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Comparing the forecastability of alternative quantitative models: a trading simulation approach in financial engineering

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Abstract

In this article, we build Box-Jenkins ARMA model and ARMA-GARCH model to forecast the returns of shanghai stock exchange composite index in financial engineering. Out-of-sample forecasting performances are evaluated to compare the forecastability of the two models. Traditional engineering type of models aim to minimize statistical errors, however, the model with minimum engineering type of statistical errors does not necessarily guarantee maximized trading profits, which is often deemed as the ultimate objective of financial application. The best way to evaluate alternative financial model is therefore to evaluate their trading performance by means of trading simulation. We find that both quantitative models are able to forecast the future movements of the market accurately, which yields significant risk adjusted returns compared to the overall market during the out-of-sample period. In addition, although the ARMA-GARCH model is better than the ARMA model theoretically and statistically, the latter outperforms the former with significantly higher trading performances.

1. Introduction

Forecasting the returns of stock markets has always been an important academic research area, while there is still debate on whether stock markets' returns are predictable at all: for instance, Campbell (1987) [1], Fama and French (1989) [2], and recently, Hjalmarsson (2010) [3], documented some out-of-sample predictability. On the other hand, however, Welch and Goyal (2007)'s comprehensive study shows little support for out-of-sample stock return predictability [4]. Among the numerous forecasting methods proposed, Box-Jenkins (1976)'s ARMA model is one of the most commonly used modeling and forecasting techniques. It is generally referred to as autoregressive moving average (ARMA) model. This methodology assumes that changes of time series data are related to its own past value. It creates an autocorrelation regression mode, usually estimated in-sample and then extended for out-of-sample

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forecasting [5]. Since data from financial markets are usually non-stationary, analysts can difference the original time series. However, it should be noted that, when applying Box-Jenkins method, the basic assumption is: the future pattern of a time series repeats its past pattern. It was well argued that this assumption can only be met for short-term, while forecasting accuracy tends to deteriorate over longer horizon. Later, the Autoregressive Conditional Heteroskedasticity Model (ARCH) by Engle 1982 [6], and the more generalized GARCH model by Bollerslev 1986's [7], were proposed to explain the conditional variance. These methods and their variants were used to explain volatility of some mature and emerging stock markets (e.g. Akgiray 1989 [8]; Kearney and Daly 1998 [9]; Tay and Zhu 2000 [10]; Khil and Lee 2002 [11]; and more recently, Teresiene 2009 [12]; Demireli, E. 2010 [13]).

Due to the lack of alternative investment opportunities, stock markets and their investments have contributed significant parts of investment portfolios in China. Although quite a few previous studies have been conducted to explore and model the price behaviors of stock markets, there has been little consensus on the choice of different models upon their merits or forecastability. Among these studies, Box Jenkins's ARMA model and its variant ARMA-GARCH model were adopted to forecast the return of the Chinese stock markets (among others, see more recently, Zhao 2008 [14], Huang and Wang 2010 [15], He 2011 [16]).

Among various financial engineering models, the one that has the minimal statistical error is often deemed optimal. However, the ultimate goal of investment is to make profit. The prediction model with minimal statistical errors does not necessarily guarantee maximized investment profits at the same time. The major objective of this study is thus to compare the forecastability of the ARMA model with the ARMA-GARCH model, which integrates ARMA and ARCH theories on China's Shanghai Composite index. However, unlike prior quantitative researches in the Chinese market, this study also applies investment simulation, where the projected investments profits are used to evaluate the predictability of these models. More specifically, investment simulation assumes that all models are applied with stock market investment strategies, and alternative models are compared based on their out-of-sample investment profitability.

2. Data and methodology

The financial data used in this research is the daily closing price of Shanghai Composite Index. The entire data set covers the period from 1st Jan 2001 to 31st May 2011, a total of 2562 days of observations. The data set is divided into two periods: the first data period is from 1st Jan 2001 to 31st May 2010 (2321 days of observations) while the second period is from 1st Jun 2010 to 31st May 2011 (241 days of observations). The first period, assigned to in-sample estimation, is used to determine the specifications of the models and to estimate their parameters. The second period is reserved for out-of-sample evaluation and performance comparison.

2.1. Autoregressive moving-average methodology

Box and Jenkins (1976) proposed a type of univariate time series model, which becomes very popular in financial markets. The model, known as ARMA model, combines a moving average process with a linear difference equation. The model is in such a way that the time series variable is explained purely by its own past values. The reason it is so popular in financial markets is that frequently the only data available is the single time series that of interest. The basic assumption is that the series of past value holds useful information about the future, which can be used for forecasting. The general form of an ARMA(p,q) model can be written as:

$$Y_t = \alpha + \sum_{h=1}^{p} \beta_h Y_{t-h} + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} \tag{1}$$

If $q = 0$, this process is called an AR(p) model and if $p = 0$, it is just a simple MA(q) model. It should be noted that the equation above requires that the variable Y_t to be stationary. If, however, the variable of interest is not stationary, the sequence is said to be an integrated process, which can then be explained by an autoregressive integrated moving average (ARIMA) model. In this latter case, the non-stationary series Y_t needs to be transformed into stationary series, usually by means of differencing.

The selection of the appropriate ARIMA model in this study follows the Box-Jenkins 1976 three-stage model identification strategy. Based on the obtained identification of the ARMA model, we make a static forecast over the out-of sample periods.

2.2. ARMA-GARCH model

The ARMA model aims to correct for the autocorrelation, which is common with financial time series. However, the original ARMA mean model assumes that the variances are constant over time. While in fact, it has been well documented that variances of financial time series (e.g. the stock returns) are conditional over time. The current level of variance is conditional upon the level of previous variances, that is, the time series of variance itself is auto-correlated. This raised questions regarding the validity of the original ARMA, which ignores the existence of conditional variance.

The ARCH model of Engle 1982 has been seen as a revolution in modelling and forecasting volatility. It was further generalized by Bollerslev 1986 as the GARCH model. The GARCH type models assume that volatility changes over time in an autoregressive manner.

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{2}$$

where $\omega > 0$, $\alpha_i \geqslant 0$, $\beta_j \geqslant 0$, and $\Sigma \alpha_i + \Sigma \beta_j < 1$, in order to ensure the continuing effect of various shocks to the market. h_t is the predicted value of market volatility at time t. It has positive linear correlations with the predicted volatility values, h_{t-j} of previous several periods and the square of the residuals that reflect the extent of impact of previous market news on the market, ε_{t-i}^2. Thus, this autocorrelation-based model explained the phenomenon that variance as a measure of market volatility changes over time. In this paper, we adopt GARCH(1,1), the one most widely used among GARCH(p,q) models. For the representation of GARCH(1,1) please refer to Vasilellis and Meade (1996)[17].

If we adopt the above ARMA (p, q) model as an equation to calculate the mean value, and regard the GARCH (1,1) to be a variance measuring model, the ARMA-GARCH model has the basic form as in the following:

$$Y_t = \alpha + \sum_{h=1}^{p} \beta_h * Y_{t-h} + \sum_{i=1}^{q} \gamma_i * \varepsilon_{t-i}$$
$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{3}$$

ARMA-GARCH model has a similar modeling method as that of ARMA model: it removes the variables without significant coefficients and retains only the variables with significant coefficients in these equations. In-sample estimation was then extended to out-of-sample forecasting. It should be noted that what we ultimately forecast is Y_t, the mean return rather than ht the variance.

2.3. Benchmark for trading simulation

As usual, the performance of the stock market index during the same out-of-sample period was used as a benchmark to examine the probability of these models. Needless to say, any model that outperforms the overall market on a risk adjusted basis during that period is usually regarded as a successful model.

3. Empirical findings

3.1. Out-of-sample statistical performance

The following four statistical measures were calculated to evaluate the out-of-sample statistical performance: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

Table 1. Out-of-sample performance

	ARMA	ARMA-GARCH	Market
RMSE	0.01297	0.01281	
MAE	0.00985	0.00962	
MAPE	290	154	
Cumulative Return	8.59%	5.48%	5.67%
Standard Deviation	13.56%	16.11%	19.92%
Information Ratio	0.63	0.34	0.28

As it can be seen from the above table 1, the ARMA-GARCH model outperforms the ARMA model in all of the three statistical measures.

3.2. Out-of-sample trading performance

As explained earlier, a model that has the minimal statistical errors does not necessarily generate the highest return, which is the ultimate goal of forecasting. From that perspective, it is necessary to evaluate the merits of the models based on the performance of a trading strategy. The comparison of different models can thus be carried out based on evaluation of those merits. At this stage, the relative performance of the models is measured by risk adjusted return: the information ratio, calculated by the annualized return divided by the annualized volatility.

As shown in the above table 1, the ARMA model outperforms the ARMA-GARCH model quite significantly in terms of the return, risk and the risk adjusted information ratio. Both quantitative models are able to outperform the benchmark market during the out-of-sample period.

4. Concluding remarks and further research

In this article, we have built Box-Jenkins ARMA model and ARMA-GARCH model to forecast the returns of shanghai stock exchange composite index. Out-of-sample forecasting performance is evaluated to compare the forecastability of the two models. From a statistical point of view, the ARMA-GARCH model outperform with all of the three commonly used statistical measures.

Traditional engineering type of models aim to minimize statistical errors, however, the model with minimum forecasting errors in statistical term does not necessarily guarantee maximized trading profits, which is often deemed as the ultimate objective of financial application. The best way to evaluate alternative financial model is therefore to evaluate their trading performance by means of trading simulation.

We found that both ARMA and ARMA-GARCH models were able to forecast the future movements of the market, which yields significant risk adjusted returns compared to the overall market during the out-of-sample period. In addition, although the ARMA-GARCH model is better than the ARMA model theoretically and statistically, the latter outperforms the former with significantly higher trading measures.

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