IL NUOVO CIMENTO Vol. 24 C, N. 1 Gennaio-Febbraio 2001

Estimation of directional sea wave spectra from radar images: A Mediterranean Sea case study(*)

G. Corsini, R. Grasso, G. Manara and A. Monorchio

Dipartimento di Ingegneria dell'Informazione - Università di Pisa - PISA, Italy

(ricevuto il 25 Novembre 1999; revisionato l' 11 Ottobre 2000; approvato il 17 Novembre 2000)

Summary. — An inversion technique for estimating sea wave directional spectra from Synthetic Aperture Radar (SAR) images is applied to a set of ERS-1 data relevant to selected Mediterranean areas. The approach followed is based on the analytical definition of the transform which maps the sea wave spectrum onto the corresponding SAR image spectrum. The solution of the inverse problem is determined through a numerical procedure which minimises a proper functional. A suitable iterative scheme is adopted, involving the use of the above transform. Although widely applied to the ocean case, the method has not been yet extensively tested in smaller scale basins, as for instance the Mediterranean sea. The results obtained demonstrate the effectiveness of the numerical procedure discussed for retrieving the sea wave spectrum from SAR images. This work provides new experimental data relevant to the Mediterranean Sea, discusses the results obtained by the above inversion technique and compares them with buoy derived sea truth measurements.

PACS 92.10.Hm – Surface waves, tides, and sea level.
PACS 07.07.Df – Sensors (chemical, optical, electrical, movement, gas, etc.); remote sensing.
PACS 84.40.Xb – Telemetry: remote control, remote sensing, radar.
PACS 07.05.Pj – Image processing algorithms.

1. – Introduction

The analysis and the interpretation of SAR images of the sea surface strongly relies on the comprehension of the interaction between electromagnetic waves and the sea waves. Although many interpretative models have been proposed in the literature [1, 2], a model exhaustively explaining the several and complex phenomena which characterise the formation of SAR images is not yet available. In this context, the estimation of the

(*) Paper presented at the Workshop on Synthetic Aperture Radar (SAR), Florence, 25-26 February, 1998.

sea spectrum from SAR images represents a very difficult task. One of the greatest difficulties arises from the time-dependent nature of the sea surface [3]. However, spectral analysis as applied to SAR images has demonstrated, under particular hypotheses, the possibility of evaluating some typical parameters of the sea surface, as for instance the two-dimensional wave power spectrum. Some authors have proposed functional relationships (SAR modulation transfer function) between the spectrum of the sea surface and the spectrum of SAR images. In particular, although in some specific conditions the SAR modulation transfer function can be approximated by a quasi-linear expression [4], in the most general case a non-linear transform is needed [5]. The relationship between the sea wave and SAR image spectrum must be inverted to provide an estimate of the sea wave directional spectrum from the corresponding SAR image. The inversion is accomplished by an iterative algorithm, which is based on the minimisation of a proper cost functional [5, 6]. This work is concerned with the application of the above inversion procedure to ERS-1 SAR PRI images of selected Mediterranean Sea areas. These areas have been chosen for the possibility of comparing the results obtained by the inversion algorithm with sea truth measurements provided by buoys.

2. – Models for SAR image spectrum of sea surface

The sea state can be statistically characterised by its wave spectrum [7], which describes the spectral power density as a function of the sea waves propagation vector $\vec{K}$ (two-dimensional vector which is defined on a horizontal reference plane, corresponding to the average plane of the sea surface).

The main problem consists in the definition of the relationship between the variance of the SAR image intensity and the variance of the wave height, with respect to the reference plane. In the most general case, this relationship can be expressed by the following form [5, 8]:

$$S_{\mathrm{sar}}(\vec{K}) = F_{\mathrm{sar}}\big[S_{\mathrm{h}}(\vec{K}), \vec{K}\big], \tag{1}$$

where $S_{\mathrm{h}}(\vec{K})$ and $S_{\mathrm{sar}}(\vec{K})$ denote the significant height variance spectrum of the ocean surface and the SAR image intensity variance spectrum respectively. In (1), F_{sar} is a mapping function, generally depending on the wave vector $\vec{K}$ and on the sea wave spectrum $S_{\mathrm{h}}(\vec{K})$.

An orthogonal reference frame (x, y) will be assumed in the following, where x denotes the sensor flight direction (azimuth), and y, perpendicular to the flight direction, is the SAR antenna bearing direction (range), projected onto the reference surface plane.

2·1. *Quasi-linear approximation.* – According to the theory in [4], the relationship in (1) can be reduced, in the framework of a quasi-linear approximation, to the following expression:

$$S_{\mathrm{sar}}(\vec{K}) = \exp\left[-\frac{K_x^2}{2K_\sigma^2}\right]\big|R_{\mathrm{sar}}(\vec{K})^2\big|S_{\mathrm{h}}(\vec{K}) = F_{\mathrm{sar}}^{\mathrm{ql}}(\vec{K})S_{\mathrm{h}}(\vec{K}). \tag{2}$$

The exponential term in (2), namely the *dynamic transfer function*, accounts for the attenuation introduced by the mapping function with respect to sea wave systems propagating along the azimuthal direction (x-axis). It can be also associated with the *smearing* effect; the latter is due to the fact that the elementary scatterers inside the radar resolution cell exhibit different and, sometimes, random radial velocities. Consequently, they are mapped onto the corresponding image with an undesired variable shift along the azimuthal co-ordinate, with a loss of azimuthal resolution [9]. Quantitatively, the attenuation introduced by the dynamic transfer function depends on the parameter K_σ, which can be expressed as [4]

$$K_\sigma = \frac{\pi}{2} \frac{V}{R\sqrt{H_\mathrm{s}}}. \tag{3}$$

In (3), H_s is the significant wave height of the sea surface as derived from the spectrum, V is the sensor velocity and R the distance between the antenna and the centre of the scene. The quantity $K_\sigma/(2\pi)$ is defined as the azimuthal cut-off frequency.

$R_\mathrm{sar}(\vec{K})$ in (2) is usually referred to as the *SAR modulation transfer function*. It is important to note that the long ocean waves can be imaged by a SAR as a consequence of a local (real or apparent) variation of the scene reflectivity. In particular, $R_\mathrm{sar}(\vec{K})$ analytically describes the process through which the variations of the surface height and the different speeds of the various surface portions, determine a modulation of the SAR image intensity. The resultant effect can be represented as the coherent sum of three terms, corresponding to the most important mechanisms, which contribute to the formation of the SAR image *i.e. tilt modulation, hydrodynamic modulation* and *velocity bunching*:

$$\left|R_\mathrm{sar}(\vec{K})\right|^2 = \left|R_\mathrm{t}(\vec{K}) + R_\mathrm{h}(\vec{K}) + R_\mathrm{vb}(\vec{K})\right|^2. \tag{4}$$

In (4), $R_\mathrm{t}(\vec{K})$ accounts for reflectivity changes due to the variation of the electromagnetic wave local incidence angle with respect to the normal to the macroscopic sea surface. $R_\mathrm{t}(\vec{K})$ is referred to as the *tilt modulation transfer function* [5].

The term $R_\mathrm{h}(\vec{K})$ in (4) is related to the *hydrodynamic modulation*: long waves interact in a non-linear way with short waves, determining a non-uniform distribution of the long wave roughness. In particular, short waves can ripple the surface much more either near the crests of long waves, or near the thoughts, depending on the phase displacement between short waves and long waves. This phase displacement gives rise to a different scattering coefficient on the two sides of the sea wave (up-wave and down-wave behaviour). The univocal definition of $R_\mathrm{h}(\vec{K})$ is yet a subject matter, and the only certain characteristic is its dependence on those waves propagating in the ground range direction. A simplified form of the so-called hydrodynamic modulation transfer function, $R_\mathrm{h}(\vec{K})$, has been proposed in [6].

The two afore-mentioned effects become predominant in SAR images, when sea waves propagate along the range direction. However, we note that the two effects are present in all active microwave sensors including real aperture radars; in particular, the coherent sum of these two effects is associated with the real aperture radar transfer function [10].

In order to explain how waves travelling in a direction that is parallel to the flight axis are imaged by a SAR system, we need to introduce the velocity bunching model. Velocity bunching theory accounts for the orbital velocity of the water particles (fig. 1).

This velocity is a periodical function with the same period of long waves height distribution; consequently, due also to the presence of the Doppler effect, a periodic gathering of the contributions occurs, corresponding to the same variation of intensity in the final SAR image.

Fig. 1. – Instantaneous orbital velocity of water particles: the point P moves along cycloidal trajectories.

This phenomenon is typical of SAR systems; it implies that an object, with a non-null radial velocity, is represented in the corresponding image with a shift along the azimuth, with respect to its actual position. This effect can be taken into account by the velocity bunching modulation transfer function $R_{\mathrm{vb}}(\vec{K})$ [5].

Equation (4) suggests a coherent summation of the three contributions due to the mechanisms described above. Since both tilt modulation and hydrodynamic modulation depend on those waves travelling in the range direction, the two mechanisms can strongly interact with each other. In particular, they can positively or negatively interfere, depending on whether they are radially moving toward or away from the platform. Moreover, since the tilting effect becomes predominant for small incidence angles, it is convenient to choose smaller angles of incidence on the scene in order to maximise this effect and, at the same time, to minimise the hydrodynamic effect, which is not yet fully understood. The problem of defining the limits of the quasi-linear approximation has been widely discussed in the literature. In this context, Alpers and Rufenach have proposed a parameter [3] that determines the non-linearity degree of the process originating the SAR image from the ocean surface:

$$c = 0.25\sqrt{g}\frac{R}{V}\sqrt{\sin^2\theta\sin^2\phi + \cos^2\theta}\,|\cos\phi|\sqrt{K^3}H_{\mathrm{s}}, \tag{5}$$

where the angle ϕ identifies the propagation direction of the sea wave measured from the azimuth direction. For $c \geq 0.3$ the SAR imaging process is non-linear.

The fundamental parameters affecting the total transfer function in the quasi-linear approximation $F^{\mathrm{ql}}_{\mathrm{sar}}(\vec{K})$, are

i) the ratio R/V, between the distance from the antenna to the centre of the scene and the platform velocity;

ii) the nominal incidence angle θ, with respect to the vertical direction;

iii) the polarisation of the incident electromagnetic wave;

iv) the significant wave height H_{s} of ocean waves.

As far as the dependence on H_{s} is concerned, it is important to note that in all SAR systems an increase of H_{s} determines a reduction of the azimuthal cut-off spatial frequency. Consequently, the global transfer function not only depends on the SAR system employed but also on the sea-state of the observed scene. It is also worth pointing out that the only radar parameter influencing the dynamic transfer function is the ratio R/V. As a consequence, no changes in the transfer function is introduced by a variation of the nominal incidence angle which, in SAR systems of practical use, assumes standard values between 20° and 30°. On the contrary, the polarisation state of the incoming electromagnetic wave becomes of importance.

2˙2. *Non-linear transform.* – The model developed in [5] allows us to evaluate the ocean wave spectrum from the SAR image spectrum, in more general situations than those for which the quasi-linear approximation applies. It should be noted that the phenomena and the effects discussed in the above paragraph are of fundamental importance in the definition of this model, based on the determination of a non-linear relationship between the SAR image spectrum and the ocean wave spectrum. The non-linear model contains, as a particular case, the quasi-linear approximation previously discussed above. The most general non-linear relationship can be expressed by the following equation [5]:

$$S_{\mathrm{sar}}(\vec{K}) = \exp\left[-\frac{K_x^2}{2K_\sigma^2}\right]\sum_{n=1}^{\infty}\sum_{m=2n-2}^{2n}\left(K_x\frac{R}{V}\right)^m S_{n,m}^{\mathrm{sar}}(\vec{K}), \tag{6}$$

where n denotes the order of non-linearity of the ocean spectrum, which is considered as the system input, while m represents the non-linearity order with respect to the R/V parameter, related to the velocity bunching phenomenon. The several terms of the summation can be determined by considering the definitions of the spatial autocorrelation of the surface velocities $f^v(\vec{r}) = \langle v(\vec{x}+\vec{r})v(\vec{x})\rangle$, the spatial autocorrelation of the scattering coefficient $f^R(\vec{r}) = \langle \sigma(\vec{x}+\vec{r})\sigma(\vec{x})\rangle$, and, finally, their mutual correlation $f^{Rv}(\vec{r}) = \langle \sigma(\vec{x}+\vec{r})v(\vec{x})\rangle$ [5].

If n is fixed, the expansion in (6) can be rewritten in terms of the non-linearity order m:

$$S_{\mathrm{sar}}(\vec{K}) = \exp\left[-\frac{K_x^2}{2K_\sigma^2}\right]\left[S_1^{\mathrm{sar}}(\vec{K}) + S_2^{\mathrm{sar}}(\vec{K}) + \cdots + S_n^{\mathrm{sar}}(\vec{K}) + \cdots\right]. \tag{7}$$

In particular, the first term of the summation in (7) coincides with the expression already shown for the quasi-linear approximation, being evident that the transform in (2) can be considered as a particular case of the non-linear transform in (7). However, it should also be noted that in this model, the phenomena and the effects discussed in the previous section remain of fundamental importance.

3. – Inversion algorithms

The previously obtained relationships between the ocean wave spectrum and the SAR image spectrum, allow us to define suitable inversion algorithms for estimating the sea wave spectrum. Under the simpler hypotheses in which a quasi-linear approximation can be used and after some preliminary operation typical of real data processing, as for instance *despeckle* and removal of the stationary response of the system, the inversion can be obtained by simply dividing the SAR imaging spectrum by the total transfer function $F_{\mathrm{sar}}^{\mathrm{ql}}(\vec{K})$.

Some difficulties may arise in this process, due to the fact that the transfer function also depends on the sea-state that is being considered, *i.e.* the significant wave height H_{s}. The problem can be overcome by creating an iterative inversion algorithm. Starting from a first guess of the significant wave height, the inversion can be performed by using the function $F_{\mathrm{sar}}^{\mathrm{ql}}(\vec{K})$ corresponding to the pertinent value of H_{s}. Successively, a check of the significant height associated with the spectrum obtained by this inversion is needed:

if the result differs from the assumed value, the function $F_{\mathrm{sar}}^{\mathrm{ql}}(\vec{K})$ must be updated. The process is terminated when the significant wave height value assumed for the function $F_{\mathrm{sar}}^{\mathrm{ql}}(\vec{K})$ coincides, but for a fixed error, with that calculated from the sea wave spectrum and obtained by the inversion scheme.

In the most general cases, a more complex model must be considered, for which the non-linear relation between the sea wave spectrum and the SAR image spectrum is assumed. In this framework, interesting results have been obtained by the inversion scheme suggested in [5]. The optimal estimate of the sea wave spectrum function is obtained by minimising a suitable cost functional [6]:

$$J = \int \left[S_{\mathrm{sar}}(\vec{K}) - \hat{S}_{\mathrm{sar}}(\vec{K})\right]^2 W_{\mathrm{sar}}(\vec{K})\,\mathrm{d}\vec{K} + \\ + \int \left[S_{\mathrm{h}}(\vec{K}) - \hat{S}_{\mathrm{h}}(\vec{K})\right]^2 W_{\mathrm{h}}(\vec{K})\,\mathrm{d}\vec{K} + W_\mu(\Delta\mu^2), \tag{8}$$

where

$$\Delta\mu = \int \left[\frac{1}{2} R_{\mathrm{vb}}(\vec{K})\delta S_{\mathrm{h}}(\vec{K}) + \frac{1}{2}\left|R_{\mathrm{vb}}(-\vec{K})\right|\delta S_{\mathrm{h}}(-\vec{K})\right] \mathrm{d}\vec{K}. \tag{9}$$

The minimisation can be obtained by an iterative method. In particular in (8), $\hat{S}_{\mathrm{h}}(\vec{K})$ represents a first-guess sea wave spectrum, which must be quite realistic and is usually derived by sea-truth measurements or by suitable hydrodynamic models. This spectrum is used to start the iterative process and the convergence of the process itself strongly depends on the similarity between the first-guess wave spectrum and the actual wave spectrum. Besides, $\hat{S}_{\mathrm{sar}}(\vec{K})$ is the observed SAR image spectrum, while $S_{\mathrm{sar}}(\vec{K})$ is the fitted SAR image spectrum evaluated at the generic iteration. The minimisation of the functional described above, *i.e.* the condition $\partial J/\partial S_{\mathrm{h}} = 0$, can be easily reached in the context of an iterative scheme that makes use of the quasi-linear approximation between the sea wave spectrum and the SAR image spectrum. Starting from the assumption $S_{\mathrm{h}}^1(\vec{K}) = \hat{S}_{\mathrm{h}}(\vec{K})$, for the generic step n of the iterative scheme, we evaluate the equivalent SAR image spectrum $S_{\mathrm{sar}}^n(\vec{K})$ from the best fit wave spectrum $S_{\mathrm{h}}^n(\vec{K})$ with a non-linear transform:

$$S_{\mathrm{sar}}^n(\vec{K}) = F_{\mathrm{sar}}^{\mathrm{nl}}\left[S_{\mathrm{h}}^n(\vec{K})\right]. \tag{10}$$

The solution at the step $(n+1)$ is obtained from that at the previous step n, by means of the following relationships:

$$S_{\mathrm{h}}^{n+1} = S_{\mathrm{h}}^n + \Delta S_{\mathrm{h}}^n, \tag{11}$$

$$S_{\mathrm{sar}}^{n+1} = S_{\mathrm{sar}}^n + \Delta S_{\mathrm{sar}}^n. \tag{12}$$

The variation $\Delta S_{\mathrm{sar}}^n$ used to correct the SAR image spectrum is calculated by the variation ΔS_{h}^n of the wave spectrum, by using the quasi-linear transform:

$$\Delta S_{\mathrm{sar}}^n = F_{\mathrm{sar}}^{\mathrm{ql}}\left(\Delta S_{\mathrm{h}}^n\right). \tag{13}$$

Fig. 2. – ERS-1 SAR PRI Data quick-looks. a) Orbit: 8970, Frame: 2781, Date: 3/04/93, image centre: 40°56′24″N, 13°20′24″E, Buoy coordinates: 40°52′N, 12°57′E. b) Orbit: 12792, Frame: 2781, Date: 26/12/93; image centre: 40°54′00″N, 12°29′24″E; Buoy coordinates: 40°52′N, 12°57′E. c) Orbit: 20007, Frame: 2781; Date: 13/05/95; image centre: 40°54′36″N, 8°18′36″E; Buoy coordinates: 40°32′N, 8°06′E. d) Orbit: 16579, Frame: 2727; Date: 16/09/94; image centre: 40°54′36″N, 8°18′36″E; Buoy coordinates: 43°55, 7′N, 9°49, 6′E.

Finally, the term $\Delta S_{\mathrm{h}}^{n}$ can be obtained by replacing expressions (11) and (12) into the cost function [5]. The choice of the weighting functions $W_{\mathrm{h}}(\vec{K})$ and $W_{\mathrm{sar}}(\vec{K})$ is usually made with the objective of optimising the convergence of the numerical procedure. The same observation is valid for the numerical parameter W_{μ}.

Fig. 3. – Thyrrenian Sea area close to the Isle of Ponza: a) ERS-1 SAR image spectrum; b) first-guess sea spectrum; c) best-fitted SAR image spectrum; d) estimated sea wave spectrum. (Orbit: 8970, Frame: 2781; Date: 3/04/93, Image centre: $40°56'24''$N, $13°20'24''$E).

4. – Numerical results and discussion

The algorithms previously described have been applied for estimating sea wave spectra from a set of ERS data relevant to specific areas of the Mediterranean Sea (fig. 2 shows the quick-looks of the original ERS-SAR images). Samples of the numerical results obtained are reported in this section. In order to estimate the effectiveness of the inversion algorithm, a suitable correlation coefficient between the best fitted SAR image spectrum and the ERS SAR spectrum has been introduced. It is defined as

$$\rho = \frac{\iint \left[S_{\mathrm{sar}}(\vec{K})\hat{S}_{\mathrm{sar}}(\vec{K})\right]^2 \mathrm{d}\vec{K}}{\iint \left[S_{\mathrm{sar}}(\vec{K})\right]^2 \mathrm{d}\vec{K} \iint \left[\hat{S}_{\mathrm{sar}}(\vec{K})\right]^2 \mathrm{d}\vec{K}}. \tag{14}$$

The first area under consideration contains the Isle of Ponza. In the same zone a buoy of the Italian National Hydrographic Service periodically collects sea spectrum data. The buoy coordinates are: 40°52′N, 12°57′E; it measures the height of the sea surface together with rolling and pitching with respect to the magnetic north with a sampling frequency of 1.28 Hz. In particular, fig. 3 shows the numerical results obtained by applying the iterative algorithm. The estimated ERS1-SAR image spectrum is reported in fig. 3a. The first-guess sea wave spectrum (fig. 3b) is obtained by modifying the omnidirectional Pierson sea wave spectrum by a spreading function of the following kind $\cos^n(\cdot)$, with $n = 8$. The wind friction velocity used in the Pierson model (40 cm/s) has been determined through heuristic considerations. The predicted SAR image spectrum at the final step of the iterative algorithm is shown in fig. 3c. The correlation coefficient between the images in fig. 2a and fig. 3c is equal to 0.6. The weighting functions in (8) has been defined as $W_{\mathrm{sar}}(\vec{K}) = \hat{S}_{\mathrm{sar}}(\vec{K})$ and $W_{\mathrm{h}}(\vec{K}) = 1$ over the whole domain of the wave numbers. The parameter W_μ is set equal to 0. Finally, the estimated sea wave spectrum is reported in fig. 3d. The significant wave height is equal to 4.18 m, in good agreement with the buoy derived estimation (3.9 m).

Results relevant to a second ERS1 image of the same area are shown in fig. 4. The image was recorded on December 1993. Almost at the same time, a buoy of the Italian National Hydrographic Service was acquiring data of the sea spectrum. The estimate of the ERS1 image power spectrum to be used in the inversion algorithm is depicted in fig. 4a, while the first guess of the sea wave spectrum in fig. 4b. The latter was obtained from the buoy data in the same way as the previous case relative to the Isle of Ponza area.

The optimisation procedure employs the cost function in (8), where

$$W_{\mathrm{sar}}(\vec{K}) = \gamma \exp\left[-\frac{k_x^2}{2\sigma_x^2} - \frac{k_y^2}{2\sigma_y^2}\right] \tag{15}$$

is the Gaussian weighting window proposed in [6] and $W_{\mathrm{h}}(\vec{K})$ is the weighting window proposed in [5]:

$$W_{\mathrm{h}}(\vec{K}) = \frac{\mu}{\left[b + \hat{S}_{\mathrm{h}}(\vec{K})\right]^2}. \tag{16}$$

In expression (16), μ is a coefficient accounting for the relative confidence between the observed SAR image spectrum and the first-guess wave spectrum; b is a small real positive constant, needed to avoid a division by zero in those points for which $\hat{S}_{\mathrm{h}}(\vec{K}) = 0$. The values assumed for the parameters appearing in the weighting functions are reported in table I, selected according to a convergence criterion.

The predicted SAR image spectrum at the final step of the inversion algorithm is shown in fig. 4c. The correlation coefficient between the spectra in fig. 4a and fig. 4c is equal to 0.83. The final significant wave height is equal to 3.5 m which is again in good agreement with the buoy derived estimation (3.1 m).

Fig. 4. – Thyrrenian Sea area close to the Isle of Ponza: a) ERS-1 SAR image spectrum; b) first-guess sea spectrum; c) best-fitted SAR image spectrum; d) estimated sea wave spectrum. (Orbit: 12792, Frame: 2781; Date: 26/12/93, Image centre: $40°54'00''$N, $12°29'24''$E).

The second testing zone is a Sardinian Sea area in front of Alghero. Again, a buoy of the Italian National Hydrographic Service is present in the scene, with coordinates $40°32'$N, $8°06'$E. In particular, fig. 5a shows the ERS-1 SAR image spectrum. Even this case, the first-guess sea wave spectrum has been obtained from the omnidirectional spectrum measured by the buoy, using the same spreading function as in the previous ex-

TABLE I. – *Parameters assumed in the weighting functions for the case in fig. 4.*

γ	σ_x	σ_y	μ	b	W_μ
10	0.4	0.05	1	0.1	1

Fig. 5. – Sardinian Sea area: a) ERS-1 SAR image spectrum; b) first-guess sea spectrum; c) best fitted SAR image spectrum; d) estimated sea wave spectrum. (Orbit: 20007, Frame: 2781; Date: 13/5/95, Image centre: $40°54'36''$N, $8°18'36''$E).

ample (fig. 5b). The inversion algorithm was carried out by using the weighting function in (15) and (16) with the same parameters reported in table I.

The correlation coefficient between the ERS SAR image spectrum and the best fitted one at the final step of the algorithm (fig. 5c) is equal to 0.86. The image of the estimated sea wave spectrum (fig. 5d) shows a sharp peak in a direction which is rotated of about 127° with respect the azimuth direction. It is characterised by a wavelength of about 160 m. The significant wave height obtained by integrating this spectrum is equal to 6.4 m, a value which is consistent with that measured by the buoy (6.0 m).

A third testing area is localised in the Northern Tyrrenian Sea. The SAR image relative to the selected region was recorded on September 16, 1994 (orbit 16579, frame

Fig. 6. – Numerical results relevant to Northern Tyrrhenian Sea. a) Observed SAR image spectrum concerning the examined area—min = 0.18, max = 1.14. b) Sea wave spectrum deduced by the buoy measurement and used as first guess spectrum —min = 0, max = 1490 $[\mathrm{m}^2/(\mathrm{rad/m})^2]$. c) SAR image spectrum calculated by applying the non-linear transformation to the sea spectrum in d)—min = 0, max = 2.2. d) Sea spectrum computed by the Hasselmann and Hasselmann method—min = 0, max = 2350.

2727); a portion of $3\,\mathrm{km} \times 3\,\mathrm{km}$ including a directional buoy of the Italian National Hydrographic Service was extracted from the original image which corresponds to an area of $100\,\mathrm{km} \times 100\,\mathrm{km}$. The buoy is placed at Lat $43°55.\ 7'$N e Long $9°49.\ 6'$E.

From the buoy data, recorded about 10 minutes before the satellite acquisition, a travelling wave system with direction of propagation of 57° North and spectral peak equivalent to a 128 m wave length, has been estimated. The spectral measurement has been converted into two-dimensional data, by multiplying the monodimensional data for a spreading function depending on $\cos^6(\phi - \bar{\phi})$, where $\bar{\phi}$ is the average wind direction.

Fig. 7. – Numerical results relevant to Northern Tyrrhenian Sea. a) Observed SAR image spectrum concerning the examined area—min = 0.18, max = 1.14. b) First-guess sea wave spectrum—min = 0, max = 1490 $[\mathrm{m}^2/(\mathrm{rad/m})^2]$. c) SAR image spectrum calculated by applying the non-linear transform to the sea spectrum in d)—min = 0, max = 2.2. d) Sea spectrum computed by the Hasselmann and Hasselmann method—min = 0, max = 2350.

The spectrum calculated by the above procedure is represented in fig. 6b; it has been used as a first guess for the iterative algorithm. By applying the Hasselmann algorithm to the observed SAR image spectrum (represented in fig. 6a), an estimation of the wave spectrum has been evaluated (fig. 6d), confirming the sea-truth measurements. In particular, the calculation of the significant height H_s, derived by the sea wave spectrum, gives a value of 2.9 m, very close to the value obtained by the buoy data, which is 3.0 m. Finally, the SAR image spectrum, calculated by applying the non-linear transform to the estimated wave spectrum in fig. 6d, is shown in fig. 6c. The similarity with the observed SAR image spectrum further confirms the quality of the estimation performed.

A different area of the same SAR image has been considered next. In this case no buoy measurements were available; however the algorithm has shown a good degree of convergence, and the results are shown in fig. 7. In particular, the first-guess sea wave spectrum is plotted in fig. 7b, while fig. 7d represents the estimated sea spectrum obtained from the SAR image spectrum in fig. 7a. The SAR image spectrum computed from the estimated sea spectrum of fig. 7d, is shown in fig. 7c. In this case, the significant wave height derived from the estimated sea spectrum is equal to 2.38 m.

For the two selected image portions, the weighting functions in (8) have been defined as $W_{\mathrm{sar}}(\vec{K}) = \hat{S}_{\mathrm{sar}}(\vec{K})$ and $W_{\mathrm{h}}(\vec{K}) = \mu/[b + \hat{S}_{\mathrm{h}}(\vec{K})]^2$ with μ and b as in table I. The parameter W_μ is set to zero. In both cases the correlation coefficient between the SAR image spectrum estimated from the satellite data and the best fitted SAR spectrum is greater than 0.7 confirming the good performance reached by the inversion algorithm.

5. – Concluding remarks

The inversion procedure for retrieving the sea wave power spectrum from SAR images proposed in [5, 6] has been applied to selected areas of the Mediterranean Sea. The method has been tested on a set of SAR PRI images, recorded by the European satellite ERS-1. The iterative inversion algorithm has been checked for different initialisation procedures and the results obtained have been compared in terms of the correlation coefficient between the ERS SAR image spectrum and the SAR spectrum predicted by the algorithm at its final step. High values of this correlation coefficient can be achieved by estimating the first guess sea wave spectrum from buoy derived measurements. A good agreement is obtained between the estimated value of the significant wave height and that measured by the buoy system, confirming the applicability of the method to small basins.

* * *

The results presented in this paper have been obtained in the framework of the ESA experiment ERS-1/2 n. A02.I111.

REFERENCES

[1] Ulaby F. T., Moore R. K. and Fung A. K., in *Microwave Remote Sensing* (Artech House Inc., Norwood) 1986.
[2] Kasilingam D. P. and Shemdin O. H., *J. Geophys. Res.*, **95** (1990) 16263.
[3] Alpers W. R. and Rufenach C. L., *IEEE Trans. Antennas Propagat.*, **27** (1979) 685.
[4] Monaldo F. M. and Lyzenga D. R., *IEEE Trans. Geosci. Remote Sensing*, **24** (1986) 543.
[5] Hasselmann K. and Hasselmann S., *J. Geophys. Res.*, **96** (1991) 10713.
[6] Krogstad H. E., *IEEE Trans. Geosci. Remote Sensing*, **32** (1994) 340.
[7] Apel J. R., in *Principles of Ocean Physics, Intern. Geophys. Ser.*, Vol. **38** (Academic Press, London) 1987.
[8] Krogstad H. E., *J. Geophys. Res.*, **97** (1992) 2421.
[9] Raney R. K., *IEEE Trans. Aerospace Electron. Systems*, **7** (1971) 499.
[10] Corsini G., Manara G. and Monorchio, A., *Radio Sci.*, **34** (1999) 1065.