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Weighted multi-resolution phase-unwrapping method (*)

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Summary. — The proposed method for phase unwrapping is based on a global analysis of the interferometrical phase. The underlying principle is that the interferogram is partitioned such that the unwrapped-phase function on each element can be locally modelled by the mean values of the phase difference between neighbouring pixels in azimuth and range directions. Using this local information and a least-squares algorithm (Gauss-Seidel relaxation), an approximate model of the unwrapped phase is then generated and tested by calculating the "residue image" defined as the difference between the original interferogram and the model itself. If there are residual fringes, then the result must be iteratively refined applying the method to the residue image. The accuracy of the proposed estimation depends on the dimensions of the elements and the dynamic content of the phase, *i.e.* on the "roughness" of the ground surface.

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1. – Introduction

In the last years several efforts have been made to solve the problem of phase unwrapping in SAR interferometry [1-3] . The case of recovering a phase function from high-quality data can be regarded as worked out, but when noise or aliasing corrupts the interferogram many difficulties arise.

In this article, an algorithm is proposed that is more robust to noise than classical approaches. This algorithm is based on the method described in [4] which generates a global model for the unwrapped phase, while limiting the propagation of errors due to the presence of noisy areas in the interferometrical image. The key idea is that the interferogram can be subdivided into polygonal elements, on each of which the unwrapped phase is modelled by the local mean slopes. Starting from this information, it is possible

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to locally approximate the true phase with linear functions, which in two dimensions are planes, and then a global model is reconstructed by linking the nodes of the planes found.

Suppose a 2-D continuous space and let $\phi(x, y)$ and $\phi_p(x, y)$ represent, respectively, the unwrapped and the interferometrical phase functions, where x and y indicate the azimuth and range directions. Since the interferogram contains the principal values of the phase, *i.e.* modulo 2π, we may write

$$\phi(x, y) = \phi_p(x, y) + 2\pi m_1(x, y)\,, \tag{1}$$

where $m_1(x, y)$ is an integer-valued function of azimuth and range.

If we now consider a function $\mu(x, y)$ approximating the true phase, it is possible to calculate the "residue" associated to this model as the principal value of the difference between the interferogram and the model itself:

$$R(x, y) = [\phi_p(x, y) - \mu(x, y)]_p \tag{2}$$

Combining eqs. (1) and (2) yields

$$[\phi_p(x, y) - \mu(x, y)]_p = [\phi(x, y) - 2\pi m_1(x, y) - \mu(x, y)]_p\,, \tag{3}$$

leading to

$$[\phi_p(x, y) - \mu(x, y)]_p = [\phi(x, y) - \mu(x, y) - 2\pi m_1(x, y)] + 2\pi m_2(x, y)\,, \tag{4}$$

and then

$$\phi(x, y) = [\phi_p(x, y) - \mu(x, y)]_p + \mu(x, y) + 2\pi(m_1(x, y) - m_2(x, y))\,. \tag{5}$$

Assuming the model and the residue to be continuous, then $2\pi(m_1(x, y) - m_2(x, y))$ is a continuous function of integers and, therefore, must be a constant:

$$\phi(x, y) = \mu(x, y) + [\phi_p(x, y) - \mu(x, y)]_p + 2\pi C\,. \tag{6}$$

This means that, if one is able to generate a model with a continuous residue, then the unwrapped phase is given by the sum of the model with the residue image and a constant. This constant can be calculated if the elevation of some point in the scene is known.

In the real case the interferogram is not continuous, but discrete. This implies the presence of discontinuities between near points both in the model and in the residue, but if those discontinuities are limited and have a modulo value smaller than π, it is still possible to unwrap the phase.

2. – The proposed approach

In order to determine a surface approximating the true phase function, the best solution would be the use of a finite-element based modelisation technique, dividing the domain with triangles of various shapes and dimensions that best fit the local characteristic of the interferogram. Instead of this choice, in our approach we preferred to use squared elements, mainly in the aim of improve the computational efficiency. The coarseness of the grid used for the partition of the interferogram is determined as a compromise between computational burden and precision of the result. Big squares result in a rather poor accuracy, in contrast with small squares. Once the size of the squares has been fixed, the phase model generation goes through the following steps:

– estimation of the mean slopes in azimuth and range for every square;

– estimation of the phase values at the nodes of the grid through the solution of a least mean squares problem;

– computation of the model in each point using bilinear interpolation;

– computation of the residue image.

If the obtained solution is not accurate enough, an improvement is possible iteratively applying the algorithm to the residue image with a partition of different size.

For the evaluation of the accuracy of the model, a criterion based on the number of fringes in the residue image can be used. The interferometric fringes in the interferogram are those points that are characterised by a modulo of the gradient greater than π and have therefore a discontinuity in phase. A point that belongs to a fringe therefore satisfies the following property [4]:

$$[([\phi_p(i+1,j) - \phi_p(i,j)]_p)^2 + ([\phi_p(i,j+1) - \phi_p(i,j)]_p)^2]^{1/2} \geqslant \pi\,. \tag{7}$$

The objective of the process of phase unwrapping now becomes to find the model that eliminates the fringes in the residue image, *i.e.* that eliminates the points satisfying (7). Since in real world image one has to deal with noise and other phenomena that makes this impossible, this constraint is relaxed to minimising the number of points that verify (7).

2˙1. *The computation of the phase model.* – With the approach proposed in [4] the mean slopes, in azimuth and range, are determined adjusting a bilinear model (*i.e.* a plane) to each elementary square. The algorithm searches for the plane that best fits the behaviour of the phase in every square of the grid. To this end, a general plane is constructed:

$$\mu_{\mathrm{loc}}(i,j) = \mathrm{slope}_{\mathrm{azimuth}} \times (i-i_0) + \mathrm{slope}_{\mathrm{range}} \times (j-j_0)\,, \qquad i,j \in \mathrm{ES}\,, \tag{8}$$

where i_0 and j_0 indicate the location of an arbitrary initial point and ES indicates the elementary square. The parameters $\mathrm{slope}_{\mathrm{azimuth}}$ and $\mathrm{slope}_{\mathrm{range}}$ should be chosen such that they minimise the number of fringes in the residue which is associated to the square:

$$R(i,j) = [\phi_p(i,j) - \mathrm{slope}_{\mathrm{azimuth}} \times (i-i_0) - \mathrm{slope}_{\mathrm{range}} \times (j-j_0)]_p\,, \quad i,j \in \mathrm{ES}\,. \tag{9}$$

A successive minimisation of the variance on the elementary residue allows refining the obtained slopes. This technique for estimating the local information, though robust to noise, results in a noticeable computational burden and causes the process of phase unwrapping to be slow. In order to solve this problem, we propose an alternative method for computing local slopes that, though faster and simpler, maintains the ability of limiting the propagation of errors due to the presence of noise.

For the purpose of understanding well the proposed modification, we consider for a moment the 1-D case and assume that the phase is represented by a continuous function. If the derivative of the phase function is known, the phase of a generic location can be computed by integration:

$$\phi(x) = \phi(x_0) + \int_{x_0} \phi'(x)\mathrm{d}x\,. \tag{10}$$

The derivative, $\phi'(x)$, describes the local variations and therefore the local slopes of the phase function. If we assume that the behaviour of the phase is characterised by a component with a constant slope to which a dynamic component is added that accounts for small variations, we can rewrite the first derivative as

$$\phi'(x) = \phi'_{\mathrm{cost}} + \phi'_{\mathrm{dynam}}(x)\,, \tag{11}$$

leading to

$$\phi(x) = \phi(x_0) + \phi'_{\mathrm{cost}} \times (x - x_0) + \int_{x_0} \phi'_{\mathrm{dynam}}(x)\mathrm{d}x\,. \tag{12}$$

Considering the discrete case, if we assign the mean slope of the phase function to the constant component of the first derivative, we obtain

$$\phi(x_n) = \phi(x_0) + \mathrm{meanslope} \times (n\Delta x) + w(x_0, x_n)\,, \tag{13}$$

where $w(x_0, x_n)$ represent the dynamic content of the phase and Δx is the spatial resolution.

Consider now the discrete form of (10):

$$\phi(x_n) = \phi(x_0) + \sum_{n-1} \Delta_i^x\,, \tag{14}$$

where Δ_i^x is the principal value of the phase difference between neighbouring pixels:

$$\Delta_i^x = W[\phi(x_{i+1}) - \phi(x_i)]\,, \quad i = 1, \ldots, n-1\,, \tag{15}$$

W being the wrapping operator.

The mean value of the local phase differences for all the points is given by

$$E\{\Delta^x\} = \big(\sum_{n-1} \Delta_i^x\big)/(n-1) = \big(\sum_{\mathrm{n-1}} W[\phi(x_{i+1}) - \phi(x_i)]\big)/(n-1)\,. \tag{16}$$

At this point, adding and subtracting $E\{\Delta^x\}$ to and from each term in the summation (14) yields

$$\phi(x_n) = \phi(x_0) + \sum_{n-1} (\Delta_i^x - E\{\Delta^x\} + E\{\Delta^x\}) \tag{17}$$

and

$$\phi(x_n) = \phi(x_0) + n \times E\{\Delta^x\} + \sum_{n-1} (\Delta_i^x - E\{\Delta^x\}), \tag{18}$$

where the spatial resolution is supposed to be unitary.

Comparing (18) and (13) leads to the observation that the mean of the local phase differences can be used as an estimation of the mean slope. If in (18) the dynamic term is neglected, we may write

$$\phi(x_n) \cong \phi(x_0) + n \times E\{\Delta^x\}. \tag{19}$$

Obviously, the accuracy of this approximation depends both on the dynamic content of the phase and the length of the analysed sequence.

In the 2-D case, we can now estimate the mean slope both in azimuth and in range, using the previous observations:

$$(20a) \quad \text{meanslope}_{\text{azimuth}} = E\{\Delta^x\} = \sum_i \sum_j \Delta_{ij}^x / N(M-1) =$$

$$= \sum_i \sum_j W[\phi_{i+1j} - \phi_{ij}]/N(M-1), \qquad i=1,\dots,M-1, \ \ j=1,\dots,N,$$

$$(20b) \quad \text{meanslope}_{\text{range}} = E\{\Delta^y\} = \sum_i \sum_j \Delta_{ij}^y / M(N-1) =$$

$$= \sum_i \sum_j W[\phi_{ij+1} - \phi_{ij}]/M(N-1), \qquad i=1,\dots,M, \ \ j=1,\dots,N-1,$$

where M and N are the dimensions of the rectangle on which the phase function is to be estimated. If the amount of points affected by noise or aliasing is not great compared to the total number of pixels in the image, big squares are less sensitive to noise. On the contrary, the use of small elements seems to be a better solution for capturing the details of the phase function, but results in more important effects of errors in the evaluation of local slopes. A compromise between the two possibilities could be reached using an iterative algorithm that involves a partition of different size for each iteration.

Once the mean slopes in azimuth and range are known, the phase at the vertices of the model can be computed. This is done by minimising, for every set of vertices, the difference between the phase variation on the model and the variation imposed by the slopes computed in the previous point. The solution is obtained by solving a least-mean-squares problem. From the physical point of view, this can be translated in the necessity of obtaining a continuous topographic model. Starting from the values of the phase in the vertices, one can estimate the values of the model in all the points of the domain using bi-linear interpolation.

Fig. 1. – Result showing the capability of the method in limiting error propagation.
a) Rewrapped model obtained by applying a classic least-mean-squares algorithm to a synthetic fringe pattern with a noisy area. b) The same as in a) but exploiting the good properties of the proposed method. c), d) The correspondent residual images.

2˙2. *Implementation.* – In summary, the following are the various steps necessary to perform phase unwrapping with the discussed approach. Let DIM be the size (in pixels) of the squares during a certain iteration:

1) DIM = 64;

2) divide the interferogram into squares of DIM×DIM pixel in size;

3) compute, for each square, the mean of the difference in phase between neighbouring pixels, for both the azimuth and range direction (this determines the mean slope);

4) compute the value of the phase in the vertices of the grid solving a least mean squares problem;

5) compute the phase for all the points of the model using bilinear interpolation;

Fig. 2. – Result showing the behaviour of the discussed approach in the presence of aliasing in the interferogram. A "pyramidal" phase function with an horizontal cut was used. a) The original synthetic interferogram. b) The rewrapped phase model. Also in this case the algorithm is able to limit the influence of defects in the interferometrical phase.

6) compute the residue image;

7) DIM = DIM/2;

8) unwrapping of the residue image repeating the steps 2 through 7 until DIM = 2;

9) successive unwrapping with squares growing in size until DIM=64.

Let us notice the iterative character of the algorithm. At the end of each iteration the algorithm generates an updated model and a residue image: if fringes are still present in the residue image, this will be considered as a new interferogram to be unwrapped by the same algorithm. The reconstructed phase function results from the sum of all the partial models obtained with all iterations.

In order to evaluate the accuracy of the model obtained with the steps described above, visual inspection of the residue image could be sufficient because the presence of residue fringes implicates the necessity of further processing.

3. – Experimental results

In order to illustrate the usefulness of the method, both synthetic and real data sets have been used to test the method. Figure 1 shows the results obtained by applying both a least-square algorithm and the proposed method to a synthetic fringe pattern with a noisy area simulating the presence of low coherence in real data. In order to evaluate these results, we compared the original interferogram with the rewrapped phase models. The distortions clearly visible in the image obtained using the least-square approach are strongly reduced by the described algorithm. This means that the influence of noise is strictly limited to the neighbourhood of noisy points.

The fringe patterns in fig. 2 represents what has been obtained by using an interferogram derived from a synthetic "pyramidal" phase function with an horizontal cut on a

Fig. 3. – Sequence of images illustrating the process of development of the phase model. Rewrapped phase model (left), residue image (center) and a 3-D reconstruction of the unwrapped phase (right), respectively, after the first (a), the third (b) and the fifth (c) iteration of the described approach. In d) the original interferogram is shown. It is evident how each iteration introduces more details.

face, in order to simulate an aliasing case. Also in this experiment the algorithm shows a good capability in preserving correct phase values from being corrupted by local errors propagation.

The objective of fig. 3 is to explain how the algorithm comes to the final result. In this case a real dataset has been used. A grey-levels representation of the original wrapped phase is shown in fig. 3d. It has been obtained by applying the interferometrical process to a pair of complex images acquired by the ERS-1 and ERS-2 satellites, during the tandem campaign, over the Etna volcano in Sicily (Italy). The quality of the data is overall rather high: in particular the concentric fringes corresponding to the mountain are clearly visible and appear almost free of noise. Notwithstanding these good properties, several regions in the interferogram are characterised by low coherence and inconsistencies in the phase values. For instance, in the upper part of the image fringes are damaged by noise and, close to the Etna's peak, a layover area can be observed. Moreover, in the right bottom portion of the interferogram a low-coherence area is visible, which correspond to the sea. In such a situation it is fundamental to unwrap phases by means of an algorithm able to deal with the problem of error propagation. The obtained results confirm that our approach has the capability of limiting the reduction of reliability in the final phase model, due to the presence of inconsistencies in the interferometric data. In fig. 3 the rewrapped phase model (left), the residue image (centre) and a 3-D reconstruction of the unwrapped phase (right), are shown, respectively after the first (fig. 3a), the third (fig. 3b) and the fifth (fig. 3c) iteration of the described approach. The first iteration shows clearly the effect of the applied grid: the phase model is approximated by large portions of flat surfaces (fig. 3a, on the right). As the model gets refined, more details are introduced in the model. As a consequence, fringes in the rewrapped phase image become more and more complicated. By comparing the original interferogram (fig. 3d) and the rewrapped true phase after the fifth iteration (fig. 3c, on the left), it is possible to notice the high similarity of fringes in the noise-free regions. On the contrary the result is less reliable where inconsistencies are present but, in any case, errors are confined in the noisy areas. In the residue image, after each iteration, the number of residual fringes decreases and they appear wider. This means that the significant information left in the residue image is progressively reduced. Since after the fifth iteration some large fringes are still present (fig. 3c, centre), the algorithm should be iteratively applied to the residue image in order to extract the remaining information and improve the final phase model.

4. – Discussion and conclusions

In this paper a phase unwrapping method has been described which shows a good accuracy and a noticeable robustness to noise in reconstructing the true phase model starting from the interferometric wrapped values. The algorithm is based on the reconstruction of a global unwrapped phase function exploiting local information about the shape of the function itself. The approach is iterative in the sense that an improvement of the phase model can be obtained successively applying the algorithm to the residue image: in this way it is possible to capture the details of the scene. Another important characteristic of the algorithm is the possibility for the user to keep control over the processing during the iterations and to stop the algorithm when the residue image does not contain any fringes.

Summarising, the proposed algorithm allows the user to unwrap the interferogram in an intuitive and to the point manner. The method limits error propagation, and has the advantage that no processing parameters have to be defined. Moreover, the straightforward computation of the local phase guarantees reduction of computation time in respect to other method present in literature.

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