Product disposal : a market competition perspective

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April 2020

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Keywords: Product disposal, capacity-price competition
JEL classification: D21, L13, Q02

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1 Introduction

Societies have generally become more aware of resource scarcity and production and consumption inefficiency. For example, many have criticized the regular disposal of unsold merchandise, such as food and clothing ("green criticism").[1] One luxury fashion brand faced fierce criticism after revealing that it scrapped many valuable unsold clothes and cosmetics.[2] Such a business practice, which is not exclusive to luxury products, has prevailed in many sectors. As an example that recently became famous in Japan is the "Ehomaki" sushi roll, which is consumed to celebrate the beginning of spring; it became an iconic target of criticism against product disposal because many unsold rolls are wasted every year.[3]

Perhaps the practice of discarding unsold products is unavoidable (or even rational) to some degree, however, since such products tend to be short-lived (e.g., perishable goods tend to expire quickly) and face demand uncertainty for which it is difficult to prepare precise amounts of products in advance. These factors may at least partially explain why firms regularly dispose of large amounts of unsold merchandise. The following questions still remain unanswered, however: Under what circumstances is product disposal unavoidable? Do firms dispose of unsold products too much (as criticized)? If product disposal generates inefficiency, what policy interventions can fix it?

This paper examines these positive and normative questions and attempts to evaluate the efficiency of this business practice from the point of view of economic welfare. An often-mentioned justification for product disposal is suggestive for choosing an analytical framework: product disposal is done for preventing unsold items being stolen and sold cheaply elsewhere, which may damage their intellectual property and brand value.[4] This anecdotal justification clearly suggests that products destroyed in case of unsold are more or less differentiated and have some market power. Following Maggi (1996), the present paper proposes a two-stage duopoly model with not-perfectly rigid capacity constraints: Two firms producing differentiated goods first set their production capacities and then compete in price. I extend the Maggi's model in two respects. First, by introducing demand

[1] For example, the Sustainable Development Goals (SDGs) advocated by the United Nations in 2015 propose substantial reductions in food waste and losses in their twelfth goal such that "(b)y 2030, halve per capita global food waste at the retail and consumer levels and reduce food losses along production and supply chains, including post-harvest losses."

[2] Elizabeth Paton, "Burberry to Stop Burning Clothing and Other Goods It Can't Sell," The New York Times Sep. 6, 2018.

[3] Noriko Okada, "Ehomaki sushi rolls spark controbersy," NHK World-Japan Feb. 4, 2019.

[4] Morwenna Ferrier, "Why does Burberry destroy its products and how is it justified?," The Guardian Jul. 20, 2018.

uncertainty, the two firms have to set their sales capacities before demand uncertainty is resolved. At the next stage, given information about each other's capacity and precise market size, they compete in price and determine sales quantities. Second, ex post capacity adjustment can be two directions, while the Maggi's model allows only ex post additional production. Facing the realization of high demand, the firms may expand the sales capacity through second-stage production, which is more costly than the first-stage capacity-building production. In addition, if low demand is realized, they may choose not to sell their entire capacities and scrap unsold capacity with incurring disposal costs, which is product disposal in the present model. Although the present model is highly stylized, it can be analytically solvable and describe all sets of equilibria.

Although the paper's goal is to examine the effect of competition among firms on their decisions about the pre-committed production (sales capacities), sales quantities, and product disposal, the analysis begins with a monopoly case. The monopoly setting completely abstracts firms' strategic behavior from the model, but is useful to understand the intuition of product disposal and a complete set of parameter configurations with which one of the following three cases emerges: (i) neither capacity expansion nor product disposal (capacity destruction) occurring regardless of demand realization; (ii) only capacity expansion occurring when high demand is realized; and (iii) only product disposal occurring when low demand is realized. We then proceed to the duopoly case. Our main findings are as follows. First, product disposal occurs if the sum of the unit disposal and the initial-stage production costs is sufficiently lower than demand variability *and* the net marginal cost of the second-stage production in excess of the sales capacity. Thus, capacity flexibility constraints substantially reduce product disposal even though flexibility is just one direction (only capacity expansion). Second, regarding the ratio of product disposal to capacity constraints (disposal intensity), the monopolist does not yield any inefficiency associated with product disposal, which implies that by fixing standard production distortion by monopoly, the social planner actually increases product disposal. Third, in the duopoly case, we show that a Bertrand equilibrium emerges when product disposal occurs, which contribute to *increasing* disposal intensity. This tendency is more pronounced as the degree of product differentiation decreases.

The oligopoly model with pre-committed capacity constraints has long been studied. Among others, Kreps and Scheinkman (1983) show that price competition with capacity constraints yields Cournot outcomes under some conditions. However, Maggi (1996) shows that relaxing the assumption about perfectly rigid capacity constraints and introducing product differentiation (a standard way to avert the Bertrand paradox) allow competition equilibria to range from Bertrand to the Cournot depending on the degree of rigidity of capacity constraints.[5] Mine is a variant of the Maggi model but departs from it by introducing stochastic demand, which enables capacity expansion and product disposal to emerge. In contrast, without demand uncertainty, Maggi (1996) demonstrates that firms always sell their entire sales capacities at equilibrium regardless of their competition modes. Staiger and Wolak (1992) study demand uncertainty in an oligopoly setting with strictly rigid capacity constraints; however, their interest is firms' collusive behavior and its sustainability in a repeated game context. This paper's interest is completely different, examining

The rest of this paper is organized as follows. The next section describes the model's setup. Section 3 solves the model in a monopoly setting. We also conduct welfare analysis by considering the social planner's problem. Section 4 examines how the model's implications are altered from the monopoly case by introducing two symmetric firms competing in price. Section 5 concludes.

2 Setup

2.1 Preferences

Consider an economy with two sectors: sector 0 provides a homogenous good (good 0) while sector 1 horizontally differentiated goods. While sector 0 is perfectly competitive, sector 1 is oligopolistic. For analytical simplicity, I assume only two symmetric risk-neutral firms in sector 1. A representative consumer has quasi-linear utilities of the form $U = q_0 + u(\mathbf{q})$, where q_0 is consumption of good 0 and $\mathbf{q}$ is the vector of consumption of sector 1 goods. The sub-utility $u(\mathbf{q})$ takes a form that yields the following linear demand for firm i's good:

$$q_i = a - p_i + bp_j, \quad b \in (0,1) \tag{1}$$

where $i, j = \{1, 2\}$ and $i \neq j$. As is standard, assuming that the economy's population is large enough for some workers to be employed in sector 0, this sector absorbs all income effects.

I introduce demand uncertainty for sector 1. Parameter a is assumed to be stochastic: It takes two states, "high" and "low", such that $a_H = \bar{a} + \mu$ and $a_L = \bar{a} - \mu$, where $\mu > 0$ is the deviation from the center value $\bar{a}$. Assuming each state may occur evenly, $\bar{a}$ is also the mean value of a and μ is the standard deviation of a.

[5] In addition, Maggi (1996) shows that only unique pure strategy equilibrium exists, unlike Kreps and Scheinkman (1983) in which mixed-strategy equilibria may emerge.

2.2 Production and Sales

All goods are produced with constant returns to scale technologies from labor only. Good 0 serves as the numeraire good in the model, and the competitive wage equals the marginal product of labor in good 0. Units are chosen such that the wage equals 1, which prevails across the economy assuming the free mobility of labor.

Supplying good 1 to the market is assumed to have two stages. In the first stage, risk-neutral firms produce good 1 at constant marginal cost c, and its true demand is unknown to them, so they must determine the output level only with the knowledge about the distribution of demand levels.

In the second stage, the true value of a and each firm's stock level of good 1 (sales capacity) become known to all firms. Firms sell good 1, incurring c_s for each unit of sales. Following Maggi (1996), I assume that the firms can expand their sales capacities in the second stage by paying constant marginal cost c_e, which is no less than c (i.e., $c \leq c_e$). Furthermore, the firms may not sell all the goods produced in the first stage and must scrap these unsold units of good 1, incurring unit disposal cost c_d.

These capacity expansion and capacity destruction (product disposal) not only reflect the real world (e.g., overtime work and spot procurement from outside the firm as capacity expansion and disposal of unsold merchandise as capacity destruction) but also defines the sales capacity constraint flexibility set in the first stage, which may also characterizes goods or industries. For instance, products required to use highly specialized inputs can have high c_e while those do not contain toxic chemical materials, like food and garments, may have low c_d. The second-stage production and disposal costs control the rigidity of sales capacity constraints and influence firms' strategies with respect to production and sales.

3 A Preliminary Examination

To understand the fundamentals of product disposal, I start with the monopoly case to abstract the effect of strategic interactions among firms. In the case of monopoly, the product demand can be simplified to $q = a - p$. As is standard, the model is solved by backward induction.

3.1 The Second Stage

In the second stage, the monopolistic firm determines the sales quantity, given the output level of good 1 at k. The marginal sales cost (MSC) depends on sales quantity:

$$\text{MSC} = \begin{cases} c_s - c_d & \text{for} \quad q < k \\ c_s & \text{for} \quad q = k \\ c_s + c_e & \text{for} \quad q > k. \end{cases}$$

As long as the sales is not bounded to the first-stage production of k and product disposal occurs, increasing sales by one unit saves the unit disposal cost. Thus, the MSC is $c_s - c_d$. When sales quantity simply equals sales capacity, product disposal does not occur and the MSC is c_s. Once sales quantity surpasses k, the MSC becomes $c_s + c_e$.

The combination of sales capacity constraints and demand shocks leads to several production and sales plans. However, the following three plans can immediately be excluded: (i) product disposal in high demand, (ii) sales-capacity addition in low demand, and (iii) product disposal in low demand *and* sales-capacity addition in high demand. The first two cases are straightforward. Both production and disposal are costly, and additional production after demand shocks are observed is costlier than the sales-capacity building in the first stage. Thus, the firm never chooses sales capacity associated with product disposal in high demand or product addition in low demand. The third exclusion may be somewhat elusive, however, for the following reason. Suppose that sales capacity set in the first stage is not binding in *both* demand states. Then, while a unit increase in sales capacity would raise the *ex post* profit in high demand by c_e to save product addition by one unit, it would reduce the *ex post* profit in low demand by c_d by generating a unit product disposal. Thus, if $c_e > c_d$, the firm chooses sales capacity that does not require additional production in high demand. If $c_e < c_d$, the opposite is true. Therefore, in what follows, I will examine conditions that enable one of the following three cases: (i) neither product disposal nor product addition occurs, (ii) sales capacity expanding in high demand, and (iii) unsold goods being discarded in low demand.

No Disposal and No Additional Production (NN)

Letting k denote the sales capacity set in the first stage, the profit in each demand state is expressed by

$$\pi_s(k) = k(a_s - k - c_s), \quad s = \{H, L\}.$$

For product disposal not to occur, the marginal revenue in the low-demand state evaluated at k needs to be greater than MSC:

$$a_L - 2k \geq c_s - c_d. \tag{2}$$

Likewise, for product addition (capacity expansion) not to occur, the marginal revenue in high demand evaluated at k is no greater than MSC:

$$a_H - 2k \leq c_s + c_e. \tag{3}$$

Both no-disposal and no-expansion conditions suggest that demand variability μ cannot be too large.

No Disposal and Additional Production (NA)

The profit in low demand takes the same form as the NN case: $\pi_L(k) = k(a_L - k - c_s)$. Given high demand, the firm sets the sales quantity by maximizing

$$\max_q q\,[a_H - q - (c_s + c_e)] + c_e k,$$

which leads to the sales quantity and profit in high demand such that

$$q_H = \frac{a_H - (c_s + c_e)}{2}, \quad \pi_H(k) = \frac{1}{4}\,[a_H - (c_s + c_e)]^2 + c_e k.$$

The profit linearly increases as the first-stage capacity k increases, which implies that increases in the sales capacity reduces costly additional production in the second stage.

The condition for no disposal is the same as (2). The condition for additional production to occur is the complement set of (3):

$$a_H - 2k > c_s + c_e. \tag{4}$$

Disposal and No Additional Production (DN)

This is a mirror image of the NA case. Sales quantity in high demand equals the sales capacity, k, and the profit is $\pi_H(k) = k(a_H - k - c_s)$. The firm determine sales quantity in low demand by solving

$$\max_q q\,[a_L - q - (c_s - c_d)] - c_d k,$$

which leads to the sales and profit in low demand such that

$$q_L = \frac{a_L - (c_s - c_d)}{2}, \quad \pi_L(k) = \frac{1}{4}[a_L - (c_s - c_d)]^2 - c_d k.$$

Each unit of increasing first-stage production reduces the profit by c_d due to increasing product disposal.

For production disposal to occur, we need the following condition:

$$a_L - 2k < c_s - c_d, \tag{5}$$

which is the complement set of (2). The condition for excluding additional production in the second stage is the same as (3).

3.2 The First Stage

In the first stage, the firm sets its sales capacity, k, to maximize the expected profits. In the NN case, profit maximization is

$$\max_k \frac{1}{2}k(a_L - k - c_s) + \frac{1}{2}k(a_H - k - c_s) - ck \Leftrightarrow \max_k k(\bar{a} - k - c_s - c).$$

Hence, the firm optimizes k using information about the average demand size, the unit sales cost, and the unit cost for first-stage production. The first-order condition (FOC) gives the optimal sales capacity that the firm does not alter in the second stage:

$$k_n = \frac{\bar{a} - (c + c_s)}{2}.$$

Substituting k_n into (2), the condition for no disposal is as follows.

$$c_d + c \geq \mu. \tag{6}$$

Note that $c_d + c$ is interpreted as the total unit cost for product disposal (we may refer to it as the "long-run" unit cost for product disposal because c and c_d occur in the fist and second-stages, respectively).[6] When the long-run disposal cost surpasses the size of demand fluctuations, the firm would avoid product disposal.

The condition for no additional production in (3) turns to be

$$c_e - c \geq \mu. \tag{7}$$

This condition is also intuitive. The net cost for capacity expansion in the second stage is $c_e - c$ since if the firm instead produces that margin in the first stage, it would cost c. In this sense, $c_e - c$ represents the "long-run" unit cost for capacity expansion, and condition (7) states that when the "long-run" capacity expansion cost is greater than the

[6] Of course, the immediate or "the short-run" marginal cost for dumping unsold output is c_d.

size of demand fluctuations, the firm would never expand its sales capacity even after high demand is observed.

In the capacity-expansion case (the NA case), profit maximization is given by

$$\max_k \frac{1}{2}k(a_L - k - c_s) + \frac{1}{8}[a_H - (c_s + c_e)]^2 + \frac{c_e}{2}k - ck.$$

The FOC gives the optimal sales capacity such that

$$k_a = \frac{a_L - (2c + c_s - c_e)}{2}.$$

Substituting this result into (4), the condition for additional production is

$$c_e - c < \mu, \tag{8}$$

which is the complement set of (7). The no-disposal condition is derived by substituting k_{NA} into (2):

$$c_d + c \geq c_e - c. \tag{9}$$

I now turn to the disposal case (the DN case). The first-stage optimization is formulated as follows.

$$\max_k \frac{1}{8}[a_L - (c_s - c_d)]^2 - \frac{c_d}{2}k + \frac{1}{2}k(a_H - k - c_s) - ck.$$

The FOC gives the optimal sales capacity such that

$$k_d = \frac{a_H - (2c + c_s + c_d)}{2}.$$

Applying this result to (5), the condition for disposal to occur is given by

$$c_d + c < \mu. \tag{10}$$

Substituting k_{DN} into (3), the condition for additional production not to occur is

$$c_d + c < c_e - c, \tag{11}$$

which is the complement set of (9).

Figure 1 presents the parameter configurations that determine the optimal production-and-sales strategy in a (c_d+c, c_e-c) plane. Only when both long-run disposal and additional production costs are greater than the standard deviation of demand, μ, neither product disposal nor capacity expansion occurs. Otherwise, the firm exerts either production disposal or additional production to adjust demand fluctuations. For product disposal to be chosen as the optimal strategy, the long-run disposal cost has to be lower than the standard deviation of demand *and* the long-run marginal cost for capacity expansion. I summarize these findings in the following proposition.

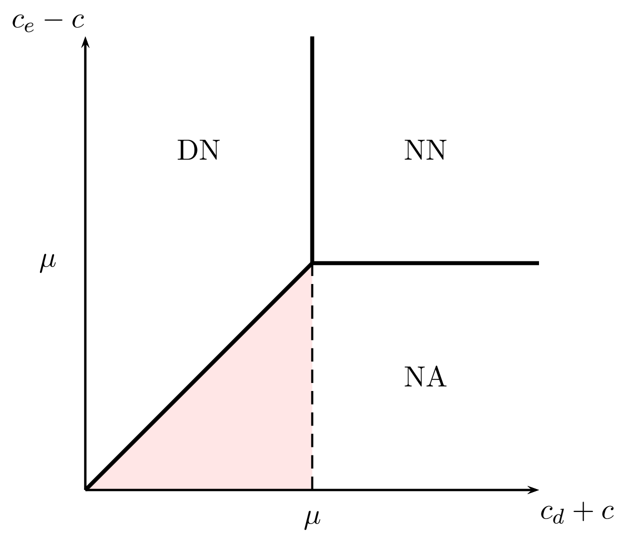

Figure 1: The Configuration of Production and Disposal

Proposition 1. *In a monopoly economy in which sales capacity must be set before demand uncertainty is resolved (the first stage) and costly sales capacity expansion is feasible after demand uncertainty is resolved (the second stage), product disposal occurs only when the sum of the marginal disposal cost and the marginal cost for first-stage production is lower than (i) the standard deviation of variable demand and (ii) the net marginal cost for the second stage production in excess of the sales capacity.*

Interestingly, second-stage production makes the condition for product disposal more stringent. To see this, suppose that the second-stage production is infeasible (sales capacity constraints are perfectly rigid). Then, the condition of $c_d + c < \mu$ is necessary and sufficient for product disposal to occur. However, once the second-stage production becomes available and the long-run marginal cost for the second-stage production, $c_e - c$, is lower than the long-run unit disposal cost, $c_d + c$, the firm is willing to adjust demand fluctuations by capacity expansion rather than product disposal; product disposal would not occur under such a parameter configuration. Figure 1 indicates the parameter range in which second-stage production makes product disposal less profitable by a colored triangle area. The following proposition captures this finding.

Proposition 2. *As the production cost in the second-stage declines, the sales capacity constraints set in the first-stage loosen. As a result, the firm will choose a smaller sales capacity in the first-stage and as a result, the possibility of product disposal reduces.*

Table 1: Summary Results: Monopoly Case

	Sales capacity	Sales quantity	Expected profits
NN	$k_n = \frac{\bar{a}-(c+c_s)}{2}$	$q_{nH} = q_{nL} = k_n$	$\bar{\pi}_n = k_n^2$
NA	$k_a = \frac{a_L-(2c+c_s-c_e)}{2}$	$q_{aH} = \frac{a_H-(c_s+c_e)}{2}$ $q_{aL} = k_a$	$\bar{\pi}_a = \frac{q_{aH}^2+k_a^2}{2}$
DN	$k_d = \frac{a_H-(2c+c_s+c_d)}{2}$	$q_{dH} = k_d$ $q_{dL} = \frac{a_L-(c_s-c_d)}{2}$	$\bar{\pi}_d = \frac{q_{dL}^2+k_d^2}{2}$

Table 1 summarizes the optimized sales capacity (first-stage production), sales in the second-stage, and profits.

3.3 Production Disposal and Social Welfare

This section focuses on the case that the firm selects the disposal strategy (the DN case) and examines both positive and normative aspects of the disposal strategy. The amount of product disposal is given by $D \equiv k_d - q_{dL}$, where q_{dL} is the sales quantity in low demand. Simple algebra leads to

$$D_m = \mu - (c_d + c). \tag{12}$$

We, then, define the disposal intensity as the ratio of disposal to sales capacity (output in the first-stage):

$$DI_m \equiv \frac{2\mu - 2(c_d + c)}{\bar{a} + \mu - (c_s + c_d + 2c)}.$$

The effects of the model's key parameters on disposal intensity are summarized as follows:

Proposition 3. *Ceteris paribus, as average market size increases ($\bar{a} \uparrow$), production cost increases ($c \uparrow$), and as disposal cost increases ($c_d \uparrow$), disposal intensity decreases. In contrast, more variable demand ($\mu \uparrow$) and higher sales cost ($c_s \uparrow$) both raise disposal intensity.*

Proof. See the Appendix. □

All these results are intuitive. Product disposal depends only on demand variability and long-run disposal cost. An increase in the average demand size raises the sales capacity and lowers disposal intensity. Sales cost increases similarly work but in the opposite direction. An increase in the disposal cost urges the firm to decrease the sales capacity and increase the sales in the low state. As a result, disposal intensity decreases. An increase in first-stage production cost lowers sales capacity, but does not affect sales quantity in the low state. Disposal intensity decreases. Finally, as demand becomes more variable, sales amount in low demand falls whereas the firm increases its sales capacity because it also cares about high demand. Consequently, disposal intensity increases.

The effect of demand variability on firm behavior is interesting. The firm's expected profit is expressed by

$$\bar{\pi}_d = \frac{k_d^2 + q_{dL}^2}{2}, \tag{13}$$

which increases as the standard deviation of demand fluctuations increases (see the Appendix). Hence, the risk-neutral firm strictly prefers demand uncertainty when optimally choosing the sales capacity at which product disposal may occur. We record this point as the following proposition.

Proposition 4. *Suppose that the firm optimally chooses the sales capacity that yields product disposal in the case of demand shortage. If the demand becomes more volatile in the manner of mean-preserving spread, then, the firm's average profit increases.*

Proof. See the Appendix. □

Perhaps surprisingly, increasing demand uncertainty and the concomitantly increasing product disposal can result in even higher consumer surplus if consumers are risk-neutral because the welfare gain from increasing supply in the high-demand state always overweighs the welfare loss due to shrinking supply in the low-demand state. Along with Proposition 4, we conclude that as demand uncertainty increases in a mean-preserving spread manner, social welfare (defined as the sum of the firm's profit and consumer surplus) improves when the firm optimally chooses a product disposal strategy. This result is recorded in the following proposition.

Proposition 5. *Suppose that all economic agents are risk-neutral and the firm optimally chooses a sales capacity that generates product disposal in the case of demand shortage. If demand becomes more volatile in the manner of mean-preserving spread, then product disposal increases and, on average, social welfare increases.*

Proof. See the Appendix. □

3.4 Policy Intervention and Efficiency

To what extent does the monopolist that optimally chooses a disposal strategy distort efficiency? To answer this question, we replace the monopolist with the social planner (the government) as a maximizer of expected social welfare. To avoid unnecessary redundancy, the parameter configurations that determine which of the three strategies (NN, NA, and DN) is chosen are the same as in the monopolist case. With this in mind, I focus on the product disposal case (DN).

Consider the second stage after low demand is observed. For a given sales capacity k, the social planner maximizes

$$\max_{q_L}(a_L - q_L)q_L - (c_s - c_d)q_L - c_d k + \frac{q_L^2}{2},$$

where the last term is consumer surplus. As is standard, this maximization problem leads to the marginal cost pricing $p_L = c_s - c_d$ and $q_L = a_L - c_s + c_d$. Hence, the social welfare in the low-demand state is given by

$$W_L(k) = \frac{(a_L - c_s + c_d)^2}{2} - c_d k.$$

When high demand is observed, k is binding and the social planner's pricing is $p_H = a_H - k$, as in the monopolist case. The social welfare in the high-demand state is

$$W_H(k) = k(a_H - k - c_s) + \frac{k^2}{2}.$$

In the first stage, the social planner chooses k to maximize the expected social welfare:

$$\max_k \bar{W}(k) = \max_k \frac{1}{2}\left[W_L(k) + W_H(k)\right] - ck,$$

leading to the socially optimal sales capacity k_D^* as follows:

$$k_{DN}^* = a_H - (c_s + c_d + 2c).$$

The amount of disposal and disposal intensity are given by

$$d^* = 2\left[\mu - (c_d + c)\right] \quad \text{and} \quad DI^* = \frac{2\mu - 2(c_d + c)}{\bar{a} + \mu - (c_s + c_d + 2c)},$$

respectively. Hence, the social planner chooses the exactly the same disposal intensity as the monopolistic firm. The social planner can fix the monopolist's output distortion, doubling the supply of good 1 (as is standard). However, with the same disposal intensity, the social planner proportionally increases product disposal. The monopoly power itself, then, does not distort product disposal. We record this finding in the following proposition.

Proposition 6. *The social planner's ex ante choice of disposal intensity, defined by the ratio of product disposal to sales capacity is the same as the monopolist's choice. The monopolist's disposal intensity is thus socially optimal.*

From Proposition 6, we derive the following corollary:

Corollary 1. *Any* ex ante *taxes and subsidies that affect the disposal intensity worsens social welfare on average.*

Proof. See the Appendix. □

Corollary 1 states that product-disposal reductions by policy intervention before demand uncertainty is resolved (*ex ante* policy intervention) are incompatible with social-welfare maximization. However, since unsold merchandize is discarded in low demand, one may think that *ex post* policy interventions (e.g. product disposal taxes or sales subsidies) may increase sales quantity, resulting in less product disposal and higher social welfare. However, such policies generally cannot avoid the time-inconsistency problem: if the firm correctly expect policy interventions when low demand is realized, it is highly likely to change the first-stage capacity setting.

4 Duopoly

We now proceed to a duopoly model with two symmetric firms facing the demand in (1). Unlike the monopoly case, the choice of competition mode–Bertrand (price) or Cournot (quantity) competition–matters. The present model is a variant of the oligopoly model with capacity constraints, and in this respect, I relay heavily on Maggi (1996) in which either Bertrand or Cournot competition is endogenously determined depending on the rigidity of pre-committed capacities. However, in Maggi (1996), predetermined capacity constraints are always binding at equilibrium because product demand is deterministic. In contrast, the present model frees capacity constraints from always being binding by incorporating stochastic demand, enabling product disposal and capacity expansion.

4.1 Second-Stage Subgame

As in the monopolist case, we start with the subgame in the second stage. Regardless of the realized demand state, if capacity constraints are not binding (either due to capacity expansion or product disposal), Bertrand (price) competition emerges. By contrast, if each

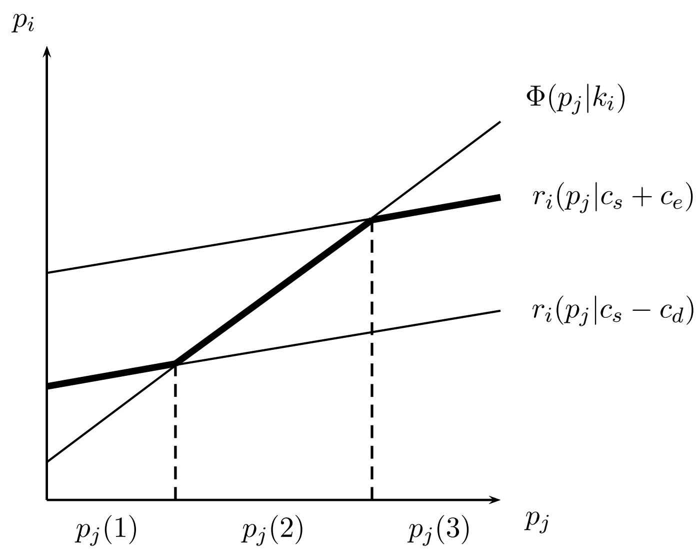

Figure 2: Firm i's Best Response

firm sells its entire predetermined capacity at the market-clearing price, the second-stage competition mode is essentially Cournot, as Kreps and Scheinkman (1983) point out.

In general, for firm i's profit of $\pi_i = (p_i - x)q_i$, where x is the cost of generating one unit of sales, firm i's reaction function in Bertrand competition is defined by $r_i^b(p_j|x) \equiv \arg\max_{p_i} \pi_i$. Using the demand function in (1), the explicit form is given by $r_i^b(p_j|x) = (a + bp_j + x)/2$.[7] This Bertrand reaction function holds when firms either expands capacity or dispose of products. To clarify this point, firm i's capacity-expansion optimization in the second stage is $\max_{p_i}[p_i - (c_s + c_e)]q_i + c_e k_i$ and its product disposal is $\max_{p_i}[p_i - (c_s - c_d)]q_i - c_d k_i$. In both optimization, the pre-committed capacity k_i does not affect firm i's best-reaction prices.

If the firm neither expands capacity nor disposes of products, it sells precisely its entire capacity. Hence, the price-setting subgame is reduced to a capacity-setting fist-stage game so that Cournot competition emerges (Kreps and Scheinkman, 1983). We can define price combinations that exactly satisfy firm i's capacity constraints set in the first stage by $p_i \equiv \Phi_i(p_j|k_i)$ (status quo prices). The explicit form of this "status quo" curve is given by $\Phi_i(p_j|k_i) = a - k_i + bp_j$.

How does firm i react to the rival firms' price p_j? Figure 2 illustrates two Bertrand reaction functions, $r_i^b(p_j|c_s + c_e)$ and $r_i^b(p_j|c_s - c_d)$, and the status quo curve $\Phi(p_j|k_i)$ for

[7] I omit the subscripts $s = \{H, L\}$ for the demand states since discussion here is commonly applicable to both demand states. I will reintroduce the subscripts when I discuss the full-game.

given k_i in a (p_j, p_i) plane. It is easy to verify that $0 < \partial r_i^b/\partial p_j < \partial \Phi_i/\partial p_j < 1$.[8] Relegating rigorous exposition to the Appendix, I here present the intuition about firms i's reaction. When the rival price p_j is low ($p_j(1)$), the residual demand for firm i is low. Thus, firm i's best response is the Bertrand reaction price $r_i(p_j|c_s - c_d)$, which is above the status quo price $\Phi_i(p_j|k_i)$ so that product disposal occurs. When the residual demand is small, firm i's rational behavior is selling only a part of its capacity and maintaining a high price rather than selling up to capacity at a low price. When the rival price falls in the middle rage $p_j(2)$, firm i's demand is relatively large. Hence, the best response is selling the entire capacity at the market-clearing price $\Phi(p_j|k_i)$. Finally, if the rival price p_j is high ($p_j(3)$), the residual demand is large enough for firm i to expand its sales capacity. The best response coincides with the Bertrand reaction price $r_i(p_j|c_s + c_e)$.

In summary, firm i's reaction curve in the second-stage subgame, $R_i(p_j|k_i)$, is as follows:

$$R_i(p_j|k_i) = \begin{cases} r_i^b(p_j|c_s - c_d) & \text{for} \quad p_j < \frac{2k_i - a + (c_s - c_d)}{b} \\ \Phi_i(p_j|k_i) & \text{for} \quad \frac{2k_i - a + (c_s - c_d)}{b} \leq p_j \leq \frac{2k_i - a + c_s + c_e}{b} \\ r_i^b(p_j|c_s + c_e) & \text{for} \quad p_j > \frac{2k_i - a + c_s + c_e}{b}, \end{cases}$$

which is depicted as the bold line in Figure 2. Firm i's best-response price to p_j monotonically increases and always has a slope less than 1. Firm j's best response to p_i is symmetry around a 45 degree line. Hence, the two best reaction functions necessarily intersect only once: The second-stage subgame has a unique equilibrium with a pure strategy for each demand state.

4.2 Capacity-setting Game: Status Quo

Given these subgame reaction functions, I move to first-stage capacity setting. As in the monopolist case, the three cases of NN, NA, and DN exhaust all possible equilibria in the duopoly. First, the subgame best-response functions intersect at the segment in which both pre-committed k_i and k_j are binding in both demand states ($p_j(2)$ in Figure 2). Neither capacity expansion nor product disposal occurs (NN). Since neither firms alter its capacities in the second stage, optimization is reduced to a capacity-choice problem in the first-stage:

$$\max_{k_i} \frac{1}{2}(p_{iL} - c_s)k_i + \frac{1}{2}(p_{iH} - c_s)k_i - ck_i,$$
$$\Leftrightarrow \max_{k_i} \left[E(p_i) - (c + c_s)\right] k_i,$$

[8]To illustrate Figure 2, I assume that the second-stage parameters are configured such that the Bertrand reaction functions intersect with the status quo curve for non-negative p_j, namely, $k_i > (a - x)/2$. This is just made for explanatory convenience.

where $E(p_i) \equiv (p_{iL} + p_{iH})/2 = \bar{a}/(1-b) - k_i/(1-b^2) - bk_j/(1-b^2)$ is the average price. Firm i's optimal capacity choice is given by

$$k_i(k_j) = \frac{1}{2}\left[(1+b)\bar{a} - (1-b^2)(c+c_s) - bk_j\right],$$

which leads to the sales capacity in symmetric equilibrium such that

$$k_n = \frac{(1+b)[\bar{a} - (1-b)(c+c_s)]}{2+b}. \tag{14}$$

The equilibrium price in each state is $p_{ns} = (a_s - k_n)/(1-b)$ for $s = \{H, L\}$. The firms respond to demand fluctuations by adjusting market-clearing prices.

Under what conditions is the equilibrium of the NN case sustainable? The firms' sales capacities are binding in both demand states, so deviations from the equilibrium price in the high-demand state need to be price-cutting by capacity expansion. Likewise, when the low-demand state realizes, possible deviations from the equilibrium price are price-raising by product disposal. The condition for no-expansion is that, given that the rival firm stays on k_n, firm i's marginal revenue from capacity expansion evaluated at k_n is no greater than the marginal cost for capacity expansion $c_s + c_e$ in the high-demand state. Using k_n in (14), the explicit form of this condition is given by

$$c_e - c \geq \frac{\mu}{1-b}. \tag{15}$$

In a similar vein, deviations from the equilibrium price in the low-demand state in order for disposal not to occur, firm i's marginal revenue evaluated at k_n is no less than the marginal cost for product disposal $c_s - c_d$:

$$c_d + c \geq \frac{\mu}{1-b}. \tag{16}$$

These no-deviaition conditions are more stringent than the counterpart conditions in the monopoly case (see (6) and (7)). When the two products are independent, i.e. $b = 0$, (15) and (16) are identical to the non-deviation conditions in monopoly, and as the two goods become more substitutable ($b \uparrow$), either capacity expansion or product disposal is more likely to occur. Intuitively, in duopoly, each firm attempts to steal rents from the rival firm by capacity expansion with price-cutting in high demand or product disposal with price-raising in low demand. The more substitutable the two goods become ($b \uparrow$), the easier the rent sealing is. Thus, the NN case is less sustainable in duopoly than in monopoly. This result is recorded in the following proposition.

Proposition 7. *Capacity expansion in the high-demand state and product disposal in the low-demand state are more likely to emerge in duopoly than in monopoly. This tendency increases in less-differentiated products.*

4.3 Capacity Expansion

Next, the subgame best-response functions in the high-demand state intersect at the segment in which both firms expand their sales capacity ($p_j(3)$ in Figure 2). The pre-committed capacities must be binding in the low-demand state ($p_j(2)$ in Figure 2) for the following reason. Suppose not. Then, the second stage profit-maximization in the low state is written by either capacity expansion ($\max_{p_i}[p_i - (c_s + c_e)]q_i + c_e k_i$) or product disposal ($\max_{p_i}[p_i - (c_s - c_d)]q_i - c_d k_i$). In case of capacity expansion, the firm can raise the second-stage profits by increasing capacity without altering the equilibrium price. The firm continues to increase its capacity until no room exists for capacity expansion, which contradicts with non-biding capacity in the low state. In case of product disposal, the firm can raise the second-stage profits by cutting capacity. However, this necessarily reduces profit in the high state. Thus, if profit gains in the low state overweighs profit loss in the high state, the firm continues to cut its capacity until capacity constraints are binding in the high state, which contradicts with capacity expansion in the high state. If profit gains in the low state is overweighed by profit loss in the high state, the firm must increase initial capacity and product disposal disappears, which is also contradiction. Hence, when the firm expands capacity in the high-demand state, its pre-committed capacity must be binding when the low-demand state is realized.

The maximized profit in the high-demand state is expressed by $\pi_{aH}(k_i) = (q_{aH})^2 + c_e k_a$, where q_{aH} is the Bertrand sales quantity such that

$$q_{aH} = \frac{a_H - (1-b)(c_s + c_e)}{2-b}.$$

The Bertrand price is given by

$$p^b_{aH} = \frac{a_H + (c_s + c_e)}{2-b}.$$

When low demand is realized, the sales capacity is binding, so capacity-choice optimization in the first stage is

$$\max_{k_i} \frac{1}{2}(p_{iL} - c_s)k_i - ck_i + \frac{\pi_{aH}(k_i)}{2} \Leftrightarrow \max_{k_i}[p_{iL} - (2c + c_s - c_e)]k_i,$$

which gives firm i's reaction function such that

$$k_i(k_j) = \frac{1}{2}\left[(1+b)a_L - (1-b^2)(2c+c_s-c_e) - bk_j\right].$$

The sales capacity in symmetric equilibrium is

$$k_a = \frac{(1+b)[a_L - (1-b)(2c+c_s-c_e)]}{2+b}.$$

The Cournot price in the low state is given by

$$p_{aL} = \frac{a_L + (1-b^2)(2c+c_s-c_e)}{(1-b)(2+b)}.$$

For capacity expansion to occur, the marginal revenue from capacity expansion in the high-demand state evaluated at k_a is greater than the marginal cost in capacity expansion c_s+c_e. It is easy to verify that this condition is the complement set of (15): $c_e - c \leq \mu/(1-b)$ (see the Appendix). Likewise, for product disposal not to occur, the marginal revenue from product disposal evaluated at k_a needs to be greater than the marginal cost in product disposal $c_s - c_d$. This no-disposal condition is derived as $c_e - c < c_d + c$, which is the same as in the monopoly case.

4.4 Product Disposal

Finally, the subgame best-response functions in the high state intersect at the segment in which both firms are capacity constrained ($p_j(2)$ in Figure 2) while they intersect at the segment of $p_j(1)$ in the low state. The reason for the capacity-constrained firms in high demand is analogous to that for capacity constrained-firms in low demand in the NA case. The second-stage equilibrium price in the low state is independent from the pre-committed capacity in case of product disposal. Thus, if capacity expansion occurred in the high state, the firm could monotonically increase profits in both states by expanding its capacity in the first-stage. Product expansion cannot occur. Product disposal in the high state also yields room for profit gains by capacity cutting in the first stage. Hence, the pre-committed capacity must be binding in the high state.

The maximized profit in the low-demand state is given by $\pi_{dL}(k_i) = (q_{dL})^2 - c_d k_i$, where q_{dL} is the Bertrand sales quantity such that

$$q_{dL} = \frac{a_L - (1-b)(c_s - c_d)}{2-b}. \tag{17}$$

The Bertrand price is given by

$$p_{dL} = \frac{a_L + (c_s - c_d)}{2-b}.$$

The pre-committed capacities are binding in the high-demand state, so capacity-choice optimization in the first stage is

$$\max_{k_i} \frac{1}{2}(p_{iH} - c_s)k_i - ck_i + \frac{\pi_{dL}(k_i)}{2} \Leftrightarrow \max_{k_i}[p_{iH} - (2c + c_s + c_d)]k_i.$$

Firm i's reaction functions is

$$k_i(k_j) = \frac{1}{2}[(1+b)a_H - (1-b^2)(2c + c_s + c_d) - bk_j],$$

and the sales capacity in symmetric equilibrium, thus, is given by

$$k_d = \frac{(1+b)[a_H - (1-b)(2c + c_s + c_d)]}{2+b}. \tag{18}$$

The Cournot price in the high state is given by

$$p_{dH} = \frac{a_H + (1-b^2)(2c + c_s + c_d)}{(1-b)(2+b)}.$$

For product disposal to occur, the marginal revenue from product disposal in the low-demand state evaluated at k_d is lower than the marginal cost in product disposal $c_s - c_d$. It is easy to verify that this condition is the complement set of (16): $c_d + c \leq \mu/(1-b)$ (see the Appendix). Likewise, for capacity expansion not to occur, the marginal revenue from capacity expansion evaluated at k_d needs to be lower than the marginal cost in capacity expansion $c_s + c_e$. This no-expansion condition is derived as $c_e - c > c_d + c$, which is the same as in the monopoly case.

The disposal intensity in duopoly is given by

$$DI_d = 1 - \frac{q_{dL}}{k_d}, \tag{19}$$

where q_{dL} and k_d are given by (17) and (18), respectively. Although the explicit form of DI_d is more involved than the counterpart in monopoly, it is easy to verify that the characteristics in Proposition 3 holds.

More interesting (and complicated) question is whether market competition may contribute to decreasing disposal intensity (product disposal per output). First, when $b = 0$, the two products are independent so that each firm behaves as a monopolist. This can be seen by setting $b = 0$ in (17) and (18), leading to the monopolistic solutions (see the DN case in Table 1). Next consider $b > 0$. Although it is difficult to evaluate the sign of $\partial DI_m/\partial b$ without any restrictions on b, around $b = 0$, we can obtain

$$\left.\frac{\partial DI_m}{\partial b}\right|_{b=0} = a_L(2c + c_s + c_d) - a_H(c_s - c_d).$$

Thus, if the unit sales cost is lower than the unit disposal cost, the sign is unambiguously positive, which means that price competition in a differentiated duopoly increases disposal intensity relative to monopoly. Suppose that the unit sales cost is large enough to let $c_s - c_d$ be positive. The duopoly disposal intensity is lower than the monopoly disposal intensity if

$$\frac{a_H}{a_L} > \frac{2c + c_s + c_d}{c_s - c_d}$$

holds. The left-hand side, the ratio of market size in the two demand states, is interpreted as variability of demand.[9] The right-hand side is the ratio of the long-run (first-stage) unit cost of capacity building to the short-run (second-stage) unit sales cost, which increases as the first stage unit production cost c or the unit disposal cost c_d increases. Therefore, else equal, duopolistic competition tends to reduce disposal intensity with high demand variability and higher long-run disposal costs ($c_d + c$). These results are summarized in the following proposition.

Proposition 8. *Duopolistic competition in differentiated goods does not always reduce disposal intensity (product disposal per output) from the monopoly benchmark. If the unit sales cost is lower than the unit disposal cost, the disposal intensity in duopoly is higher than in the monopoly. Otherwise, duopolistic competition lowers disposal intensity if demand variability is sufficiently high relative to the ratio of the long-run capacity building cost to the short-run sales cost.*

In summary, I conclude that duopolistic competition reduces the likelihood of firms' choice of the status quo strategy. Either capacity expansion or product disposal is more likely to occur and this tendency is enhanced as the products become better substitutes (Proposition 7). The welfare effect is obviously positive because firms adjust first-stage sales capacities more flexibly, in addition to standard welfare gains from becoming less monopolistic. Product disposal becomes more frequent. However, with demand uncertainty, product disposal itself is not irrational. Furthermore, duopolistic competition leads to either higher or lower disposal intensities depending on the relative size of demand variability to production costs. Although difficult to obtain clearcut conclusions, it is fair to say that when product demand is highly variable, duopolistic competition contributes to lowering disposal intensity, relative to benchmark monopoly (Proposition 8).

[9] It is easy to verify that a_H/a_L increases as μ increases.

5 Concluding Remarks

This paper investigates the effect of competition among firms on their decisions about the pre-committed production (sales capacities), sales quantities, and product disposal in imperfect competition. To obtain benchmark results, the analysis begins with a monopoly case. Using the benchmark, I describe a complete set of parameter configurations with which one of the following three cases emerges: (i) neither capacity expansion nor product disposal occurring regardless of demand realization; (ii) only capacity expansion occurring when high demand is realized; and (iii) only product disposal occurring when low demand is realized. The present paper finds that the flexibility in capacity constraints, such as responding to high demand by spot production, substantially reduces the likelihood of product disposal. In addition, the paper confirms that disposal intensity chosen by a monopolist is socially optimal, which implies that even the social planner who maximizes the expected social welfare before demand uncertainty is resolved, cannot reduce product disposal.

With these benchmark results, the paper examines the effects of duopolistic competition in differentiated goods on product disposal. Main findings are as follows. First, duopolistic competition increases the likelihood that either capacity expansion or product disposal occurs. This tendency is enhanced as the two goods are better substitutable. In this sense, competitive markets observe product disposal more frequently. Second, duopolistic competition leads to either higher or lower disposal intensities depending on the relative size of demand variability to production costs.

There are issues requiring further investigation. First, the present model features oligopolistic competition in differentiated goods, which eliminates the possibility of mixed-strategy equilibria. Nevertheless, analytical solutions are somewhat complex which reduces the model's tractability. The assumption of linear demand is hopefully relaxed to check the robustness of the obtained results. All these are left for future work.

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A Proofs

A.1 Proposition 2

$\frac{\partial DI}{\partial \bar{a}} < 0$ and $\frac{\partial DI}{\partial c_s} > 0$ are immediate. Since the sales quantity is positive,

$$q_L > 0 \quad \Leftrightarrow \quad \bar{a} > \mu + c_s - c_d \tag{A.1}$$

holds, which proves

$$\frac{\partial DI}{\partial c} = -\left[\bar{a} + \mu - (2c + c_d + c_s)\right] + 2\mu - 2(c + c_d) = -(\bar{a} - \mu + c_d - c_s) < 0.$$

To obtain $\frac{\partial DI}{\partial \mu} > 0$ and $\frac{\partial DI}{\partial c_d} < 0$, subtracting $c + c_s$ from the both sides of (A.1) yileds

$$\bar{a} - (c + c_s) > \mu - (c_d + c) > 0$$

which proves

$$\frac{\partial DI}{\partial \mu} = \bar{a} + \mu - (2c + c_d + c_s) - \left[\mu - (c + c_d)\right] = \bar{a} - (c + c_s) > 0,$$
$$\frac{\partial DI}{\partial c_d} = -\left[\bar{a} + \mu - (2c + c_d + c_s)\right] + \mu - (c + c_d) = -\bar{a} + c + c_s < 0.$$

A.2 Proposition 4

Since $\partial k_d/\partial \mu = 1/2$ and $\partial q_{dL}/\partial \mu = -1/2$,

$$\frac{\partial \bar{\pi}_d}{\partial \mu} = 2k_d \frac{\partial k_d}{\partial \mu} + 2q_{dL}\frac{\partial q_{dL}}{\partial \mu} = k_d - q_{dL} > 0.$$

A.3 Proposition 5

The average social welfare is defined by the sum of the average profits and the average consumer surplus. In the DN case, the average social welfare is

$$\bar{W}_d = \bar{\pi}_d + \frac{1}{2}\left[\frac{k_d^2 + q_{dL}^2}{2}\right] = \frac{3}{4}\left[\frac{k_d^2 + q_{dL}^2}{2}\right].$$

Thus, from Proposition 4, we can immediately conclude that $\partial \bar{W}_d/\partial \mu > 0$.

A.4 Corollary 1

How about a disposal tax? The government chooses t_d to maximize the following (average) social welfare:

$$\begin{aligned}\bar{W}_d(t_d) &= \bar{\pi}_d(t_d) + \frac{1}{2}\left[\frac{k_d^2(t_d) + q_{dL}^2(t_d)}{2}\right] + \frac{1}{2}t_d\left[k_d(t_d) - q_{dL}(t_d)\right] \\ &= \frac{3}{4}\left[k_d(t_d)^2 + q_{dL}(t_d)^2\right] + \frac{1}{2}t_d\left[k_d(t_d) - q_{dL}(t_d)\right]\end{aligned}$$

Noting that $\partial k_d/\partial t_d = -1/2$ and $\partial q_{dL}/\partial t_d = 1/2$, the derivative of $\bar{W}_d$ with respect to t_d is given by

$$\begin{aligned}\frac{\partial \bar{W}_d(t_d)}{\partial t_d} &= \frac{3}{2}\left[k_d\frac{\partial k_d}{\partial t_d} + q_{dL}\frac{\partial q_{dL}}{\partial t_d}\right] + \frac{1}{2}\left[k_d - q_{dL}\right] + \frac{1}{2}t_d\left[\frac{\partial k_d}{\partial t_d} - \frac{\partial q_{dL}}{\partial t_d}\right] \\ &= \frac{3}{4}\left[-k_d + q_{dL}\right] + \frac{1}{2}\left[k_d - q_{dL}\right] - \frac{1}{2}t_d = \frac{q_{dL} - k_d}{4} - \frac{1}{2}t_d.\end{aligned}$$

Since $d = k_d - q_{dL} > 0$, we have

$$\left.\frac{\partial \bar{W}_d(t_d)}{\partial t_d}\right|_{t_d=0} < 0$$

B Technical Note

B.1 Cournot Competition in Differentiated Goods

Suppose that direct demand function for firm i is given by

$$q_i = a - b_1 p_i + b_2 p_j, \tag{B.1}$$

where p_j it the price of rival firm's good. The corresponding inverse demand is

$$p_i = \alpha - \beta_1 q_i - \beta_2 q_j,$$

where $\alpha = a/(b_1 - b_2)$, $\beta_1 = b_1/(b_1^2 - b_2^2)$, and $\beta_2 = b_2/(b_1^2 - b_2^2)$.

In Cournot competition, firms compete in quantities. Firm i choose q_i to maximize $(\alpha - \beta_1 q_i - \beta_2 q_j)q_i - xq_i$, taking as given q_j. The Cournot reaction function of firm i to output q_j is $(\alpha - x - \beta_2 q_j)/2\beta_1$. By symmetry, the Cournot equilibrium is

$$q^c = \frac{\alpha - x}{2\beta_1 + \beta_2}, \quad p^c = \frac{\beta_1 \alpha + (\beta_1 + \beta_2)x}{2\beta_1 + \beta_2}.$$

However, for deriving firm i's reaction function, it is convenient to consider profit-maximizing firm i that chooses p_i with maintaining the rival firm's quantity q_j constant. Note that such choice of p_i necessarily affects p_j (the demand for firm j which is analogous to (B.1) and

$$\frac{\partial p_j}{\partial p_i} = \frac{b_2}{b_1}.$$

Given the marginal production cost x, firm i's profit-maximization with respect to p_i (instead of q_i) yields firm i's reaction function in *price* space. Noting that $\partial q_i/\partial p_i = -b_1 + b_2 \partial p_2/\partial p_1 = -b_1 + b_2^2/b_1$, the first-order condition is given by

$$\begin{aligned}
&q_i + p_i \frac{\partial q_i}{\partial p_i} = x \frac{\partial q_i}{\partial p_i} \\
\Leftrightarrow & a - b_1 p_i + b_2 p_j + \frac{b_2^2 - b_1^2}{b_1} p_i = \frac{b_2^2 - b_1^2}{b_1} x \\
\Leftrightarrow & ab_1 + b_1 b_2 p_j + (b_2^2 - 2b_1^2) p_i = (b_2^2 - b_1^2) x \\
\Leftrightarrow & p_i^c = \frac{1}{2b_1^2 - b_2^2} \left[ab_1 + b_1 b_2 p_j^c + (b_1^2 - b_2^2) x \right]. \qquad \text{(B.2)}
\end{aligned}$$

The slope of this reaction function is $\frac{\partial p_j}{\partial p_i} = (2b_1^2 - b_2^2)/b_1 b_2$ is greater than 1 since $b_1 > b_2$ is assumed.

Due to the symmetric structure of the model, we can analogously derive firm j's reaction function. In equilibrium, $p_i = p_j$, substituting this into (B.3), we obtain

$$p_i^c = p_j^c = \frac{ab_1 + (b_1^2 - b_2^2)x}{(b_1 - b_2)(2b_1 + b_2)}, \quad q_i^c = q_j^c = \frac{(b_1 + b_2)\left[a - (b_1 - b_2)x\right]}{2b_1 + b_2}.$$

In Bertrand competition, $\partial q_i/\partial p_i = -b_1$. Thus, firm i's reaction function is derived by

$$\begin{aligned}
&q_i + p_i \frac{\partial q_i}{\partial p_i} - \frac{\partial q_i}{\partial p_i} x = 0 \\
\Leftrightarrow & a - b_1 p_i + b_2 p_j - b_1 p_i + b_1 x = 0 \\
\Leftrightarrow & p_i^b = \frac{a + b_1 x + b_2 p_j^b}{2b_1}. \qquad \text{(B.3)}
\end{aligned}$$

As is standard, the slope of this reaction function is greater than 1 in a (p_i, p_j) plane since $b_1 > b_2$ is assumed. It is straightforward to check the slope of the Bertrand-reaction function is steeper than that of the Cournot-reaction function.

By symmetry, the price and sales quantity in the Bertrand equilibrium is as follows:

$$p_i^b = p_j^b = \frac{a + b_1 x}{2b_1 - b_2}, \quad q_j^b = q_j^b = \frac{b_1 \left[a - (b_1 - b_2)x\right]}{2b_1 - b_2}. \tag{B.4}$$

It is straightforward to check $p_i^b < p_i^c$ since

$$\begin{aligned}
&\frac{ab_1 + (b_1^2 - b_2^2)x}{2b_1^2 - b_2^2 - b_1 b_2} > \frac{a + b_1 x}{2b_1 - b_2} \\
\Leftrightarrow &[ab_1 + (b_1^2 - b_2^2)x][2b_1 - b_2] - [a + b_1 x][2b_1^2 - b_2^2 - b_1 b_2] > 0 \\
\Leftrightarrow &2ab_1^2 - ab_1 b_2 + (2b_1 - b_2)(b_1^2 - b_2^2)x - 2ab_1^2 + ab_2^2 + ab_1 b_2 - b_1[2b_1^2 - b_2^2 - b_1 b_2]x > 0 \\
\Leftrightarrow &(2b_1^3 - 2b_1 b_2^2 - b_1^2 b_2 + b_2^3)x + ab_2^2 - (2b_1^3 - b_1 b_2^2 - b_1^2 b_2)x > 0 \\
\Leftrightarrow &b_2^2 \left[a - (b_1 - b_2)x\right] > 0,
\end{aligned}$$

so that the more differentiated the goods are ($b_2 \downarrow$), the smaller is the difference between the Cournot and Bertrand prices, and in the extreme sitution of independent goods ($b_2 = 0$) the difference is zero. The type of competition becomes less important, the less related the goods are.

Consider first profits in Cournot competition:

$$\begin{aligned}
\pi_i^c &= (p_i^c - x)q_i^c \\
&= \left[\frac{ab_1 + (b_1^2 - b_2^2)x}{2b_1^2 - b_2^2 - b_1 b_2} - x\right] q_i^c \\
&= \left[\frac{ab_1 + (b_1^2 - b_2^2)x - (2b_1^2 - b_2^2 - b_1 b_2)x}{2b_1^2 - b_2^2 - b_1 b_2}\right] q_i^c \\
&= b_1 \left[\frac{a - (b_1 - b_2)x}{2b_1^2 - b_2^2 - b_1 b_2}\right] q_i^c = b_1(b_1^2 - b_2^2) \left[\frac{a - (b_1 - b_2)x}{2b_1^2 - b_2^2 - b_1 b_2}\right]^2 = \frac{b_1}{b_1^2 - b_2^2} (q_i^c)^2
\end{aligned}$$

Then, profits in Bertrand competition is

$$\begin{aligned}
\pi_i^b &= (p_i^b - x)q_i^b \\
&= \left[\frac{a + b_1 x}{2b_1 - b_2} - x\right] q_i^b \\
&= \left[\frac{a - (b_1 - b_2)x}{2b_1 - b_2}\right] q_i^b = b_1 \left[\frac{a - (b_1 - b_2)x}{2b_1 - b_2}\right]^2 = \frac{(q_i^b)^2}{b_1}
\end{aligned}$$

Since the goods are substitutes ($\beta_2 > 0$), low prices mean low profits, and Cournot profits are larger than Bertrand profits: $\pi^c > \pi^b$.

$$
\begin{aligned}
&\frac{b_1}{b_1^2 - b_2^2}(q_i^c)^2 > \frac{(q_i^b)^2}{b_1} \\
\Leftrightarrow &\frac{b_1^2}{(b_1+b_2)(b_1-b_2)} > \left[\frac{b_1(2b_1+b_2)}{(b_1+b_2)(2b_1-b_2)}\right]^2 \\
\Leftrightarrow &\frac{b_1+b_2}{b_1-b_2} > \left[\frac{2b_1+b_2}{2b_1-b_2}\right]^2 \\
\Leftrightarrow &2b_2^3 > 0,
\end{aligned}
$$

which is true as long as $b_2 > 0$.

B.2 The second-stage subgames

Consider the case of $x = c_s - c_d$ first. When firm i reacts to firm j's price with the Bertrand prices, firm i's profits is expressed by a function of p_j as follows:

$$
\begin{aligned}
\pi_i^b(p_j|k_i) &= [r_i^b(p_j|k_i) - x]q_i - c_d k_i \\
&= \left(\frac{a+bp_j+x}{2} - x\right)\left(a + bp_j - \frac{a+bp_j+x}{2}\right) - c_d k_i = \frac{1}{4}(a - x + bp_j)^2 - c_d k_i,
\end{aligned}
$$

which is a quadratic function of p_j and monotonically increasing in the domain of $p_j > 0$. When firm i reacts to firm j's price with the status quo prices, firm i's profits is expressed by a function of p_j as follows:

$$
\pi_i^c(p_j|k_i) = [\Phi_i(p_j|k_i) - c_s]\,k_i = (a - k_i - c_s + bp_j)k_i,
$$

which is a linear function of p_j with the slope bk_i.

I claim that $\pi^b - \pi^c \geq 0$ for all p_j. To see this,

$$
\begin{aligned}
&\frac{1}{4}[a - (c_s - c_d) + bp_j]^2 - c_d k_i - (a - k_i - c_s + bp_j)k_i \\
=&\frac{1}{4}[a - (c_s - c_d) + bp_j]^2 - [a - (c_s - c_d) + bp_j]k_i + k_i^2 \\
=&\left[\frac{a - (c_s - c_d) + bp_j}{2} - k_i\right]^2 \geq 0,
\end{aligned}
$$

which reveals that π^c is a tangent of π^b and the tangent point is given by

$$
(p_j, p_i) = \left(\frac{2k_i - a + (c_s - c_d)}{b}, k_i + c_s - c_d\right).
$$

At the tangent point, firm i's capacity is entirely sold. Thus, for $p_j > [2k_i - a + (c_s - c_d)]/b$, firm i cannot have products for disposal, which restricts the rival price range where firm i

optimally disposes of its product such that $p_j < [2k_i - a + (c_s - c_d)]/b$, which coincides with $p_j(1)$ in Figure 2.

For the case of $x = c_s + c_e$, $\pi^b - \pi^c \geq 0$ can be analogously shown. Likewise, π^c is a tangent of π^b in the case of $x = c_s + c_e$ and the tangent point is given by

$$(p_j, p_i) = \left(\frac{2k_i - a + c_s + c_e}{b}, k_i + c_s + c_e\right).$$

At the tangent point, firm i's capacity is entirely sold. Thus, only for the range of $p_j > [2k_i - a + c_s + c_e]/b$, capacity expansion is relevant. This is $p_j(3)$ in Figure 2. In sum, firm i's reaction schedule in the second-stage subgame, $R_i(p_j|k_i)$, is as follows:

$$R_i(p_j|k_i) = \begin{cases} r_i^b(p_j|c_s - c_d) & \text{for} \quad p_j < \frac{2k_i - a + (c_s - c_d)}{b} \\ \Phi_i(p_j|k_i) & \text{for} \quad \frac{2k_i - a + (c_s - c_d)}{b} \leq p_j \leq \frac{2k_i - a + c_s + c_e}{b} \\ r_i^b(p_j|c_s + c_e) & \text{for} \quad p_j > \frac{2k_i - a + c_s + c_e}{b}. \end{cases}$$