Localizing an atom via quantum interference

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We show that a three-level Λ-type atom interacting with a classical standing-wave field resonantly coupling one transition and a weak probe laser field resonantly coupling the second transition can be localized provided the population of the upper state is observed.

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The subwavelength localization of an atom using laser-induced schemes has been actively studied [1–9]. Several models have been proposed using, for example, the measurement of the phase shift due to an off-resonant standing-wave field [1–3], the entanglement between the atom's position to its internal state [4], and others [5,6]. Recently, Zubairy and co-workers [7–9] have proposed two simple localization schemes using either the measurement of Autler-Townes split spontaneous emission in a three-level system [7,8] or the resonant fluorescence in a two-level system [9]. The main advantage of these schemes is that the localization of the atom occurs immediately in the subwavelength domain of the standing-wave field as spontaneous emission is recorded during the atom's motion in the standing-wave field.

In this article we describe a related method for localizing an atom in a standing-wave field. We use a three-level Λ-type atom that interacts with two fields, a probe laser field and a classical standing-wave coupling field. If the probe field is weak then the measurement of the population in the upper level can lead to subwavelength localization of the atom during its motion in the standing wave. The degree of localization is dependent on the parameters of interaction, especially on the detunings and the Rabi frequencies of the atom-field interactions.

The atomic system under consideration is shown in Fig. 1. It consists of three atomic levels in a Λ-type configuration. The atom is assumed to be initially in state $|0\rangle$. The transition $|1\rangle \leftrightarrow |2\rangle$ is taken to be nearly resonant with a classical standing-wave field aligned along the x direction. In addition, the atom interacts with a probe laser field near resonant with the $|0\rangle \leftrightarrow |2\rangle$ transition. We assume that the center-of-mass position of the atom is nearly constant along the direction of the standing wave. Hence, we apply the Raman-Nath approximation [10] and neglect the kinetic part of the atom from the Hamiltonian. Then, the Hamiltonian of the laser-driven part of the system in the interaction picture and the rotating wave approximation reads

$$H = \Omega |0\rangle\langle 2| e^{-i\Delta_0 t} + g(x)|1\rangle\langle 2| e^{-i\Delta_1 t} + \text{H.c.} \qquad (1)$$

Here $\Omega = -\vec{\mu}_{02} \cdot \hat{\varepsilon}_a E_a$, $g(x) = G \sin(kx) (G = -\vec{\mu}_{12} \cdot \hat{\varepsilon}_b E_b)$ are the Rabi frequencies of the probe and coupling fields, respectively, with $\vec{\mu}_{nm}$ $(n,m = 0-2)$ being the dipole matrix element of the $|n\rangle \leftrightarrow |m\rangle$ transition. The unit polarization vector and the amplitude of the probe (coupling) field are denoted by $\hat{\varepsilon}_a$ $(\hat{\varepsilon}_b)$ and E_a (E_b), respectively. The Rabi frequency $g(x)$ is position dependent with G being its constant part. The Rabi frequencies are taken to be real. Also, $\Delta_0 = \omega_{20} - \omega_a$ $(\Delta_1 = \omega_{21} - \omega_b)$ is the field detuning from resonance with the $|0\rangle \leftrightarrow |2\rangle$ $(|1\rangle \leftrightarrow |2\rangle)$ transition, where $\omega_{nm} = \omega_n - \omega_m$. Finally, ω_a (ω_b) is the probe (coupling) field angular frequency and $k = \omega_b / c$ is the wavenumber of the classical standing-wave coupling field.

To simplify matters, we will assume that the probe laser field is weak, allowing a perturbative solution to be sought. The dynamics of the system is described using a probability amplitude approach with the statevector of the complete system at time t being written as

$$|\psi(t)\rangle = \int dx f(x)|x\rangle [a_0(x,t)|0\rangle + a_1(x,t)|1\rangle + a_2(x,t)|2\rangle], \qquad (2)$$

with $a_0(x,t=0) = 1$, $a_1(x,t=0) = a_2(x,t=0) = 0$ as the initial conditions. Here $a_n(x,t)$ is the time- and position-dependent probability amplitude of the atom being in level $|n\rangle$ and $f(x)$ is the center-of-mass wave function of the atom.

We are interested in the conditional position probability distribution [7], i.e., the probability of the atom having position x in the standing-wave field when the atom is found in its internal state $|2\rangle$. Thus, taking the appropriate projections we find that the conditional position probability distribution is given by

$$F(x,t|b) = |\mathcal{N}|^2 |f(x)|^2 |a_2(x,t)|^2, \qquad (3)$$

with $\mathcal{N}$ being a normalization factor. Therefore, the problem reduces to determining the squared amplitude of the prob-

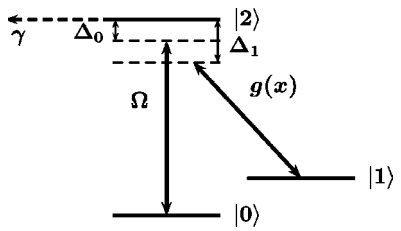

FIG. 1. A schematic diagram of the system under consideration. The atom interacts with a nearly resonant standing-wave field that couples the $|1\rangle \leftrightarrow |2\rangle$ transition and a probe laser field that couples the $|0\rangle \leftrightarrow |2\rangle$ transition.

ability amplitude $a_2(x,t)$. This can also be measured in the laboratory using standard spectroscopic methods [11,12].

We define the slowly varying probability amplitudes $b_n(x,t)$ as $b_0(x,t)=a_0(x,t)$, $b_1(x,t)=a_1(x,t)e^{i(\Delta_1-\Delta_0)t}$, $b_2(x,t)=a_2(x,t)e^{-i\Delta_0 t}$. Substituting Eqs. (1) and (2) into the time-dependent Schrödinger equation, we obtain the following equations for the time evolution of the reduced probability amplitudes:

$$i\dot{b}_0(x,t)=\Omega b_2(x,t), \tag{4}$$

$$i\dot{b}_1(x,t)=(\Delta_0-\Delta_1)b_1(x,t)+g(x)b_2(x,t), \tag{5}$$

$$i\dot{b}_2(x,t)=\left(\Delta_0-i\frac{\gamma}{2}\right)b_2(x,t)+\Omega b_0(x,t)+g(x)b_1(x,t), \tag{6}$$

where γ denotes the decay outside the system and has been added phenomenologically in Eq. (6). A proper quantum mechanical inclusion of this decay process leads to the same result in Eq. (6) [13].

The solution of Eqs. (4)–(6) is obtained by means of time-dependent perturbation theory. Assuming that the coupling laser-atom interaction is weak so that $\Omega \ll G,\gamma$ is satisfied, we have $b_0(x,t)\approx 1$. Then the long-time solution of Eq. (6) is given by

$$b_2(x,t\to\infty)=-\frac{\Omega(\Delta_0-\Delta_1)}{(\Delta_0-\Delta_1)\Delta_0-g(x)^2-i\gamma(\Delta_0-\Delta_1)/2}. \tag{7}$$

Therefore the conditional position probability distribution is given by

$$F(x,t\to\infty|b)=|\mathcal{M}|^2|f(x)|^2\frac{\Omega^2(\Delta_0-\Delta_1)^2}{[(\Delta_0-\Delta_1)\Delta_0-g(x)^2]^2+\gamma^2(\Delta_0-\Delta_1)^2/4}. \tag{8}$$

As $f(x)$ is assumed to be nearly constant over many wavelengths of the standing-wave field, the conditional position probability distribution is determined by the filter function

$$W(x)=\frac{\Omega^2(\Delta_0-\Delta_1)^2}{[(\Delta_0-\Delta_1)\Delta_0-G^2\sin^2(kx)]^2+\gamma^2(\Delta_0-\Delta_1)^2/4}. \tag{9}$$

Equation (9) shows that the conditional position probability distribution depends on two controllable detunings, the probe laser detuning and the detuning of the coupling standing-wave field. We note that the filter function of Eq. (9) has the same form as that of Zubairy and co-workers [7,8]. However, there are two major differences between our scheme and that of Zubairy and co-workers [7,8]. First, in the previously proposed scheme [7,8] the atom needs to be prepared in an excited state, however, in our scheme the atom can be in its ground state for localization to occur. This simplifies the demands on initial-state preparation. Second, as we will see below, localization occurs by fixing the two controllable atom-field detunings to certain values. However, in the scheme of Zubairy and co-workers [7,8] one of the detunings is the vacuum field-atom detuning, which is hard to control.

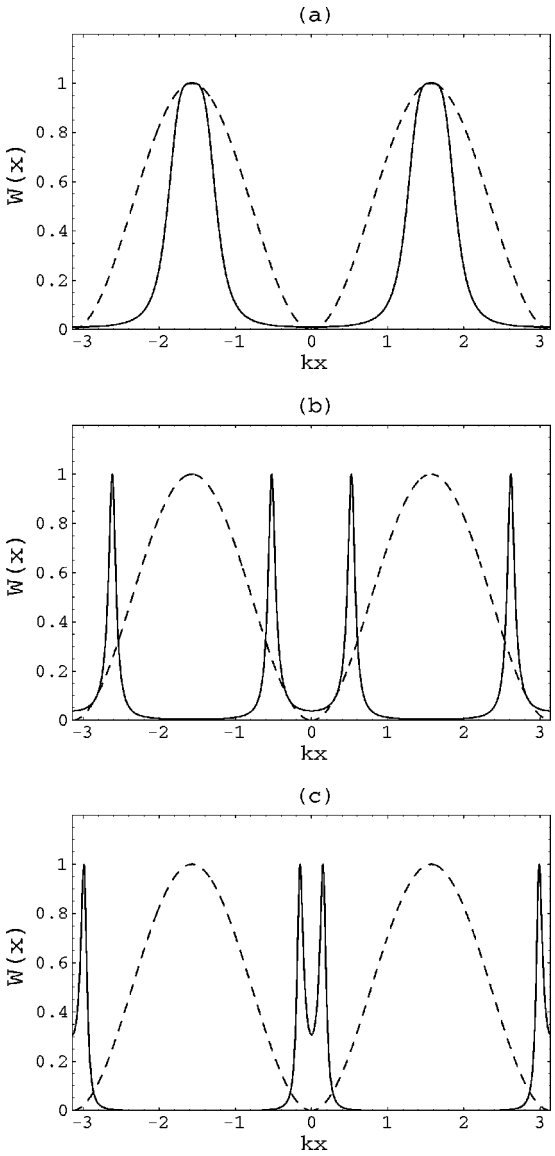

FIG. 2. The filter function $W(x)$ (in arbitrary units) as a function of kx for the parameters $G=1$, $\Delta_1=0$, $\gamma=0.2$ and, (a) $\Delta_0=1$, (b) $\Delta_0=0.5$, and (c) $\Delta_0=0.15$. The dashed curve is a sine-squared function illustrating the position-dependent standing-wave field Rabi frequency. All parameters are measured in arbitrary units.

The maxima of the filter function are found when the probe laser detuning satisfies the equation

$$\Delta_0=\frac{\Delta_1}{2}\pm\frac{1}{2}\sqrt{\Delta_1^2+4G^2\sin^2(kx)}, \tag{10}$$

which means that the maxima are located at

$$kx=\pm\sin^{-1}\left(\frac{\sqrt{\Delta_0(\Delta_0-\Delta_1)}}{G}\right)+n\pi, \tag{11}$$

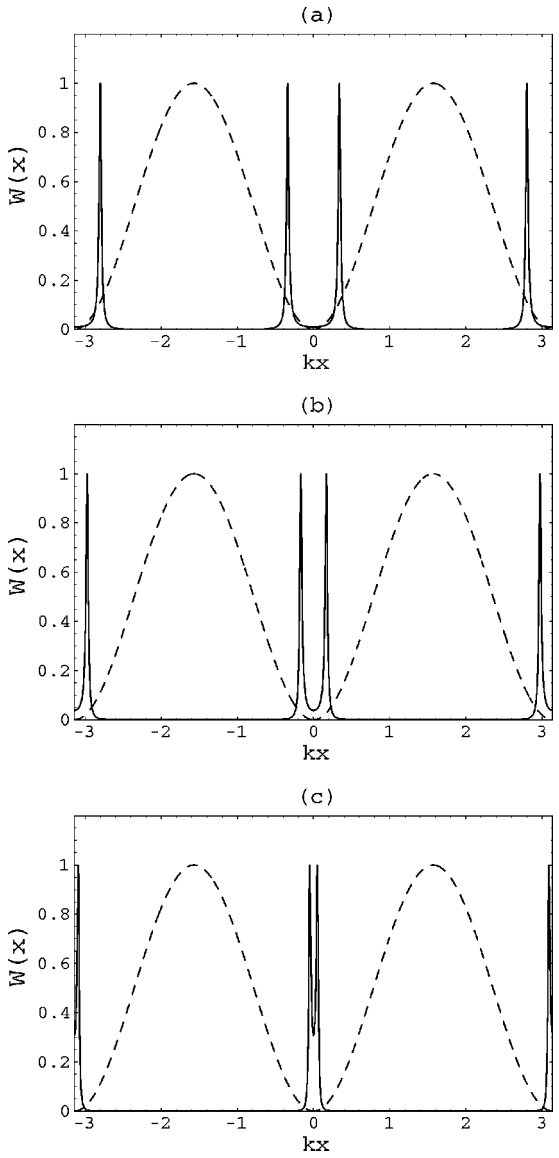

FIG. 3. The influence of the coupling-field strength is illustrated. The parameters are the same as Fig. 2 but with $G=3$.

where n is an integer. For given Δ_0, Δ_1, and G, the width of any peak, which characterizes the degree of localization, is given by

$$a=\left|\sin^{-1}\left(\frac{\sqrt{(\Delta_0+\gamma/2)(\Delta_0-\Delta_1)}}{G}\right)-\sin^{-1}\left(\frac{\sqrt{(\Delta_0-\gamma/2)(\Delta_0-\Delta_1)}}{G}\right)\right|. \tag{12}$$

Therefore, the degree of localization depends on the detunings Δ_0, Δ_1, and the Rabi frequency of the coupling field G.

In Fig. 2 we present the results for the conditional position probability distribution for the standing-wave coupling field on resonance with the $|1\rangle\leftrightarrow|2\rangle$ transition and three different values of the probe-field detuning. It is immediately seen that localization occurs in the system. The degree of localization depends crucially on the probe laser detuning.

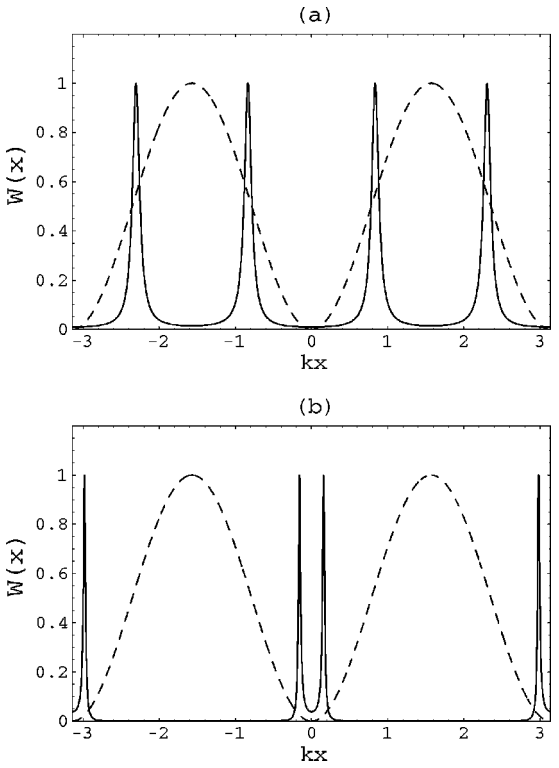

FIG. 4. The influence of the coupling-field detuning is illustrated. The parameters are the same as Figs. 2(a) and 2(b) but with $\Delta_1=0.5$.

As this detuning becomes smaller (and closer to the zero value of the coupling-field detuning), the localization becomes more pronounced. In addition, atomic localization is crucially dependent on the standing-wave coupling-field intensity. In Fig. 3 we show the same results as in Fig. 2 but with three times larger the Rabi frequency of the coupling field. The increase of the coupling-field intensity leads to stronger localization of the atom. We note that in Fig. 3(c) the localization is larger than $\lambda/100$. Finally, as also noted by Qamar *et al.* [8], the localization depends on the detuning of the standing-wave coupling field. This is shown in Fig. 4, where the same results as in Figs. 2(a) and 2(b) are displayed, but with nonzero coupling-field detuning. The localization of the atom is much stronger for the chosen value of the detuning than for a zero detuning as in Fig. 2.

Subwavelength atomic localization in our scheme is a quantum interference effect in this Λ-type atom. This quantum interference can be understood either in the bare states or in the dressed (dark and bright) states of the system [14,15]. In the dressed-state picture, a particular superposition of the two lower states is formed (the dark state) that under certain conditions, is not coupled to any other state of the system. The same quantum interference has lead to many interesting phenomena ranging from coherent population trapping [11–13] and electromagnetically induced transparency [16] to measurement of photon statistics of a quantized radiation field [17] and coherent destruction of quantum tunneling [18].

In summary, we have proposed a simple localization scheme for an atom in a standing-wave field that allows us to

determine its position with high precision. Our scheme is related to those proposed by Zubairy and co-workers [7–9] but is based on the measurement of the upper-state population of a Λ-type atom as the atom moves in the standing-wave field. As there is a plethora of experimentally accessible atoms that can be modeled as three-level Λ-type systems [11,12,15,16], our proposal simplifies a possible experimental implementation of quantum-interference-induced subwavelength atomic localization.

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