Kingston University London

Mathematics eAssessment using Numbas: Experiences at Kingston with a partially "flipped" classroom

1. Background & motivation
2. Approach adopted
3. Findings
4. Discussion

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Surely it's just Linear Algebra, why change anything?

There's a lot of change underway at Kingston

Building programmes.

Revised Academic Framework:

- 30 credit modules,
- rationalised courses,
- greater emphasis on feedback,
- fewer summative assessments…

Pegg, Ann (2013). *'We think that's the future': Curriculum reform initiatives in higher education*. HEA.

Aims?

- Coverage
  - Introductory Linear Algebra:
    - Matrices, Gaussian Elimination, Eigenvectors
- Efficient and Effective Engagement
  - over 4 weeks, ~100 students,
  - despite timetable & classroom constraints
  - using appropriate techniques/tools
    - eAssessment
    - Matlab

The “flipped” classroom approach?

- Hopes
  - Changing this

  - Into this?

- Fears
  - Engagement is lost

  - Chaos ensues!

It's not exactly new…

1967

- Hartley J., Cameron A. *Some observations on the efficiency of lecturing*. Educ. Rev. 1967

1981

- Gibbs G., *Twenty terrible reasons for lecturing*, SCED Occasional Paper 8. 1981

2014

- *"The More I Lecture, The Less I Know If They Understand*." 6th February 2014 (online)
  - "The lecturer is prone to self-deception … egocentrism and confirmation bias"

What did we do? Partial Flip + eAssessment.

- *Partial* as there was no structured offline interaction
  - *c.f.* Eric Mazur's *Peer Instruction* approach: "concepts-in-the-classroom"
- Formative eAssessment
  - with a miniscule marks incentive (~1%)
  - to encourage students to self-test
  - leading to *summative* eAssessment

How did we use eAssessment? Why use Numbas?

- Formative eAssessment
  - National HE STEM Programme
  - Numbas (Newcastle & mathcentre.ac.uk)
  - Random parameters encourage students to

    learn the *method*

    rather than

    learning the *question*

Stage 1: Advance material

- Gapped notes (Word & PDF) with separate formative eAssessment

Gaussian Elimination

How do we solve a 2x2 system of linear equations?

Example 4

$$\begin{aligned} 2x_1 + 2x_2 &= 4 \quad (1) \\ 4x_1 - 3x_2 &= 1 \quad (2) \end{aligned}$$

Which operations did we use?

We added a multiple of equation (1) to the equation (resulting equation by a non-zero constant to get the solution for x_2.

<u>None of these operations changed the solution.</u>

Exercises

Use Gaussian elimination to solve the following linear systems:

1. $$\begin{aligned} x_2 + x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 4 \\ x_1 - x_2 + x_3 &= 5 \end{aligned}$$

2. $$\begin{aligned} 2x_1 + 4x_2 - x_3 &= -5 \\ x_1 + x_2 - 3x_3 &= -9 \\ 4x_1 + x_2 + 2x_3 &= 9 \end{aligned}$$

3. $$\begin{bmatrix} 1 & -5 & 1 \\ 10 & 0 & 20 \\ 5 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \end{bmatrix} \quad \begin{bmatrix} 3 \end{bmatrix}$$

James Denholm-Price & Peter Soan | 1st May 2014

Formative eAssessment using Numbas with feedback

Previous | **Question 2** | Next

Question 1
2 marks.

Question 2
Unanswered. ✖

Question 3
11 marks.

Total 0/19

End Exam

Inverse of a 2×2 matrix

Suppose M is a 2×2 matrix and $\det(M) = \Delta \neq 0.$

Then M is invertible and:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow M^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} \frac{d}{\Delta} & -\frac{b}{\Delta} \\ -\frac{c}{\Delta} & \frac{a}{\Delta} \end{pmatrix}$$

Applying this to these examples we obtain:

b)

$$A^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ \frac{3}{8} & \frac{1}{4} \end{pmatrix}$$

c)

$$B^{-1} = \begin{pmatrix} \frac{5}{28} & \frac{1}{28} \\ \ldots & \ldots \end{pmatrix}$$

Matlab & fractions

- Matlab makes (some) answers easy

  …even the ones requiring rational input

  …but then they’re learning Matlab too :-)
- However key methods like Gaussian Elimination aren’t so badly affected

Stage 2: In-class discussion.

- Topics for further discussion were identified by electronic voting

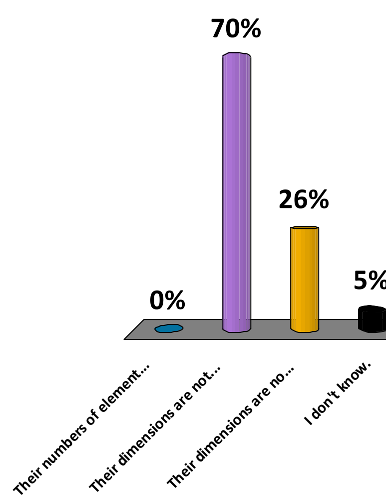
Which of the following is the best answer to why (3x2) and (2x2) matrices can't be added to, or subtracted from, one-another?

A. Their numbers of elements is not equal.

B. Their dimensions are not identical.

C. Their dimensions are not compatible, e.g. $(n \times \boldsymbol{m})$ and $(\boldsymbol{m} \times p)$.

D. I don't know.



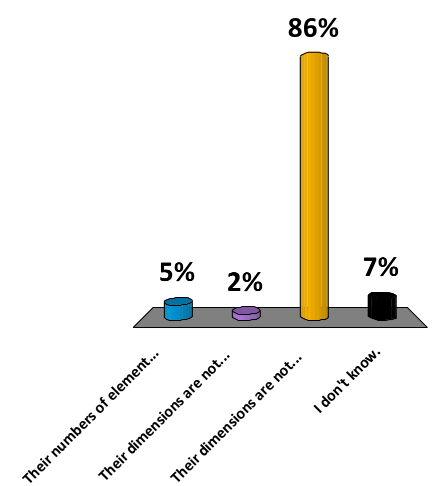
Which of the following is the best answer to why (3x2) and (3x2) matrices can't be multiplied together?

A. Their numbers of elements is not equal.

B. Their dimensions are not identical.

C. Their dimensions are not compatible, e.g. $(n\times\boldsymbol{m})$ and $(\boldsymbol{m}\times p)$.

D. I don't know.



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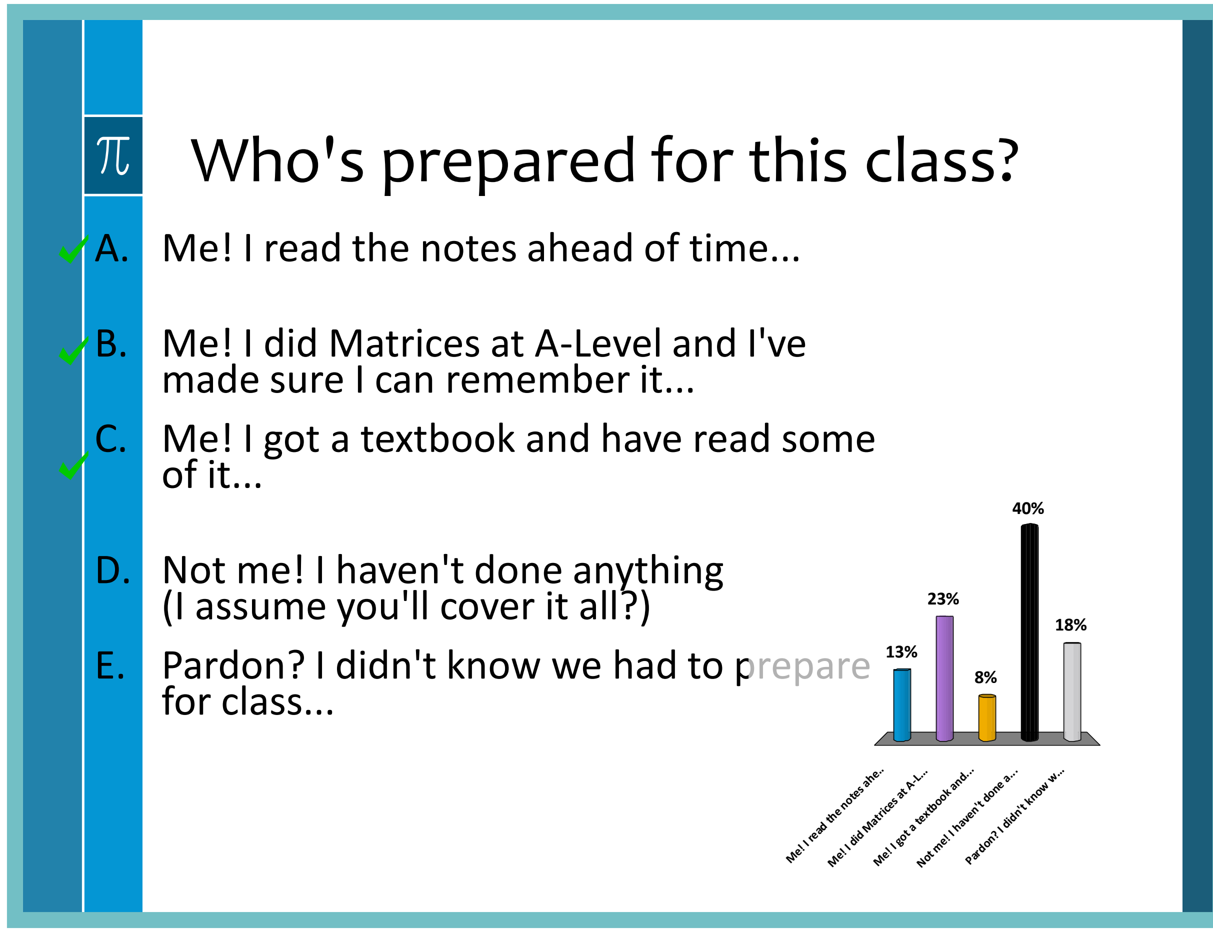
Did students prepare for class?

- Little prep-work, but evidence in marks is blurred by spread of student A-level experience



$$\left(\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 1 & \square & \square \\ 0 & 0 & \square & \square \end{array}\right)$$

From this you should find:

$z =$ $\square$

d)

From the second row of the reduced matrix you find an equation involving only y and z and using your value for z we find:

$y =$ $\square$

Then using the first row we have the equation :

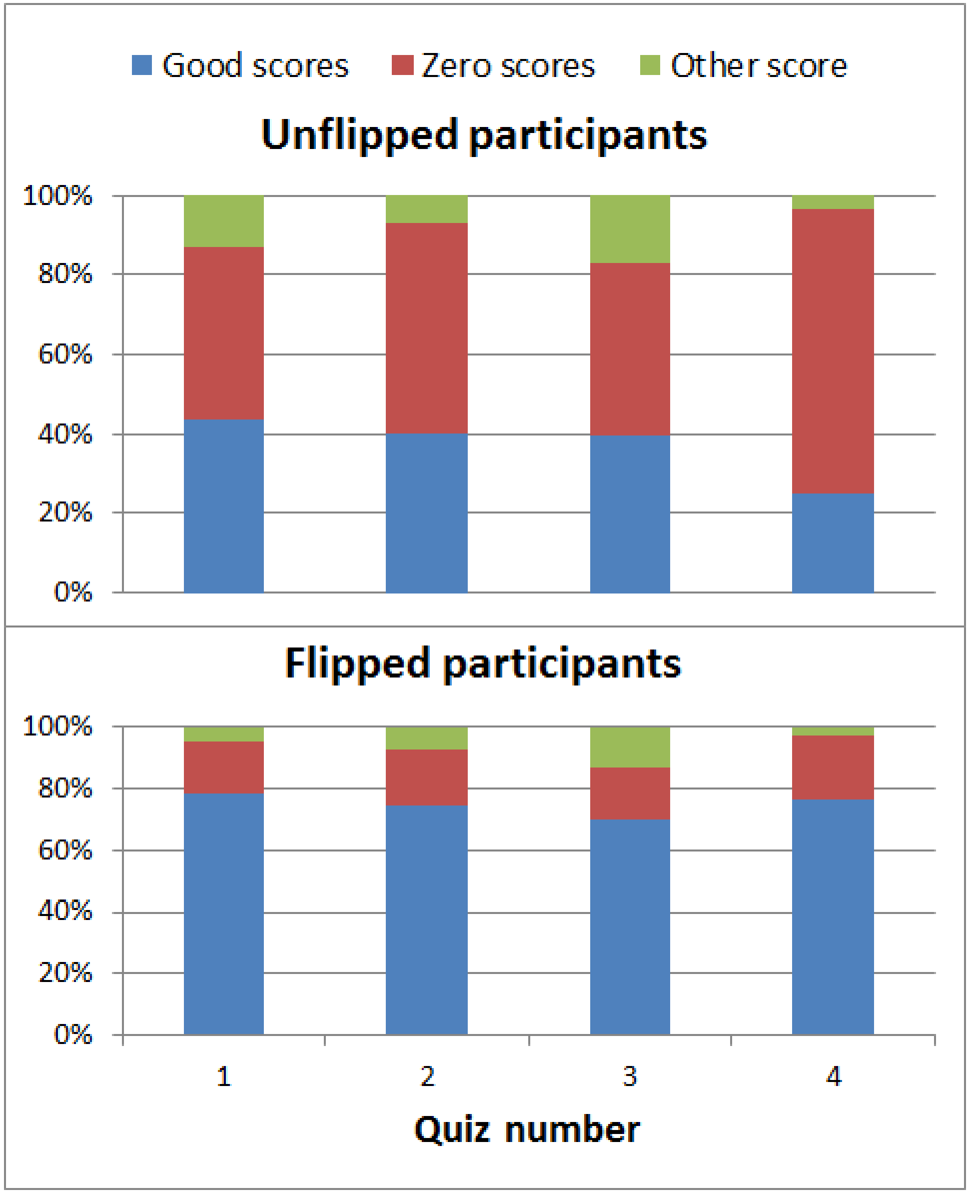
$$x + 3y + 3z = 1$$

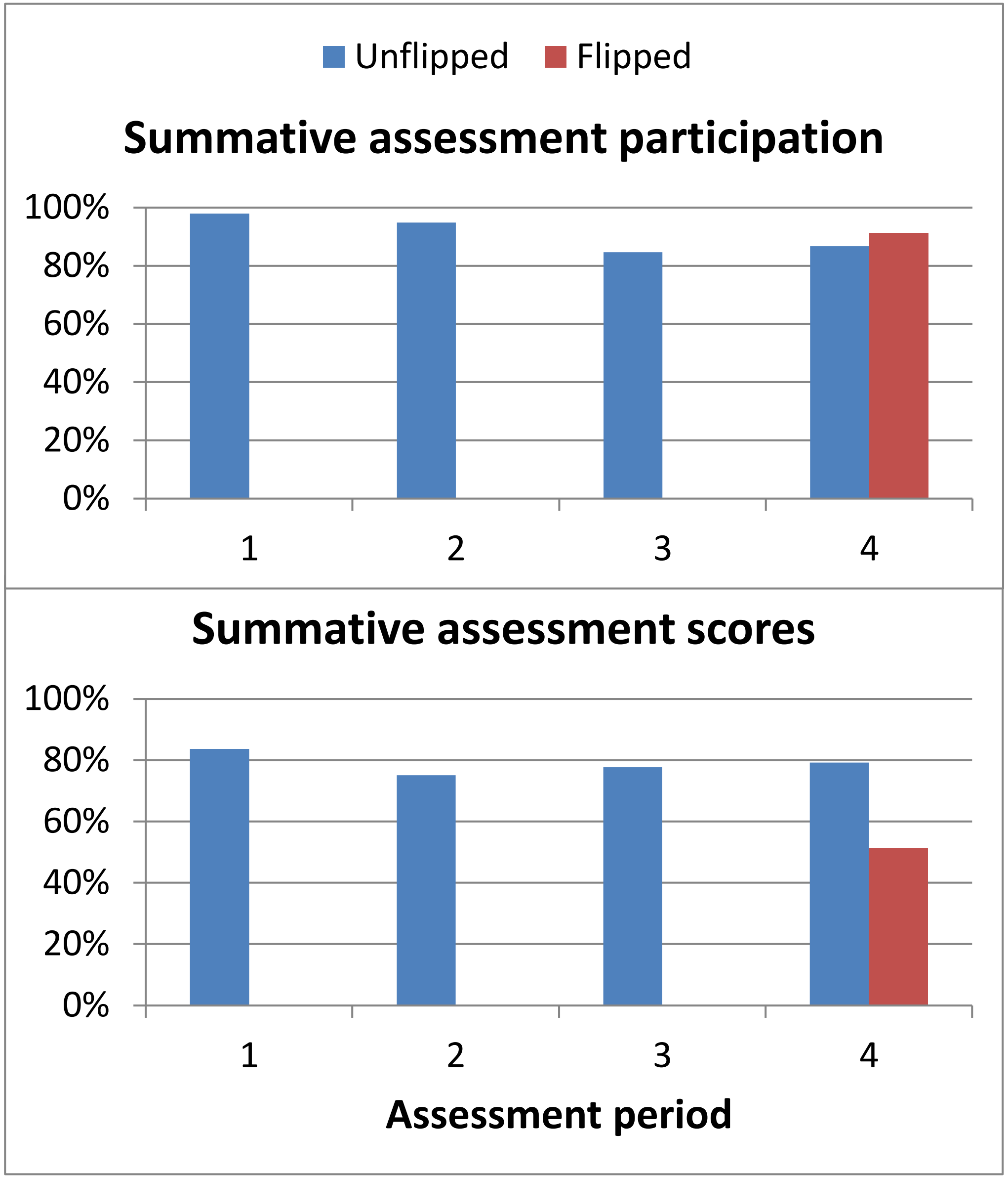
Using this you can now find x:

$x =$ $\square$

Formative Participation

- Formative engagement, *e.g.* numbers doing quizzes 1–4 and/or doing well.



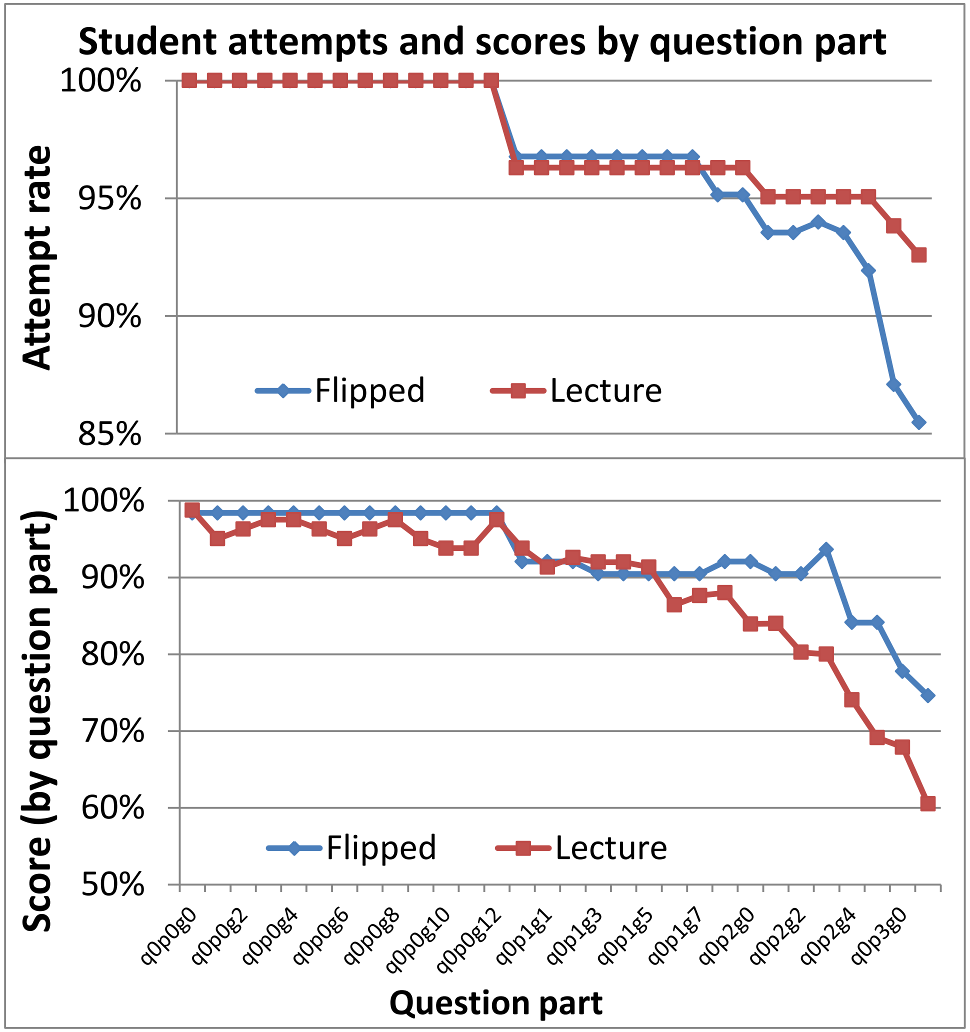


Summative assessment

- *Not a cohort effect!*
- *E.g.* Calculus module scores
  2013 = 72%
  2014 = 75%

Fatigue?

- Gaussian Elimination question
  - 30 discrete parts
  - Flipped group drops-out quicker *but* scores better



Confounding factors

- Matlab encourages surface learning?
- Too high expectations?
- Test fatigue?

Lessons

- Scaffolding to smooth the transition into a flipped approach
  - Managing student expectations and assessment literacy
- Investigate confounding factor (test fatigue) in our measure of success
- Link eAssessment directly to preparatory material
- Turn “gapped notes” into interactive e-resources?

Questions

- Is the “flipped classroom” appropriate
  - for Linear Algebra and Matlab?
  - for mathematics in general?
  - in higher education?
  - for 1st year?

Thanks for listening

☺

- With thanks to
  - Bill Foster, Christian Perfect, Anthony Youd from Newcastle University for Numbas
  - Michael Grove from Birmingham University for HESTEM