraint is introduced:

$$k(x_1,y_1) - k(x_2,y_2) = r\left(k(x_1,y_1) - k(x_2,y_2)\right), \tag{12}$$

where $r(a)$ is the closest integer to a, and (x_1,y_1) and (x_2,y_2) are neighboring pixels. The fact that $\{k\}$ is treated as a real-valued field while constraining its differences to be integer is crucial for the correct behavior of the algorithm. A functional is introduced to model constraint (12):

$$V[k] = \sum \left[k(x_1,y_1) - k(x_2,y_2) - r\left(k(x_1,y_1) - k(x_2,y_2)\right)\right]^2. \tag{13}$$

The posterior energy for field $\{k\}$ is then given by

$$U[k] = R[g + 2\pi k] + \lambda V[k], \tag{14}$$

where R is a smoothing functional, and the parameter λ controls the smoothing performed by the algorithm. Minimization of U can be rapidly solved by a Newtonian descent automaton which starts from the initial point $k^{(0)} = -g/2\pi$.

([1]) Note the difference between the scalar $\{k\}$ field used in this section and the vector **k** field introduced in sect. **2**; the latter is to be added to the phase gradient vector field.

This algorithm has shown to be capable of filtering noise (even in the case that it does not lead to inconsistencies) and to interpolate the reconstructed phase over regions of invalid information.

We remark that this algorithm does not work under the assumptions i) and ii) stated in subsect. **3**˙1, but its performances are influenced by the peculiar way in which the integer nature of PU is taken into account. Indeed, the parameter λ controls both the noise reduction and the closeness to integer values for field $\{k\}$. In subsect. **3**˙5 we will describe a new regularization scheme which dispels the two assumptions described above and fully exploits the integer nature of PU.

3˙4. *Phase unwrapping method based on network programming.* – Costantini [14] proposed an interesting algorithm based on network programming. He focuses on the integer vector field $\mathbf{k}$ which must be added to wrapped phase gradient in order to recover the true phase gradient (see eq. (6)). A cost functional is introduced as a weighted sum of absolute values of field $\mathbf{k}$:

$$\mathcal{C}[\mathbf{k}] = \sum c_1(x,y)\,|k_x(x,y)| + \sum c_2(x,y)\,|k_y(x,y)|\,. \tag{15}$$

The solution of phase unwrapping is assumed to be the integer field $\mathbf{k}$ which minimizes $\mathcal{C}$ among those verifying the constraint (7). Due to the form of cost functional $\mathcal{C}$, this problem is recognized to be equivalent to the problem of finding the minimum cost flow on a network, for the solution of which there exist very efficient techniques.

We observe that the performance by this algorithm is completely dependent on the choice of weights $\{c\}$. In particular, choosing constant weights leads often to unsatisfactory results. A version of this algorithm, developed by Eineder *et al.* [15], includes a pre-processing stage where the weights are fixed according to information such as the amplitude image a, the residues density r and the estimated raw slope s. For each pixel and for each quality a, r, s an indicator ia, ir, is is set to one if the corresponding quality is lower than a threshold ta, tr, ts, otherwise the indicator is set to zero. Two quantities are calculated, $ca = 99 \times ia + 1$ and $crs = 99 \times ir \times is + 1$. Finally the weights are generated as $c = \max(ca, csr)$. The authors report optimal values of thresholds for unwrapping interferograms from alpine regions [15]. It seems to us that the problem of determining weights is here only changed into the problem of finding optimal thresholds values. A fully automatic strategy to fix the weights requires, in our view, further investigation. The issue is addressed later in this paper (see sect. **5**).

3˙5. *Stochastic relaxation algorithm for phase unwrapping.* – The problem of finding the most probable absolute phase field when only a wrapped field corrupted by noise and undersampling is available, is a typical "maximum *a posteriori*" (MAP) estimate problem. When the probabilistic relation among phase values in neighboring pixels is described as a Markov random field, it is well known that the probability distribution can be made to correspond to the Gibbs distribution of a thermodynamic physical system characterized by a given energy and temperature, if the energy function is properly chosen.

The convergence to the MAP field is not guaranteed by deterministic search methods, which can get trapped into local maxima. On the contrary, probabilistic relaxation methods, such as the stochastic annealing, where the "temperature" is progressively lowered, simulating the annealing processes by which physical systems can be driven to their minimum energy state, can be proved to converge to the searched MAP field.

Our algorithm formulates PU as a constrained optimization problem for the integer vector field $\mathbf{k}$ in eq. (6) [16]. Such field must satisfy the constraint (7), hence, the following functional is introduced:

$$C[\mathbf{k}] = \sum_{(x,y)} \left\{ k_x(x, y+1) - k_x(x,y) - k_y(x+1, y) + k_y(x,y) - I(x,y) \right\}^2, \tag{16}$$

where $I(x, y)$ is the quantity defined in eq. (5). One can easily check that the constraint (7) is equivalent to $C[\mathbf{k}] = 0$. The solution of the phase unwrapping problem is assumed as the phase field corresponding to the vector $\mathbf{k}$ which minimizes $R[\mathbf{A}+2\pi\mathbf{k}]$ among those verifying the constraint $C[\mathbf{k}] = 0$, where $R[\cdot]$ is the functional in eq. (10). To solve the optimization problem, $\mathbf{k}$ is modeled as a Markov random field (MRF) with probability function

$$P_{\beta,\lambda}(\mathbf{k}) = \frac{\exp\left[-H[\mathbf{k}]\right]}{\sum_{\mathbf{k}'} \exp\left[-H[\mathbf{k}']\right]}, \tag{17}$$

where

$$H[\mathbf{k}] = \beta \left(R[\mathbf{A} + 2\pi\mathbf{k}] + \lambda C[\mathbf{k}] \right). \tag{18}$$

The optimization problem is solved by sampling the MRF in the limit $\beta, \lambda \to \infty$, using the simulated annealing technique [17].

Some remarks are in order. During the processing, only integer numbers of 2π-cycles are added to the wrapped phase field. Therefore, the algorithm removes only the noise that leads to inconsistencies; filtering of the reconstructed surface is deferred to a subsequent stage. As a final remark, we note that this algorithm does not require the estimation of parameters which control the compromise between regularity and fidelity to the data.

3˙6. *Experiments*. – In this section we present the results we have obtained by using the algorithms described above on simulated phase surfaces.

In fig. 1 (a) we report a smooth Gaussian simulated phase surface on a 100×100 grid. In fig. 1 (b) the corresponding residue map is depicted. This test surface exhibits aliasing conditions due to topography alone, as can be seen from the regular location of the residues.

The surface unwrapped by the LMS method is reported in fig. 1 (c); the reconstruction errors due to the mentioned bias in the slope estimation are evident. The result obtained by unwrapping the same surface by the algorithm based on Tikhonov regularization is shown in fig. 1 (d); it is very close to the one from LMS. The output of the algorithm described in subsect. **3**˙3 is depicted in fig. 1 (e); we observe spreading of errors from inconsistencies. We used $\lambda = 1$ for both the algorithms described in subsects. **3**˙2 and **3**˙3. The stochastic algorithm, instead, perfectly reconstructed the input surface (fig. 1 (f)). We did not compare with the algorithm described in subsect. **3**˙4 because it would have required fixing the weights according to some arbitrary criterion.

We have tested the robustness of the algorithms in noisy conditions. In fig. 2 (a), the same simulated phase surface is shown as those illustrated in 1 (a), with added interferometric noise [18], with coherence equal to 0.6. The corresponding residue map is shown in fig. 2 (b). Now we have residues due to both noise- and topography-generated

Fig. 1. – Comparison between global methods on a smooth surface: (a) Gaussian surface (100×100 pixels); (b) residue map; (c) reconstructed surface by LMS; (d) reconstructed surface by the method described in subsect. **3**˙2; (e) reconstructed surface by the method described in subsect. **3**˙3; (f) reconstructed surface by the method described in subsect. **3**˙5.

aliasing. Figures 2 (c), (d), (e), and (f) show the surfaces reconstructed respectively by LMS, Tikhonov regularization, Parallel Bayesian approach, and Stochastic algorithm. We observe that the output from the algorithm described in subsect. **3**˙2 is a smoothed version of the output from LMS. Both the parallel algorithm and the stochastic one show robustness with respect to noise.

Fig. 2. – Comparison between global methods on a noisy surface: (a) Synthetic surface (100×100 pixels) obtained by adding noise corresponding to coherence $\gamma = 0.6$ to the surface depicted in fig. 1 (a); (b) residue map; (c) reconstructed surface by LMS; (d) reconstructed surface by the method described in subsect. **3**˙2; (e) reconstructed surface by the method described in subsect. **3**˙3; (f) reconstructed surface by the method described in subsect. **3**˙5.

4. – Local algorithms

Local phase unwrapping algorithms are generally aimed at removing locally the effects of the residues. For instance, a classical solution consists in placing so-called ghost lines or cutlines, *i.e.* arbitrary lines connecting residues of opposite sign, on which it is assumed that the phase is in aliasing conditions, and then performing the integration along paths avoiding these lines [19, 3, 20, 21]. In this way, the phase gradient is locally allowed to

Fig. 3. – Principal Phase Derivative numbering. The pixels are here represented by the black dots, while the gradients are represented by the segments joining them.

take any value greater than π in magnitude.

In the case of close residue couples, removal of the effects of the residues can equivalently be accomplished by correcting locally the phase gradients by adding or subtracting one phase cycle to their wrapped value. In the following, we restrict to this kind of inconsistencies.

4˙1. *Neural network approach.* – The purpose of our neural network approach is to see whether it is possible to recognize local phase gradient configurations in a neighborhood of any residue having at least another adjacent residue of opposite sign, so as to apply the proper corrections, by analyzing a series of simulated examples. To this end, we exploit the generalization properties of a feed-forward multi-layer perceptron (MLP), trained with the back-propagation scheme [22].

The input layer of our network is given by 24 units, each receiving one of the 24 phase gradient values present in a window of 4×4 pixels, according to a conventional numbering shown in fig. 3. Such windows are extracted from the interferogram, centered on every minimal circulation giving a residue by the integral (5). The output layer is composed by 24 binary units, whose values are intended as flags which give indications about whether to apply (1) or not (0) the correction to each phase gradient site, consisting in not wrapping its value, in order to restore the circulation and eliminate the residue. The hidden layer of the network contains instead a number of neurons different for each experiment, in order to test its learning and generalization capabilities.

The network has been trained on target flags, which determine whether the wrapped values of the corresponding input principal phase gradients coincide (0) or not (1) with the absolute phase gradients. In this way, the network should learn to recognize, by looking at the local configurations of principal phase gradients, the phase gradient sites where the "wrapping rule" stated above is actually violated.

The selected supervised training of the MLP, performed over a data set of patterns extracted randomly from simulated phase fields, was meant to find a mapping function that couples to any given input wrapped phase gradient configuration a certain output

Fig. 4. – Residues left on the phase field after application of the MLP as a function of the coherence level on the image, for different coherence levels in the training datasets: $\diamondsuit$ training on $\gamma = 0.6$, $\circ$ training on $\gamma = 0.75$, $\times$ training on $\gamma = 0.9$.

combination of corrections.

4˙2. *Results*. – An experiment was performed in order to assess the real possibilities of this approach in various noise and topography conditions. A series of simulated absolute phase fields was generated, constituted by cones (to reproduce all the azimuthal exposition angles) and ramps of various heights and slopes, with added noise of various coherence conditions ($\gamma = 0.6$, 0.75, and 0.9). Several training sessions were performed, with a neural network having a hidden layer of 100 neurons. The graph in fig. 4 reports the performance of the network trained on various noise levels, with respect to the elimination of the residues in the central circulation of every 4×4 window. As can be seen, MLPs trained on simulated fields of a given coherence level tend to better succeed in eliminating residues on phase fields characterized by coherence equal to or higher than that it has been trained on. This could be expected, since higher coherence entails residue couples configurations easier to solve. The graph in fig. 4 shows that the performances settle to the removal of around 70–80% of the initial number of residues.

It can be argued that larger windows would perhaps contain more information, so as to improve the residue removal capabilities of the neural network. However, an increase in the input and output vector sizes, say to a 6×6 window, would entail having 60 neurons in input and 60 in output, which means increasing the computational load with respect to the case with 24 input/output units. Moreover, to properly train such a complex network, a much higher number of simulated examples would be necessary. This seems to set an intrinsic limit to the local methods based on the determination of the true cutline from the local configuration of the wrapped phase field. It seems impractical to extract this information, with the neural approach, at least within the limits of an acceptable complexity. Additional *a priori* information is therefore needed for a correct phase unwrapping.

Fig. 5. – The general scheme of the weight generation procedure described in the text.

5. – Weight map determination for phase unwrapping

It has been shown how the ill-posedness of the phase unwrapping mathematical problem makes it impossible to have a "universal" algorithm, *i.e.* one which works in all situations: the most powerful global algorithms, which make use of regularization methods, work on assumptions about the actual smoothness of the solution, and therefore fail on surfaces which do not meet those characteristics; local, residue-joining procedures, although fast, are able to eliminate only a fraction of the phase residues on the interferogram, if a reasonable level of complexity is to be maintained. Alternatively, local algorithms can also be made to join and eliminate all the residues on an image, but the criteria for choosing coupling orders, and the actual paths followed to join distant residues, remain arbitrary to a large extent.

Fig. 6. – Performances of the WMCF algorithm working with various weight maps derived via the method described in the text. (a) percentage of wrongly-unwrapped pixels; (b) total dynamics of the error surface, in 2π phase cycles. The legend reports the input information layers used: the symbols mean coherence (γ), amplitude of the master and slave images (A_1 and A_2), phase gradient amplitude ($|\nabla\phi|$), and residue density ($|R|$).

In practice, then, all phase unwrapping algorithms have to rely on external information to reach good performances. In this view, an automatic methodology has been worked out to produce weight maps for use in conjunction with phase unwrapping algorithms. It is the subject of the next sections.

5˙1. *Automated methodology*. – Our methodology for weight map generation is a flexible and automatic framework, easily extendable to any number of "information layers". Any number of such layers con be considered for input, with the only constraint that an *a priori* relationship of each information layer with respect to weight values has to be established. The general scheme is shown in fig. 5.

In this way, each information layer is converted into a *feature*. Each pixel is thus associated with a feature vector, of as many components as the number of input layers. Such vectors can be represented in a multi-dimensional feature space, in which each axis is ordered according to the *a priori* relationships. To render the method more robust and less data-dependent, the vectors are clusterized in a self-consistent way by means of a self-organizing map (SOM) neural network [23], into a predefined, arbitrary number of weight levels. The final map is then generated by assigning each pixel to its nearest cluster in the feature space, and associating a weight value given by the Euclidean distance of that cluster from the origin, in the same space.

5˙2. *Experimental results*. – Maps obtained in the way described in the preceding section were used in conjunction with the weighted version of the minimum-cost flow algorithm, introduced in subsect. **3**˙4, to test various performance parameters as a function of both the number of input layers and of output weight levels.

The testing was made over simulated interferometric data, so that it was possible to compare the obtained unwrapped phase with the original, simulated absolute phase field. Testing was performed by calculating both the total number of wrongly-unwrapped pixels, and the overall dynamics (in 2π-phase cycles) of the error. The main results are

Fig. 7. – Information layers used for the unwrapping of the 1024×1024 real interferogram; (a) wrapped phase, (b) coherence, (c) master image amplitude, (d) phase gradient.

reported in the graphs in fig. 6.

It can be seen how both the percentage of wrongly-unwrapped pixels, and the dynamics of the unwrapping error, in phase cycles, show a significant improvement as the number of levels in the weight map is increased from 2 to 5. An improvement can also be noted as the number of input information layers is increased.

The automatic methodology has been also applied to a real dataset consisting of a ERS-1/ERS-2 tandem pair of images([2]), acquired on June 5-6, 1996. Experiments were

([2]) A pair of images acquired by the ERS-1 and ERS-2 satellites, respectively, over the same earth location, with the time interval of 24 hours, is conventionally termed a "tandem" pair.

Fig. 8. – Results of the MCF PU applied to the interferogram in fig. 7 (a), using a 3-levels weight mask derived from 3 information layers, namely master amplitude, coherence, and phase gradient. (a) Weight mask; (b) k-field (ghost lines); (c) unwrapped phase surface; (d) absolute phase surface simulated from a DEM of the area; (e) unwrapped phase surface obtained with LMS unwrapping.

performed with various numbers of input layers and of mask levels. The best result was chosen by comparing the dynamics of the unwrapped phase with the known height dynamics found on the area, properly scaled by the factor taking into account the baseline and the observation geometry.

Figure 7 reports the layers used for the unwrapping of a 1024×1024 pixel scene, centered over the southern Apennines area of Valle del Sele. The area covered is about 40×40 km wide. Figure 8 shows the best results obtained with a 3-levels weight mask derived from the 3 information layers of coherence, master amplitude, and phase gradient. A visual comparison can be made between the unwrapped phase surface (fig. 8 (c)) and the corresponding absolute phase surface as simulated from a Digital Elevation Model (DEM) of the area (fig. 8 (d)). As a reference, the surface obtained through the classical LMS algorithm is shown in fig. 8 (e), and shows considerable artifacts due to the strong aliasing conditions.

6. – Conclusions

In this paper we have analyzed the performances and the limits of global and local methods in the unwrapping of noisy and undersampled phase fields, such as those typically encountered in SAR interferometry.

We have presented a short review of the known global methods and a comparison of their performances and limitations when applied to the wrapped phase fields affected by heavy noise and undersampling. A method based on stochastic annealing has been also presented, which is able to recover the absolute phase field in rather extreme situations.

It has been shown how, in local methods, in order to eliminate ambiguities in the recovered phase field, one has to avoid arbitrary assumptions in the identification of the cutlines connecting pairs of residues of opposite sign. It has been shown that, even with a neural approach, it is impractical to deduce a minimum set of true interconnecting rules from the observed values in the local wrapped phase field. A neural approach such as the one presented in the paper has a valid chance of application as a quick pre-processing "filter", intended to rapidly eliminate the most common type of residue couple, *i.e.* the close dipoles due mainly to noise.

In order to introduce additional *a priori* information needed for the correct recovery of the absolute phase field, an automated methodology has been presented. Tests on simulated and real data have been reported, which show improvements in the yield of weighted unwrapping algorithms such as the minimum cost flow, when run with weight maps derived from several input information layers, and with more than two weight values.

* * *

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