# Statistical convective down motion driven by random inputs of localized buoyancy in a homogeneous sea

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**Summary.** — A model for the dynamics of dense water plumes in a homogeneous sea initially at rest, suddenly perturbed on the air-sea surface by a series of random buoyancy inputs localized on small space and time scales, is presented here. A Lagrangian representation allows the time evolution for a single, mixing plume able to carry down dense water mass to be obtained. Moreover scaling laws are found for long times, which depend on the surface air-sea interaction statistics involved and on the forcing time scale: in this way it is shown that the asymptotic time evolution of the plumes is the result of surface heterogeneous buoyancy forcing inputs.

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## 1. – Introduction

Sudden cooling and evaporative events occur on the sea surface during the winter in several regions, inducing buoyancy instabilities and convective mixing, an important mechanism responsible for deep water formation (DWF). A recent review and references are given in Maxworthy (1997) and in Marshall and Schott (1998). The known experimental observations of these phenomena in an open sea (Marshall *et al.*, 1994; 1998; Schott *et al.*, 1993; 1994; Medoc, 1970) can be summed up in four phases: a preconditional (or doming formation) phase, a violent mixing (or plume formation) phase, a plume mixing (or rotating chimney formation) phase, a chimney baroclinic instability (or cone formation) phase until the chimney finally breaks off as the external forcing dies down. The dynamics of plumes during the second above-mentioned phase has in recent years been the focus of observational, laboratory, numerical and theoretical studies (Fernando *et al.*, 1991; Coates *et al.*, 1995; Maxworthy and Narimousa, 1994; Marshall *et al.*, 1993; 1994; 1998). These studies suggest some scale relations for the convective layer in a homogeneous sea relating the layer depth $h$, its

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velocity ($\vec{\mathbf{u}} = (u, w)$) and the reduced gravity $g'$ to the assumed basic physical quantities controlling the whole phenomenon (*i.e.* the surface buoyancy flux $B_0$, assumed as homogeneous and constant, and the time $t$): $h \simeq (B_0 t^3)^{1/2}$, $u \sim w \sim (B_0 t)^{1/2} \sim (B_0 h)^{1/3}$, $g' = -b \sim (B_0 t^{-1})^{1/2} \sim B_0^{1/3} h^{-1/3}$; a self-similarity process is assumed for the plume dynamics (Send *et al.*, 1995).

The object of the present work is to discuss the plume formation process in a homogeneous sea when the surface buoyancy flux is not assumed to be homogeneous or constant but a statistical set of buoyancy inputs; the existence, in such a case, of scale laws and their consistency with the observed phenomenology and the Marshall scale laws are examined. These transient phenomena are indeed fundamental factors in the comprehension of the chimney dimension and its critical depth.

For the sake of simplicity we neglect the Earth's rotation effect by studying a bidimensional case (vertical plane), in Boussinesq approximation. The Coriolis force $f$ is thus disregarded. This is in any case a good step toward understanding plume evolution because experimental and numerical studies demonstrate that the convective layer deepening is a small space and time scale phenomenon ($h$ much less than the sea depth, $t \ll f^{-1}$) and is not controlled by the Earth's rotation.

The sea is assumed to be initially still, but is then perturbed on the surface by sudden transversal dry (or cool) winds: these phenomena in fact occur under strong meteorological perturbations (mistral, monsoon) that appear as a large series of random events occurring in a short period of time. The result of the winds on the sea surface is assumed to be equivalent to sudden space and time non-homogeneous surface cooling; in fact the analysis of field data, as obtained by Schott and Leaman (1991) in the Gulf of Lions, suggests that the turbulent plumes are generated by the non-uniformities of the cooling effects. But MEDOC observations (Schott *et al.*, 1996) show decorrelation between plumes on a 2 km horizontal range. This allows us to follow the dynamics of a single plume space uncorrelated with the others but forced by localized small inputs.

Here two limiting cases are analysed:

*a*) deterministic evolution of a single plume due to a sudden single localized buoyancy input (DE);

*b*) its evolution under the effect of many random localized external buoyancy inputs occurring during a finite time (SE).

Both a Eulerian and a Lagrangian representation of the plume formation process are presented; the asymptotic time evolution of the single plumes, able to carry the dense water mass down and to mix it under different air-sea interaction statistics, is described. The Marshall scale laws are identified again in the case (DE), or when a very rapid linear multiplicative white noise or a constant buoyancy input are given to the sea surface. A new behaviour and new scale laws are found when some different interaction statistics are assumed: in such cases the plume time evolution is ruled by the spread buoyancy initial conditions, which dominate the single input of the buoyancy parcel.

## 2. – The model

Consider a bidimensional, homogeneous model for a vertical plane $(x, z)$ of a sea in Boussinesq approximation (Phillips, 1966; Garrett *et al.*, 1996). Its dynamics is described by two coupled equations (derived from the Navier-Stokes and continuity equations) for

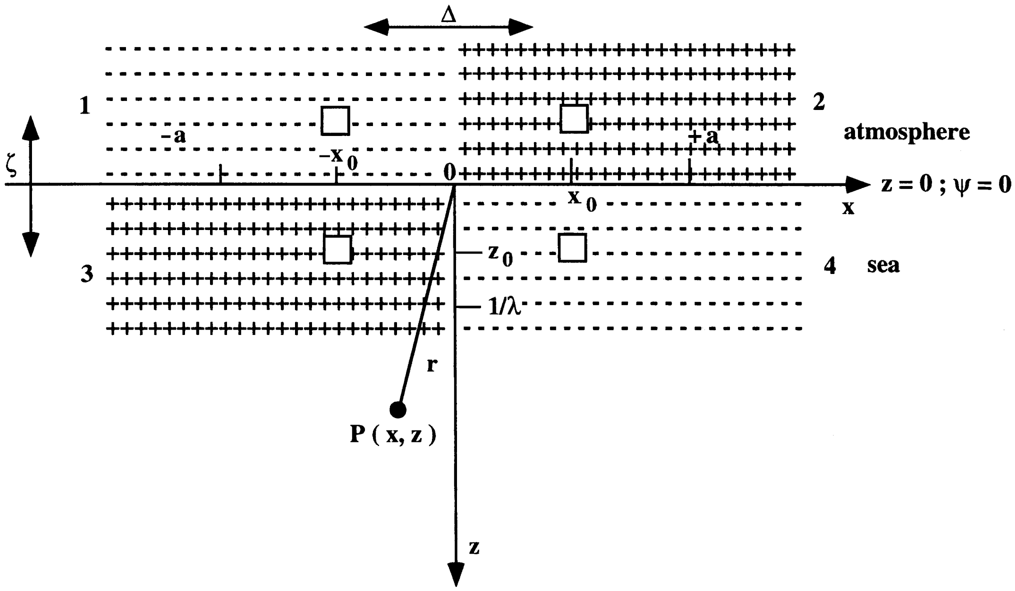


Fig. 1. – The spread vorticity sources forming the spread "quadrupoles" by the "image charge method". The squares are the charges $\pm |dq(x_0, z_0)|$; at $P(x, z)$ the streamfunction $d\psi$ generated by everyone of them is a function of $r_i(x - x_i, z - z_i)$ for $i = 1, \dots 4$.

the buoyancy $b = -g\frac{\varrho - \varrho_0}{\varrho_0}$ and the vorticity $\Delta^2\psi = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$ (defined through the streamfunction $\psi(x, z, t)$ so that $u = -\frac{\partial \psi}{\partial z}$, $w = \frac{\partial \psi}{\partial x}$) (Pedlosky, 1979; Phillips, 1969; Gill, 1982). In the coordinates system described in fig. 1 these equations are

(1a) $$\frac{\partial b}{\partial t} - \frac{\partial \psi}{\partial z}\frac{\partial b}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial b}{\partial z} = b_a \delta(t)\, e^{-\lambda z} e^{-x^2/2a^2},$$

(1b) $$\frac{\partial \Delta^2 \psi}{\partial t} - \frac{\partial \psi}{\partial z}\frac{\partial \Delta^2 \psi}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial \Delta^2 \psi}{\partial z} = -\frac{\partial b}{\partial x},$$

with the boundary and initial conditions

(1c) $$\psi(x, 0, t)) = \psi(x, H, t) = 0,$$

(1d) $$\frac{\partial \psi}{\partial x}(x, z, t = 0) = \frac{\partial \psi}{\partial z}(x, z, t = 0) = 0.$$

Here $g$ is the gravity acceleration, $\varrho_0, \varrho$ are the unperturbed and perturbed densities, $u, w$ are the horizontal and vertical components of the velocity field. The buoyancy input is schematized here as a sudden event (a single Dirac function $\delta(t)$) localized in a single point ($x = 0$), but spread over a range $a$ as a Gaussian and is also penetrating; $\lambda^{-1}$ is the penetration depth. In order to examine the effect of the buoyancy loss, we neglect the effect of the transversal wind stress.

In order to compare the results of our model with the classical Marshall laws, it is necessary to rewrite our model as a function of the surface buoyancy flux; it is possible to do this, if we note that in a sea with a flat surface (without doming due to preconditioning) an evaporation or cooling event corresponds to giving a vertical velocity $w_0$ to the sea surface such that $w = w_0 + \frac{\partial \psi}{\partial x}$, where $\frac{\partial \psi}{\partial x}$ is the vertical velocity taken in the moving surface reference frame and $w_0$ is its velocity in the laboratory reference; this velocity is connected with the lowering of the surface position because of the evaporation and cooling event, which differs from zero only in the instant $t = 0$ as shown in the footnote [1]. The consequence is that this model corresponds to a single localized buoyancy flux $B_0 = b_0 w_0 = \frac{b_a^2}{\lambda g'} \delta(t)\, e^{-\lambda z} e^{-x^2/2a^2}$ such that its time and space average is [1]

$$\langle B_0 \rangle = \frac{\lambda}{at} \int_0^t \mathrm{d}t' \int_{-L}^{L} \mathrm{d}x \int_0^H \mathrm{d}z\, B_0 \simeq \frac{|b_a|}{\lambda t}\,. \tag{2}$$

Such a singular forcing is a very idealized one, entailing an instantaneous response of the sea surface to an external atmospheric event: it may be thought of as identically a very fast external forcing or a sudden surface cooling event due to a past, longer atmospheric event.

Equations (1a)-(1d) form a non-linear system, whose symmetries in space and time allow us to say that:

– The buoyancy input is symmetric around $x = 0$ and decreases down to a thickness of 30 m $\leqslant \lambda^{-1} \leqslant$ 100–200 m; the penetration depth may have a large spatial range extending from the radiative penetration depth to the thermal boundary layer depth, depending on the heat diffusion coefficient $k$. We will actually show later how in our model it coincides with the initial convective source depth. We analyse the non-diffusive limiting case, such that $\lambda^{-1} \simeq 30$ m; but in order to compare our results with the experimental ones, a larger $\lambda^{-1}$ has to be chosen.

– The consequence is the formation of a horizontal buoyancy gradient which is antisymmetric around the perturbation centre; two opposite space-diffused, time-dependent, vorticity sources appear around $x = \pm a$. The velocity field is such that its horizontal component is antisymmetric around the perturbation centre and its vertical component is symmetric around it.

– The buoyancy suddenly changes locally because of the surface input, and is then driven by the velocity field along the streamlines.

We thus start to analyze the initial evolution of the system for $t \simeq 0$: $b(x, z, 0) =$

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[1]

$$w_0 = \frac{\mathrm{d}z}{\mathrm{d}t}\bigg|_{z=0} = \frac{\mathrm{d}z}{\mathrm{d}(\varrho - \varrho_0)} \frac{\mathrm{d}(\varrho - \varrho_0)}{\mathrm{d}t}\bigg|_{z=0} = -\frac{\mathrm{d}z}{\mathrm{d}(\varrho - \varrho_0)} \frac{\varrho_0}{g} \frac{\mathrm{d}b}{\mathrm{d}t}\bigg|_{z=0} =$$

$$-\frac{1}{\dfrac{g}{\varrho_0} \dfrac{\mathrm{d}(\varrho - \varrho_0)}{\mathrm{d}z}\Big|_{z=0}} \frac{\mathrm{d}b}{\mathrm{d}t}\bigg|_{z=0} = \frac{-b_a e^{-x^2/2a^2}}{\lambda |b_a| e^{-x^2/2a^2}} \delta(t) = \frac{\delta(t)}{\lambda}\,.$$

$b_a e^{-\lambda z} e^{-x^2/2a^2}$ is thus fixed by the initial input of buoyancy by integration of eq. (1a) for short times, when the streamfunction $\psi$ and its space derivatives are approximately zero, whilst $\psi(x, z, t)$ is the solution of the linearly time increasing equation

$$\Delta^2 \psi = -\int_0^t \frac{\partial b}{\partial x}\bigg|_{t \simeq 0} \mathrm{d}t' = \frac{b_a x}{a^2} e^{-\lambda z} e^{-x^2/2a^2} t$$

with the boundary condition $\psi(x, 0, t) = 0$. This may be identified as a Poisson problem: the vorticity field is generated by a space distributed source, namely a linearly time increasing «charge density». The boundary conditions (1c), (1d) and the symmetry properties of the vorticity sources allow the problem to be transformed into a bidimensional Laplace problem, soluble by the «image charge method, well known in electrostatics (see fig. 1). The «charge density» may thus be thought of as a continuous envelope of space spread vorticity "quadrupoles", centred on the origin, whose "charges", symmetrically arranged around the axes, have intensity

$$|\mathrm{d}q(x_0, z_0, t)| = \int_0^t \left| -\frac{\partial b}{\partial x}(x_0, z_0) \right| \mathrm{d}t'\, \mathrm{d}x_0\, \mathrm{d}z_0 = \int_0^t \mathrm{d}t'\, |\mathrm{d}q(x_0, z_0)|\, \mathrm{d}x_0\, \mathrm{d}z_0,$$

and whose sizes are $\zeta = 2z_0$, $\Delta = 2x_0$. For short times, each of these generates a streamfunction such as $\mathrm{d}\psi = |\mathrm{d}q| \dfrac{x}{|x|} \log \dfrac{r_2 r_3}{r_1 r_4}$, where $r_i$ $(i = 1, \ldots 4)$ are the distances of the generic point $P(x, z)$ from the four "quadrupole charges" as shown in fig. 1: here it is evident that

$$r_1 = \left(\left(x + \frac{\Delta}{2}\right)^2 + \left(z + \frac{\zeta}{2}\right)^2\right)^{1/2},$$

$$r_2 = \left(\left(x - \frac{\Delta}{2}\right)^2 + \left(z + \frac{\zeta}{2}\right)^2\right)^{1/2},$$

$$r_3 = \left(\left(x + \frac{\Delta}{2}\right)^2 + \left(z - \frac{\zeta}{2}\right)^2\right)^{1/2},$$

$$r_4 = \left(\left(x - \frac{\Delta}{2}\right)^2 + \left(z - \frac{\zeta}{2}\right)^2\right)^{1/2},$$

so that, for $x, z, \to 0$, $\mathrm{d}\psi \propto \dfrac{|\mathrm{d}q|\zeta\Delta}{(\zeta^2 + \Delta^2)^2} |x| z$ and for $\dfrac{\Delta}{x}$, $\dfrac{\zeta}{z} \to 0$, $\mathrm{d}\psi \propto \dfrac{|\mathrm{d}q| x}{|x|} \dfrac{\zeta\Delta |x| z}{(x^2 + z^2)^2}$ decaying as $r^{-4}$ for long distances. If we consider every vorticity source as independent of the others, near each one the streamlines are closed lines (such as ellipses whose focus is near the source) that at a certain distance become hyperbolae $|x| z = \text{const}$, whose asymptotes coincide with the axes; a net downward velocity is thus observed around $x = 0$; an antisymmetric horizontal velocity $u(x) = -u(-x)$ is thus also obtained near the axes. But it is possible intuitively to see how the envelope of all the closed streamfunctions $\mathrm{d}\psi$ generated by every "quadrupole" (whose charge is $\mathrm{d}q$) is destroyed by interference: the streamfunctions are actually additive only near the origin where every $\mathrm{d}\psi$ assumes a hyperbolic form; we are thus justified in thinking of our model streamfunction for short times such that the non-linear terms are negligible:

$$\psi = \int_0^t \int_0^L \int_0^H -\frac{\partial b}{\partial x}(x_0, z_0)\,\frac{\Delta\zeta|x|z}{(\zeta^2+\Delta^2)^2}\,\mathrm{d}x_0\,\mathrm{d}z_0\,\mathrm{d}t'.$$

The integrated, concentrated "quadrupole charges", whose intensity is $|q| = \int_q |\mathrm{d}q|(x_0, z_0) = -\int_0^L \int_0^H \frac{\partial b}{\partial x}(x_0, z_0)\,\mathrm{d}x_0\,\mathrm{d}z_0 = \frac{b_a x}{|x|\lambda}$ are localized at $\bar{z} = \pm\,\zeta/2 = \pm\,\frac{1}{\lambda}\log 2$, $\bar{x} = \pm\,\Delta/2 = \pm a\sqrt{2\log 2}$, so that $\int_0^{\bar{z}} \mathrm{d}z\; b(0, z) = \frac{1}{2}\int_0^H \mathrm{d}z\; b(0, z)$; $b(\bar{x}, \bar{z}) = \frac{b(0, \bar{z})}{2}$; this is, for the Boussinesq approximation, a conserved quantity during the following evolution; its position will be the initial condition for its following evolution.

2·1. *Lagrangian representation of one plume formation.* – The hyperbolic streamfunction form, good for short times, is valid for all the times. If we follow the buoyancy along its path, for the continuity equation we have

(4) $b(x(t), z(t), t) =$

$$= b\left(x(0) + \int_0^t u(x(0), z(0), t')\,\mathrm{d}t',\; z(0) + \int_0^t w(x(0), z(0), t')\,\mathrm{d}t',\; 0\right).$$

The initial convective process formation is described in fig. 2. The buoyancy is dragged along the streamfunction, like the vorticity sources; as the latter, situated initially at $(x_0, z_0)$, move downwards along the isolines $|x|z$, they produce increasingly new streamlines depending on their position (figs. 2, 3). The significant part of the

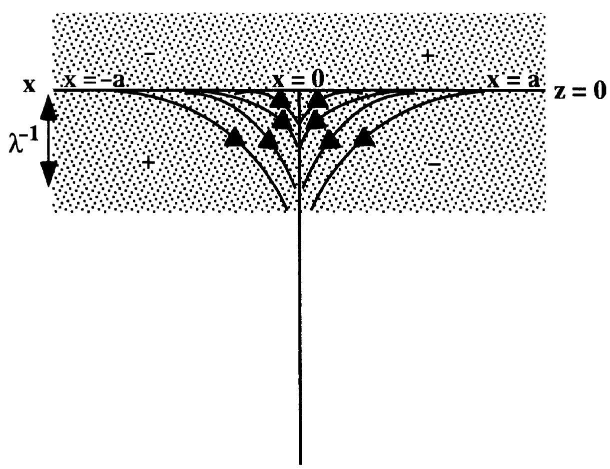


Fig. 2. – The diffuse vorticity quadrupoles $\mathrm{d}q = -(\partial b/\partial x)(x_0, z_0)\,\mathrm{d}x_0\,\mathrm{d}z_0$ and the hyperbolic streamfunctions driving them during the initial convective process.

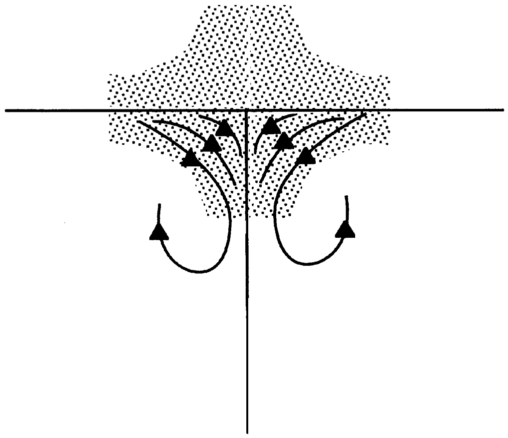


Fig. 3. – The diffuse vorticity quadrupoles driven by the streamfunctions and the new streamlines generated.

streamfunction generated by the whole spread vorticity source is

$$\psi(x,\, z) = \int_q \mathrm{d}q(\Delta,\, \zeta) \int_0^t \frac{\zeta\Delta}{(\zeta^2+\Delta^2)^2}\, |x| z\, \mathrm{d}t', \tag{3a}$$

where $\int_0^L \int_0^H -\frac{\partial b}{\partial x}\, \mathrm{d}x_0\, \mathrm{d}z_0 = \int_q \mathrm{d}q(\Delta,\, \zeta)$.

It is possible to introduce a transformation from the Eulerian $(x, z, t)$ to the Lagrangian $(x(0), z(0), t)$ variables (Salmon, 1999): every fluid particle retains the same values of $(x(0), z(0))$ the whole time; along the hyperbolic streamlines the convective derivative is $\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t}$, and $\frac{\partial}{\partial x} = a_{11}\frac{\partial}{\partial x(0)} + a_{12}\frac{\partial}{\partial z(0)}$; here $a_{12}=0$ and, for long times, $a_{11}\simeq 1$.

The path followed by the vorticity sources may be identified as the quadrupole size evolution, such that

$$\Delta(t) = \bar{x} + \int_0^t u(x(0),\, z(0),\, t')\,\mathrm{d}t', \qquad \zeta(t) = \bar{z} + \int_0^t w(x(0),\, z(0),\, t')\,\mathrm{d}t' \tag{5}$$

and

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2} = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\psi}{\partial x}\right)_0 \simeq \int_q \mathrm{d}q(\Delta,\, \zeta)\, \frac{\Delta\zeta}{(\zeta^2+\Delta^2)^2}\, z_{z=\zeta/2}. \tag{6}$$

Each infinitesimal vorticity source $\mathrm{d}q$ moves along a hyperbola, as shown in fig. 2, so that $|x|z \simeq \mathrm{const} > 0$; during its motion it does not change, so that $\mathrm{d}q = \mathrm{d}q(\bar{x}, \bar{z})$. As it seeps $\zeta \gg \Delta$, but $\zeta\Delta = \mathrm{const} = C$ so that eq. (6) becomes

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2} \simeq \int_q \mathrm{d}q(\bar{x},\, \bar{z})\, \frac{C}{\zeta^3} = \frac{|b_a|}{\lambda}\, \frac{C}{\zeta^3}\,. \tag{7}$$

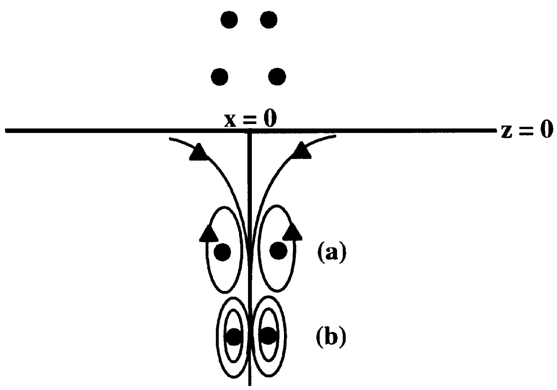


Fig. 4. – The whole vorticity sources ever closer to the axes $x = 0$ for long times: a) at time $t$; b) at time $t + \Delta t$.

If we now follow the water parcel giving rise to all the vorticity, we find that it moves along the stream, but remains unchanged. It is thus reasonable to assume that every $\mathrm{d}q$ vorticity source moves asymptotically along the same path independently of the others, so that the whole point-sized "charge" integrated $q$, of order $|b_a|\lambda^{-1}$, eventually makes its contribution to the convective velocity. The long-time convective process is described in fig. 4. In the simple case considered so far of one single sudden buoyancy input the integration of eq. (7) for the scaled variable $\zeta' = \lambda\zeta$ gives, if we rename $\zeta' = \zeta$ (see the appendix),

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \frac{|b_a|Ct}{[1 + C|b_a|t^2]^{1/2}}\,, \tag{8}$$

tending to $(C|b_a|)^{1/2}$ for long times.

This result shows that the plume evolution is able, as in the classical theory, to carry the water mass down, so as to mix it. So, if we could know the value of $|b_a|$ in an experimental or field observation we would measure a constant velocity of the water plume as a function of this quantity; but the Marshall scale laws are functions of the surface buoyancy flux ([2]) whose values are generally calculated by an averaging process (daily, weekly or monthy) from satellite or meteorological observations; these scaling laws are thus recognized if, by using eq. (2), we remember that $|b_a| = \langle B_0\rangle t$, so that, for long times, the scaled variable $\zeta$ scales as $\zeta \simeq (C\langle B_0\rangle t^3)^{1/2}$, and $\frac{\mathrm{d}\zeta}{\mathrm{d}t} \simeq (C\langle B_0\rangle t)^{1/2}$.

([2]) Remember that (Gill, 1982) the buoyancy flux is defined as

$$B_0 = -\frac{g\alpha Q}{\varrho_0 c_\mathrm{w}} + g\beta s\frac{E - P}{\varrho_0}\,,$$

where $Q$ is the heat flux, $\alpha = \frac{-1}{\varrho}\left(\frac{\partial\varrho}{\partial T}\right)_{p,\,T}$, $\beta = \frac{1}{\varrho}\left(\frac{\partial\varrho}{\partial s}\right)_{p,\,T}$, $E$ is the evaporation rate and $P$ the rainfall rate, $s$ the salinity, $c_\mathrm{w}$ the water specific heat.

The impulsive nature of the event described through our model, such that the generally measured averaged value $\langle B_0 \rangle$ is in inverse relation to the time, gives such a $t^{1/2}$-dependence of the velocity. A deeper analysis of these results will be discussed in the conclusive section.

## 3. – Stochastic approach

This delta-function approach is idealized because it assumes a single sudden variation of the surface buoyancy whatever the meteorological conditions may be (in any case a single storm): it is only presumptive about the plume dynamics whose time scale is experimentally longer than the external time variability. It is possible to verify for a 100–500 m plume that for a turbulent velocity of 5–10 cm/s (as experimentally measured, *e.g.* at the Medoc region) the time scale of the variation of the plume is about 2000 s as its lifetime is $\sim 2$h while the time scale of the cat's paws of wind is about 3–10 s. That means that during the convective stage we do not have a single variation of the surface buoyancy, but many, each of which is related to a cat's paw: the plume dynamics is the effect of an integration over the fast time variation and has to be dependent on the air sea statistics. In order to compare many kinds of air-sea interaction statistical models, we analyse in particular a Wiener process, a linear white noise process and a softer one (deterministic with noise) in the separate cases of high and low frequency of the external event.

3·1. *Wiener.* – If we assume that many random "delta" events (Gardiner, 1983) impinge on the same region, so that at random times $0 \leqslant t_1 \leqslant \ldots t_n < T \rightarrow \infty$ we have a lot of inputs $b_a$ changing randomly so that their increments $\mathrm{d}b_a \sim \varepsilon^{1/2} \mathrm{d}\mathbf{W}(t)$ are independent, the generalized equation (7) scaled with $\lambda$ becomes $\frac{\mathrm{d}^2 \int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{C\varepsilon^{1/2} \int \mathrm{d}\mathbf{W}^{(t)}}{\zeta^3}$ (3). Physically this model shows a series of different evaporation or cooling events, each able to change the absolute buoyancy: given $b_a = b_a^i$ at the time $t = t_i$, we have $b_a = b_a^i + \mathrm{d}b_a$ (such that $\langle \mathrm{d}b_a \rangle = 0$, but $\langle b_a \rangle \sim \langle \mathbf{W}(t) \rangle \neq 0$) at the time $t = t_{i+1}$; namely every density increment may vary in size, but the final result is in any case an evaporation or cooling process such that the buoyancy is always negative and $\langle b_a \rangle = \langle \bar{b}_a \rangle + \varepsilon^{1/2} \int \langle \mathrm{d}\mathbf{W}(t) \rangle$. Each of these random events is an external forcing dragging the plume downward: it is added randomly to the former events so that the plume path $\left( \int \mathrm{d}\zeta \right)$ is randomly changed. On mathematical grounds, we can consider $\zeta = \zeta(\sqrt{\varepsilon})$; if $\varepsilon \ll 1$, then $\zeta \simeq \zeta_0 + \sqrt{\varepsilon}\zeta_1 \ldots$, so that we can have a set of differential equations for every order. At the 0th-order in $\sqrt{\varepsilon}$ ($\varepsilon = 0$, so that only the single event at $t = 0$ is considered) we have $\zeta_0 = 1$. Its integration at the first order in $\sqrt{\varepsilon}$ gives $\zeta_1 = C\varepsilon^{1/2} \int^t \mathrm{d}t' \int^{t'} \mathrm{d}t'' \int^{t''} \mathrm{d}\mathbf{W}(\tilde{t})$ whose mean value is zero; if we square and average this

(3) The latter process is defined as a stochastic one whose mean value is zero, and whose variance increases with time so that $\int^t \int^s \langle \mathrm{d}\mathbf{W}(t') \, \mathrm{d}\mathbf{W}(t'') \rangle = \min[t, s]$. The parameter $\varepsilon$ has the dimensions of $\frac{b_a^2}{t}$, that is the mean square variability over a time such that $\tau^{-1}$ is the process probability over unit time, *i.e.* the process variability $\langle \Delta b_a^2 \rangle^{1/2}$ over unit time (we have chosen an ideal process whose probability distribution is diffusive with a constant diffusion coefficient).

expression, we have (see the appendix)

$$\frac{\mathrm{d}}{\mathrm{d}t}[\langle\zeta^2\rangle-\langle\zeta\rangle^2]^{1/2}=\frac{5C\varepsilon^{1/2}}{4\sqrt{3}}t^{3/2}. \tag{9}$$

It should be noted that the integration leading to this equation is made only after elevation to a second power, so that the time evolution of the vorticity sources is given by their space dispersion, as led by the external process time dispersion.

In order to explain this result, and to compare it with the similarity scaling laws, it is possible to define a time scale $\tau$ (time scale between buoyancy inputs); if we use dimensional arguments, a scale stochastic buoyancy flux may be defined as

$$[\langle B_0^2\rangle-\langle B_0\rangle^2]^{1/2}=\langle\Delta b_a^2\rangle^{1/2}\overline{w}=\frac{\langle\Delta b_a^2\rangle^{1/2}}{\lambda\tau} \tag{10}$$

(where $\langle\Delta b_a^2\rangle^{1/2}=(\varepsilon\tau)^{1/2}$ is the mean variability between buoyancy inputs for unit time) and if we scale the time as $t=\frac{t}{\tau}$ as well as every length as $\zeta'=\lambda\zeta$, eq. (9) becomes (by renaming $\zeta=\zeta'$)

$$\frac{\mathrm{d}}{\mathrm{d}t}[\langle\zeta^2\rangle-\langle\zeta\rangle^2]^{1/2}\simeq[\langle B_0^2\rangle-\langle B_0\rangle^2]^{1/2}t^{3/2}. \tag{10a}$$

In such a case, the plume evolution is ruled by the spread surface buoyancy flux, dominating the single DE buoyancy parcel dynamics and leading to an increasingly accelerated crash on the bottom. A similarity process can be recognized, but with new scale relations.

If $\varepsilon\gg1$, a $\frac{1}{\varepsilon}$ expansion does not show any physical solution because only a non-physical, instantaneous plume deepening seems possible, but a suitable time scale transformation leads to the time power law shown in eqs. (9), (10a) (Bouché, 2000).

3·2. *Linear multiplicative white noise.* – If we assume a process such as (defined in Gardiner, 1983, p. 103) (4)

$$\mathrm{d}b_a\sim b_a\,\varepsilon^{1/2}\,\mathrm{d}\mathbf{W}(t), \tag{11}$$

eq. (7), scaled with $\lambda$, becomes $\frac{\mathrm{d}^2\int\mathrm{d}\zeta}{\mathrm{d}t^2}=\frac{C\varepsilon^{1/2}\int b_a(t')\,\mathrm{d}\mathbf{W}(t')}{\zeta^3}$, where $b_a(t)=b_a(t_0)\cdot$ $\cdot\, e^{\sqrt{\varepsilon}\int\mathrm{dW}-\frac{\varepsilon}{2}t}$; for $\varepsilon\ll1$ (so that $b_a(t)\simeq b_a(t_0)(1+\sqrt{\varepsilon}\int\mathrm{d}\mathbf{W})$), if we perform the same expansion as before, we can integrate till the first order in $\sqrt{\varepsilon}$, square and average and get (see the appendix)

$$\frac{\mathrm{d}}{\mathrm{d}t}[\langle\zeta^2\rangle-\langle\zeta\rangle^2]^{1/2}\simeq\frac{5}{2}\left[\frac{C\varepsilon b_a^2(t_0)}{2\sqrt{3}}\right]^{1/2}t^{3/2}. \tag{12}$$

(4) This is a kind of multiplicative white noise, where the constant $\varepsilon$ is $\tau^{-1}$.

If we scale the time as $t = t/\tau$, and the buoyancy flux as $B_0 = \frac{b_a(t_0)}{\lambda\tau}$, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}[\langle\zeta^2\rangle - \langle\zeta\rangle^2]^{1/2} \simeq B_0 t^{3/2}, \tag{12a}$$

so that the convective velocity scales with $B_0 t^{1/2}$ instead of $(B_0 t)^{1/2}$.

For $\varepsilon \gg 1$, a $\frac{1}{\varepsilon}$ expansion gives (see the appendix)

$$\left\langle\left(\frac{\mathrm{d}^2\int\mathrm{d}\zeta}{\mathrm{d}t^2}\right)^2\right\rangle^{1/2} \simeq \frac{Cb_a(t_0)}{(\langle\zeta^2\rangle^{1/2})^3}. \tag{13}$$

We recognize here the same equation as DE, and the same scaling laws.

3·3. *Softer stochastic air-sea interaction.* – So far we have analysed only discontinuous cases; furthermore, a softer superficial density changing process may be realistic. If the air-sea interaction is simulated by

$$\mathrm{d}b_a \sim a(t)\,\mathrm{d}t + \varepsilon^{1/2} b_a\,\mathrm{d}\mathbf{W}(t), \tag{14}$$

where $a(t)$ (5) is a generic time function, we have (Gardiner, 1983, p. 112)

$$b_a(t) = e^{\sqrt{\varepsilon}\int_0^t \mathrm{dW} - \frac{\varepsilon}{2}t}\left[b_a(t_0) + \int_0^t e^{-\sqrt{\varepsilon}\int_t^{t'}\mathrm{dW} + \frac{\varepsilon}{2}t'} a(t')\,\mathrm{d}t'\right]; \tag{15}$$

if $\varepsilon \gg 1$, the plume evolution is deterministic and its deepening velocity time powers are produced wholly by the rate $a(t)$; if $\varepsilon \ll 1$, by an $\varepsilon$ expansion we find at the 0th-order that the plume time behaviour depends on $a(t)$ and not on the stochastic term. But the non-linearity of the equations is such that it is generally not possible to be sure about the uniqueness of the solution and many possible temporal evolutions are possible. In fact at the next order, small amplitude variance oscillations, whose frequency is dependent on time, appear in any case (see the appendix).

3·4. *Constant surface buoyancy flux.* – It is in any case interesting to see how this equation leads naturally to the Marshall laws in the classical case of constant surface buoyancy flux; in fact, if

$$\frac{\mathrm{D}b}{\mathrm{D}t} = \frac{\partial B_0}{\partial z} \tag{16}$$

with $B_0 = B_a e^{-\lambda z} e^{-\frac{x^2}{2a^2}}$, the function $b(x, z, t)$ is not a conserved quantity any more; but for Boussinesq approximation, it is possible to say that the quantity

$$q(t_0) = \int_0^{t_0}\mathrm{d}t' \int_0^{L/2}\mathrm{d}x_0 \int_0^H \mathrm{d}z_0\,\mathrm{d}q(x_0, z_0)$$

remains anyway constant during the next times: $q = q(t)$ changes, but $q(t_0)$ is conserved. It is so possible to follow its evolution along the hyperbolic lines, so that the

(5) $a(t)$ is a function giving a deterministic finite velocity to the buoyancy variability.

plume evolution may be described by the equation

$$\frac{d^2 \zeta}{dt^2} = |q(t_0)| \frac{C}{\zeta^3}, \tag{17}$$

where $t_0$ is the time during which the buoyancy flux is applied; the scaled variable $\zeta$ thus behaves as $\zeta \simeq (CB_a t_0)^{1/2}(t - t_0)$. If the time is scaled with $t_0$, for long times we have

$$\zeta \simeq (CB_a t_0^3)^{1/2} t \tag{18}$$

and

$$\frac{d\zeta}{dt} \simeq (CB_a t_0)^{1/2}. \tag{18a}$$

## 4. – Conclusions

In order to check the postulated generation of turbulent plumes as a result of time and space surface heterogeneous buoyancy forcing inputs, we investigated the asymptotic time evolution of a single plume due to an external forcing, localized in space and horizontally decaying as a Gaussian, by testing various time distribution functions for the air-sea interaction. In every case we assumed that the global effect of the diffuse vorticity sources can be viewed as a destructive interference by the closed streamlines in their vicinity, so that only the hyperbolae near the origin can be summed: the consequence is that the vorticity sources are driven by the latter; as the vorticity approaches the central axes, the mixing closed lines are increasingly in phase, so that their interference is no longer destructive and their turbulent effect increases constantly.

The vorticity sources generate an increasingly diminishing horizontal scale convective motion, whose vorticity is conserved; so if a single $\delta(t)$ event (DE) sweeps the localized region, the sinking velocity, driven along the hyperbola, will become constant and very slow; *i.e.* the single plume will be able to carry the water mass down, but, as it goes, it will give rise to convective mixing too, so that its velocity becomes constant. Only a continuous recurrence of the event can push down the heavy water with an accelerated motion. For long times, the DE plume evolution satisfies the Marshall scaling laws, as in the classical case in which the time constant surface buoyancy flux is given if every length is scaled with $\lambda$. It shows that the long-time behaviour of the plumes is not really a collective effect, but depends only on the external source and its statistics. When a small-amplitude random Wiener process, such as a linear white noise, is assumed for the air-sea interaction, a self-similar behaviour may be identified for long times, but new scaling laws are found for the convective velocity and depth if the time is scaled with a time scale related to the input probability. This assumes a statistics, like the simple ones analysed here, specific to a random walk process for which a constant jump probability per unit time is defined. It is possible to predict that, if the air-sea interaction statistics is such that it is not possible to define such a time scale, we will no longer find any self-similar behaviour. It must in any case be noted that the plume evolution and scaling laws are defined through the mean square deviation, ruled by the spread buoyancy inputs: this hides the single buoyancy parcel dynamics because it is ruled by a higher time power; therefore, the plume dynamics and scaling laws are produced by the external statistics. Moreover, if the air-sea interaction is simulated by a Wiener process, the convective quantities scale with the scaled surface buoyancy flux variability, while they scale with the scaled mean buoyancy flux if a linear multiplicative white noise is used. At the continuous limit ($\tau \to 0$) of a multiplicative white noise the Marshall scale laws are in any case recognized again. This kind of statistics is specific to a random walk process, so that every $b_a$ is independent of the others and no memory of the past is assumed. The Wiener process is an extremely limiting case, generally assumed when thermodynamically non-homogeneous, far from equilibrium conditions are imposed. Both appear as idealized statistics that are no longer reliable in our real case of a storm during which strong winds blow over the sea surface; the statistics involved depends on the characteristics of the real process, and only a deep analysis of the experimental atmospheric data allows us to infer it. But it is important to note how the time evolution depends on the air-sea interaction statistics involved. If a softer stochastic air-sea interaction is assumed, the plume evolution is no longer defined: it depends on the rate $a(t)$, but the noise introduces small-amplitude variance oscillations whose frequency is dependent on the time. Moreover, it is possible to infer that, if the air-sea interaction statistics is such that it is not possible to define a time scale $\tau$ any more, we will no longer find any self-similar behaviour. Both the plume evolution and the scaling laws are defined except for the constant $C$: it is the hyperbolic isoline along which the vorticity source runs; its value is undefined because of the roughness of the model: in fact, $C = C(t)$, because during the vorticity source dragging, newer and newer stream-functions, just as new smaller $C$ hyperbolae, are generated; but for $\Delta \to 0$ it is reasonable to neglect its variation, as compared with the vertical movement $\zeta$ (Bouché, 2000). We may conclude that, for long times, the plume time evolution is not really a collective effect, but depends only on the external source and its statistics. A forthcoming analysis is planned for the observed distance among the developed plumes and their dimensions.

## Appendix

### Calculations for one plume evolution

a) *DE*

If we multiply both members of eq. (7) by $\frac{d\zeta}{dt}$, we have

$$\frac{1}{2}\frac{d}{dt}\left(\frac{d\zeta}{dt}\right)^2 = \int_q dq \frac{C}{2}\frac{d}{dt}\left(-\frac{1}{\zeta^2}\right), \tag{A.1}$$

so that

$$\frac{d\zeta}{dt} = \left[\int_q dq\, C\left(\frac{1}{\zeta(0)^2} - \frac{1}{\zeta^2}\right)\right]^{1/2}.$$

The following integration and some algebra, together with the boundary conditions $\zeta(0) = \bar{z}$, $\dot{\zeta}(0) = 0$, lead to eq. (8).

b) *Wiener process*

If we again write eq. (7) scaled with $\lambda$ as

$$\frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{C\varepsilon^{1/2}\int \mathrm{d}\mathbf{W}}{\zeta^3}, \tag{A.2}$$

so that the whole "force" is the sum of the infinitesimal "forces" given by the single buoyancy inputs, we can transform it into the coupled system

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{C\varepsilon^{1/2}\int \mathrm{d}\mathbf{W}}{\zeta^3}, \tag{A.3a}$$

$$\frac{\mathrm{d}\int \mathrm{d}\zeta}{\mathrm{d}t} = v. \tag{A.3b}$$

Here $\zeta = \zeta(\sqrt{\varepsilon})$, $v = v(\sqrt{\varepsilon})$ so that, if $\varepsilon$ is small, it is possible to write $\zeta = \zeta_0 + \sqrt{\varepsilon}\zeta_1 + \ldots$ and to transform the coupled equation system (A.3a), (A.3b) into a series of coupled systems for every $\sqrt{\varepsilon}$-order:

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = C\int \mathrm{d}\mathbf{W}\,\frac{\mathrm{d}^{i-1}}{\mathrm{d}\varepsilon^{(i-1)}}\,\frac{1}{\zeta^3}\bigg|_{\varepsilon=0}, \qquad \frac{\mathrm{d}\int \mathrm{d}\zeta_i}{\mathrm{d}t} = v_i. \tag{A.4}$$

At the 0th-order, $\frac{\mathrm{d}v_i}{\mathrm{d}t} = 0$; for the boundary conditions the only solution is $\zeta_0 = \bar{z}$; at the first order, we have

$$\frac{\mathrm{d}v_1}{\mathrm{d}t} = \frac{C\int \mathrm{d}\mathbf{W}}{\bar{z}^3},$$

so that (if we remember that $\bar{z} \sim 1$)

$$\zeta_1 = C\int^t \mathrm{d}\left[\int^{t'} \mathrm{d}t''\int^{t''} \mathrm{d}\bar{t}\int^{\bar{t}} \mathrm{d}\mathbf{W}(\bar{\bar{t}})\right], \tag{A.5}$$

whose mean value is zero. It is easy to verify that

$$\langle\zeta^2\rangle - \langle\zeta\rangle^2 = \varepsilon\langle\zeta_1^2\rangle, \tag{A.6}$$

so that if we square and average, remembering that $\varepsilon t = \langle\int^t \mathrm{d}\mathbf{W}(t')\int^t \mathrm{d}\mathbf{W}(t')\rangle$, we obtain eq. (9). If the value of $\varepsilon$ is large, the last series expansion is no longer possible; if the opposite expansion in $\varepsilon^{-1/2}$ is performed, the only possible solution of $\frac{C\int \mathrm{d}\mathbf{W}}{\zeta_0^3} = 0$ is $\zeta \sim \mathrm{O}(\infty)$.

c) *Linear multiplicative white noise*

If again

$$\frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{C\varepsilon^{1/2}\int^t b_a(t')\,\mathrm{d}\mathbf{W}(t')}{\zeta^3}, \tag{A.7}$$

we have

$$\text{(A.8)}\qquad \frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{\varepsilon^{1/2} C b_a(t_0)}{\zeta^3}\, e^{\sqrt{\varepsilon}\int \mathrm{d}\mathbf{W} - \frac{\varepsilon}{2}(t-t_0)}.$$

If $\varepsilon \ll 1$, so that $b_a(t) \simeq b_a(t_0)(1+\sqrt{\varepsilon}\int \mathrm{d}\mathbf{W})$, the same series expansion as before, at the first order in $\sqrt{\varepsilon}$, leads to the equation

$$\text{(A.8a)}\qquad \frac{\mathrm{d}^2\int \mathrm{d}\zeta_1}{\mathrm{d}t^2} = \frac{C b_a(t_0)\int^t \mathrm{d}\mathbf{W}(t')}{\zeta_0^3}.$$

If we integrate this equation, square and average as before, we get the results shown in eq. (19). If $\varepsilon \gg 1$, $b_a(t) \simeq b_a(t_0)\, e^{-\frac{\varepsilon t}{2}}$, so that

$$\text{(A.9)}\qquad \frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} \simeq \frac{C\varepsilon^{1/2}}{\zeta^3}\, b_a(t_0)\int^t e^{-\varepsilon t'/2}\, \mathrm{d}\mathbf{W}(t'),$$

whose mean is zero; but if we square and average this equation, we have

$$\text{(A.10)}\qquad \left\langle\left(\frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2}\right)^2\right\rangle \simeq \frac{C^2 \varepsilon b_a(t_0)\int^t e^{-\varepsilon t'}\, \mathrm{d}t'}{\langle\zeta^6\rangle}$$

that, in the limit $\varepsilon \gg 1$, leads to eq. (13).

d) *Soft stochastic model*

If $\varepsilon \gg 1$, eq. (A.1) becomes

$$\text{(A.11)}\qquad \frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{C b_a(0)\, e^{-\varepsilon t/2}}{\zeta^3}\left[1 + b_a^{-1}(0)\int^t e^{\varepsilon t'/2} a(t')\, \mathrm{d}t'\right].$$

After a very short time $t = 2/\varepsilon$, the first term on the right-hand side of this equation goes to zero; the plume evolution depends mainly on the soft cooling or evaporative rate $a(t)$; if this is a polynomial, its integration gives terms such as $\left(\frac{2}{\varepsilon}\right)^n t^m$ + decaying terms, so that

$$\frac{\mathrm{d}^2\int \mathrm{d}\zeta}{\mathrm{d}t^2} = \frac{C}{\zeta^3}\Sigma_k a_k \left(\frac{2}{\varepsilon}\right)^k t^m.$$

If a $\frac{1}{\varepsilon}$ expansion is done, at the 0th-order this equation is equal to zero, so that $\zeta_0 = \bar{z}$. At the next orders the above-mentioned conclusions may be drawn.

If $\varepsilon \ll 1$ an $\varepsilon$ expansion of eq. (A.11) gives, at the 0th-order,

$$\frac{\mathrm{d}^2\int \mathrm{d}\zeta_0}{\mathrm{d}t^2} = \frac{C b_a(0)}{\zeta_0^3}\left[1 + b_a^{-1}(0)\int a(t')\, \mathrm{d}t'\right].$$

At the first order

$$\frac{\mathrm{d}^2\int \mathrm{d}\zeta_1}{\mathrm{d}t^2} = \alpha(t)\,\zeta_1 + \zeta_1'(t)\,,$$

where

$$\alpha(t) = -\frac{3Cb_a(0)}{\zeta_0^4}\left[1 + b_a^{-1}\int^t \alpha(t')\,\mathrm{d}t'\right]$$

and

$$\zeta_1'(t) = \frac{Cb_a(0)}{\zeta_0^3}\left[\left[1 + b_a^{-1}\int^t \alpha(t')\,\mathrm{d}t'\right]\int^t \mathrm{d}\mathbf{W}(t') + b_a^{-1}(0)\int^t \mathrm{d}t'\,\alpha(t')\int^{t'} \mathrm{d}\mathbf{W}(\bar{t})\right].$$

It may be transformed into a linear coupled system

(A.12a) $$\frac{\mathrm{d}v_1}{\mathrm{d}t} = \alpha(t)\,\zeta_1 + \zeta_1',$$

(A.12b) $$\frac{\mathrm{d}\zeta_1}{\mathrm{d}t} = v_1,$$

whose solution is

$$v_1 \sim Ae^{i\omega(t)}\left[1 + A^{-1}\int^t e^{-i\omega(t')\zeta_1'(t')}\,\mathrm{d}t'\right],$$

where

$$\omega(t) \sim \frac{(3Cb_a(0))^{1/2}}{\zeta_0^2(t)}\left[1 + b_a^{-1}(0)\int^t \alpha(t')\,\mathrm{d}t'\right]^{1/2}.$$

But $\zeta_1' = \zeta_1'(\mathrm{d}\mathbf{W})$, so that $\langle\zeta_1'\rangle = 0$, and

$$\langle\zeta^2\rangle - \langle\zeta\rangle^2 \sim \varepsilon\langle\zeta_1^2\rangle\,.$$

If we integrate eq. (A.12b) and square it, simple calculations lead to the conclusions discussed at the end of subsect. 3·3.

* * *

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