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Modified BMIA/CAG method for the electromagnetic analysis of large-scale problems of random rough-surface scattering(*)

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Summary. — An efficient technique based on two-dimensional Fast Fourier Transform (FFT) and linear interpolation is presented for the evaluation of the scattering by a rough terrain surface which is of interest in remote-sensing applications characterized by a very large correlation length. Such technique, where introduced in a BMIA/CAG method, can reduce the computation time appreciably.

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1. – Introduction

Over the past few years, theoretical and empirical models have been developed in an attempt to predict the scattering of rough soil surfaces modeled as a homogeneous half-space medium with a rough interface and an effective dielectric constant. These models have very low computational cost but can be considered sufficiently accurate only in a limited region. Recently, enhanced computational capabilities of digital computers have increased the interest in Monte-Carlo simulations of random rough-surface scattering. In this kind of study the scattered field intensity is averaged over one hundred of typical realizations of the assumed scenary built in conformity with its statistical behavior. Obviously, it is very important to minimize the computation time for the analysis of each realization and several techniques have been developed for the fast computation of deterministic profile scattering. These techniques are based on the modification of the classical method of moments (MoM) to allow a fast evaluation of the reaction integral and, when an iterative solver is used, a fast matrix-vector multiplication. Examples include the impedance matrix localization (IML) technique [1], the complex multipole

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Fig. 1. – Geometry of the problem.

beam approach (CMBA) [2], the fast multipole (FMM) [3], the matrix decomposition algorithm (MDA) [4], the adaptive integral method (AIM) and the banded matrix iterative approach/canonical grid method (BMIA/CAG) [5, 6].

Specifically, the BMIA/CAG method is one of the most efficient when applied to the studying of rough profiles. The majors feature of BMIA/CAG is that it separates the near-field interaction from the far-field one. The latter one is efficiently evaluated through a Taylor series expansion. On the contrary, the near-field interaction requires the evaluation of several reaction integrals and their storage in a banded matrix form. Even if the above-mentioned method is fast and reliable to analyze very large one-dimensional terrain profile, some problems can arise in evaluating and storing the banded matrix when applied to two-dimensional surfaces.

In this paper, we propose an efficient technique to evaluate the near-field interaction, which employs a Fast Fourier Transform algorithm and linear interpolation. When introduced into a BMIA/CAG method, it can reduce the computation time appreciably. Furthermore, because the proposed technique allows a very fast evaluation of the elements of the near-field interaction banded matrix, we can avoid the storage of these elements and evaluate them every time the conjugate gradient solver needs. This allows to drastically reduce the request of dynamic memory and to analyze rough surfaces with very large correlation length.

The accuracy of the proposed method is explored for a one-dimensional profile by comparing the results with that produced by a plain BMIA/CAG technique and a classical Method of Moments formulation.

2. – Formulation

Figure 1 shows an example of a two-dimensional rough surface S illuminated by a tapered plane wave $\vec{E}^{\mathrm{i}}$. The surface is considered as a perfect electric conductor with a random height profile $z = f(x, y)$. The incident field induces surface currents $\vec{J}$ on the rough surface that produce an electric scattered field $\vec{E}^{\mathrm{s}}$. The latter can be computed

from the surface currents by $\vec{E}^{\mathrm{s}} = -j\omega\vec{A} - \nabla\phi$, with the magnetic vector potential $\vec{A}$ and the scalar potential ϕ defined as

$$\left\{ \begin{array}{c} \vec{A}(\vec{r}) \\ \phi(\vec{r}) \end{array} \right\} = \frac{j}{4\pi} \int\int_S \left\{ \begin{array}{c} -j\mu\vec{J}(x',y') \\ \nabla\cdot\vec{J}(x',y')/\omega\varepsilon \end{array} \right\} G(\vec{r},\vec{r}')\,\mathrm{d}S'\,, \tag{1}$$

where $G(\vec{r},\vec{r}') = \exp[-jkR]/R$ and $R = |\vec{r}-\vec{r}'|$. When the incident wave is tapered, the illuminated rough surface can be confined to a surface $L_x \times L_y$, and we can derive an integrodifferential equation for $\vec{J}$ by enforcing the boundary condition $\widehat{n}\times\left(\vec{E}^{\mathrm{i}} + \vec{E}^{\mathrm{s}}\right) = 0$ on S, obtaining

$$\widehat{n}\times\vec{E}^{\mathrm{i}}\Big|_S = \widehat{n}\times\left(j\omega\vec{A} + \nabla\phi\right)\Big|_S\,. \tag{2}$$

Equations (1) and (2) represent the so-called electric-field integral equation (EFIE). A method of moments is applied to obtain a linear system of equations for the surface current that makes use of the set of triangular basis functions introduced by Glisson [7]. The approach combines the advantages of triangular patch modeling and the EFIE formulation, and gives origin to an algorithm which is simple and efficient.

Following the procedure described in [7], the rough surface is modeled by triangular patches. The surface current density on the surface is then approximated as

$$\vec{J}(\vec{r}) = \sum_{n=1}^{N} I_n \vec{j}_n(\vec{r})\,, \tag{3}$$

where N is the number of interior edges and $\vec{j}(\vec{r})$ is the vector basis function defined on the adjacent triangles associated with the n-th edge, and it is given as

$$\vec{j}_n(\vec{r}) = \begin{cases} \vec{j}_n^{+} = \dfrac{l_n}{2A_n^{+}}\vec{\rho}_n^{+}\,, & \vec{r}\in T_n^{+}\,, \\ \vec{j}_n^{-} = -\dfrac{l_n}{2A_n^{-}}\vec{\rho}_n^{-}\,, & \vec{r}\in T_n^{-}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{4}$$

in which l_n is the length of the edge, $A_n^{\pm}$ is the area of the triangle $T_n^{\pm}$, and $\vec{\rho}_n^{\pm}$ is the position vector, as shown in fig. 2.

The surface divergence of $\vec{j}_n$ is found as

$$\nabla\cdot\vec{j}_n(\vec{r}) = \begin{cases} +\dfrac{l_n}{A_n^{+}}\,, & \vec{r}\in T_n^{+}\,, \\ -\dfrac{l_n}{A_n^{-}}\,, & \vec{r}\in T_n^{-}\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{5}$$

When the expansions for $\vec{J}(\vec{r})$ and $\nabla\cdot\vec{J}(\vec{r})$ are used in (1) and a Galerkin weighted residual method is employed a $N\times N$ system of linear equations is obtained, which may

Fig. 2. – Triangle pair and geometrical parameters associated with interior edge.

be written in matrix form as

$$\mathbb{Z}\,\mathbf{I} = \mathbf{V}\,. \tag{6}$$

Even if the analysis is restricted to a finite region, the latter is usually very large in terms of wavelength, and a direct application of the method of moments (MoM) is not possible because of large computational times and storage requirements. To overcome this limitation for each observation point $\vec{r}$ on the rough surface we can define a distance $r_d = |\vec{r} - \vec{r}'|$ that separate two regions: a near-interaction region and a weak-interaction region [5]. Under these hypothesi we can split the integral in eq. (1) as

$$\begin{aligned}\left\{ \begin{array}{c} \vec{A}(\vec{r}) \\ \phi(\vec{r}) \end{array} \right\} = \frac{j}{4\pi} \int\int_S \left\{ \begin{array}{c} -j\mu\vec{J}(x',y') \\ \nabla\cdot\vec{J}(x',y')/\omega\varepsilon \end{array} \right\} G(\vec{r},\vec{r}')\, U(r_d - d_{xy})\, \mathrm{d}S' + \\ + \frac{j}{4\pi} \int\int_S \left\{ \begin{array}{c} -j\mu\vec{J}(x',y') \\ \nabla\cdot\vec{J}(x',y')/\omega\varepsilon \end{array} \right\} G(\vec{r},\vec{r}')\, U(d_{xy} - r_d)\, \mathrm{d}S'\,,\end{aligned} \tag{7}$$

where $U(x)$ is the Heaviside step function and $d_{xy} = \sqrt{(x-x')^2 + (y-y')^2}$. In the last equation the first integral represents the strong interaction, while the second one represents the weak interaction. Accordingly, the impedance matrix $\mathbb{Z}$ in eq. (6) can be considered as the sum of a strong-interaction matrix $\mathbb{Z}^{\mathrm{s}}$ and a weak-interaction matrix $\mathbb{Z}^{\mathrm{w}}$ (*i.e.* $\mathbb{Z} = \mathbb{Z}^{\mathrm{s}} + \mathbb{Z}^{\mathrm{w}}$).

Since in the weak-interaction region $h = |f(x,y) - f(x',y')| \ll d_{xy}$ we can approximate the square root appearing in the Green's function by using $\widetilde{M}$ terms of the relevant Taylor series with respect to the height, resulting

$$G(d_{xy}, h) \simeq \sum_{m=0}^{\widetilde{M}} a_m(d_{xy}) \exp[-jkd_{xy}]/d_{xy}\, h^{2m}\,, \tag{8}$$

where $a_0 = 1$, $a_1 = (1 + jkd_{xy})/2$, $a_2 = (3 + 3jkd_{xy} - k^2 d_{xy}^2)/8$, ...

Furthermore, since the surface height is known, we can project the integration domain on the plane x-y, thus obtaining

$$\mathrm{d}S' = \sqrt{1 + \left[\frac{\partial f(x',y')}{\partial x'}\right]^2 + \left[\frac{\partial f(x',y')}{\partial y'}\right]^2}\, \mathrm{d}x'\,\mathrm{d}y' = P(x',y')\,\mathrm{d}x'\,\mathrm{d}y'\,. \tag{9}$$

Fig. 3. – Centroids of triangular elements and FFT sampling joints.

As a consequence, the weak term of eq. (7) can be written as

$$\sum_{m=0}^{\widetilde{M}} \frac{j}{4\pi} \int\int_S \left\{ \begin{array}{c} -j\mu\vec{J}(x',y') \\ \nabla\cdot\vec{J}(x',y')/\omega\varepsilon \end{array} \right\} a_m(d_{xy}) \exp[-jkd_{xy}]/d_{xy} \cdot \\ \cdot [f(x,y) - f(x',y')]^{2m} \; P(x',y')\,\mathrm{d}x'\,\mathrm{d}y' \,. \tag{10}$$

By analyzing the integrals in eq. (10) we can easily recognize that they take a two-dimensional convolution form of the type

$$\sum_{m=0}^{M} R_m(x,y) \int\int_S \left\{ \begin{array}{c} -j\mu\vec{J}(x',y') \\ \nabla\cdot\vec{J}(x',y')/\omega\varepsilon \end{array} \right\} A_m(x-x',y-y')\, T_m(x',y')\,\mathrm{d}x'\,\mathrm{d}y' \,, \tag{11}$$

where $M = (\widetilde{M}+1)^2 - 1$, and they can efficiently be evaluated by means of a two-dimensional FFT. The only limitation is due to the fact that the observation points represent an equispaced rectangular grid. Since we have chosen a triangular patch model of the rough surface, it is then necessary to oversample the FFT. For example, we can use a domain subdivision as sketched in fig. 3, where the N_c centroids of the triangular elements are pointed out by a dot, while the additional sampling points are shown as crosses. Even if the total number of sampling points is now $N_\mathrm{t} = 4.5\,N_\mathrm{c}$, the efficiency of the method is not compromised and the little added computational effort is well repaid by the accuracy in modeling the rough surface and by the efficiency in the code implementation.

For the generic i-th triangular element of centroid coordinates (x_i, y_i), we can write each term of the sum appearing in eq. (11) in a matrix form as

$$\mathbb{V}_\phi^m = \mathbb{R}^{(m)}\mathbb{A}^{(m)}\mathbb{T}^{(m)}\mathbb{J}_\phi \,, \tag{12}$$

$$\mathbb{V}_A^m = \mathbb{R}^{(m)}\mathbb{A}^{(m)}\mathbb{T}^{(m)}\mathbb{J}_A \,, \tag{13}$$

Fig. 4. – N_p parallel planes that intercept the terrain profile.

where $\mathbb{A}^{(m)} = \{A_m(x_i - x_j, y_i - y_j)\}$, $\mathbb{J}_A = \left\{-j\mu\vec{J}(x_j, y_j)\right\}$, $\mathbb{J}_\phi = \left\{\nabla \cdot \vec{J}(x_j, y_j)/\omega\varepsilon\right\}$, $\mathbb{R}^{(m)} = \{R_m(x_j, y_j)\,\delta_{i,j}\}$, $\mathbb{T}^{(m)} = \{T_m(x_j, y_j)\,\delta_{i,j}\}$, with $\delta_{i,j}$ the Kronecker delta-function, and $\mathbb{V}_A^{(m)}$ and $\mathbb{V}_\phi^{(m)}$ represent the contribution to the vector and scalar potential, respectively. By expanding the density current as in eq. (6) and by using the relationships (4) and (5), we can easily join $\mathbb{J}_A$, $\mathbb{J}_\phi$ to the global vector unknown $\mathbf{I}$ through matrices $\mathbb{H}_i$, with $i = x, y, z$, and $\mathbb{H}_\phi$, respectively. By applying the standard Galerkin weighted residual method to eq. (2) [7], we obtain for each weighting functions n

$$\sum_{m=0}^{M}\left\{\left[\mathbb{V}_A^{(m)}\right]_{i=c_n^+}\cdot\frac{\vec{\rho}_{c_n^+}}{2}+\left[\mathbb{V}_A^{(m)}\right]_{i=c_n^-}\cdot\frac{\vec{\rho}_{c_n^-}}{2}\right\}+ \tag{14}$$

$$+\sum_{m=0}^{M}\left\{\left[\mathbb{V}_\phi^{(m)}\right]_{i=c_n^+}-\left[\mathbb{V}_\phi^{(m)}\right]_{i=c_n^-}\right\}=\vec{E}^i_{\tan}(\vec{r}_{c_n^+})\cdot\frac{\vec{\rho}_{c_n^+}}{2}+\vec{E}^i_{\tan}(\vec{r}_{c_n^-})\cdot\frac{\vec{\rho}_{c_n^-}}{2}\,.$$

By introducing matrices $\mathbb{Q}_i$, with $i = x, y, z$, and $\mathbb{Q}_\phi$ we obtain

$$\mathbb{Z}^w\,\mathbf{I}=\sum_{m=0}^{M}\left\{\sum_{i=x,y,z}\mathbb{Q}_i\mathbb{R}^{(m)}\mathbb{A}^{(m)}\mathbb{T}^{(m)}\mathbb{H}_i\,\mathbf{I}+\mathbb{Q}_\phi\mathbb{R}^{(m)}\mathbb{A}^{(m)}\mathbb{T}^{(m)}\mathbb{H}_\phi\,\mathbf{I}\right\}\,. \tag{15}$$

It is worth noting that matrix $\mathbb{A}^{(m)}$ in the last equation is a block Toeplitz matrix, while matrices $\mathbb{R}^{(m)}$ and $\mathbb{T}^{(m)}$ are diagonal and matrices $\mathbb{H}_i$ and $\mathbb{H}_\phi$ are extremely sparse (three valued elements per row), such as $\mathbb{Q}_i$ and $\mathbb{Q}_\phi$ (two valued elements per row). The $\mathbb{Z}^w$ matrix is a full matrix with $\mathbf{O}(N^2)$ elements and usually cannot be stored for large-scale problems. On the contrary, matrix $\mathbb{A}^{(m)}$ requires the storage of $\mathbf{O}\left((\tilde{M}+1)N_t\right)$ elements, while the other matrices needed the storage of $\mathbf{O}\left(8N_\mathrm{t}+20N_\mathrm{c}\right)$ elements.

Furthermore, when the conjugate gradient (CG) method is used to solve the matrix equation (6), the $\mathbb{Z}^w\mathbf{I}$ product can be conveniently evaluated by performing, for each term m, first the product $\mathbb{T}^{(m)}\mathbb{H}_i\,\mathbf{I}$ (pre-multiplication), where $i = x, y, z, \phi$, then the product $\mathbb{A}^{(m)}$ by $\left(\mathbb{T}^{(m)}\mathbb{H}_i\,\mathbf{I}\right)$ by means of a 2D FFT, since $\mathbb{A}^{(m)}$ is a block Toeplitz matrix. Finally, the product $\mathbb{Q}_i\mathbb{R}^{(m)}\left(\mathbb{A}^{(m)}\mathbb{T}^{(m)}\mathbb{H}_i\,\mathbf{I}\right)$ is performed (post-multiplication). The latter scheme allows to evaluate the $\mathbb{Z}^w\mathbf{I}$ product by $\mathbf{O}\left(4MN_\mathrm{t}\left[\log(N_\mathrm{t})+3\right]\right)$ operations instead of $\mathbf{O}\left(N^2\right)$. For instance, to model a 50×50 square wavelength surface with rms

Fig. 5. – Relative error of the magnetic field on the conducting surface with respect to the MoM solution.

height $h = 0.5\lambda$ by using $\widetilde{M} = 2$ and 2^{18} unknowns, the proposed scheme gets only 2% of the computational time required by the standard $\mathbb{Z}^w\mathbf{I}$ product.

Concerning the strong-interaction matrix $\mathbb{Z}^s$ this is sparse and, with an appropriate numbering of the triangular patches, it can be put in a banded form. If b is the band of the $\mathbb{Z}^s$ matrix the memory requirement to store it is $\mathbf{O}(bN)$ and the LU decomposition requires $\mathbf{O}(b^2N/2)$, while the backsubstitution requires $\mathbf{O}(2bN)$. However, if the rough terrain is discretized with small triangular patches, we can suppose that the z-variation of the profile is negligible in each trial function. Therefore, the term $f(x_i, y_i) - f(x_j + x', y_j + y') \simeq f(x_i, y_i) - f(x_j, y_j) = \Delta z_{ij}$ can be considered as a constant: thus the first integral in eq. (7) takes a convolution form. This means that we can efficiently evaluate it using a 2D FFT [8]. However, when making the 2D FFT we also get all the interaction values of all pairs of trial and weighting functions whose distance along the z-direction is Δz_{ij}. This suggests the possibility of introducing a set of N_p parallel planes that intercept the terrain profile as sketched in fig. 4. Notice that a gap $\Delta z \ll \lambda$ is chosen (usually $\lambda/20 \leq \Delta z \leq \lambda/10$). Then, we can store the samples of N_p 2D FFTs evaluated with $\Delta z_{ij} = n\Delta z$, where $n = 0, 1, 2, \ldots, N_\mathrm{p} - 1$ as the columns of a matrix $\mathbb{S}$. In this way the generic element of the strong-interaction matrix $\mathbb{Z}^\mathrm{s}$ can be estimated through an interpolation applied to a pair of appropriate elements of the $\mathbb{S}$-matrix. In particular, the interpolation requires only two multiplications for each interaction, instead of one integral evaluation, as in a classic MoM method.

This means that, after the $\mathbb{S}$-matrix is evaluated, the proposed technique allows a much faster process of evaluation of the banded matrix elements. Furthermore, it is worth noting that in a Monte-Carlo method we need to evaluate the matrix $\mathbb{S}$ once for all the terrain profiles simulated (usually more than one hundred), at the beginning of the simulation. This means that the time necessary to evaluate the required N_p 2D FFTs is negligible compared to the full process time.

Furthermore, the dynamic memory required to store the $\mathbb{S}$-matrix is also negligible with respect to that needed to store the banded matrix $\mathbb{Z}^\mathrm{s}$ of the BMIA/CAG method. As matter of fact, the number of rows in the matrix $\mathbb{S}$ is equal to the band b of $\mathbb{Z}^\mathrm{s}$, and the number N_p of parallel planes (*i.e.* the number of FFTs) needed to intercept the terrain profiles is usually much smaller than the number of rows N of the $\mathbb{Z}^\mathrm{s}$ matrix.

Fig. 6. – Real and imaginary part of the magnetic field on the conducting surface normalized to the incident magnetic-field amplitude.

3. – An application to one-dimensional profiles

To evaluate the reliability of the proposed method, the scattering from a deterministic profile has been analyzed. In particular, we have considered the case of a TE polarized plane wave impinging orthogonally on a perfectly electric conductor. This is assumed as uniform along the y-direction and sinusoidally modulated along the x-direction with amplitude 0.25λ and period 1.9λ.

Figure 6 shows the real and imaginary parts of the normalized magnetic field on the conducting surface. In particular, the BMIA/CAG solution with $r_d = 2\lambda$ (dashed line) and the proposed modified BMIA/CAG solution with $\Delta_z = \lambda/20$ (dash-dotted line) are compared with a classic MoM solution (solid-line) with 0.1λ rectangular trial functions, which can be regarded as exact. Notice that for this case, the Physical Optics approximation (dotted line) is not valid [9].

To highlight the differences between the two methods, fig. 5a shows the relative error due to the BMIA/CAG (solid line) and the modified BMIA/CAG (dashed line) approximations compared to the MoM solution. No significant differences are evident between the obtained results, but the proposed formulation is about eleven times faster than the plain BMIA/CAG.

Furthermore, to emphasize the errors introduced by the proposed technique alone, we have evaluated all the elements of the impedance matrix without using the BMIA/CAG approximation. Figure 5b shows the relative error concerning the MoM solution, when $\Delta_z = \lambda/20$ or $\Delta_z = \lambda/10$ linear interpolation is used (solid line). It can be noted that the error is smaller than that introduced by the BMIA/CAG approximation (see fig. 5a).

Figure 5b also reports the relative error when a quadratic interpolation, instead of a linear, is used (dashed line). Notice that the error is not strongly reduced and the evaluation time increased (four products were needed instead of one), so the linear interpolation seems the best approach.

Figure 7 shows the normalized bistatic scattering coefficient, averaged over 100 realizations, for a random rough profile illuminated orthogonally by a plane wave (TE case). The surface length is set $L = 50\lambda$, the correlation length $l = 2\lambda$ and the rms height $h = 0.5\lambda$. In fig. 7 we report the BMIA/CAG solution with $r_d = 3\lambda$ (dotted line)

Fig. 7. – Normalized bistatic scattering coefficient of a 1D random rough surface for a surface length of 50λ, a correlation length of 2λ and a rms height of 0.5λ: (solid line) modified BMIA/CAG, (dotted line)BMIA/CAG.

compared with the proposed modified BMIA/CAG solution with $\Delta_z = \lambda/20$ (solid line). A very good agreement from the two solutions is evident but we have recorded $28.43\,\Delta t$ by using the BMIA/CAG technique and only $2.58\,\Delta t$ employing the proposed method, where $\Delta t = 60$ s on a Pentium 200 MHz.

4. – Conclusion

An efficient technique that allows a very fast evaluation of the elements of the near-field interaction banded matrix of the BMIA/CAG method has been described.

The approximation introduced in the discretization of the terrain surfaces produces an error in the valuation of the field that, when a $\Delta_z = \lambda/20$ gap is chosen, is about a fifth of that introduced by the BMIA/CAG method.

When used in a Monte-Carlo simulation of the electromagnetic scattering from a random rough surface, the proposed technique produces results which appear indistinguishable from those obtained with the plain BMIA/CAG method, but the computing is time reduced of a factor ten.

REFERENCES

[1] Canning F. X., *A New Combination Field Integral Equation for Impedance Matrix Localization (IML), IEEE AP-S Symposium*, **3** (1993) 1140-1143.

[2] Boag A., Michielssen E. and Mittra R., "Hybrid multipole beam approach to electromagnetic scattering problems," *J. Appl. Comput. Electromagn. Soc.*, **9** (1994) 7-17.

[3] Coifman R., Rohklin V. and Wandzura S., "The fast multipole method for the wave equation: a pedestrian prescription", *IEEE Antennas Propagat.*, **AP-35** (1993) 7-12.

[4] Michielssen E. and Boag A., "Multilevel evaluation of electromagnetic field for rapid solution of scattering problems", *Microwave Opt. Technol. Lett.*, **7** (1994) 790-795.

[5] Tsang L., Chan C. H., Pak K. and Sangani H., "Monte-Carlo simulations of large-scale problems of random rough surface scattering and application to grazing incidence with BMIA/canonical grid method", *IEEE Trans. Antennas Propagat.*, **AP-43** (1995) 851-859.

[6] Tsang L., Chan C. H. and Pak K., "Monte-Carlo simulations of two-dimensional random rough surface using the sparse-matrix flat-surface iterative approach", *Electronic Lett.*, **29** (1993) 1153-1154.
[7] Rao S. M., Wilton D. R. and Glisson A. W., "Electromagnetic scattering by surfaces of arbitrary shape", *IEEE Trans. Antennas Propagat.*, **AP-30** (1982) 409-418.
[8] Peters T. J. and Volakis J. L., "Application of a conjugate gradient FFT method to scattering from thin planar material plates", *IEEE Trans. Antennas Propagat.*, **AP-36** (1988) 518-526.
[9] Zaki K. A. and Neureuther A. P., "Scattering from a perfectly conducting surface with a sinusoidal height profile: TE Polarization", *IEEE Trans. Antennas Propagat.*, **AP-19** (1971) 208-214.