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Essays on the Economics of Innovation

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Contents

List of Figures

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Introduction

This dissertation is composed of three research papers on the Economics of Innovation. The central theme of the dissertation is the analysis of innovation incentives and their relation with market structure, intellectual property rights and technology sharing.

The first two chapters study the recent concern that strong intellectual property rights may restrict the access to research inputs needed to perform innovation. This problem has been dubbed the "Tragedy of the Anticommons", and may be particularly acute in the case of complex sequential innovations (innovations that require several prior inventions to be developed).

Chapter 1 presents a model of sequential innovation in which an innovator uses n patented inputs in R&D to invent a new product and the value of innovations is private information of the innovator. Substitutability between the inputs used in research goes from 0 (perfect complements) to ∞ (perfect substitutes).

The model shows that an increase in the complexity of innovation (measured by the number of inputs needed in research) will decrease the probability of innovation if the research inputs are complements, and will increase it if the inputs are substitutes. The probability of innovation will always be suboptimal, even in the case of substitutive inputs. Moreover, this probability will go to zero as complexity goes to infinity for highly complementary inputs.

A proposed solution to the Tragedy of the Anticommons has been the creation of Patent Pools (agreements between patent holders on the licensing terms of their inventions). The model shows that Patent Pools can increase innovation only if they are composed by complementary inventions.

Finally, the paper presents a policy analysis, and shows that the strength of patent policy (measured by patent length or breadth) should decrease with complexity if the inputs are complements and increase with complexity if the inputs are substitutes.

Chapter 1 complements the literature of sequential innovation and links it to the literatures of complementary monopoly and patent pools, by analyzing what happens when a given innovation depends not only on one, but on *several* prior inventions. Moreover, since the model has been carefully micro-founded, it allows to explain the economic intuition behind previous results for the first time. However, the model in Chapter 1 is static, when the nature of sequential innovation is truly dynamic: old inventions were once new ideas, and current new ideas are the stepping stones of future innovations.

With this in mind, Chapter 2 presents a dynamic extension of the first chapter. There is a sequence of innovations, and each innovation builds upon all prior inventions. The value of innovations is private information of the innovator.

Developing a dynamic model is important for several reasons. First, it will eliminate any bias stemming from the asymmetric treatment of old and new ideas. Second, patent policy will affect not only current but also future innovative activity. Third, it allows to analyze the problem of assigning resources to enabling innovations with low commercial value (basic research).

The paper analyzes innovation in three cases: patents, no-patents and patent pools. With patents, each innovator has to pay licensing fees to previous innovators. In equilibrium, the probability of innovation decreases as the sequence progresses, converging to zero in the limit. Without patents, the probability of innovation is constant and higher than in the patents case, unless the degree of appropriability of the innovation is very low. A patent pool increases the probability of innovation, in comparison with the patents case, and dynamic incentives imply a higher probability of innovation than in the static case, which strengthens previous results in the literature. The paper also shows that innovation is suboptimal in the three cases analyzed, and presents an extension for patents of finite length.

Chapter 3 analyzes the trade-off between collaboration and secrecy in industries where open source and proprietary firms co-exist. According to the traditional view in the Economics of Innovation, monopoly is necessary for innovation. Knowledge is a public good, so free-riding will lead to an under-provision of ideas. Moreover, innovation requires a fixed cost investment, and innovators will not be able to cover this cost under competition, given that profits would go to zero. These problems may be diminished if the innovator has a temporary monopoly over the use of the idea. On one hand, ideas become excludable, which reduces free-riding incentives. On the other hand, monopoly rents allow the innovator to afford the cost of innovation.

Open source appears counter to this knowledge. Innovators choose to disclose their innovations, so they are voluntarily renouncing their monopolistic advantage. Moreover, open source goods are subject to free-riding, which should lead to low quality products. However, what we observe in most cases is open source goods of high quality, successfully competing with proprietary goods. How does open source overcome these free-riding incentives?

I present a model in which ex-ante symmetric firms decide whether to be open source or proprietary and their investment in R&D. Firms sell packages composed of a primary good (like software) and a complementary private good. The only difference between both kinds of firms is that open source firms share their technological advances on the primary good, while proprietary firms keep their innovations private.

The motivations of individual developers have been analyzed in the literature, but we know less of the motivations of for-profit firms. To my knowledge, this is the first paper to include competition between for-profit open source and proprietary firms, and where the decision to become open source is endogenous. This paper also contributes to the literature of cooperation in R&D.

The most important finding is the existence of forces leading to an asymmetric market structure, even though all the firms are ex-ante symmetric. This market structure is characterized by a few large proprietary firms and many small open source firms, which is consistent with observations of recent surveys.

The model and the results are important for a variety of reasons. First, endogenizing the participation decision is crucial for understanding the motivations of commercial firms to participate in open source projects. Second, the model shows under what conditions open source can overcome free-riding and produce a good of high quality, even without coordination of individual efforts. Third, the model shows that in equilibrium there are forces leading to an asymmetric market structure. Finally, the model allows for a welfare comparison of different equilibria.

Resumen

Esta tesis está compuesta por tres estudios en el campo de la Economía de la Innovación. El tema central es el análisis de los incentivos para innovar y su relación con la estructura de mercado, los derechos de propiedad intelectual y la cooperación en investigación.

Los dos primeros capítulos estudian la reciente preocupación de que fuertes derechos de propiedad intelectual puedan restringir el acceso a insumos utilizados en I+D. Este problema ha sido llamado la "Tragedia de los Anticomunes" (Tragedy of the Anticommons), y puede ser particularmente grave en el caso de innovaciones secuenciales complejas (innovaciones que están basadas en varias invenciones previas).

El Capítulo 1 presenta un modelo de innovación secuencial en el que un innovador utiliza n insumos patentados en I+D para inventar un nuevo producto, y el valor de las innovaciones es información privada del innovador. La substituibilidad entre los insumos va de 0 (complementos perfectos) a ∞ (substitutos perfectos).

El modelo muestra que un aumento en la complejidad de la innovación (medida por el número de insumos utilizados en I+D) reduce la probabilidad de innovación si los insumos son complementarios, y la aumenta si los insumos son substitutos. La probabilidad de innovación siempre será subóptima, incluso en el caso de que los insumos sean substitutos, y convergerá a cero cuando la complejidad vaya a infinito para insumos altamente complementarios.

Una posible solución para la Tragedia de los Anticomunes es la creación de consorcios de patentes (patent pools), los cuales son acuerdos entre los dueños de patentes para licenciar conjuntamente sus invenciones. El modelo muestra que los consorcios de patentes pueden aumentar la probabilidad de innovación sólo si están compuestos por patentes complementarias.

Por último, el capítulo presenta un análisis de política de patentes, y muestra que la dureza de las patentes (medida a través de la duración

o amplitud de las patentes) debería disminuir con la complejidad de la innovación si los insumos utilizados en I+D son complementarios, y aumentar si los insumos son substitutos.

El aporte principal del Capítulo 1 es el de complementar la literatura de innovación secuencial, y vincularla con la literatura de monopolio complementario y de consorcios de patentes, al analizar lo que ocurre cuando una innovación está basada en varias innovaciones anteriores, y no sólo una. Por otra parte, dado que el modelo ha sido cuidadosamente micro-fundado, permite explicar por primera vez la intuición económica detrás de resultados anteriormente expuestos. Sin embargo, el modelo en el Capítulo 1 es estático, cuando la naturaleza de la innovación secuencial es verdaderamente dinámica: los inventos anteriores fueron alguna vez nuevas ideas, y las nuevas ideas constituyen la base de futuras innovaciones.

Con esto en mente, el Capítulo 2 presenta una extensión dinámica del primer capítulo. Hay una secuencia de innovaciones, donde cada innovación se basa en todas las invenciones previas, y el valor de las innovaciones es información privada del innovador.

El desarrollo de un modelo dinámico es importante por varias razones. En primer lugar, permite eliminar cualquier sesgo derivado del tratamiento asimétrico de las ideas nuevas y las antiguas. En segundo lugar, la política de patentes afectará no sólo a la innovación actual, sino también a las futuras innovaciones. En tercer lugar, el modelo permite analizar el problema de como asignar recursos para proteger a las innovaciones con bajo valor comercial (investigación básica).

El artículo analiza la innovación en tres casos: patentes, no-patentes y consorcios de patentes. Con patentes, cada innovador tiene que pagar licencias a los innovadores anteriores. En equilibrio, la probabilidad de innovación disminuye a medida que avanza la secuencia de innovaciones, y converge a cero en el límite. Sin patentes, la probabilidad de innovación es constante y mayor que en el caso de las patentes, a menos de que el grado de apropiación de la innovación sea muy bajo. Un consorcio de patentes aumenta la probabilidad de innovación, en comparación con el caso de las patentes, y los incentivos dinámicos implican una mayor probabilidad de innovación que en el caso estático, lo que refuerza resultados anteriores en la literatura. El artículo también muestra que la innovación es subóptima en los tres casos analizados, y presenta una extensión para el caso de patentes de duración finita.

El Capítulo 3 analiza el conflicto entre colaborar en investigación o mantener las innovaciones en secreto, en industrias donde coexisten empresas de tipo open source (software libre) y propietarias. De acuerdo al punto de vista tradicional en la Economía de la Innovación, el monopolio es necesario para la innovación. El conocimiento es un bien público, y por lo tanto el free-riding dará lugar a una generación subóptima de ideas. Por otra parte, la innovación requiere la inversión de un costo fijo y los innovadores no serán capaces de cubrir este costo en competencia, dado que los beneficios irían a cero. Estos problemas podrían verse disminuidos si se concede al innovador un monopolio temporal sobre el uso de la idea. Por un lado, las ideas se convertirían en bienes excluibles, lo que reduciría el free-riding. Por otra parte, los ingresos del monopolio permitirían al innovador afrontar el costo de la innovación.

El fenómeno open source parece contradecir este punto de vista. En open source, los innovadores revelan sus innovaciones a sus competidores, por lo que renuncian voluntariamente a sus ventajas monopolísticas. Por otra parte, open source está sujeto a free-riding, lo que debería dar lugar a productos de baja calidad. Sin embargo, lo que observamos en la mayoría de los casos es que los productos open source tienen alta calidad y que compiten con éxito con los productos propietarios. ¿De qué manera logran superarse los incentivos a hacer free-riding en open source?

En el Capítulo 3, presento un modelo en el que empresas ex-ante simétricas deciden entre ser open source o propietarias, y su inversión en I+D. Todas las empresas venden paquetes compuestos por un buen primario (como el software) y un bien privado complementario. La única diferencia entre ambos tipos de empresas es que las open source comparten sus avances tecnológicos en el bien primario, mientras que las empresas propietarias mantienen sus innovaciones en secreto.

Las motivaciones de los programadores individuales para participar en open source han sido extensamente analizadas en la literatura, pero sabemos menos de las motivaciones de las empresas con fines de lucro. Este es el primer artículo donde se incluye competencia directa entre empresas comerciales de tipo open source y propietario, y en el que la decisión de ser de un tipo u otro endógena. Asimismo, este artículo contribuye a la literatura de cooperación en I+D.

El resultado más importante es la existencia de fuerzas tendientes a generar una estructura de mercado asimétrica, a pesar de que todas las empresas son ex-ante simétricas. Esta estructura de mercado está caracterizada por unas pocas empresas propietarias de gran tamaño y muchas open source de menor tamaño, lo que coincide con los hallazgos de encuestas recientes.

El modelo y los resultados son importantes por varias razones. En primer lugar, hacer endógena la decisión de participación en open source es crucial para comprender las motivaciones de las empresas comerciales. En segundo lugar, el modelo muestra en qué condiciones el open source puede superar el free-riding y producir un bien de alta calidad, incluso sin una coordinación de los esfuerzos individuales. En tercer lugar, se muestra la existencia de fuerzas tendientes a generar una estructura de mercado asimétrica. Por último, el modelo permite una comparación de bienestar social de los distintos equilibrios.

CHAPTER 1

Anticommons in Sequential Innovation[1]

Abstract. When innovation is sequential, the development of new products depends on the access to previous discoveries. As a consequence the patent system affects both the revenues and the cost of the innovator. We construct a model of sequential innovation in which an innovator uses n patented inputs in R&D to invent a new product. We ask: (i) what is the net effect of patents on innovation as technologies become more complex (n increases)? (ii) are patent pools welfare enhancing? (iii) what is the optimal response of patent policy as technological complexity increases? The answers to these questions depend on the degree of complementarity between the inputs used in research.

1. Introduction.

Knowledge builds upon previous knowledge. This is true for most innovations nowadays, especially in hi-tech industries like molecular biology, plant biotechnology, semiconductors and software. In some cases, the innovation consists of an improvement of an older version of the same good. In other cases, the research leading to the discovery of the new good depends on the access to research tools, techniques and inputs which are previous innovations themselves.

In any case, innovation activity will in general depend on the access to previous innovations. Depending on the structure of the patent system, many of these inventions will be protected by patents. This means that patents affect not only the revenues of the innovator, but also the cost of performing an innovation.

Recent concern has arisen on the possibility that patents (or other kinds of Intellectual Property) can restrict access to research inputs, hindering innovation as a consequence. The innovator and the owners

[1]This chapter is based on Llanes and Trento (2007).

of patents on previous inventions share the revenues of the innovation. As the number of inputs needed in research increases, the innovator faces a *patent thicket* and is threatened by the possibility of *hold-up*, namely the risk that a useful innovation is not developed because of lack of agreement with the patent holders. This problem has been dubbed the *tragedy of the anticommons* (Heller 1998, Heller and Eisenberg 1998). When too many agents have exclusion rights over the use of a common resource, this resource tends to be underutilized, in clear duality with the tragedy of the commons in which too many agents hold rights of use and the resource tends to be overused.

This problem may be particularly acute in biomedical research, where there is a deep controversy over the patenting of gene fragments and research tools. Take for example the case of the MSP1 antigen (Plasmodium Falciparum Merozoite Specific Protein 1), widely recognized as the most promising candidate for an anti-malarial vaccine. A study of the Commission on Intellectual Property Rights (2002) found more than 39 patent families covering DNA fragments, methods for processing fragments, production systems, vaccine delivery systems, etc. As a consequence, a potential innovator willing to commercialize a vaccine based on MSP1 must get prior permission from the owners of these property rights.

Anticommons can arise in biotechnology as well. A good example is Golden Rice, which required payment of up to 40 licenses, depending on the country of commercialization (Graff, Cullen, Bradford, Zilberman, and Bennett 2003).

As a final example, consider the case of software patents, which cover mathematical algorithms and techniques. Software programs have become so complex that any single program may use thousands of algorithms (Garfinkel, Stallman, and Kapor 1991), possibly infringing a large number of patents. This explains the expected increase in patent litigation in this sector in the next years (think of Microsoft vs. the programmers and users of Linux), and the formation of a Patent Commons by firms involved in the Open Source community (IBM, HP, Novel, Sun, etc.).

We address these issues by constructing a model of sequential innovation in which an innovator uses n patented inputs to develop a new invention. Substitutability between the inputs goes from zero (perfect complements) to infinity (perfect substitutes) and the input sellers compete in prices but do not know the exact value of the innovation for the innovator.

We study how the probability of performing the innovation changes as technologies become more complex (n increases) and find that it decreases when the inputs are market complements and increases when they are market substitutes. Therefore, we prove that the anticommons hypothesis may hold when inputs are essential and not easy to substitute.

Then we analyze the limiting economy when $n \to \infty$. We show that the probability of innovation is always less than socially optimal unless the inputs are perfect substitutes. Moreover, the probability of innovation goes to zero when the elasticity of substitution is below a threshold level which is higher than 1.

We also analyze the creation of a patent pool as a possible solution to the tragedy of the anticommons. A patent pool is a cooperative agreement among patent holders, through which they agree on the licensing terms of a subset of their patents. The USPTO (US Patents and Trademarks Office) itself has recommended the creation of patent pools to ease the access to biotechnology research tools (Clark, Piccolo, Stanton, and Tyson 2000). We find that a patent pool reduces the cost of innovation if the inputs are complements and increases it if the inputs are substitutes. The reason is that when the inputs are complements, an increase in the price of one of them decreases the profit of the rest of firms. The pool takes this effect into account when maximizing total profits, and therefore will set a lower price for the inputs.

Finally, we find that the optimal degree of patent protection is decreasing in n if the inputs are highly complementary (i.e. have low substitutability) and increasing in n in the opposite case. This surprising result contrasts with the increase in the strength of Intellectual Property Rights on research tools in the last two decades, and can be related to an extended belief that patents are good for innovation and the rent-seeking activities of agents with vested interests.

1.1. Related literature. This paper is related to the literatures of sequential innovation, complementary monopoly and patent pools.

However, the focus of this paper is different. We analyze the effects of patents on the pricing of research inputs and examine the consequences of an increase in the complexity of innovation (measured by n) on the probability that a new good is introduced and the optimal patent policy.

There is an extended literature on Sequential Innovation (Scotchmer 1991, Green and Scotchmer 1995, Chang 1995, Scotchmer 1996), which is mainly concerned with the optimal division of profits between successive innovators. Generally, in these models, there are two innovations which have to be introduced sequentially (the second innovation cannot be introduced until the first one has), and the objective is to find the patent policy that maximizes the incentives to invest in both innovations. In this paper we generalize these models by assuming that any innovation is based on n of previous innovations, as in Boldrin and Levine (2005).

In this sense our paper is more related to the literature on complementary monopoly initiated by Cournot (1838). Cournot modeled a competitive producer of brass who must buy zinc and copper from two separate monopolists (zinc and copper are perfect complements), and showed that (i) the price of the inputs is higher than the price that a single provider would set, (ii) the total cost of the inputs is increasing in the number of inputs, (iii) in the limit, as $n \to \infty$ the cost of the inputs is such that the demand for the final good is zero.

Cournot's theory of complementary monopoly has been later extended in various directions. Bergstrom (1978) allows for a more general technology and studies the behavior of the factor market in depth. He is concerned with analyzing the duality between price and quantity competition and assumes a zero marginal cost of the inputs. Our model is similar to Bergstrom's, but we focus on a different problem (sequential innovation) and assume a positive marginal cost. As we will show, Bergstrom's assumption of zero marginal cost is not trivial, as the results depend both quantitatively and qualitatively on this assumption. Chari and Jones (2000) relate complementary monopoly to the externality problem. They show that, because agents play strategically, the market outcome in economies with complementarities is inefficient. This is true even if property rights are fully assigned. They also show that the most inefficient outcomes result from economies with a large number of agents, which is related to the case when the probability of innovation goes to zero as $n \to \infty$ in our model.

Cournot's theory has been also used by the literature on patent pools. Shapiro (2001) was the first to suggest that patent pools may be anticompetitive when they are formed by substitute patents, and pro-competitive when formed by complementary patents. Lerner and Tirole (2004) build a model to generalize Shapiro's results. They base their definition of substitutability on the shape of the payoff function of the innovator and prove that the higher the substitutability among patents, the higher the probability that the patent pool is anticompetitive.

With respect to this literature on patent pools, our paper provides two contributions. First, in addition to showing that patent pools reduce the price of the inputs when they are complements, we also show that the pool price is independent on the number of inputs, and therefore patent pools can potentially prevent the tragedy of the anticommons from happening. Second, we base our definition of complements and substitutes on the traditional cross-price derivatives, which allows us to be more precise in determining the effects of patent pools on innovation and to explain the economic intuition behind the results.

2. The model.

There are n research inputs $(x_1, ..., x_n)$ and a potential innovator who may use the n inputs in R&D in order to invent a new good. The n inputs have already been invented and are ready to be produced. We make this assumption in order to concentrate on the effects of the pricing of old ideas on innovation activity (read Section 4 for further details). The structure of Intellectual Property Rights is such that each input is protected by a patent, granting its owner a monopoly over it. Each patent is owned by a different patentee and thus each of the n inputs is supplied by a different producer. Given that the inputs are imperfect substitutes of each other, the factor market is a differentiated goods oligopoly. The input sellers compete in prices and the value of the innovation is private information of the innovator.

2.1. Technology. The innovator can perform R&D to invent a new final good according to the following CES technology:

$$y = A \left(\sum_{i=1}^{n} {x_i}^{\rho} \right)^{\frac{1}{\rho}}, \tag{1.1}$$

where y is a measure of the R&D effort, A is a scale parameter, x_i is the amount of input i used, n is the number of inputs and $\rho \in (-\infty, 1]$ is a technological parameter related to the substitutability between the inputs.

The innovator faces an indivisibility problem, meaning that a minimum amount of R&D effort is required to invent a new good. When the R&D effort is below that threshold level there is no innovation. Without loss of generality we can set the threshold level at 1, so that the indicator function for the innovation is:

$$I = \begin{cases} 1 & \text{if } y \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

The above CES specification is a simple and general way to introduce substitutability and complementarity between the inputs used in research. In our model, ideas have economic value because they are embodied in physical objects (Romer 1990, Boldrin and Levine 2002, 2005b). The innovator uses these physical objects to innovate, not the abstract ideas. Accordingly, the input decision is not discrete (to use the idea or not), but rather continuous (the research inputs can be used in variable amounts). The qualitative results of the paper would still hold if input decisions were discrete, but there would be a significative loss in tractability (read Section 5.5).

A good example of our research technology is the case of computer programs used in research labs. The lab has to pay a license fee for each copy of the program used in the innovation process, and may alter the number of copies it uses depending on their price. Other good examples are old ideas which provide component parts for a new product. In the early radio industry, for example, according to Edwin Armstrong (inventor of FM radio) "it was absolutely impossible to manufacture any kind of workable apparatus without using practically all of the inventions which were then known", like the high-frequency alternator, high-frequency transmission arc, magnetic amplifier, the crystal detector, diode and triode valves, directional aerial, etc. Similar examples can be found in semiconductors, electronics, and other industries.

We set the scale parameter A in (1.1) equal to $n^{(\rho-1)/\rho}$ in order to eliminate any returns from specialization or division of labor. Usually CES production functions exhibit a property called increasing returns to specialization (or love for variety in the case of utility functions). Following an argument similar to Romer (1987), suppose that the production function is $y = \left(\sum_{i=1}^{n} x_i{}^{\rho}\right)^{1/\rho}$, and let X be the total quantity of inputs used in production. Because of symmetry, all inputs will be used in the same quantity in the equilibrium, so $x_i = X/n$ for all i, and output will be equal to $y = n^{(1-\rho)/\rho} X$. There are positive returns to specialization because an increase in n holding X constant causes output to increase. We eliminate this effect by introducing $A = n^{(\rho-1)/\rho}$ in the production function.

The complexity of the innovation is measured by n. More complex technologies use a larger number of components or require more research tools in order to be developed. Each input is produced with a constant marginal cost of $\varepsilon > 0$. We assume that the resources used to produce inputs are sold in a competitive market, so that the private and the social cost of producing inputs coincide. The assumption of no returns to specialization guarantees that the social cost of performing the innovation does not change as technologies become more complex. In other words, there is no technological advantage or disadvantage from increases in n.

2.2. Value of the innovation and structure of the information. The social value of the innovation, v, is the total surplus generated by the new product. To focus on the factor market, we will assume that the innovator is a perfect price discriminator in the final goods market. This means that the private value for the innovator coincides with the social value of the innovation.

The value of the innovation is private information of the innovator. This may be because the innovator has better information about the characteristics of the new product or about the valuation of the consumers. The sellers of inputs only know that v has a cumulative distribution $F(v)$. Therefore $F(v)$ is the probability that the innovation has a return less or equal to v. In Section 5 we show that the assumptions of perfect price discrimination and asymmetric information can be relaxed without altering the results.

The hazard function is defined as $h(v) = f(v)/(1 - F(v))$, where $f(v)$ is the density function corresponding to $F(v)$. In order to guarantee the quasi-concavity of the maximization problem of the input producers, the following assumption will hold throughout the paper:

ASSUMPTION 1.1 (Nondecreasing hazard function). *$h(v) > 0$ and $h'(v) \geq 0$ on a support $[\underline{v}, \bar{v}]$, and $h(v) = 0$ outside of this support.*

This assumption on the hazard function is very general, and holds for most continuous distribution functions. We will analyze the meaning of the hazard function in Section 3.1. Notice that we are not restricting $\underline{v}$ nor $\bar{v}$ to be finite.

An important assumption is that the distribution of values of the innovation does not change with n. This assumption, together with the absence of returns to specialization in the R&D technology imply the following lemma:

LEMMA 1.1. *The probability that an invention is socially optimal does not depend on its complexity.*

PROOF. The probability that an innovation is socially optimal is the probability that its social value is larger or equal than its social cost. The social cost of an innovation coincides with the resources used to produce it. Therefore, the probability that an innovation is socially optimal is $Prob\,(v - \sum_{i=1}^{n} \varepsilon\, x_i \geq 0)$. Because of the symmetry in the innovation technology, $x_i = 1/n$, so this probability becomes $1 - F(\varepsilon)$, which depends on the distribution of social values of the innovation and the marginal cost of the inputs but *not* on the number of inputs used in R&D. ■

In this paper, we are interested in studying the effects of increasing technological complexity on the probability of innovation. Lemma 1.1 assures that a change in n affects this probability only through a change in the number of inputs used in research, but not through a change in the social value or cost of the innovation. In other words, we want to compare innovations with different n but the same net social value.

In Section 5 we relax these assumptions by letting the value of the innovation be a function of n and allowing returns to specialization in the R&D technology. We find that the main results of the paper are not significantly affected by a change in these assumptions.

2.3. Market interaction. The players of the game are the n input sellers and the innovator. A strategy for input seller i is a choice of price for her input. A strategy for the innovator is a function $g : \mathbb{R}_+^n \times v \rightarrow \mathbb{R}_+^n$, namely a demand x_i for each input, as a function of the price of all the inputs and the realization of the value of the innovation.

The timing of the game is as follows: (i) the input producers simultaneously set the price of their inputs, (ii) Nature extracts a value v from the distribution $F(v)$, and (iii) given prices, the innovator calculates the input mix that minimizes the cost of innovation and decides whether to innovate or not.

The equilibrium concept we use is Symmetric Subgame Perfect Equilibrium (SSPE). A set of strategies $\{p_i\}_{i=1}^n, g$ is a SSPE if it is a Nash equilibrium of every subgame of the original game, and $p_i = p$ for all i.

The payoff for input producer i is $x_i\,(p_i - \varepsilon)$ and the payoff of the innovator is $I\,v - \sum_{i=1}^n p_i\,x_i$.

2.3.1. *Innovator's Problem.* Given input prices $\{p_i\}_{i=1}^n$, the innovator solves the following Cost Minimization Problem (CMP):

$$\begin{aligned} c &= \min \sum_{i=1}^n p_i x_i \\ s.t. \qquad & n^{-\frac{1-\rho}{\rho}} \left(\sum_{i=1}^n {x_i}^\rho \right)^{\frac{1}{\rho}} \geq 1. \end{aligned}$$

The solution to this problem is the set of conditional factor demands x_i and the minimum cost of innovation c. Given c, the innovator will perform the innovation ($I = 1$) if $v \geq c$.

2.3.2. *Input Seller's Problem.* When setting the price the sellers of inputs do not know the realization of v. They only know that given $\{p_i\}_{i=1}^n$ the probability that $v \geq c$ (the probability of innovation) is $1-F(c)$. Therefore, the expected demand of input firm i is $E(x_i) =$

$(1-F(c))\,x_i$, and its Profit Maximization Problem (PMP) is:

$$\max_{p_i} \quad \Pi_i = (1 - F(c))\, x_i(p_i - \varepsilon),$$

where both c and x_i come from the CMP of the innovator.

3. Equilibrium.

In this section we solve recursively for the SSPE. Therefore, we begin by solving the Innovator's Problem (second stage of the game). The demands are those of a typical CES production function.

PROPOSITION 1.1 (Solution of the Innovator's Problem). *The conditional demand of input i and the cost of innovation are:*

$$x_i \;=\; I\, n^{-\frac{1}{1-\sigma}}\, p_i^{-\sigma} \left(\sum_{i=1}^{n} {p_i}^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}},$$

$$c \;=\; n^{-\frac{1}{1-\sigma}} \left(\sum_{i=1}^{n} {p_i}^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

where $\sigma = 1/(1-\rho)$ is the elasticity of substitution between the inputs. The innovator will introduce the new good (I=1) if $v \geq c$.

The restrictions on ρ imply that the elasticity of substitution σ goes from 0 (perfect complements) to ∞ (perfect substitutes).

Given x_i and c, the symmetric equilibrium price p solves

$$p = \underset{p_i \geq \varepsilon}{\operatorname{argmax}}\ (1 - F(c))\ x_i\,(p_i - \varepsilon)\,,$$

where $c = n^{-\frac{1}{1-\sigma}} \left(p_i^{1-\sigma} + (n-1)p^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ and $x_i = n^{-1}\, p_i^{-\sigma}\, c^{\sigma}$. It is useful to notice that in the symmetric equilibrium ($p_i = p$ for all i), $c = p$ and $x_i = 1/n$ for all i. Also, $p \geq \varepsilon$ in equilibrium because otherwise firms would be making negative profits and would find it profitable to deviate by setting a higher price.

Because of the nature of Nash equilibria, for any value of n, ε, and $\sigma < \infty$ there exists equilibria where p is so high that the probability of innovation is zero (i.e. profits are zero for all input sellers) but any deviation by a *single* input seller is not enough to make it positive. However, these are trivial equilibria coming from the definition of Nash equilibria without any intrinsic economic value. We are interested in the existence of equilibria with a positive probability of innovation ($p < \bar{v}$).

The following proposition characterizes the solution of the first stage of the game (the Input Seller's Problem).

PROPOSITION 1.2. *A SSPE with positive probability of innovation ($p < \bar{v}$) exists and is unique. The equilibrium price solves*

$$(p - \varepsilon)\, h(p) = n - \sigma\,(n-1)\,(p-\varepsilon)/p.$$

The conditional input demand is $x = 1/n$, the cost of innovation is $c = p$ and the probability of innovation is $1 - F(p)$.

PROOF. The firm wants to maximize $(1 - F(c))\, x_i\,(p_i - \varepsilon)$. The derivative with respect to price is:

$$D(p_i) = -f(c)\,\frac{\partial c}{\partial p_i}\, x_i\;(p_i - \varepsilon) + (1 - F(c))\,\left(\frac{\partial x_i}{\partial p_i}\;(p_i - \varepsilon) + x_i\right).$$

By Shepard's Lemma $\partial c/\partial p_i = x_i$, and by symmetry $c = p$, $x_i = 1/n$ and $\partial x_i/\partial p_i = -(n-1)\sigma/(n^2\, p)$. Therefore, the first order condition becomes:

$$D(p) = -f(p)\,\frac{p-\varepsilon}{n^2} + (1 - F(p))\,\left(-\frac{\sigma\,(n-1)\,(p-\varepsilon)}{n^2\, p} + \frac{1}{n}\right).$$

Now we prove that the solution cannot be ε nor $\bar{v}$ for $n < \infty$. $p = \varepsilon$ cannot be the equilibrium because $D(\varepsilon) = (1 - F(\varepsilon))/n > 0$. Also, $p = \bar{v}$ cannot be the equilibrium both if $\bar{v}$ is finite or infinite. If $\bar{v} < \infty$, then $D(\bar{v}) = -f(\bar{v})\,\left(\frac{\bar{v}-\varepsilon}{n}\right) < 0$. On the other hand, $lim_{p\to\infty} D(p) = -\infty < 0$. Therefore, the solution must satisfy $D(p) = 0$. Multiplying $D(p)$ by $-n^2/(1 - F(p))$ we get:

$$h(p)\,(p-\varepsilon) + \sigma\,(n-1)\,\frac{p-\varepsilon}{p} - n = 0. \tag{1.2}$$

We can be sure that equation (1.2) has exactly one solution because it is continuously increasing in p by Assumption 1.1, is negative when $p = \varepsilon$ and is positive when $p \to \bar{v}$ (Assumption 1.1 implies that $\lim_{p\to\bar{v}} h(p)\, p = \infty$ for finite or infinite $\bar{v}$). Therefore, the solution exists and is unique. Rearranging terms in equation (1.2) we get the desired result. ∎

Example. We will find it useful to illustrate the results with the help on an example based on the uniform distribution. This example has the advantage of providing an explicit solution for the equilibrium price. Specifically, assume that the value of the innovation, v, is uniformly distributed between 0 and 1. This means that $F(v) = v$ and $h(v) = 1/(1-v)$. The equilibrium price is:

$$p = \frac{a + \sqrt{a^2 + 4\,\sigma\,\varepsilon\,(n-1)\,b}}{2\,b}, \tag{1.3}$$

where $a = n + \varepsilon - \sigma(n-1)(1+\varepsilon)$ and $b = 1 + n(1-\sigma) + \sigma$. The cost of innovation is equal to the price and the probability of innovation is simply $1 - p$.

3.1. The meaning of the First Order Condition. It interesting to analyze the meaning of the optimality condition. In the traditional case, when there is no uncertainty, the PMP is simply to maximize $x_i\,(p_i - \varepsilon)$. In this case, the optimal price solves:

$$\frac{p_i - \varepsilon}{p_i} = \frac{1}{\eta_{x_i}},$$

where η_{x_i} is the price elasticity of the demand for inputs (in absolute value) and $(p_i - \varepsilon)/p_i$ is the Lerner index, which measures the gain over marginal cost as a proportion of price.

In our case, the PMP is to maximize $(1 - F(c))\,x_i\,(p_i - \varepsilon)$, so the optimal price solves:

$$\frac{p_i - \varepsilon}{p_i} = \frac{1}{\eta_{x_i} + h(c)\,p_i x_i},$$

where $h(c)$ is the hazard function.

Therefore, our model adds an specific term related with the probability of selling the inputs. The hazard function is $h(c) = -\frac{\partial(1-F(c))}{\partial c}\frac{1}{(1-F(c))}$, so it measures the proportional decrease in the probability of selling the inputs when total cost increases. We can interpret $1 - F(c)$ as a demand for final good and c as its price (read Section 5). Then, if we multiply the hazard function by c, we get the elasticity of the final demand in absolute value ($\eta_D = h(c)c$).

This means that the optimal price in Proposition 1.2 solves:

$$\frac{p_i - \varepsilon}{p_i} = \frac{1}{\eta_{x_i} + \eta_D\,\frac{p_i x_i}{c}},$$

so our additional term is equal to the price-elasticity of final demand times the share of input i in total expenditure. Notice that this result is very general, since we are not assuming any specific demand function for the inputs, nor any distribution for the values of the innovation.

3.2. Elasticity of substitution. The price of the inputs and the cost of innovation in equilibrium depend on the elasticity of substitution, the complexity of the innovation and the marginal cost of the inputs. In the following subsections we will analyze the comparative statics of the above equilibrium.

PROPOSITION 1.3. *The cost of innovation is decreasing in σ.*

PROOF. Equation (1.2) provides an implicit function of p in terms of σ. We can calculate $\partial c/\partial \sigma$ using the implicit function theorem (remember that p=c in the symmetric equilibrium):

$$\frac{\partial c}{\partial \sigma} = -\frac{(n-1)(p-\varepsilon)/p}{h(p) + h'(p)\,(p-\varepsilon) + \sigma\,(n-1)\varepsilon/p^2}$$

It is easy to see that this derivative is always negative (the numerator and the denominator are positive). The result follows. ■

Figure 1 depicts the cost of innovation (i.e. the price of the inputs) as a function of σ for the uniform distribution and for $n = 10$ and $\varepsilon = 0.1$.

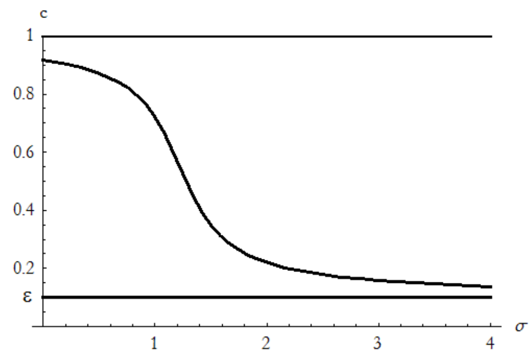


FIGURE 1. Cost of innovation as a function of σ.

The cost of innovation is monotonically decreasing in σ because of increased competition as the inputs become more substitutable. As $\sigma \to \infty$ price converges to marginal cost ε, which is the standard Bertrand price competition result with homogeneous goods.

3.3. Complements and Substitutes. We will classify inputs in market complements and substitutes according to the sign of the cross-price derivative of expected demand which, in this setting, is equivalent to analyzing the cross-price derivative of expected profit. This classification is equivalent to the one used in game theory, where the actions of two agents are said to be complements (substitutes) when an increase in the action of one of them implies a decrease (increase) in the payoff of the other agent. In our model, the actions are prices and the payoff is expected profit. Notice that this is an equilibrium definition since it is based on the best response of the innovator.

DEFINITION 1.1 (Market complements and substitutes). *Input j is a market complement (substitute) of input i if* $\frac{\partial E(x_i)}{\partial p_j} < 0$ $\left(\frac{\partial E(x_i)}{\partial p_j} > 0\right)$.

An increase in the price of input j has two effects on the expected demand of input i. On one hand, the conditional demand of input i increases (substitution effect). On the other hand, the probability of innovation decreases because the inputs are more expensive to the innovator (innovation effect). The sign of the cross-price derivative depends on which of the two effects is stronger. The cross-price derivative is:

$$\frac{\partial E(x_i)}{\partial p_j} = (1 - F(c))\frac{\partial x_i}{\partial p_j} + \frac{\partial(1 - F(c))}{\partial p_j}x_i.$$

The first effect is related to the standard substitution effect of consumer demand theory. Remember that the Cost Minimization Problem is equivalent to the Expenditure Minimization Problem and in this case there are no wealth effects of price changes (the conditional factor demands are equivalent to Hicksian demands). In principle, the derivative $\partial x_i/\partial p_j$ could be positive, negative or zero. However, the property of negative semidefiniteness of the matrix of cross-price derivatives (which implies that every input must at least have one technical substitute), together with the symmetry of the production function, implies that this derivative is non-negative. The inputs will be technical substitutes ($\partial x_i/\partial p_j > 0$) except in the case of perfect complements, where $\partial x_i/\partial p_j = 0$.

The second effect is due to the fact that the demand for innovations is downward sloping. The cost of the inputs used in research affects the profitability of innovation. Therefore, an increase in the price of any input will lower the probability of innovation. This effect is negative, except in the case of perfect substitutes, when it is zero.

Now that our definition of complementarity and substitutability is clear, we can be precise in our exposition. In what follows when we say that inputs are complements or substitutes, we mean that they are *market* complements or substitutes. We will see that the distinction between complements and substitutes is crucial for the predictions of the model.

The following lemma shows the value of σ that makes the cross-price derivative equal to zero.

LEMMA 1.2. *The cross-price derivative $\partial E(x_i)/\partial p_j$ is zero in the symmetric equilibrium if and only if $\sigma = \sigma^*$, where σ^* is the argument that solves $h\left(\frac{\sigma}{\sigma-1}\varepsilon\right) = \sigma - 1$.*

PROOF. The cross-price derivative is:

$$\frac{\partial E(x_i)}{\partial p_j} = (1 - F(c))\,\frac{\partial x_i}{\partial p_j} - f(c)\,x_i.$$

By Shepard's Lemma, $\partial c/\partial p_j = x_j$. Imposing symmetry, $x_i = x_j = 1/n$ and $\partial x_i/\partial p_j = \sigma/(n^2\,p)$. Rearranging terms we get:

$$\frac{\partial E(x_i)}{\partial p_j} = \frac{1}{n\,(1 - F(p))}\left(\frac{\sigma}{p} - h(p)\right).$$

This will be zero in the symmetric equilibrium only when $h(p) = \sigma/p$. Introducing this into the first order condition (1.2) and rearranging we get $(p - \varepsilon)/p = \sigma^{-1}$ or $p = \sigma\varepsilon/(\sigma - 1)$. Plugging this value of p in $h(p) = \sigma/p$ we get the desired result. ■

The following proposition classifies inputs in complements and substitutes according to Definition 1.1. It is interesting to see that this distinction depends on the values of σ and ε, but not on the value of n.

PROPOSITION 1.4. *In the symmetric equilibrium, inputs are complements when $\sigma < \sigma^*$ and substitutes when $\sigma > \sigma^*$.*

PROOF. We know from Lemma 1.2 that the cross-price derivative is zero when $\sigma = \sigma^*$ and that its sign depends on $\sigma/p - h(p)$. The latter expression is increasing in σ because p is decreasing in σ from

Proposition 1.3 and h is non-decreasing in p from Assumption 1.1. The result follows. ■

Interestingly, the value of σ which divides inputs in complements and substitutes has to be larger or equal than 1. To see this, suppose that $\sigma^* < 1$. This means that $h(\varepsilon\sigma^*/(\sigma^* - 1)) < 0$, which is not possible. In the case of the uniform distribution, for example, inputs are complements when $\sigma < (1+\varepsilon)/(1-\varepsilon)$ and substitutes when $\sigma > (1+\varepsilon)/(1-\varepsilon)$.

3.4. Patent Pools. Until now, the research inputs were priced non-cooperatively. In this subsection we analyze what happens when all inputs are priced cooperatively, either by a collective institution such as a patent pool or by a single patent holder (monopolist) that owns all the patents. Proposition 1.5 shows the equilibrium price in this case. The difference with the previous case is that now the patent holder maximizes joint-profits and therefore takes into account the cross-price effects between expected demands.

Proposition 1.5 (Patent Pool). *The equilibrium price when all the inputs are priced cooperatively, p^*, is the argument that solves $h(p)(p-\varepsilon) = 1$.*

Proof. Given the symmetric input demands, the pool wants to sell a symmetric bundle. Therefore $x_i = 1/n$ and $p_i = p$ for all i and the pool wants to maximize total profits $n\,(1 - F(p))\,(p - \varepsilon)$. The first order condition is $n(-f(p)\,(p-\varepsilon) + 1 - F(p)) = 0$. Rearranging terms we get the desired result. ■

Notice that p^* depends only on the functional form of h and the value of ε, but not on the values of σ or n. The following proposition compares the cost of innovation when the inputs are priced individually, c, with that of a patent pool, p^*.

Proposition 1.6. *The cost of innovation when the inputs are priced non-cooperatively, c, is equal to that of a patent pool, p^*, when the cross-price derivative is zero ($\sigma = \sigma^*$), it is larger when the inputs are complements ($\sigma < \sigma^*$) and it is smaller when the inputs are substitutes ($\sigma > \sigma^*$).*

PROOF. We know from the proof of Lemma 1.2 that when $\sigma = \sigma^*$, the cross-price derivative is zero and $\sigma = p\,h(p)$. Replacing this in (1.2) and rearranging we get $h(p)(p-\varepsilon) = 1$, which is the cooperative result. Given that p is decreasing in σ, whereas p^* is independent of σ, $p > p^*$ when $\sigma < \sigma^*$ and $p < p^*$ when $\sigma > \sigma^*$. ■

The difference between cooperative and non-cooperative pricing is that in the first case the firms take into account the effect of an increase in the price of one input on the demand for the rest. When $\sigma = \sigma^*$ this effect is zero so the price of the pool coincides with that of the non-cooperative equilibrium. When $\sigma < \sigma^*$ the effect is negative, so the pool knows that an increase in price will decrease the demand for the rest and will set a price smaller than the uncoordinated input sellers. The opposite happens when $\sigma > \sigma^*$.

In the case of the uniform distribution, the pool price is $p^* = (1 + \varepsilon)/2$. Figure 2 compares this price with the non-cooperative price for $\varepsilon = 0.1$ and $n = 5$.

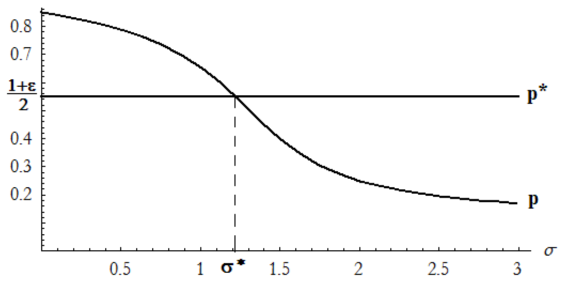


FIGURE 2. Cooperative and non-cooperative pricing.

3.5. Increasing complexity. Proposition 1.7 shows that the sign of the effect of an increase in the complexity of the innovation (n) depends on whether the inputs are complements or substitutes.

PROPOSITION 1.7. *The cost of innovation increases as innovation becomes more complex if the inputs are complements ($\sigma < \sigma^*$) and decreases if the inputs are substitutes ($\sigma > \sigma^*$).*

PROOF. We are looking for the effect of a unit increase in n, but it will suffice to determine the sign of $\partial c/\partial n$. Equation (1.2) provides an implicit function of c in terms of n. Therefore, we can calculate $\partial c/\partial n$ using the implicit function theorem:

$$\frac{\partial c}{\partial n} = \frac{1 - \sigma\,(p-\varepsilon)/p}{h'(p)\,(p-\varepsilon) + h(p) + \sigma(n-1)\varepsilon/p^2}$$

We know that the denominator is always positive. Therefore, the sign of this derivative depends on the sign of the numerator.

From equation (1.2) we get the following relation in equilibrium $\sigma\,(p-\varepsilon)/p = (n-h(p)\,(p-\varepsilon))/(n-1)$. Introducing this in the numerator and operating, it becomes $(h(p)\,(p-\varepsilon)-1)/(n-1)$. We know that $h(p)\,(p-\varepsilon) = 1$ when $\sigma = \sigma^*$ from the proof of Proposition 1.6. Given that $h(p)\,(p-\varepsilon)$ is increasing in p, it is decreasing in σ. Therefore, the numerator is positive when $\sigma < \sigma^*$ and it is negative when $\sigma > \sigma^*$. The result follows. ■

The probability of innovation is simply $1 - F(c)$, so it moves in an opposite direction to the cost:

$$\frac{dPr}{dn} = -f(c)\,\frac{dc}{dn}.$$

As before, the effect on the probability of innovation of an increase in the complexity of innovation depends on the substitutability between the inputs. If inputs are complements, then the probability decreases as n increases. If inputs are substitutes, then the probability increases as n increases.

Figure 3 shows what happens in the uniform distribution example as the complexity of the innovation increases from $n = 5$ to $n = 15$, for $\varepsilon = 0.1$. The cost schedules cross when $\sigma = 1.22$, which is exactly $\sigma^* = (1+\varepsilon)/(1-\varepsilon)$. This means that the cost of innovation increases if the inputs have low substitutability and decreases in case of high substitutability.

Proposition 1.7 is the most important result of the paper. It says that patents are very harmful when innovation is sequential and the research inputs are essential or difficult to substitute, but do not pose an important problem when inputs are easily replaceable.

Figure 4a shows cost as a function of n for complementary inputs and $\varepsilon = 0.1$ in the case of the uniform distribution. As innovation becomes more complex, the cost of innovation increases and converges to 1 when $n \to \infty$. This means that the probability of innovation

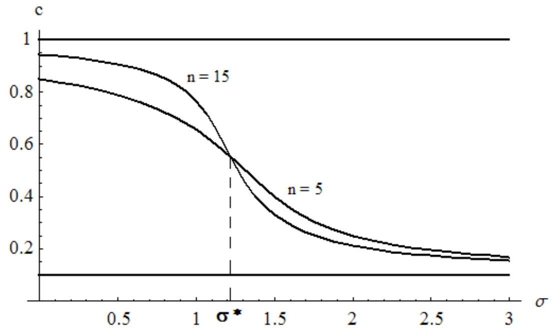
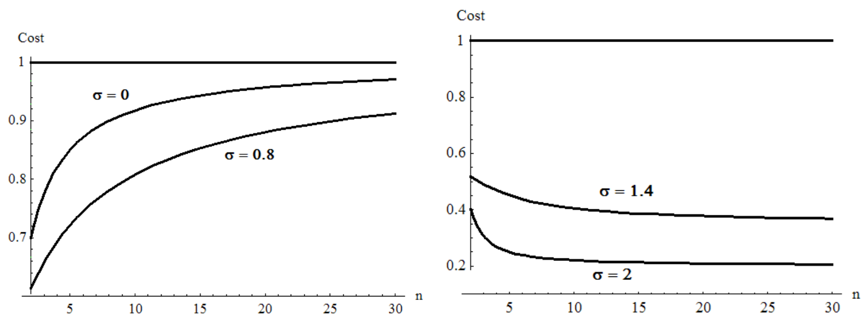


FIGURE 3. Effects of an increase in the complexity of innovation.

decreases with n and converges to 0. Convergence is faster when σ gets closer to zero. When the substitutability between the inputs is very low (σ close to zero), the probability of innovation is very small even for simple innovations (low n).

Figure 4b shows that the conclusions change when the research inputs are substitutes. In this case the cost of innovation decreases when the complexity of innovation increases (i.e. the probability of innovation increases with n).



(A) Complements. (B) Substitutes.

FIGURE 4. Cost of innovation as a function of n.

3.6. The Tragedy of the Anticommons Revisited. The model presented in this paper gives a formal treatment to the tragedy of the anticommons (Heller 1998, Heller and Eisenberg 1998). An anticommon arises when multiple owners have the right to exclude each other from using a scarce resource, causing its inefficient under-utilization.

This problem is symmetric to the tragedy of the commons, where multiple owners have the right to use a scarce resource, but nobody has exclusion rights and resources tend to be overused.

In our model, the scarce resource is the net social value of the innovation $(v - \varepsilon)$ to be shared between the innovator and the patent holders. Each patent holder decides the selling price of her input. It is interesting to notice that when $\sigma \leq 1$ all the inputs are essential to perform the innovation so all the input sellers can potentially impede the innovation by setting a high price.

We show in Proposition 1.6 that when the inputs are market complements the cost of gathering all the inputs with fragmented property rights is larger than what it would be if there was a sole owner of all the inputs or the inputs were priced cooperatively by a collective institution like a patent pool. Moreover, according to Proposition 1.7, this problem gets worse as technologies become more complex, requiring more and more inputs in order to be developed. These results hold not only for perfect complementarity between the inputs, but whenever the elasticity of substitution is not sufficiently large to compensate the negative effect of price changes on the probability of innovation.

The result in Proposition 1.6 is consistent with the findings of Shapiro (2001) and Lerner and Tirole (2004). The difference is that since we use the standard definition of complements and substitutes, we can be more precise in explaining the reasons why the cost of gathering a bundle of patents increases or decreases with the formation of the pool. It all depends on the relation between input demands. If the joint profit of the rest of firms decreases with an increase in the price of one firm (the input is a complement of the rest of inputs), a pool will consider it optimal to set a lower price when taking this effect into account. The opposite happens when the inputs are substitutes.

3.7. High complexity and Monopolistic Competition. It is interesting to analyze the equilibrium of the economy when $n \to \infty$ for two reasons. First, $n \to \infty$ represents innovations that are highly complex and therefore require a *large number* of inputs to be developed. The innovator faces a patent thicket and has to gather inputs from many patentees. We know how the probability of innovation changes as n increases, but it is interesting to determine in what cases it will go to 0 or $1 - F(\varepsilon)$. Second, in this limiting economy there is an infinite number of input sellers, so the effect of a price change by a single firm has a infinitesimal impact on the cost of innovation, and the market becomes monopolistically competitive.

Proposition 1.8 characterizes equilibria with positive probability of innovation ($p < \bar{v}$). In this case there are values of σ for which there is no equilibrium with positive probability of innovation.

PROPOSITION 1.8. *A SSPE with $p < \bar{v}$ exists only when $\sigma > \hat{\sigma}$ where $\hat{\sigma} = \frac{\bar{v}}{\bar{v}-\varepsilon}$. The equilibrium price and cost of innovation are $p = \frac{\sigma}{\sigma-1}\,\varepsilon$.*

PROOF. Dividing the first order condition (1.2) by n, we get:

$$h(p)\,(p-\varepsilon)\,\frac{1}{n} + \sigma\left(\frac{n-1}{n}\right)\left(\frac{p-\varepsilon}{p}\right) - 1 = 0.$$

As $n \to \infty$, the term with the hazard function goes to zero. This is because each firm becomes negligible and does not affect the probability of innovation on its own. It is clear that the equilibrium price of the limiting economy solves:

$$\sigma\left(\frac{p-\varepsilon}{p}\right) - 1 = 0.$$

Therefore, $p = \frac{\sigma}{\sigma-1}\,\varepsilon$, which is between ε and $\bar{v}$ only when $\sigma > \frac{\bar{v}}{\bar{v}-\varepsilon}$. ■

It is interesting to comment on three characteristics of the equilibrium. First, any $p \geq \bar{v}$ is an equilibrium for any value of σ in this limiting economy. If $p \geq \bar{v}$ the probability of innovation is zero, but if a single input seller deviates, its impact on the cost of innovation is infinitesimal, so the probability of innovation (i.e. expected profits) remains unchanged. Therefore, there are no profitable deviations when $p \geq \bar{v}$.

Second, the equilibrium quantity x_i goes to zero as $n \to \infty$. This is because the number of inputs is increasing towards infinity but the total quantity of inputs required is keeping constant, given our assumptions on the innovation technology.

Finally, it is easy to show that $1 \leq \hat{\sigma} < \sigma^*$. The first inequality follows trivially from the fact that $\hat{\sigma} = \bar{v}/(\bar{v} - \varepsilon)$. Therefore, $\hat{\sigma} = 1$ only when $\bar{v} \to \infty$ or $\varepsilon = 0$. For the second inequality, it is enough to compare the equilibrium price when $\sigma = \hat{\sigma}$ with the equilibrium price when $\sigma = \sigma^*$, since price is decreasing in σ. When $\sigma = \hat{\sigma}$, price is equal to $\bar{v}$. When $\sigma = \sigma^*$ we know that the equilibrium price solves $h(p)(p - \varepsilon) = 1$. If $p = \bar{v}$, then $h(p)(p - \varepsilon) \to \infty$, which is much larger than 1. For $h(p)(p - \varepsilon)$ to decrease and approach 1, p has to decrease. This means that equilibrium price is larger with $\hat{\sigma}$ and therefore $\hat{\sigma} < \sigma^*$.

Figure 5 shows the cost schedule as a function of σ when $\bar{v} = 1$ and $\varepsilon = 0.1$. The equilibrium of the limiting economy does not depend on the distribution of v, but it depends on the upper bound of the support of the distribution.

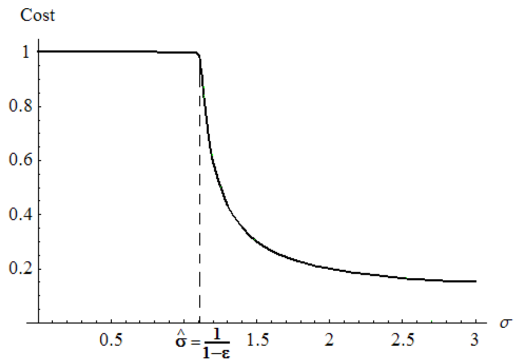


FIGURE 5. Cost of innovation in the limiting economy.

The equilibrium price is the same than Dixit and Stiglitz's (1977) monopolistic competition model. When inputs are substitutes, firms set a mark-up over marginal cost equal to $1/(\sigma - 1)$. This means that the pricing inefficiency decreases as n increases, but it does not disappear even when $n \to \infty$.

For complements, the outcome depends on whether σ is greater or less than $\hat{\sigma} = \bar{v}/(\bar{v} - \varepsilon)$. When $\sigma > \bar{v}/(\bar{v} - \varepsilon)$, firms set a mark-up just like in the substitutes case. When $\sigma \leq \bar{v}/(\bar{v} - \varepsilon)$, the only equilibria have $p \geq \bar{v}$ and so the probability of innovation is zero. In this case, as n increases the inefficiency due to monopoly pricing increases and it is at its maximum when $n \to \infty$.

4. Sequential Innovation, Patent Policy and Welfare.

In order to focus on the effects of the pricing of old ideas on the discovery of a new one, we have assumed that old ideas have already been invented when the new invention is considered. In this case, the optimal patent policy would be to remove patents completely: with patents the market price of the inputs is always above their marginal cost, introducing a wedge between the social and private cost of the innovation.

However, in many cases patent policy also affects the incentives to discover those research inputs. Given the sequential character of innovation, old ideas are past innovations themselves, so patent policy affects not only the cost, but also the revenues of the innovation.

A thorough treatment of this issue would require the development of a dynamic model, which we leave for further research. However, a slight modification of our basic model will allow us to extract some useful intuitions for the design of optimal patent policy.

Patent policy will be represented by a continuous policy parameter $\phi \in [0, 1]$, which represents patent length. Any new product is protected by a patent for a proportion ϕ of the useful life of the invention. After that, any producer can enter the market and produce the new good.

In the final goods sector, this means that the innovator remains a (perfectly discriminating) monopolist for a proportion ϕ of time. After that, free entry and perfect competition lead to zero profits. In both cases (perfect price discrimination and perfect competition), the quantity of final good produced is optimal, so there is no inefficiency coming from monopoly pricing in the final goods market. The profit of the innovator is $\phi\, v$, where v is the social value of the innovation, just as before.

In the inputs market, assuming that previous ideas were introduced sequentially, only a proportion ϕ of patents has not yet expired. This means that the innovator has to pay a non-competitive price for $\phi\, n$ inputs (those still protected by patents) and a competitive price equal to marginal cost for the other $(1-\phi)\, n$ inputs (those for which patents have already expired).

The policy parameter ϕ can also be interpreted as the novelty requirement, the patent breadth or the strength with which IP law is enforced in courts. In this case, there is a probability ϕ that the innovator is granted a patent and that the patent can be successfully defended in court. At the same time, only a proportion ϕ of previous inventors has been granted a patent.

The above assumptions guarantee that the patent regime does not affect the social value of the innovation (i.e. the total consumer surplus from the new good). In order for the patent system not to affect the social cost of the inputs either, it should lead to an optimal combination of the inputs for the innovation, which requires all inputs to be used in the same quantity. This condition holds when the inputs are perfect complements, given that conditional demands are constant and independent of price, and when the inputs are perfect substitutes, given that the price of the inputs will be equal to ε independently of the patent policy. On the other hand, when the inputs are imperfect substitutes patent policy will affect the social cost of innovation. If some inputs are protected by patents and others are not ($0 < \phi < 1$), the innovator will tend to use the cheaper unprotected inputs in a higher proportion. As a consequence, the social cost of the inputs used in R&D will be higher than ε.

This argument implies that the patent policy that maximizes the probability of innovation also maximizes expected social welfare only when the inputs are perfect complements or perfect substitutes. In the intermediate cases, the patent policy not only affects the probability of innovation but also its social cost. Therefore, when $0 < \sigma < \infty$, the policy that maximizes the probability will not maximize expected social welfare in general.

Given that it is not possible to obtain a general solution for the optimal policy in the cases of imperfect substitutability, we will first focus in the perfect complements and perfect substitutes cases. Then, we will analyze intermediate cases with the aid of numerical simulations.

4.1. Perfect Complements. Under perfect complementarity, the probability that revenues ($\phi\, v$) exceed the cost of the innovation is $1 - F(c/\phi)$ and the conditional input demands are constant and equal to $1/n$, regardless of whether the inputs are protected by patents or not. The cost of innovation is

$$c = \sum_{i=1}^{\phi n} \frac{1}{n}\, p_i + (1-\phi)\, \varepsilon.$$

The Profit Maximization Problem (PMP) of the input sellers becomes:

$$\max(1 - F(c/\phi))\,\frac{1}{n}\,(p_i - \varepsilon).$$

The following proposition characterizes the equilibrium given patent policy ϕ.

PROPOSITION 1.9. *The cost of innovation in the symmetric equilibrium solves*

$$h(c/\phi)\,(c - \varepsilon) = n\,\phi^2 \tag{1.4}$$

and its derivative with respect to patent policy ϕ is

$$\frac{dc}{d\phi} = \frac{h'\,c\,(c - e) + 2\,n\,\phi^3}{\phi\,(h'\,(c - \varepsilon) + h\,\phi)}. \tag{1.5}$$

PROOF. The first order condition of the input seller's PMP is:

$$-f\left(\frac{c}{\phi}\right)\,\frac{1}{\phi}\,\frac{\partial c}{\partial p_i}\,\frac{p_i - \varepsilon}{n} + \left(1 - F\left(\frac{c}{\phi}\right)\right)\frac{1}{n} = 0.$$

It is easy to prove that in a symmetric equilibrium, $p - \varepsilon = (c - \varepsilon)/\phi$. Imposing symmetry, using Sheppard's Lemma and rearranging we get expression (1.4). Equation (1.5) follows from a simple application of the implicit function theorem. ■

The following lemma presents a simplification which allows us to find the optimal patent policy.

LEMMA 1.3. *The patent policy that maximizes the probability of innovation solves $\frac{\partial c}{\partial \phi} = \frac{c}{\phi}$.*

PROOF. The problem of maximizing $1 - F(c(\phi)/\phi)$ is equivalent to the problem of minimizing $c(\phi)/\phi$, given that F is non-decreasing. The first order condition for the latter is

$$\frac{c'(\phi)\,\phi - c(\phi)}{\phi^2} = 0.$$

Rearranging this expression, we get our desired result. ■

PROPOSITION 1.10. *The optimal patent policy ϕ^* solves*

$$h\left(\frac{2\varepsilon}{\phi}\right) = \frac{n\,\phi^2}{\varepsilon}$$

and its derivative with respect to the complexity of the innovation is

$$\frac{\partial \phi^*}{\partial n} = -\frac{\phi^4}{2(h'\,\varepsilon^2 + n\,\phi^3)},$$

which is always negative.

PROOF. From Lemma 1.3 we know that the optimal policy solves $\frac{\partial c}{\partial \phi} = \frac{c}{\phi}$. Using (1.5) we have that:

$$\frac{h'\,c\,(c-e) + 2\,n\,\phi^3}{\phi\,(h'\,(c-\varepsilon) + h\,\phi)} = \frac{c}{\phi}.$$

Simplifying we get:

$$h(c/\phi)\,c = 2\,n\,\phi^2. \tag{1.6}$$

Combining this last expression with (1.4) we get that at the optimal policy the cost of innovation is $c = 2\varepsilon$. Introducing this in (1.6), we get our first result. The second result follows from an application of the implicit function theorem. ■

From Proposition 1.10 it follows that the optimal policy is larger than zero and less than one for $\varepsilon > 0$. This means that some protection is always desirable, even when the research inputs cannot be substituted at all. However, the most important result in Proposition 1.10 is that the degree of patent protection should decrease when technologies become more complex.

4.2. Perfect Substitutes. Proposition 1.10 shows the effects of increases in complexity on the optimal policy when the inputs are perfect complements. However, this result changes when the inputs are perfect substitutes. In this case, the cost of innovation is ε for any level of patent protection. This means that the patent policy should be directed at maximizing the revenues from the innovation, so $\phi^* = 1$ regardless of the complexity of the innovation.

4.3. Imperfect Substitutes. A comparison of the results with $\sigma = 0$ and $\sigma \to \infty$ gives an intuition of what happens for intermediate values of σ. There will be a $\bar{\sigma}$ such that the optimal policy decreases for lower degrees of substitutability, and increases in the opposite case. Given that it is not possible to get an explicit formula for $\bar{\sigma}$, we resort to a numerical analysis. Figure 6 shows the optimal patent policy as a function of σ for $n = 10$ and $n = 20$ ($\varepsilon = 0.1$). We can see that the optimal policy decreases as n goes from 10 to 20 for $\sigma < 1.137$, and increases in the opposite case, which verifies our previous conjecture.

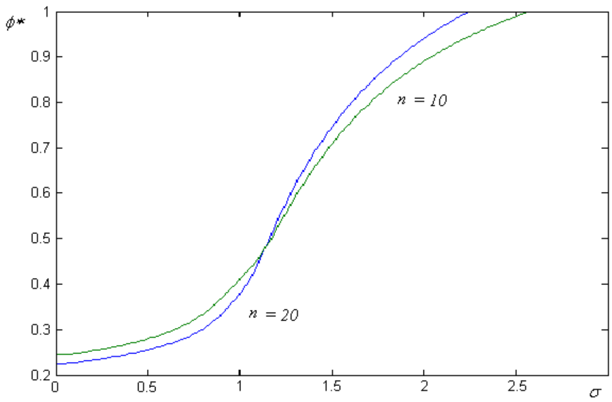


FIGURE 6. Optimal policy for different values of n.

5. Extensions.

In this section we analyze the consequences of relaxing some of the basic assumptions of the model.

5.1. Social value and cost depend on complexity. Until now, we have assumed that the distribution of values of the innovation and the social cost of the inputs do not depend on n, and that there are no returns from specialization. Under these assumptions, a change in n only changes the number of producers from whom the innovator has to buy the research inputs in order to innovate, but does not change the probability that the innovation is socially valuable.

However, it could be argued that the revenues of the innovator or the cost of the inputs are increasing or decreasing in n, or that a higher number of inputs has a positive impact in the R&D technology due to a higher division of labor. All these changes have equivalent effects on the probability of innovation so we will concentrate on changes in the distribution of returns of the innovation.

Let the return of the innovation be $a(n)\, v$, with $a'(n) \geq 0$ or $a'(n) \leq 0$ and $\lim_{n\to\infty} a(n) = a_\infty > 0$. v has a cumulative distribution $F(v)$ as before. Notice that we are not setting an upper bound on a_∞. All we require is that if a is non-increasing it does not go to zero as

$n \to \infty$. This is because if $a_\infty = 0$ then the distribution of values of the innovation will collapse to zero and the innovation will never be profitable when n is very large by assumption.

The probability of innovation is $1 - F(c/a)$, and in the symmetric equilibrium $c = p$ and $x = 1/n$. The equilibrium price of the inputs (i.e. the cost of innovation) solves:

$$(p - \varepsilon)\, h(p/a)/a = n - \sigma\,(n-1)\,(p-\varepsilon)/p$$

but we are more interested in the ratio $k = c/a$, which enters in the probability of innovation. Replacing in the previous equation we have:

$$(k - \varepsilon/a)\, h(k) = n - \sigma\,(n-1)\,(k-\varepsilon/a)/k. \tag{1.7}$$

This equilibrium is equivalent to the one in Proposition 1.2, thinking of $k = c/a$ as the cost of innovation and ε/a as the social cost of the inputs. We can prove the same theorems as before with respect to the difference between complements and substitutes, the welfare effects of patent pools and $\partial k/\partial\sigma$. However, $\partial k/\partial n$ will be different because now ε/a is a function of n.

Using the implicit function theorem on the equilibrium relation (1.7) we get:

$$\frac{\partial k}{\partial n} = \frac{\frac{h(k)\,(k-\varepsilon/a)-1}{n-1} - \frac{n\,a'}{a}\,\frac{\varepsilon/a}{k-\varepsilon/a}}{h'(k)\,(k-\varepsilon/a) + h(k) + \sigma(n-1)(\varepsilon/a)/k^2}$$

As before, the sign of this derivative depends only on the sign of the numerator, but now there is an additional term which shifts the threshold value of σ that divides positive and negative changes in k. This threshold value will be to the left of σ^* when $a'(n) > 0$ and to the right of σ^* when $a'(n) < 0$.

Two important remarks are in order. First, if $a'(n)$ is large then the last term in the numerator will determine the sign of the derivative. In this case, the effect of changes in n on revenues completely overcomes the effect on the pricing of inputs, and $\partial k/\partial n$ has the opposite sign of $a'(n)$ irrespective of the value of σ. Second, even for small $a'(n)$, when $a'(n) > 0$ and $\sigma \to \infty$ the derivative is always positive. Therefore when $a'(n)$ is small and positive, there are two regions where $\partial k/\partial n$ is positive: one with low values of σ and another with large values of σ.

According to the previous analysis, assuming that the return of the innovation depends on n has an effect on the derivative of the probability of innovation with respect to n. Next, we will show that this assumption has no significant effect on the analysis of the equilibrium as $n \to \infty$.

The equilibrium price solves:

$$h(p)\,(p-\varepsilon)\,\frac{1}{a\,n} + \sigma\left(\frac{n-1}{n}\right)\left(\frac{p-\varepsilon}{p}\right) - 1 = 0.$$

When $n \to \infty$, the first term will go to zero because $a_\infty > 0$. Therefore, the equilibrium price is the same as before, $p = \frac{\sigma}{\sigma-1}\,\varepsilon$, which is less than the maximum possible revenue ($a_\infty\,\bar{v}$) only if $\sigma > a_\infty\bar{v}/(a_\infty\bar{v}-\varepsilon)$. When $\sigma \leq a_\infty\bar{v}/(a_\infty\bar{v}-\varepsilon)$, on the other hand, there is no equilibrium price such that the probability of innovation is positive.

The probability of innovation is $1 - F(p/a_\infty)$. There are two possible cases. If $a_\infty < \infty$ then the probability of innovation is less than optimal, just as in the basic model. If $a_\infty = \infty$ then the probability of innovation will go to 1 for $\sigma > 1$ and 0 for $\sigma \leq 1$, which is the same as assuming $\varepsilon = 0$ in the basic model.

5.2. No Asymmetric Information. Another assumption of the basic model is that there is asymmetric information on the value of the innovation (read Gallini and Wright 1990, Bessen 2004, for good discussions of why this assumption makes sense). However, our results do not depend on the existence of asymmetric information. All that is needed for the results is a downward sloping demand for innovations.

An alternative interpretation could be that there is a continuum of innovators with decreasing returns to their innovations. Suppose that the innovators are indexed by the return to their innovations, which ranges between $\underline{v}$ and $\bar{v}$. Now, $F(v)$ is the measure of innovations with a return less or equal to v. Also, assume that the innovations do not compete against each other in the final goods market and that the input sellers cannot price discriminate between the innovators. It is easy to see that all the previous results translate directly into this setting. All that changes is that now $1 - F(c)$ is not the probability of innovation but the measure of innovations performed.

A second alternative would be to assume that there is a continuum of perfectly competitive innovators and that the inputs are not used in research, but in the production of every unit of final good. This description is closer to Cournot's (1838) theory of complementary monopoly. The production function of output is $y = \left(\sum_{i=1}^{n} n^{\rho-1} x_i{}^{\rho}\right)^{1/\rho}$ and the demand of the final good is $y = 1 - F(p_y)$, where y and p_y are the quantity and price of the final good. In this case, the input demands in Proposition 1.1 still solve the CMP, but c becomes a marginal cost per unit of final good. Competition leads price to marginal cost ($p_y = c$), but c still remains above ε. $1 - F(c)$ is now the equilibrium quantity of final good, but all the previous results still hold. However, now there is a welfare loss from the anti-competitive pricing in the inputs market, which is approximately equal to $(c - \varepsilon)(F(c) - F(\varepsilon))/2$.

5.3. No price discrimination. We can also relax the assumption that the innovator is a perfect price discriminator. Dropping this assumption introduces a wedge between the social and private values of the innovation. This means that the distribution of values of innovation changes, and that now there is also an inefficiency in the final goods sector. Assume that the social value of the innovation is still distributed according to $F(v)$, with probability density function $f(v)$. The private value of the innovation is now v_p, which is less than the social value of the innovation. With a linear demand for the final good, for example, the private return of the innovation would be $v_p = v/2$, which has a probability density function given by $2\,f(2v_p)$. The qualitative results are the same as before. All that changes is that now the probability of innovation decreases for each value of σ, and so the values of σ^*, $\hat{\sigma}$ and $\bar{\sigma}$ increase. Also, the optimal patent protection is lower for each value of σ and n than in the case of perfect price discrimination.

5.4. Uncertain return of the innovation. We have also assumed that the innovator is the only one that knows the value of the innovation. In this section we ask what happens if v is also unknown to the innovator. Formally, we do this by changing the timing of the game: (i) the input producers simultaneously set the price of their inputs, (ii) given prices, the innovator calculates the input mix that minimizes the cost of innovation and decides whether to innovate or not, and (iii) Nature extracts a value v for the innovation from the distribution $F(v)$.

We begin by solving the second stage of the game. The innovator decides what would be the optimal combination of inputs to perform the innovation in case he decides to perform it. This leads to the same cost of innovation and conditional demands as before. Then, the innovator decides whether to perform the innovation or not, in order to maximize expected profits $E(v) - c$. The innovation will be performed if $E(v) \geq c$ and will not be performed otherwise. If $E(v) < \varepsilon$, then the innovation will never be performed, so we assume that $E(v) \geq \varepsilon$. We also assume that the innovator will perform the innovation if $E(v) = c$.

The uncertainty has now passed from the input sellers to the input producer. The problem of the input sellers is deterministic, they know $E(v)$ and they know that if the price is higher than $E(v)$ the innovation will not be performed (i.e. their demands will be zero). Now, the inputs are always market substitutes unless $\sigma = 0$. It is easy to show that the innovation will always be performed, and that the elasticity of substitution only affects the distribution of payoffs between the input sellers and the innovator.

Lemma 1.4 shows that input demands are discontinuous at a certain price, and Proposition 1.11 proves that in the symmetric equilibrium $c \leq E(v)$ so the innovation is always performed.

LEMMA 1.4. *Input demands are discontinuous at*

$$p_i = \left(nE(v)^{1-\sigma} - \sum_{j \neq i} {p_j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

PROOF. The demand for inputs is positive if the cost of innovation is not larger than the expected value of the innovation, that is:

$$n^{-\frac{1}{1-\sigma}} \left(\sum_{i=1}^{n} {p_i}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \leq E(v).$$

Rearranging terms, we get the condition on the price of the input:

$$p_i \leq \left(nE(v)^{1-\sigma} - \sum_{j \neq i} {p_j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

If p_i is larger than this value, then the innovation is not performed and the demand for all inputs is zero. ■

The input sellers want to maximize profits $x_i\,(p_i - \varepsilon)$. Proposition 1.11 states the solution of the game.

PROPOSITION 1.11. *The equilibrium price when the return of the innovation is uncertain for the innovator is:*

$$p = \begin{cases} \frac{\sigma(n-1)}{\sigma(n-1)-n}\varepsilon & \text{if } \sigma > \frac{n}{n-1}\frac{E(v)}{E(v)-\varepsilon}, \\ E(v) & \textit{otherwise.} \end{cases}$$

PROOF. After imposing symmetry, the derivative of $x_i\,(p_i - \varepsilon)$ with respect to p_i becomes:

$$D(p) = \frac{1}{n}\left(-\frac{\sigma(n-1)}{n}\frac{p-\varepsilon}{p} + 1\right).$$

Lemma 1.4 implies that if the derivative with respect to price is positive at $p = E(v)$, this is a symmetric equilibrium, as firms are making positive profit, do not want to lower price ($D \geq 0$), and would have a zero profit if they would rise price. This happens when $\sigma \leq \frac{n}{n-1}\frac{E(v)}{E(v)-\varepsilon}$.

When $\sigma > \frac{n}{n-1}\frac{E(v)}{E(v)-\varepsilon}$, on the other hand, the equilibrium price solves the unrestricted first order condition $D(p) = 0$. ■

5.5. Input decision is discrete. Suppose now that input decisions are discrete (zero or one). What matters for innovation now is whether old ideas are used in research (for which you have to pay a license fee) or not. In this case, the qualitative results of the paper would be left unchanged, but the description of technology would be more complex, and we would have to resort to reduced forms for the payoffs of the innovator (like in Lerner and Tirole 2004). As a consequence, we would not be able to use the traditional definition of complements and substitutes, and the model would lose descriptive power.

Consider first the perfect complements case. The innovation is performed if all previous ideas are used in R&D, and is not performed if at least one of them is not used. This corresponds to $\sigma = 0$ in our model, so all previous conclusions still hold. In the perfect substitutes case, the innovator needs to use $m < n$ inputs in R&D, and any of the n old ideas is equally good. In this case, all the inputs would be priced at marginal cost (Bertrand competition), which corresponds to $\sigma \rightarrow \infty$ in our model. For intermediate cases, the value of the innovation depends on the number of ideas finally used. The degree of substitutability/complementarity depends now on the shape of the payoff function. Nevertheless, the intuition behind our previous results would be left unchanged: When inputs are more substitutable, competition leads to lower prices, and when inputs are more complementary, each input becomes more necessary and therefore prices will tend to increase.

6. Conclusions.

Innovation in hi-tech industries is sequential (each innovation builds on previous innovations) and complex. In this context patent protection affects the expected profits of the innovator in two ways: (i) by increasing her expected revenues because of the monopoly power it grants over the innovation; (ii) by increasing the cost of innovation, since the innovator must pay each of the previous patented innovations on which her new good is built.

In this paper we constructed a model of a complex and sequential innovation to analyze how the probability that an innovation is privately profitable changes as technologies become more complex and the inputs used in research are patented. We found that the results depend on the substitutability of the research inputs.

When the inputs are complements, the profitability of the innovation is decreasing in the technological complexity. In the limit (when $n \rightarrow \infty$), when the degree of substitutability is below a threshold level, which is higher than 1, the innovation is never profitable. This paper therefore gives a formal treatment of the tragedy of the anticommons.

On the other hand, when the inputs are substitutes, the profitability of the innovation is increasing in technological complexity. Even in this case, when $n \rightarrow \infty$, the cost of gathering all the inputs for the innovation is always too high from a social point of view and thus the probability of innovation is suboptimal.

Since we used a very general model not relying on strong assumptions, our findings generalize the results of the literature on complementary monopoly, mainly concerned with perfect complementarity.

We also studied what happens when inputs are priced cooperatively, either by a collective organization as a patent pool or by a single owner of all the inputs. We found that the cost of the innovation decreases with respect to the non-cooperative pricing, when inputs are market complements, while it increases when inputs are market substitutes. This result is in line with the intuition of Shapiro (2001) and the model of Lerner and Tirole (2004). In this sense the contribution of this paper has been to use a more precise definition of complementarity and substitutability. This has allowed us to study with greater detail what is the intuition behind this result.

Finally we studied the welfare implications of the patent system. We find that, when research inputs are complements, the optimal degree of patent protection is decreasing in the complexity of the innovation. This is the exact opposite of what we observe in the real world: the complexity of technology is increasing but patents are becoming stronger every day. Not only they have been recently extended to sectors previously lacking protection (sexually reproduced plants, software, business methods, products and processes of biotechnology, including plants and animals). Also patent length has been increasing over the years, and an patent systems are being created in countries where they did not previously exist. We think this is a contradiction worth to be studied further.

Bibliography

Bergstrom, T. C. (1978): "Cournot Equilibrium in Factor Markets," Discussion paper, University of Michigan working paper.

Bessen, J. (2004): "Holdup and Licensing of Cumulative Innovations with Private Information," *Economics Letters*, 82(3), 321–26.

Boldrin, M., and D. K. Levine (2002): "The Case against Intellectual Property," *American Economic Review (Papers and Proceedings)*, 92(2), 209–212.

——— (2005a): "The economics of ideas and intellectual property," *Proceedings of the National Academy of Sciences*, 102(4), 1252–1256.

——— (2005b): "Intellectual Property and the Efficient Allocation of Surplus from Creation," *Review of Economic Research on Copyright Issues*, 2(1), 45–66.

Chang, H. F. (1995): "Patent Scope, Antitrust Policy, and Cumulative Innovation," *RAND Journal of Economics*, 26(1), 34–57.

Chari, V., and L. Jones (2000): "A reconsideration of the problem of social cost: Free riders and monopolists," *Economic Theory*, 16(1), 1–22.

Clark, J., J. Piccolo, B. Stanton, and K. Tyson (2000): "Patent pools: a solution to the problem of access in biotechnology patents?," Discussion paper, United States Patent and Trademark Office.

Commision on Intellectual Property Rights (2002): "Integrating Intellectual Property Rights and Development Policy: Report of the Commision on Intellectual Property Rights," London.

Cournot, A. (1838): *Researches Into the Mathematical Principles of the Theory of Wealth*. Irwin (1963).

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297–308.

Gallini, N. T., and B. D. Wright (1990): "Technology Transfer under Asymmetric Information," *RAND Journal of Economics*, 21(1), 147–60.

Garfinkel, S., R. Stallman, and M. Kapor (1991): "Why patents are bad for software.," *Issues in Science and Technology*, 8(1), 50–55.

Graff, G. D., S. E. Cullen, K. J. Bradford, D. Zilberman, and A. B. Bennett (2003): "The public-private structure of intellectual property ownership in agricultural biotechnology," *Nature Biotechnology*, 21(9), 989–995.

Green, J. R., and S. Scotchmer (1995): "On the Division of Profit in Sequential Innovation," *RAND Journal of Economics*, 26(1), 20–33.

Heller, M. A. (1998): "The Tragedy of the Anticommons: Property in the Transition from Marx to Markets," *Harvard Law Review*, 111(3), 621–688.

Heller, M. A., and R. S. Eisenberg (1998): "Can Patents Deter Innovation? The Anticommons in Biomedical Research," *Science*, 280(5364), 698–701.

Lerner, J., and J. Tirole (2004): "Efficient Patent Pools," *American Economic Review*, 94(3), 691–711.

Llanes, G., and S. Trento (2007): "Anticommons and optimal patent policy in a model of sequential innovation," Working Paper 07-68 (38), Department of Economics, Universidad Carlos III de Madrid.

Romer, P. M. (1987): "Growth Based on Increasing Returns Due to Specialization," *American Economic Review*, 77(2), 56–62.

——— (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98(5), 71–102.

Scotchmer, S. (1991): "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives*, 5(1), 29–41.

——— (1996): "Protecting Early Innovators: Should Second-Generation Products Be Patentable?," *RAND Journal of Economics*, 27(2), 322–331.

Shapiro, C. (2001): "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting," in *Innovation policy and the economy*, ed. by A. B. Jaffe, J. Lerner, and S. Stern, vol. 1, pp. 119–150. MIT Press for the NBER.

CHAPTER 2

Dynamic Incentives in a Model of Sequential Innovation[1]

ABSTRACT. We study the problem of introducing a sequence of innovations when their value is private information. We study three cases: patents, no-patents and patent pools. With patents, each innovator has to pay licensing fees to previous innovators. We find that the probability of innovation decreases as the sequence progresses, converging to zero in the limit. Without patents, the probability of innovation is constant and higher than in the patents case, unless the degree of appropriability of the innovation is very low. A patent pool increases the probability of innovation, in comparison with the patents case, and dynamic incentives imply a higher probability of innovation than in the static case, which strengthens previous results in the literature. We also show that innovation is suboptimal in the three cases analyzed, and extend the model for patents of finite length.

1. Introduction

In Llanes and Trento (2007) we analyzed the problem of introducing an innovation based upon n existing inventions. We introduced a model in which all prior inventions are protected by broad patents, and the innovator has to pay licensing fees to n patent holders in order to innovate. Substitutability among prior inventions goes from 0 (perfect complements) to ∞ (perfect substitutes), and this determines the degree of competition between patent holders. The new idea has random value, which is private information of the innovator.

Our main findings were: (i) when innovations become more complex, the probability of innovation decreases (increases) if prior inventions are complements (substitutes), (ii) the formation of a patent pool (a cooperative agreement among patent holders, through which they agree on the licensing terms of their patents) increases (decreases) the

[1]This chapter is based on Llanes and Trento (2008).

probability of innovation if prior inventions are complements (substitutes), (iii) when $n \to \infty$, the probability of innovation goes to zero for innovations based on highly complementary prior inventions, and is positive but less than optimal in the other cases, and (iv) when prior inventions are complementary, the strength of patent policy (patent breadth or length) should decrease with complexity, in order to maximize expected welfare.

The paper provides a rigorous analysis of the problem known as the tragedy of the anticommons: when too many agents have exclusion rights over the use of a common resource, this resource tends to be underutilized, in clear duality with the tragedy of the commons in which too many agents hold rights of use and the resource tends to be overused. Additionally, the model shows that the formation of a patent pool can mitigate the perverse effects of patents when innovations are highly complementary.

Llanes and Trento (2007) complements the literature of sequential innovation and links it to the literatures of complementary monopoly and patent pools, by analyzing what happens when a given innovation depends not only on one, but on several prior inventions (for more details read Llanes and Trento 2007 and references therein). However, the model in that paper is static, when the nature of sequential innovation is truly dynamic: old inventions were once new ideas, and new ideas are the stepping stones of future innovations.

Developing a dynamic model is important for several reasons. First, it will eliminate any bias stemming from the asymmetric treatment of old and new ideas. Second, patent policy will affect not only current but also future innovative activity. Third, it will allow us to analyze the problem of assigning resources to enabling innovations with low commercial value (basic research).

In this paper we extend the analysis of Llanes and Trento (2007) to a dynamic framework. There is a sequence of innovations $n = 1, 2, \ldots$ and each innovation builds upon all prior inventions. Each innovation has a commercial value (the profit it generates as a final good), which is random and private information of the innovator, and a deterministic cost of R&D to be developed. We are mainly interested in determining if an anticommons problem arises also in this dynamic version of the model.

In the basic model, all innovations are protected by a patent of infinite breadth and length. Patents affect the innovator in two ways: on one hand, the innovator has to pay licensing fees to all previous inventors, but on the other hand, she will collect licensing revenues from all subsequent innovators, in case they decide to innovate. Therefore, it is not clear what is the net effect of patents on innovation as the sequence of innovations progresses. We compare this basic model to the case in which patents are completely removed from the economy, and analyze the effects of the creation of a patent pool.

We find that with patents, innovation becomes harder and harder with more complex innovations. The probability of innovation goes to 0 as $n \to \infty$. The probability of innovation is higher than in the static case, but not enough to stop the tragedy of the anticommons from happening.

In the no-patents case, on the other hand, the probability of innovation is constant and depends on the degree of appropriability of the commercial value of the innovation on the final goods sector. The no-patents case will provide higher innovation than the patents case unless the innovator can appropriate a very small fraction of the value of the innovation.

When ideas are protected by patents, the formation of a patent pool increases the probability of innovation for all innovations. Interestingly, the probability of innovation with a pool is constant and higher than what it would be in the static case. This result strengthens the findings of Shapiro (2001), Lerner and Tirole (2004), and Llanes and Trento (2007) for static models. The comparison between the pool and the no-patents case depends once again on the degree of appropriability of the value of the innovation in the latter case.

We solve for the innovation policy that maximizes the expected welfare of the sequence of innovations. We find that innovation is sub-optimal in the three cases. In the no-patents case, there is a dynamic externality: innovators do not take into account the impact of their decision on the technological possibilities of future innovators. In the two other cases, the inefficiency is caused by asymmetric information and market power: patent holders do not know the value of the innovation, which generates a downward sloping expected demand for the use of their ideas, and market power implies a price for old ideas above marginal cost.

Our paper is related to Hopenhayn, Llobet, and Mitchell (2006), which also presents a model of cumulative innovation with asymmetric information. However, the focus of that paper is different. Innovations are substitutive: the introduction of a new product automatically implies the disappearance of old versions in the market. Patents impede subsequent innovations for the duration of the patent. The question they study is how to allocate monopoly power in the final goods market to successive innovations. A trade-off arises because the promise of property rights to the first innovator limits what can be offered to the second innovator. In our paper, innovations are complementary: all prior inventions are necessary to introduce a new idea. We study what is the effect of intellectual property rights on the pricing of old ideas. The problem is that granting too many rights on sequential innovations implies an increase in licensing fees, hindering innovation as a consequence. Therefore, the two papers offer complementary analysis of the process of sequential innovation when the value of innovations is private information.

2. Innovation with patents

There is a sequence of innovations $n = 1, 2, \ldots$. Innovation n cannot be introduced until innovation $n-1$ has been introduced. Formally, the model is a multi-stage game with uncertain end. At each stage, an innovator may introduce an innovation. If the innovation is performed, the game continues and further innovations may be introduced. If the innovation fails to be performed, the game ends and no other innovation can be introduced (we will relax this assumption in Section 8). We will see that the probability that the game continues is determined endogenously.

The innovation process is deterministic. At stage n, the innovator may introduce the new idea by incurring in an R&D cost of ε. The new idea is based on $n-1$ previous ideas. These previous ideas are protected by patents, which means that the innovator has to pay licensing fees to the $n-1$ previous innovators (patent holders), in case she wants to introduce the innovation. The cost of innovation is the sum of the cost of R&D and the licensing fees paid to patent holders.

Each idea has a commercial value v_n, which represents the revenues obtained by selling the new product in the final goods market. In order to concentrate on the effects of patents on innovation activity, we will assume that the innovator is a perfect price discriminator in the final goods market, which means that the commercial value of the innovation is equal to the total consumer surplus generated by the new product.

The value of the innovation is private information of the innovator. Patentees only know that v_n is drawn from a uniform distribution between 0 and 1, with cumulative distribution function $F(v_n) = v_n$.

The new idea will be protected by a patent of infinite length, which means that the innovator can request licensing fees from all subsequent innovators (we will relax this assumption in Section 9). The total revenues of the innovation equal the commercial value of the innovation plus the future licensing revenues.

The timing of the game within each stage is the following: (i) the $n-1$ patent holders set licensing fees p_n^i, (ii) Nature extracts a value for v_n from distribution $F(v_n)$, (iii) the innovator decides whether to innovate ($I_n = 1$) or not ($I_n = 0$).

At each stage, patent holders only care about maximizing the expected future licensing revenues. Let J_n^i denote the expected future licensing revenues of innovator i at stage n, given that stage n has been reached. Then,

$$\begin{aligned} J_n^i &= Pr_n\, p_n^i + Pr_n\, Pr_{n+1}\, p_{n+1}^i + \dots \\ &= \sum_{m=n}^{\infty} p_m^i \prod_{k=n}^{m} Pr_k, \end{aligned}$$

where Pr_n is the probability that the n^{th} innovation is introduced, given that all prior innovations have been introduced. Notice that the probabilities Pr_n work as intertemporal discount factors, which arise endogenoulsy from the specification of the model.

J_n^i can also be expressed in a recursive way:

$$J_n^i = Pr_n\,(p_n^i + J_{n+1}^i),$$

This means that with probability Pr_n the innovation is performed, and the patent holder gets the licensing fee from the innovator plus the continuation value of her expected licensing revenues.

The innovator's payoff is $I_n(v_n + J_{n+1}^n - c_n - \varepsilon)$, where $c_n = \sum_{i=1}^{n-1} p_n^i$ is the sum of licensing fees paid to previous innovators.

We will focus on Markov strategies. A strategy for player i specifies an action conditioned on the state, where actions are prices and the state is simply the number of previous innovations introduced. The equilibrium concept is Markov Perfect Equilibrium.

Perfectness implies that future prices will be determined in following subgames, as the result of a Nash equilibrium. Thus, players understand that no action taken today can influence future prices and probabilities. Current actions only affect the probability that the following stage is reached, through the effect of current prices on the probability of innovation. We have just proved the following lemma:

LEMMA 2.1. *J_m^i for $m > n$ does not depend on any action taken at stage n.*

The game is solved recursively. The solution to the innovator's problem is straightforward. Given v_n and c_n, the innovator forecasts J_{n+1}^n, and decides to innovate ($I_n = 1$) if the revenues from the innovation exceed the cost of innovation:

$$I_n = \begin{cases} 1 & \text{if } v_n + J_{n+1}^n \geq c_n + \varepsilon, \\ 0 & \text{otherwise,} \end{cases}$$

which implies that the probability of innovation is $Pr_n = 1 + J_{n+1}^n - c_n - \varepsilon$.

Patent holders want to maximize their expected licensing revenues from stage n onwards. They know their decisions do not affect J_{n+1}^i (they can only affect the probability that stage $n+1$ is reached), and decide a licensing fee p_n^i, taking the decision of the other patent holders as given. The patent holder's problem is:

$$\max_{p_n^i} J_n^i = Pr_n \, (p_n^i + J_{n+1}^i),$$

which leads to a price equal to $p_n^i = (1-\varepsilon)/n$.

Imposing symmetry, $p_n^i = p_n$ and $J_n^i = J_n$ for all i. Replacing prices and probability in J_n, we get $J_n = \left(\frac{1-\varepsilon}{n} + J_{n+1}\right)^2$. Rearranging this equation, $J_{n+1} = \sqrt{J_n} - \frac{1-\varepsilon}{n}$, which is a decreasing sequence, converging to 0 as $n \to \infty$.

The sequence of probabilities of innovation is:

$$Pr_{n+1} = \left(Pr_n - \frac{1-\varepsilon}{n}\right)^{1/2},$$

which is also a decreasing sequence converging to 0 as $n \to \infty$. This means that innovation gets harder and harder with more complex innovations (those that are based on more previous innovations).

3. Innovation without patents

Suppose that a policy reform completely removes patents. This change has two effects on innovation. First, the revenues of the innovator in the final goods sector will decrease as a result of imitation. Specifically, assume that the innovator can only appropriate a fraction $\phi \in [0, 1]$ of the consumer surplus generated by the innovation. Second, innovators will not pay licensing fees to previous innovators, nor will they charge for the use of their ideas in subsequent innovations. Therefore, $c_n = 0$ and $J_n = 0$ in the previous model.

At each stage: (i) nature extracts a value of the innovation v_n, and (ii) the innovator decides to innovate or not. The innovator will innovate if $\phi\, v_n \geq \varepsilon$ and will not innovate otherwise. Thus, the probability of innovation is constant and equal to $1 - \varepsilon/\phi$ if $\phi > \varepsilon$. If $\phi \leq \varepsilon$, then the probability of innovation is zero.

4. Patent pools

In this section we analyze what happens when licensing fees are set cooperatively by a collective institution like a patent pool. At each stage, the pool maximizes the future expected revenues of *current* patent holders. The pool will set a symmetric price for all current patent holders. Once an innovation is performed, the innovator becomes a member of the pool in all subsequent stages. In the first stage there is no pool because no innovation has been introduced (the pool plays from stage 2 onwards).

The probability of innovation is $Pr_n = 1 + J_{n+1} - (n-1)p_n - \varepsilon$, and the pool's problem is

$$\max_{p_n} J_n = Pr_n\,(p_n + J_{n+1}).$$

The difference with respect to the non-cooperative case is that the pool recognizes cross-price effects, and therefore is encouraged to set lower prices than in the no-pool case.

The first order condition of the pool is $-(n-1)(p_n + J_{n+1}) + 1 + J_{n+1} - (n-1)p_n - \varepsilon = 0$. A higher J_{n+1} fosters innovation in two ways. First, it increases the future revenues of the innovator. Second, it encourages the pool to set a lower price, because it increases the loss of current patent holders if the sequence of innovations is stopped.

The equilibrium price is $p_n = \frac{1-\varepsilon}{2(n-1)} - \frac{n-2}{2(n-1)}J_{n+1}$, which is equal to the static price a pool would set *minus* an additional term arising from the pool's concern of keeping future revenues.

The probability of innovation becomes $Pr_n = \frac{1-\varepsilon}{2} + \frac{n}{2}J_{n+1}$. Introducing price and probability in J_n, we get

$$J_n = \frac{1}{n-1}\left(\frac{1-\varepsilon}{2} + \frac{n}{2}J_{n+1}\right)^2.$$

This is a first order non-linear difference equation. J_n is decreasing in n and converges to 0 as $n \to \infty$.

The sequence in terms of probabilities is

$$Pr_n = \frac{1-\varepsilon}{2} + \frac{1}{2}Pr_{n+1}^2.$$

which is a constant sequence such that $Pr_n = 1 - \sqrt{\varepsilon}$ for $n \geq 2$. To determine Pr_1 we need J_2, which is equal to $(1-\sqrt{\varepsilon})^2$. Then, $Pr_1 = \min\{1, 2(1-\sqrt{\varepsilon})\}$.

5. Comparison

Figure 1 shows the evolution of the probability of innovation in the three cases studied above: infinitely lived patents, no-patents and patent pool. The cost of R&D is $\varepsilon = 0.2$ and we consider $\phi = 1$ (full appropriation) and $\phi = 0.3$ (the innovator appropriates 30% of consumer surplus) for the no-patents case.

Comparing the patent and no-patent cases, we can see that patents increase the probability of the first innovations but decrease the probability of further innovations. The number of innovations for which patents increase the probability depends on ϕ. For example, when $\phi = 1$, patents only increase the probability of the first innovation. Nevertheless, even when $\phi = 0.3$ the probability increases only for the first two innovations. For patents to increase the probability of several innovations, it is necessary that ϕ is very small and close to ε.

When ideas are protected by patents, the formation of a patent pool increases the probability of innovation. Figure 1 shows that the probability of innovation with patent pools is always larger than the patents case. Moreover, with a pool the probability of innovation does not go to zero as $n \to \infty$. The comparison with the no-patents case depends on ε and ϕ. If ϕ is low, a patent pool increases the probability of all innovations. When ϕ is high, the pool increases the probability of the first innovation, and decreases the probability of all posterior innovations.

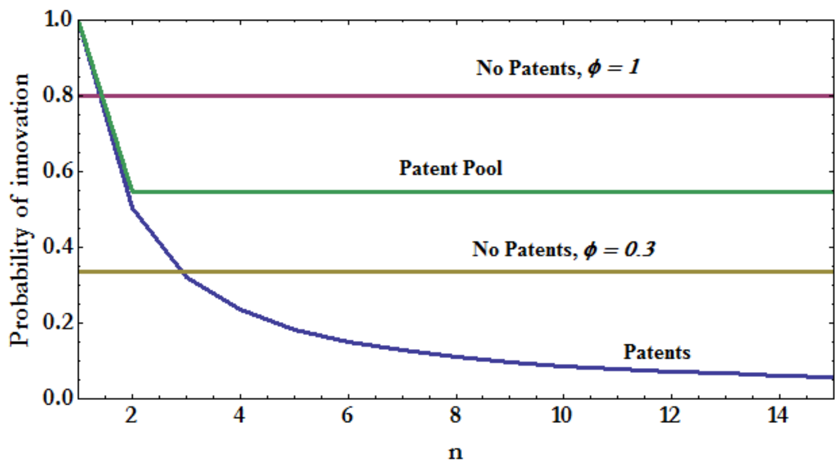


FIGURE 1. Comparison of Equilibria.

6. Socially Optimal Innovation

The relevant measure of welfare is the expected social value generated by the sequence of innovations. The social value of an innovation is equal to the increase in consumer surplus minus the cost of the resources spent in R&D. Therefore, at stage n, the social value generated is $v_n - \varepsilon$ if an innovation is performed, and 0 otherwise. Let W be the expected social value. Then,

$$\begin{aligned} W &= \sum_{n=1}^{\infty} E(v_n - \varepsilon \,/\, I_n = 1)\, Pr(I_n = 1) + E(0 \,/\, I_n = 0)\, Pr(I_n = 0) \\ &= \sum_{n=1}^{\infty} E(v_n - \varepsilon \,/\, I_n = 1) \prod_{m=1}^{n} Pr_m \\ &= \sum_{n=1}^{\infty} \int_{w_n}^{1} \frac{v_n - \varepsilon}{1 - w_n}\, dv_n \prod_{m=1}^{n} (1 - w_m) \\ &= \sum_{n=1}^{\infty} \left(\frac{1 + w_n - 2\varepsilon}{2} \right) \prod_{m=1}^{n} (1 - w_m), \end{aligned}$$

where w_n is the smallest v_n such that the innovation is performed. In the cases studied above, $w_n = \varepsilon/\phi$ when there are no patents and $w_n = c_n + \varepsilon - J_{n+1}$ with patents or patent pools.

Suppose now that the decision of whether to innovate or not is taken by a centralized agency or social planner. The social planner has to determine $\{w_n\}_{n=1}^{\infty}$, namely what is the minimum value of v_n she would require to perform the innovation at stage n. The planner may decide to perform an innovation even when the realization of v_n is less than ε, if the expected gain from future innovations exceeds the current loss in terms of welfare.

Proposition 2.1 (Socially optimal innovation). *In order to maximize expected social welfare, the innovation should be performed at stage n if and only if $v_n \geq w_n^*$, where*

$$w_n^* = \begin{cases} 0 & \text{if } \varepsilon \leq E(v_n) = 1/2, \\ \sqrt{2\,\varepsilon - 1} & \text{if } \varepsilon > E(v_n) = 1/2. \end{cases}$$

Proof. Because previous decisions are irrelevant once a stage is reached, the social planner's problem at stage n is exactly the same as the problem at stage $n+1$, which means that $w_n = w$ for all n. The social planner wants to maximize

$$\begin{aligned} W &= \sum_{n=1}^{\infty} \left(\frac{1 + w - 2\,\varepsilon}{2} \right) (1 - w)^n, \\ &= \left(\frac{1 + w - 2\,\varepsilon}{2} \right) \frac{1 - w}{w} \end{aligned}$$

The first order condition is $-(1+w^2-2\varepsilon)/(2w^2) \leq 0$, with equality if $w \geq 0$. The value that equates the first order condition, $w^* = \sqrt{2\varepsilon - 1}$, makes sense only when $\varepsilon \geq 1/2$. On the other hand, when $w \to 0$, the first order condition converges to $\text{sign}\,(2\varepsilon - 1)\infty$, which means that $w^* = 0$ only if $\varepsilon < 1/2$. ∎

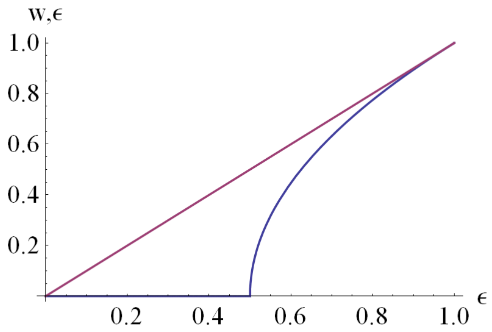


Figure 2. Socially Optimal Innovation.

Proposition 2.1 implies that innovation will be suboptimal in the three cases studied above, unless $\varepsilon = 0$. There are three reasons why this may be so: externalities, market power and asymmetric information.

The externality is best described by analyzing the no-patents case. Without patents, the innovator will perform the innovation when $v_n \geq \varepsilon/\phi$. Given that $w_n^* \leq \varepsilon$, the innovator may decide not to perform an innovation when it is socially optimal to do so, even if $\phi = 1$. There is a dynamic externality: the innovator ignores the effect of her decision on the technological possibilities of future innovators. This effect is well known in the literature of sequential innovation (Scotchmer 1991, Hopenhayn, Llobet, and Mitchell 2006), and is similar to the one found in the literature of moral hazard in teams (see for example Holmstrom 1982), where each agent internalizes only his reward from the effort exerted.

With respect to the patents and patent pool cases, the inefficiency arises from a different source: market power and asymmetric information. Because patentees care about the stream of future licensing revenues, they internalize the effect of today's decision on future innovation. However, asymmetric information implies a downward sloping demand for innovations, and market power implies inefficient pricing of patents, which leads to suboptimal innovation. As the number of holders of rights on innovation increases, the inefficiency due to market power increases.

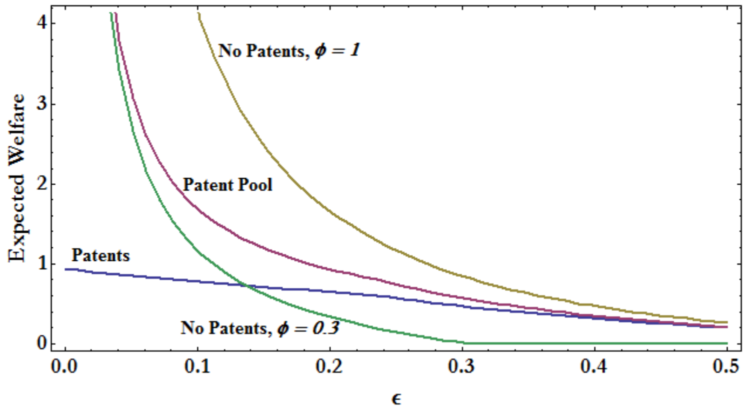


FIGURE 3. Comparison of expected welfare.

6.1. Static versus dynamic incentives. Previous models of complementary monopoly, sequential innovation and patent pools were static (Shapiro 2001, Lerner and Tirole 2004, Boldrin and Levine 2005, Llanes and Trento 2007). It is interesting to ask what changes when we add the dynamic dimension.

To see what happens in the static case, assume only one innovation is being considered. The innovation uses $n-1$ old ideas, which have already been invented. If the innovation is performed, the innovator obtains a value v from a uniform distribution between 0 and 1, and has to incur in a cost ε in R&D. The probability of innovation is $Pr = 1 - \varepsilon - c_n$, with patents or patent pool and $Pr = 1 - \varepsilon/\phi$ without patents.

With patents, the patent holder's problem is to maximize $Pr\, p^i$, the equilibrium price is $\frac{1-\varepsilon}{n}$ and the probability of innovation is $\frac{1-\varepsilon}{n}$. We have shown that in the dynamic model, the probability is $\frac{1-\varepsilon}{n} + J_{n+1}$, with $J_{n+1} > 0$. This extra term arises because the innovator gets licensing revenues from future innovators. Dynamic incentives imply a higher probability of innovation, but the increase is not enough to prevent the probability from converging to 0 as $n \to \infty$.

A patent pool would consider cross-price effects, which would lead to a price of $\frac{1-\varepsilon}{2\,(n-1)}$ and a probability of innovation of $\frac{1-\varepsilon}{2}$. The probability of the corresponding dynamic model is $\frac{1-\varepsilon}{2} + (n-1)J_n$, with $J_{n+1} > 0$. In this case, the extra term arises not only due to the future licensing revenues of the innovator, but also because the pool is concerned with keeping the future licensing revenues of current patent holders.

With respect to the no-patents case, the profit-maximizing decision is the same as in the dynamic case. This means that innovators will perform the innovation if $\phi\, v_n \geq \varepsilon$, which leads to a probability of $Pr = 1 - \varepsilon/\phi$. However, in the dynamic case innovation is suboptimal even when $\phi = 1$, which contrast with the static case, where innovation is socially optimal because there is no intertemporal link between innovations and therefore there is no externality.

7. Externalities and Optimal Transfers

In the previous section we have shown that sequential innovation is suboptimal because of the presence of externalities and asymmetric information. Without patents, current innovators do not take into account the effect of their decisions on the innovation possibilities of future innovators. The solution to this problem would require intertemporal transfers between innovators. Patents provide a way to transfer resources from future innovators to current innovators, but we have shown that with patents, market power leads to high licensing fees for old innovations, and therefore to low innovation. In this section we ask how close can the government get to the social optimum when it does not have information on the value of innovations.

To do this, we will use a simplified 2-period version of the general model. In the first period, innovator 1 has the option of introducing an innovation with value v_1 and cost ε. If innovator 1 decides to perform the innovation, in the second period, innovator 2 can introduce an innovation with value v_2 and cost ε. Innovator 1 does not know v_2.

To determine the social optimum, we have to assume the social planner knows v_1 at stage 1, and v_2 at stage 2. It is likely to think that the government would have reduced information on v_n, but assuming the social planner does not know v_n would imply that innovation decisions without patents give higher welfare than the social optimum, which does not make sense. Later we will analyze government policy, and we will assume that the government does not know v_n.

7.1. Optimal innovation. Let us begin by finding the optimal innovation policy in this 2-stage model. At stage 2 the value and cost of the first innovation are sunk. Therefore, the second innovation should be performed if $v_2 \geq \varepsilon$, and should not be performed otherwise. Consider now the first innovation. The social planner will introduce this innovation if

$$\begin{aligned} v_1 + Pr(v_2 \geq \varepsilon)\, E(v_2 - \varepsilon / v_2 \geq \varepsilon) &\geq \varepsilon, \\ v_1 + (1-\varepsilon)\left(\frac{1-\varepsilon}{2}\right) &\geq \varepsilon. \end{aligned}$$

which leads to a probability of innovation $Pr_1^* = \min\{1, \frac{(1-\varepsilon)(3-\varepsilon)}{2}\}$.

Without patents, the probability of introducing the second innovation is $Pr_2 = 1 - \varepsilon$, which is optimal. However, the probability of introducing the first innovation is also $Pr_1 = 1 - \varepsilon$, which is less than optimal. The reason is the same as in Section 6: the first innovator does not take into account the effect of her decision on the innovation possibilities of the second innovator.

With patents, innovator 1 sets a licensing fee p_1 to try to extract part of the surplus of innovator 2 (in this 2-period model, the patent and patent pool cases are the same). The probability of innovation of innovator 2 is $Pr_2 = 1 - \varepsilon - p_1$. Innovator 1 maximizes:

$$\max_{p_1} v_1 - \varepsilon + (1 - \varepsilon - p_1)\, p_1,$$

which leads to a price $p_1 = (1 - \varepsilon)/2$. The probabilities of innovation are $Pr_1 = \min\{1, \frac{(1-\varepsilon)(5-\varepsilon)}{4}\}$ and $Pr_2 = (1 - \varepsilon)/2$, so $Pr_1 \leq Pr_1^*$ and $Pr_2 < Pr_2^*$. This is due to the combined effects of asymmetric information and market power.

Therefore, the 2-period model presents a simplified version of the general model but still allows to capture the externality and asymmetric information problems.

7.2. Second Best Innovation: Optimal Transfers. One way to correct the dynamic externality would be to allow for transfers from future innovators to current innovators. We have seen that patents fail to convey appropriate incentives because of asymmetric information. In this subsection we analyze what is the optimal transfer a government should set to maximize expected welfare when it does not have information on the value of innovations, and we compare it with that of the patents case.

Assume that the government does not know v_1 nor v_2. In this case, the government cannot make the transfer depend on the realization of v_2, and it will be impossible to reach the optimum. The best the government can do is to set a transfer equal to t if innovator 2 innovates, and 0 otherwise.

In this case, innovator 1 will innovate if $v_1 + Pr_2\, t \geq \varepsilon$, and innovator 2 will innovate if $v_2 \geq \varepsilon + t$. The government wants to maximize expected welfare:

$$\begin{aligned} W &= Pr_1 \Big(E(v_1 - \varepsilon / v_1 + Pr_2\, t \geq \varepsilon) + Pr_2\, E(v_2 - \varepsilon / v_2 \geq \varepsilon + t) \Big), \\ &= \frac{1}{2}(1-\varepsilon)(2-\varepsilon-t)(1-\varepsilon+t(1-t-\varepsilon)). \end{aligned}$$

Solving this problem we get that the second best transfer with asymmetric information is:

$$t^* = \frac{3 - 2\varepsilon - \sqrt{6 - 6\varepsilon + \varepsilon^2}}{3},$$

where $t^* < p_1$. Therefore, even if the government does not know v_2, it will set a lower transfer than the licensing fee of innovator 1 with patents. This is due to the combined effects of asymmetric information and market power with patents.

8. Ongoing innovation

In this section we analyze what happens if the sequence of innovations does not stop after one innovation fails to be performed. There is a sequence of innovations $n = 1, 2, \ldots$, just as before, but now there can be many trials until an innovation is successful.

Innovator n, j is the j^{th} innovator trying to introduce innovation n ($j-1$ previous innovators tried to introduce innovation n without success). The innovator has to pay licensing fees to $n-1$ patentees (the $n-1$ previous successful innovators), and obtains a draw v_{nj} from the same distribution as before. If the revenues from the innovation are higher than the cost, innovator n, j will introduce the innovation, and in the next stage, innovator $n+1, 1$ will try to introduce innovation $n+1$. If revenues are lower than cost, innovator n, j fails to introduce innovation n, which will then be tried by innovator $n, j+1$ in the following stage. This innovator will face the same $n-1$ patent holders and will have a new draw of the value of innovation $v_{n,j+1}$.

For this model, we need to be more specific about the time dimension. Specifically, assume that stages correspond with time periods. At each period only one trial for one innovation is performed. The discount factor of innovator and patent holders is β. At stage $n+j$ the game is summarized by a state $\{n, j\}$.

Let J^i_{nj} be the expected future licensing revenues of patentee i at trial j of innovation n, given that stage $n+j$ has been reached under state $\{n, j\}$. Expressed in a recursive way:

$$J^i_{nj} = Pr_{nj}\,(p^i_{nj} + \beta J^i_{n+1,1}) + (1 - Pr_{nj})\beta J^i_{n,j+1},$$

where Pr_{nj} is the probability that innovation n is introduced in trial j. With probability Pr_{nj}, the patent holder gets the price p^i_{nj} plus the continuation value of J of the first trial of the next innovation, $J^i_{n+1,1}$, appropriately discounted by β. With probability $1 - Pr_{nj}$, the innovation will not be introduced and the patent holder gets the continuation value of J corresponding to the next trial of the current innovation.

The profit of the innovator is $I_{nj}\,(v_{nj} + \beta J_{n+1,1} - c_{nj} - \varepsilon)$.

Just as before, subgame perfection implies that the patent holders take $J^i_{n+1,1}$ and $J^i_{n,j+1}$ as given when deciding p^i_{nj}. The profit maximizing price is $p^i_{nj} = (1 - \varepsilon + \beta J^i_{n,j+1})/n$. In a symmetric equilibrium, $p^i_{nj} = p_n$ and $J^i_{nj} = J_n$ for all i, j.

Replacing in the probability of innovation, we get $Pr_n = \frac{1-\varepsilon}{n} + \beta J_{n+1} - \frac{n-1}{n}\beta J_n$, and introducing in the expression for J^i_{nj}:

$$J_n = \frac{1}{1-\beta}\left(\frac{1-\varepsilon}{n} + \beta J_{n+1} - \frac{n-1}{n}\beta J_n\right)^2.$$

Rearranging this expression:

$$J_{n+1} = \frac{1}{\beta}\left(\sqrt{(1-\beta)J_n} - \frac{1-\varepsilon}{n}\right) + \frac{n-1}{n}J_n,$$

which is a decreasing sequence converging to 0 as $n \to \infty$.

The sequence in terms of probabilities is:

$$Pr_{n+1} = \frac{1-\beta}{\beta}\left(Pr_n + \frac{n-1}{n}\frac{\beta}{1-\beta}Pr_n^2 - \frac{1-\varepsilon}{n}\right)^{1/2},$$

which is also a decreasing sequence converging to 0 as $n \to \infty$. Therefore, the main conclusions of the basic model still hold under when innovation does not stop when a single innovation fails.

9. Finite Patents

In this section we analyze what happens if patents have finite length. Each stage corresponds to one period and only one innovation is attempted at each period. If the innovator decides to introduce the innovation, she obtains a patent for L periods. This means that the innovator has to pay patents for L previous innovations, but also charges licenses to L future innovators.

The main difficulty of the present analysis is that now the identity of the patent holders matters. The price and future expected licensing revenues will be different for different patent holders, depending on how long will it take for her patent to expire.

The innovator will introduce the innovation if the revenues from innovation are larger than the cost:

$$v_n + \sum_{m=n+1}^{n+L} p_m^n \prod_{k=n+1}^{m} Pr_k \geq \sum_{i=n-L}^{n-1} p_n^i + \varepsilon,$$

which means that the probability of innovation is

$$Pr_n = 1 + \sum_{m=n+1}^{n+L} p_m^n \prod_{k=n+1}^{m} Pr_k - \sum_{i=n-L}^{n-1} p_n^i - \varepsilon.$$

The L current patent holders differ in their objective functions. Let J_n^i be the future expected revenues of patent holder i at stage n, given that stage n has been reached. Then,

$$J_n^i = Pr_n(p_n^i + J_{n+1}^i).$$

The patent holder charging a license for the last time is patent holder $n-L$, so $J_{n+1}^{n-L} = 0$. The patent of $n-L+1$, on the other hand, will last for one more period, so $J_{n+1}^{n-L+1} = Pr_{n+1}\, p_{n+1}^{n-L+1}$. In this way, we can construct the future expected revenues of the L patent holders.

The profit maximization problem is

$$\max_{p_n^i} J_n^i = Pr_n(p_n^i + J_{n+1}^i).$$

The first order condition is $-p_n^i - J_{n+1}^i + Pr_n = 0$, so $p_n^i + J_{n+1}^i = Pr_n$ and $J_n^i = Pr_n^2$ for all i. This also implies that $p_n^{n-L} = Pr_n$.

We are interested in stationary equilibria, which means that $Pr_n = Pr$ for all n. This, together with the first order condition, implies that $p_n^i = Pr(1 - Pr)$ for $i \geq n - L$. Replacing in the probability of innovation, we get:

$$\begin{aligned} Pr &= 1 + \sum_{m=n+1}^{n+L} p_m^n \prod_{k=n+1}^{m} Pr_k - \sum_{i=n-L}^{n-1} p_n^i - \varepsilon \\ &= 1 + \sum_{m=1}^{L-1} Pr(1 - Pr)Pr^m + PrPr^L - (L-1)Pr(1 - Pr) - Pr - \varepsilon. \end{aligned}$$

Solving for Pr, we get:

$$Pr = \frac{L + 1 - \sqrt{(L-1)^2 + 4L\varepsilon}}{2L}$$

which is the stationary equilibrium probability of innovation.

Figure 4 shows the probability of innovation as a function of the patent length for $\varepsilon = 0.2$. We can see that the probability of innovation decreases with L, which means that patents hurt more than benefit the innovator. This is because the innovator has to pay licenses that are certain to the patent holders, but the future licensing revenues are uncertain, as they depend on future innovations being performed.

It is also interesting to see that $Pr \to 0$ when $L \to \infty$ and $Pr \to 1 - \varepsilon$ when $L \to 0$, which correspond to the previously analyzed patents and no-patents cases (with $\phi = 1$).

9.1. Revenues Depend on Patent Length. We have assumed that the revenues from selling the new product in the final goods market are independent of patent length. In this subsection we analyze what happens when we relax this assumption. Assume the revenues of the innovator are $\phi(L)\, v_n$, with $\phi'(L) \geq 0, \phi''(L) \leq 0, \lim_{L \to 0} \phi(L) = \underline{\phi}$ and $\lim_{L \to \infty} \phi(L) = 1$. Here, $\underline{\phi}$ is the fraction of social surplus the innovator would appropriate without any patent protection, due to trade secrets or first mover advantages.

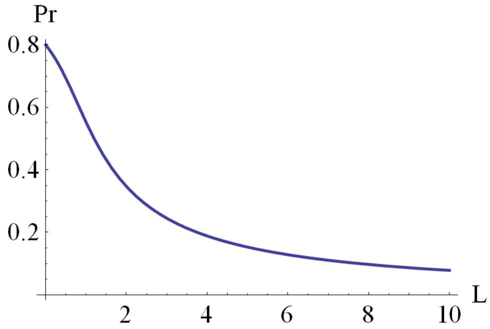


FIGURE 4. Probability of Innovation and Patent Length.

In this case, the innovator will innovate if

$$\phi(L)\,v_n + \sum_{m=n+1}^{n+L} p_m^n \prod_{k=n+1}^{m} Pr_k \geq \varepsilon + \sum_{i=n-L}^{n-1} p_n^i.$$

Applying a similar procedure as that of the previous case, we obtain the probability of innovation in the stationary equilibrium:

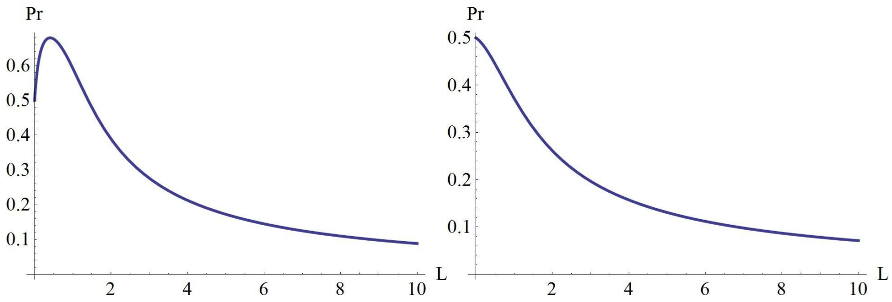
$$Pr = \frac{L+1-\sqrt{(L-1)^2+4L\varepsilon/\phi(L)}}{2L}.$$

The effect of patent length on the probability of innovation depends on the functional form of $\phi(L)$. Let $\phi(L) = 1 - \frac{1-\phi}{(L+1)^\gamma}$, where γ measures the speed at which revenues grow when L increases. Figure 5a shows that when ϕ is more concave ($\gamma = 1$), the probability of innovation first increases and then decreases with patent length. The optimal length is positive and finite (in this case $L = 1$). Figure 5b shows that for a lower degree of concavity of $\phi(L)$ it is optimal to completely remove patents. Therefore, the results do not change significantly when the revenues in the final goods sector depend on patent length.

10. Conclusions

The contribution of this paper to the literature of sequential innovation is twofold. First, it extends the analysis of anticommons to a dynamic framework. Second, it models the exact role of externalities, asymmetric information and market power in generating an inefficient innovative effort under different patent policy arrangements.

We develop a model of sequential innovation to analyze the dynamic incentives of innovators and patent holders. There is a sequence



(A) $\underline{\phi} = 0.2, \varepsilon = 0.1, \gamma = 1.$ (B) $\underline{\phi} = 0.2, \varepsilon = 0.1, \gamma = 0.1.$

FIGURE 5. Probability of innovation as a function of patent length.

of innovations $n = 1, 2, \ldots$ such that each innovation builds upon all prior inventions. Each innovation has a commercial value (the profit it generates as a final good), which is random and private information of the innovator, and a deterministic cost of R&D to be developed. We study three cases: patents, no-patents and patent pools.

With patents, each innovator has to pay licensing fees to previous innovators, but will also collect licensing revenues from future innovators. We find that the probability of innovation decreases as the sequence of innovations progresses, converging to zero in the limit. This is because the accumulation of rights on innovation increases the total license payments to previous innovators, but expected future licensing revenues decrease as a consequence of the lower probability of innovation.

Without patents, innovators do not have to pay previous innovators, but they will not be able to charge future innovators either. At the same time, imitation will be easier so the innovator will only appropriate a fraction of the commercial value of the innovation. In this case, each innovator will perform the innovation if the value she appropriates is larger than the cost of R&D. The probability of innovation will be constant and higher than in the patents case, unless the degree of appropriability of the innovation is very low.

When innovations are protected by patents, the formation of a patent pool increases the probability of innovation. As in Llanes and Trento (2007), the patent pool takes into account cross-price effects between the different innovators, and therefore is encouraged to set lower licensing fees, but dynamic incentives imply a higher probability of innovation than in the static case, which strengthens previous results in the literature.

We find the socially optimal innovation policy and show that innovation is suboptimal in the three cases studied. In the no-patents case, there is a dynamic externality: innovators do not take into account the impact of their decision on the technological possibilities of future innovators. In the two other cases, the inefficiency is caused by asymmetric information and market power: patent holders do not know the value of the innovation, which generates a downward sloping expected demand for the use of their ideas, and market power implies a price for old ideas above marginal cost.

We extend the model to an ongoing sequence of innovations (cases where the sequence of innovations does not stop when one innovation is not performed). We find that the probability of innovation with patents is decreasing and goes to zero in the limit, so the Tragedy of the Anticommons is still verified under this alternative specification.

We also extend the model to patents of finite length, and find that the probability of innovation converges to some positive value in the limit. However, we also find that the optimal patent length (the one that maximizes the probability of innovation) is very small. When patent length does not affect the revenues of the innovator in the final goods market, the optimal patent length is zero. When patent length affects these revenues, innovators should be granted a patent for only one or two periods, depending on the exact effect of patents on the commercial value of the innovation.

Bibliography

Boldrin, M., and D. K. Levine (2005): "The economics of ideas and intellectual property," *Proceedings of the National Academy of Sciences*, 102(4), 1252–1256.

Holmstrom, B. (1982): "Moral Hazard in Teams," *The Bell Journal of Economics*, 13(2), 324–340.

Hopenhayn, H., G. Llobet, and M. Mitchell (2006): "Rewarding Sequential Innovators: Prizes, Patents, and Buyouts," *Journal of Political Economy*, 114(6), 1041–1068.

Lerner, J., and J. Tirole (2004): "Efficient Patent Pools," *American Economic Review*, 94(3), 691–711.

Llanes, G., and S. Trento (2007): "Anticommons and optimal patent policy in a model of sequential innovation," Working Paper 07-68 (38), Department of Economics, Universidad Carlos III de Madrid.

——— (2008): "Dynamic incentives in a model of sequential innovation," Department of Economics, Universidad Carlos III de Madrid.

Scotchmer, S. (1991): "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives*, 5(1), 29–41.

Shapiro, C. (2001): "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting," in *Innovation policy and the economy*, ed. by A. B. Jaffe, J. Lerner, and S. Stern, vol. 1, pp. 119–150. MIT Press for the NBER.

CHAPTER 3

Industry Equilibrium with Open Source and Proprietary Firms

Abstract. I present a model of industry equilibrium to study the co-existence of Open Source (OS) and Proprietary (P) firms. Two novel aspects of the model are: (1) participation in OS arises as the optimal decision of profit-maximizing firms, and (2) OS and P firms may (or may not) co-exist in equilibrium. Firms decide their type and investment in R&D, and sell packages composed of a primary good (like software) and a complementary private good. The only difference between both kinds of firms is that OS share their technological advances on the primary good, while P keep their innovations private. The main contribution of the paper is to determine conditions under which OS and P co-exist in equilibrium. Interestingly, this equilibrium is characterized by an asymmetric market structure, with a few large P firms and many small OS firms. This finding is consistent with the observations of recent surveys.

1. Introduction

Collaboration in research enhances the chances of discovery and creation. This is true not only for scientific discoveries, but also for commercial innovations. However, innovators face incentives to limit the access of competitors to their innovations. According to the traditional view in the economics of innovation, innovators innovate because they obtain a monopolistic advantage over their competitors. Therefore, innovators should prevent other from gaining access to their discoveries, either by keeping them secret or by protecting them with patents.

This view contrasts with the Open Source (OS) development model, which has been intensively used in the software industry since the 1990s and in other industries at various points in time, as documented in the next section. In OS, developers voluntarily choose to disclose their technological improvements so that they can be copied, used and improved by other innovators free of charge, as long as further advances are also kept OS. But if everybody has access to the same technologies, then how do developers benefit from their collaborations? What do they receive in exchange for renouncing their monopolistic advantage? The answer is that OS producers collect most of their revenues from a complementary good (servicing and support in the case of software) that they also sell. Still, this leaves open the question of how can OS firms coexist and compete with Proprietary (P) firms, as the latter earn revenue from both the sale of software and the complementary services? Existing literature has yet to address this question, which is instead the main focus of this paper.

I present a model of industry equilibrium with endogenous technology sharing. Firms decide whether to become OS or P, how much to invest in product development, and the price of their products. For firms electing the OS regime, a contractual arrangement (such as the General Public License) forces them to share their improvements to the main product if they want to benefit from the contributions of other OS firms. P firms, on the other hand, develop their products on their own. Both kinds of firms sell a complementary good, the quality of which depends on the individual investment in the development of the main good. Consumers value the quality of both goods (vertical differentiation) but also have idiosyncratic tastes for the products of different firms (horizontal differentiation).

Depending on parameter values, there are equilibria with both kinds of firms and equilibria with only OS firms. When the consumer valuation of the complementary good is low in comparison with the valuation of the primary good, the equilibrium has both kinds of firms. In this case, the market structure is asymmetric with few large P firms and many small OS firms. This finding is consistent with the observations of recent surveys. Seppä (2006) compares both kinds of firms, and finds that OS firms tend to be younger and generally smaller than P firms. Bonaccorsi and Rossi (2004) show that the most important motive for firms to participate in OS projects is that it allows small firms to innovate.

A second result of the paper is the characterization of product quality under either regime. Investment in R&D may be very small in an OS firm, due to the presence of free-riding. However, P firms do not share their technological advances, generating a duplication of effort. As a consequence, either model may yield higher product quality in equilibrium.

The model shows that when both kinds of firms coexist the products of P firms are of higher quality than those of OS firms. On the other hand, when all firms are OS two things are possible: OS may prevent the entry of a higher quality good or it may result in a product of higher quality than that a potential P firm. The latter is the case when the consumers' valuation of the complementary good is high enough relative to the valuation of the main good.

Another interesting finding is that, even when the quality of OS is lower than that of P, average quality increases with the number of OS firms because this stimulates research by P firms. Investments in quality by P firms are strategic substitutes, so when a P firm becomes OS, it decreases its investment in quality and the other P firms respond by increasing theirs.

Welfare will in general be sub-optimal because of the public good problem in OS and the duplication of effort of P firms. In Section 4, I perform a welfare comparison of the different market equilibria depending on the number of firms in OS. The equilibrium may have too many or too few firms in OS. The latter will happen when average quality increases with the number of OS firms. In this case, a policy stimulating OS projects would increase social welfare. When all firms are OS in equilibrium, on the other hand, there is no other industry structure with larger welfare (a decrease in the number of OS firms would decrease social welfare).

The above model assumes symmetric consumer preferences for OS and P products. In other words, the substitutability between two OS products is the same than the substitutability between an OS and a P product. However, given that OS firms sell the same primary good, their products are likely to be more similar than those of P firms. I modify the basic model to allow for a higher cross-price elasticity between OS products by introducing two idiosyncratic taste shocks: one for the main good and one for the complementary good. OS firms share the first shock, so they are differentiated only through the complementary good. In this case, I find that if the substitutability between OS products is high enough, there are equilibria with only P firms, and also multiple equilibria.

The model and the results are important for a variety of reasons. First, endogenizing the participation decision is crucial for understanding the motivations of commercial firms to participate in OS projects. Second, the model shows there are forces leading to an asymmetric market structure, even though all firms are ex-ante symmetric. Third, the model shows under what conditions OS can overcome free-riding and produce a good of high quality, even without coordination of individual efforts. Fourth, the model shows that even when the quality of OS software is lower than the quality of P software, an increase in the number of OS firms may increase total average quality. Finally, the model allows a welfare comparison of the different equilibria.

1.1. The Software Industry and OS in Detail. There are clear antecedents of OS in the history of technological change and innovation. Nuvolari (2005), for instance, describes two nineteenth century episodes with similar characteristics: the iron industry in Cleveland (UK) and the development of the Cornish pumping engine. In both episodes, inventors shared their improvements with the rest, which led to a fast technical advance (see Allen 1983, Nuvolari 2005, for detailed quantitative analyses). One of the characteristics in common with OS is the presence of complementarities. Iron industry entrepreneurs were also owners or had mining rights of the mines in the Cleveland district. Improvements in the efficiency of blast furnaces lead to an increase in the value of the iron ore deposit. In the case of the Cornish engine, technical advances were publicized by mine managers, stimulated to do so by the owners of these mines.

OS has been used to develop software since the early years of computer science, but gained special revelance in the 1990s, with the success of Linux, Apache and Sendmail, among other programs. Software programmers started to develop software as OS to avoid the restrictions imposed by P firms on the access to the source code.

OS projects have significant market shares. According to IDC, in the second quarter of 2007 the market shares of server operating systems installed in new computer servers were: Microsoft 38.2%, Unix 31.7%, Linux 13.6%, and other 16.5%. This shows that Linux has a significant market share in the market for server operating systems. However, there are reasons to think that Linux's market share is underestimated by IDC. First, the measurement is a flow, not a stock. Second, the operating system is very often changed by users in the years following the acquisition of a computer server and Linux is considered to run better on old computers. It is also interesting to notice that most Unix systems nowadays are also OS. If we sum the shares for Unix-like systems (Unix plus Linux), we get that OS operating systems have the largest share in the server operating systems market.

The participation of individual developers in OS is still very important, but the same is true for commercial firms. In the case of embedded Linux, for example, 73.5% of developers work for commercial firms and contribute 90% of the total investment in code (Henkel and Tins 2004). Lakhani and Wolf (2005) show that 55% of OS developers contribute code at work, and these programmers contribute 50% more hours than the rest. Lerner, Pathak, and Tirole (2006) show that around 30% of OS contributors work for commercial firms (however, they cannot identify non-US commercial contributors). Moreover, they show that commercial firms are associated with larger and more dynamic OS projects (commercial contributors have four times more sensitivity to the growth of the project).

Firms participate in OS projects because they sell goods and services complementary to the code. For instance, IBM sells consultancy services and specialized complementary software, HP sells personal computers and computer servers and Red Hat sells training and support services.

The presence of complementarities in OS has been documented by Henkel and Tins (2004) and Dahlander (2004). Henkel and Tins present a survey of embedded Linux developers and show that 51.1% of developers work for manufacturers of devices, chips or boards and 22.4% work for specialized software companies. Dahlander finds that the dominant trend for appropriating the returns of innovation in OS is the sale of a complementary service.

The sale of a complementary service can indeed be profitable. The case of Red Hat is illustrative. According to its financial statements, in fiscal year 2007 Red Hat invested $71 million in R&D, and got over $400 million in revenues for its subscription and training services. However, competition is large within the market for support services. Table 1 shows the license and maintenance revenues of the top 5 vendors of Linux in 2003. These firms collaborate in software development but are competitors in the market for related support services.

	Total Revenue	Market Share (%)
Red Hat	69.3	53.5
Novell	29.0	22.4
Turbolinux	11.2	8.6
MandrakeSoft	4.4	3.4
Red Flag	2.7	2.1
Other	13.0	10.0
Total	129.6	100.0

Table 1. Revenues of top 5 Linux vendors in 2003 (millions of dollars). Source: IDC, 2005.

OS licenses are the instruments guaranteeing the access of developers to the source code. Some licenses allow further modification of the source code without imposing any restriction on developers. Restrictive OS licenses, on the other hand, require the disclosure of further improvements to the source code when programs are distributed (programmers are still allowed to keep their innovations private if the program is for personal use). The most popular OS license is the General Public License (GPL), which is a restrictive license. The GPL is used by Linux, MySQL, Perl and Java, for example. It is true that some OS contributors disclose improvements to the source code even when these modifications are for personal use. However, restrictive licenses are the most important means for the success of OS projects. For example, the survey by Henkel and Tins (2004) finds that the main reason why developers disclose their contributions to the code is because they are forced to do so by the GPL.

1.2. Related Literature. Previous papers have been mainly concerned with explaining why individual developers contribute to OS projects, apparently for free (see Rossi 2004, Lerner and Tirole 2005, for recent surveys). Some of the initial answers have been altruism, personal gratification and career concerns. The motivations of commercial OS firms, on the other hand, have been studied less extensively.

Lerner and Tirole (2005) present a description of OS and identify directions for further research. Some of the questions related with the present paper are: (i) what are the incentives of for-profit firms to participate in OS, (ii) what development model provides higher quality and welfare, and (iii) what is the influence of the competitive environment in OS. More importantly, these authors remark that direct competition between P and OS firms has received little attention.

Most papers addressing competition between the two paradigms are duopoly models of a profit maximizing P firm and a community of not-for-profit OS developers, selling at marginal cost (Kuan 2001, Mustonen 2003, Bitzer 2004, Gaudeul 2005, Casadesus-Masanell and Ghemawat 2006, Economides and Katsamakas 2006). Introducing profit-seeking OS firms is important because it allows to perform an analysis of their product development decisions. To my knowledge, the only papers with profit maximizing OS firms are Bessen (2006) and Schmidtke (2006). However, these papers do not deal with direct competition between the two paradigms.

This paper is also related to the literature of cooperation in R&D. Kamien, Muller, and Zang (1992) show that free-riding incentives are so strong that a joint venture where firms share R&D but do not coordinate their R&D levels has a lower total investment than the individual investment of each one of these firms when there is no cooperation in R&D. I show that this result can be reversed if the consumer valuation of the complementary good is high enough in comparison with her valuation of software. Bloch (1995) builds a model of endogenous association in oligopolies. Firms decide to enter the association sequentially, and compete in quantities after the association is formed. Bloch shows that in equilibrium two competing association of firms are formed. However, firms do not decide their optimal investments in R&D, so this model cannot be used to analyze the free-riding incentives created by association.

2. The Model

2.1. Technology. There are n firms selling packages composed of a primary good (which is potentially OS) and a complementary private good. Firms may improve the quality of both goods by investing in a single R&D technology. Let x_i be the investment in R&D of firm i. The cost of the investment is $c\,x_i$, which is a fixed cost, and the marginal cost of producing packages is zero.

The quality of the primary good depends on the investment of all firms in the project. For P firms, quality is simply $a_i = \ln(x_i)$. For OS firms, quality is $a_{os} = \ln(\Sigma_{i \in os}\, x_i)$.

The quality of the complementary good is $b_i = \ln(x_i)$ for all firms. There is a learning effect: firms improve the quality of their complementary good when they participate more in the development of the primary good. For example, if a software firm participates more in an OS project, it gains valuable knowledge and expertise and then can offer a better support service.

2.2. Preferences. There is a continuum of consumers. Each consumer has income y and buys only one package. Consumer j's indirect utility from consuming package i is:

$$v_{ij} = \alpha\, a_i + \beta\, b_i + y - p_i + \varepsilon_{ij}, \tag{3.1}$$

where α is the valuation of quality of the primary good, β is the valuation of quality of the complementary good, p_i is price, and ε_{ij} is an *idiosyncratic* shock (unobservable by firms) representing the heterogeneity in tastes between consumers. This specification for preferences allows for vertical (a_i and b_i) and horizontal (ε_{ij}) product differentiation.

Each consumer *observes* prices and qualities and then chooses the package that yields the highest indirect utility. The total mass of consumers is 1, so aggregate demands are equivalent to market shares. To obtain closed-form solutions for the demands I make the following assumption, which corresponds to the multinomial logit model (McFadden 1974, Anderson, De Palma, and Thisse 1992):

ASSUMPTION 3.1. *The idiosyncratic taste shocks ε_{ij} are i.i.d. according to the double exponential distribution:*

$$\Pr(\varepsilon_{ij} \le z) = \exp\left[-\exp\left[-\nu - z/\mu\right]\right]$$

where ν is Euler's constant ($\nu \approx 0.5772$) and μ is a positive constant.

Under Assumption 3.1, the market share (demand) of firm i is:

$$s_i = \frac{\exp\left[\,(\alpha\, a_i + \beta\, b_i - p_i)\,/\,\mu\,\right]}{\sum \exp\left[\,(\alpha\, a_i + \beta\, b_i - p_i)\,/\,\mu\,\right]}. \tag{3.2}$$

The ε_{ij} have zero mean and variance $\mu^2\pi^2/6$, hence μ measures the degree of heterogeneity between consumers. I will show that the equilibrium depends on two important relations:

$$\delta = \frac{\alpha+\beta}{\mu}, \qquad \gamma = \frac{\alpha}{\alpha+\beta}.$$

δ measures the relative importance of vertical vs. horizontal product differentiation and γ represents the relative importance of the primary good vs. the complementary good (γ can also be interpreted as the degree of public good of the investment in R&D).

To guarantee the existence of a symmetric equilibrium we need enough horizontal differentiation relative to vertical differentiation. I will assume $\mu \geq \alpha+\beta$, which is a sufficient condition. Thus $\delta \in [0,1]$.

2.3. Game and Equilibrium Concept. The model is a two-stage non-cooperative game. The players are the n firms. In the first stage firms decide their type (OS or P), and in the second stage they make their investment and price decisions (x_i, p_i). It can be shown that, in the case of logit demands, equilibrium decisions would be exactly the same if they were taken sequentially (first x_i and then p_i).

I want to focus on the decisions of firms, so I will abstract from the decisions of consumers. Given investments (quality) and prices, each consumer chooses her optimal package. These decisions are summarized by consumer demands (s_i) and embedded into the firms' payoffs: $\pi_i = s_i\, p_i - c\, x_i$.

The equilibrium concept is Subgame Perfect Equilibrium. I will only analyze symmetric equilibria, i.e. all firms deciding to be of the same type in the first stage will play the same equilibrium strategy in the second stage.

3. Solution of the Model

3.1. Second Stage. Let n_{os} be the number of firms deciding to be OS in the first stage. In the second stage, firms choose p_i and x_i

to maximize $\pi_i = s_i\, p_i - c\, x_i$, taking as given the demands and the decisions of other firms.

Working with the first order conditions and imposing symmetry we get the optimal price:

$$p_i = \frac{\mu}{1 - s_i}, \tag{3.3}$$

and the optimal investment in R&D for OS and P firms:

$$x_{os} = \frac{\alpha + \beta}{c}\, s_{os} \left(1 - \gamma \frac{n_{os} - 1}{n_{os}(1 - s_{os})}\right), \tag{3.4}$$

$$x_p = \frac{\alpha + \beta}{c}\, s_p. \tag{3.5}$$

The term inside the parenthesis of (3.4) represents free-riding: assuming $s_{os} = s_p$, OS have less incentives to invest than P because they can appropriate a smaller fraction of their investment. In other words, there is a public good problem given that OS are sharing their technological advances.

From (3.2), we can get the ratio of market shares s_{os}/s_p. Introducing equations (3.3) to (3.5), taking logs and rearranging terms we get:

$$(1 - \delta)\, \ln\left(\frac{s_{os}}{s_p}\right) + \frac{1}{1 - s_{os}} - \frac{1}{1 - s_p} = \delta\, \ln\left(1 - \gamma \frac{n_{os} - 1}{n_{os}(1 - s_{os})}\right) + \delta\, \gamma\, \ln(n_{os}). \tag{3.6}$$

This equation says that the difference in market shares depends on the resolution of the conflict between *free-riding* and *duplication of effort*. To see this, notice that the left hand side is increasing in s_{os} and decreasing in s_p, so the difference in market shares will increase if the right hand side does. The first term on the right hand side is just the difference between x_{os} and x_p (free-riding). The second term is a multiplicative effect due to the elimination of the duplication of effort in OS (collaboration effect).

The second-stage equilibrium is completely characterized by (3.6) and the condition that the sum of the market shares is equal to 1:

$$n_{os}\, s_{os} + (n - n_{os}) s_p = 1. \tag{3.7}$$

Proposition 3.1. *A second-stage equilibrium exists and is unique. Given n_{os}, the equilibrium market shares solve (3.6) and (3.7).*

All proofs are in the Appendix.

3.2. Second-Stage Comparative Statics. In Lemma 3.1 I present a simple condition to determine which kind of firm will have higher market share in a second-stage equilibrium.

Lemma 3.1. *$s_p > s_{os}$ if and only if $\gamma > \hat{\gamma}(n_{os}, n)$, and $s_p < s_{os}$ in the opposite case, where $\hat{\gamma}(n_{os}, n)$ is increasing in n_{os} and n and solves:*

$$\gamma \frac{n_{os}^{\gamma}}{n_{os}^{\gamma} - 1} \frac{n_{os} - 1}{n_{os}} = \frac{n-1}{n}.$$

The comparison between OS and P quality is equivalent to the comparison between market shares: if $s_{os} > s_p$, then the value of OS packages is higher than the value of P packages, and vice versa.

Lemmas 3.2 and 3.3 analyze the effects of changes in δ and γ on s_{os} (the effects on s_p have the opposite sign).

Lemma 3.2. *s_{os} is increasing in δ if $\gamma < \hat{\gamma}(n_{os}, n)$, and decreasing in δ in the opposite case.*

Lemma 3.2 has a clear interpretation. When δ increases, vertical differentiation gets more important relative to horizontal differentiation. This means that investing in R&D has a larger effect on demand, which benefits firms with higher quality products. If $\gamma < \hat{\gamma}$, then the firms with a higher quality product are the OS firms, and therefore, their market share increases relative to the market share of the P firms. The opposite happens when $\gamma > \hat{\gamma}$.

Lemma 3.3. *There exists $\gamma_d \in (0, \hat{\gamma})$ such that s_{os} is increasing in γ for $\gamma < \gamma_d$, and decreasing in γ for $\gamma > \gamma_d$.*

Lemma 3.3 implies that the graph of s_{os} with respect to γ is hump-shaped. For low values of γ, collaboration dominates free-riding (investment is mostly private), so s_{os} is increasing in γ. For high values of γ, free-riding dominates collaboration and s_{os} is decreasing in γ.

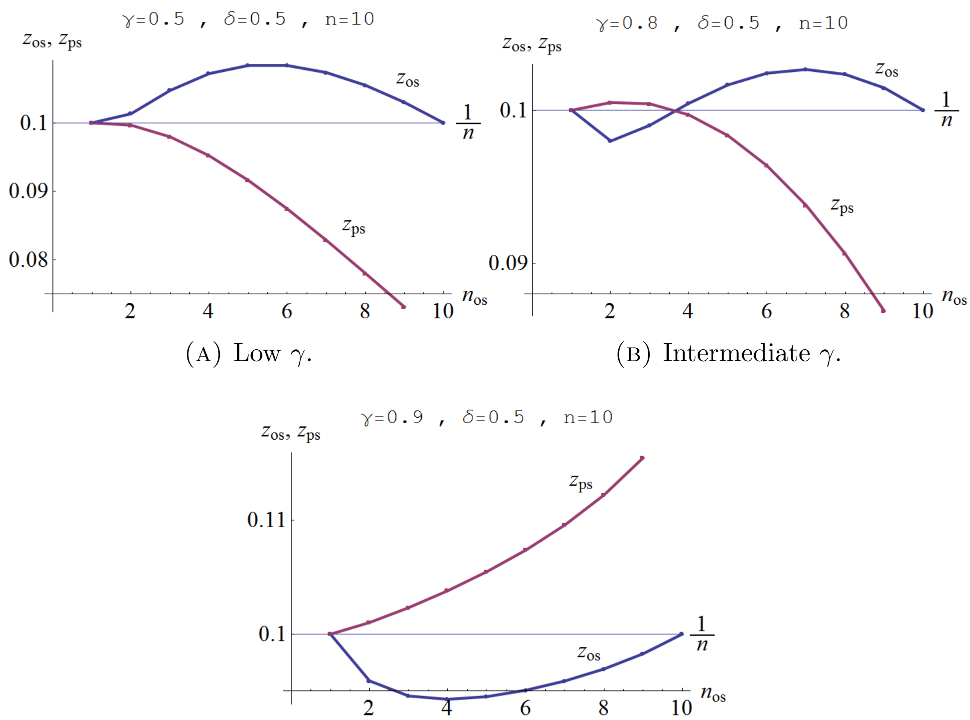
The effects of changes in n_{os} are more difficult to be determined.

An increase in n_{os} has three effects: (i) free-riding tends to decrease the market share of OS relative to P, (ii) collaboration tends to increase the market share of OS relative to P, and (iii) a level effect on the equilibrium market shares may increase or decrease both market shares.

With respect to (i) and (ii), we know from Lemma 3.3 that collaboration dominates free-riding when γ is low, and free-riding dominates the collaboration when γ is high. To understand (iii), suppose that for a given n_{os}, P firms have higher market shares. If some P becomes OS, it will change a large market share for a small market share. The difference will be distributed among all firms and hence both kinds of firms will tend to increase their market shares (positive level effect). The opposite will happen if P firms initially have lower market shares.

The following lemma, together with Lemma 3.1 can be used to describe the effects of changes in n_{os}.

LEMMA 3.4. *If* $s_p(n_{os}) < s_{os}(n_{os})$*, then* $s_p(n_{os}-1) > s_p(n_{os}) > s_p(n_{os}+1)$ *(*s_p *is decreasing in* n_{os}*).*



(A) Low γ. (B) Intermediate γ.

(C) High γ.

FIGURE 1. Effects of changes in n_{os} on market shares.

There are three cases, depending on the value of γ, which are represented in Figures 1a to 1c. When γ is low ($\gamma \leq \hat{\gamma}(2,n)$) collaboration is stronger than free-riding and the level effect is negative. This implies that s_p decreases with n_{os} (by Lemma 3.4) and s_{os} increases for small n_{os} (collaboration is stronger than the level effect) and decreases for large n_{os} (level effect is stronger than collaboration). When γ is high ($\gamma > \hat{\gamma}(n-1,n)$), free-riding is more important than collaboration and the level effect is positive. Thus, s_p increases and s_{os} decreases and then increases with n_{os}. For intermediate values of γ ($\hat{\gamma}(2,n) < \gamma \leq \hat{\gamma}(n-1,n)$), collaboration dominates for small n_{os} and free-riding dominates for large n_{os}, and the level effect is positive for small n_{os} and negative for large n_{os}.

3.3. First Stage. In the first stage of the game, firms decide whether to be OS or P, taking as given the decisions of the rest of firms and forecasting their equilibrium payoffs in the second stage. Let $\pi(n_{os})$ be the second stage equilibrium payoffs when n_{os} firms decide to be OS. Replacing the second stage equilibrium values of prices and investments for both kinds of firms we get:

$$\pi_{os}(n_{os}) = \mu \frac{s_{os}}{1-s_{os}} \left(1 - \delta(1-s_{os}) + \delta\gamma \frac{n_{os}-1}{n_{os}}\right), \tag{3.8}$$

$$\pi_p(n_{os}) = \mu \frac{s_p}{1-s_p} \left(1 - \delta(1-s_p)\right), \tag{3.9}$$

where $s_{os} = s_{os}(n_{os})$ and $s_p = s_p(n_{os})$ are the second stage equilibrium market shares. Comparing equations (3.8) and (3.9), we can see the direct effect of collaboration in profits, which is the saving in the investment cost of OS firms (third term inside the parenthesis of the first equation).

A number n_{os} of firms in OS is an equilibrium if and only if $\pi_{os}(n_{os}) \geq \pi_p(n_{os}-1)$ and $\pi_p(n_{os}) \geq \pi_{os}(n_{os}+1)$. The first inequality says that firms deciding to be OS cannot gain by deviating and becoming P. The second inequality is a similar condition on the decision of being P. The equilibrium conditions can be summarized by the function

$f(n_{os}) = \pi_{os}(n_{os}) - \pi_p(n_{os}-1)$:

$$(3.10) \qquad f(n_{os}) = \mu \frac{s_{os}}{1-s_{os}} \left(1 - \delta(1-s_{os}) + \delta\gamma \frac{n_{os}-1}{n_{os}}\right) - \mu \frac{\tilde{s}_p}{1-\tilde{s}_p} \left(1 - \delta(1-\tilde{s}_p)\right),$$

where $s_{os} = s_{os}(n_{os})$ and $\tilde{s}_p = s_p(n_{os} - 1)$. Using this function, the equilibrium conditions can be restated as $f(n_{os}) \geq 0$ and $f(n_{os}+1) \leq 0$.

The equilibrium may be such that both kinds of firms co-exist (interior equilibrium) or all firms choose to be of the same kind. $n_{os} = 0$ is always an equilibrium. For $n_{os} = 1$ to be an equilibrium we need $f(2) \leq 0$. Likewise, for $n_{os} = n$ to be an equilibrium we need $f(n) \geq 0$.

Proposition 3.2 (Existence and uniqueness of equilibrium). *An equilibrium for the complete game exists and is unique.*

Simulations show that $f(2) \geq 0$ for any γ and δ. This means that it is always profitable to begin an OS project in the case an OS project does not exist. Suppose that all firms are P and two of them decide to become OS. If the market share increases as a result of this deviation, then it is obvious that the deviation is profitable. Suppose instead that the market share of these two firms decreases as a result of their collaboration. This means that the two firms had a large cut in their investment in code. However, their market shares will not decrease as much, because their investment efforts are now pooled together. As a consequence, the profit of these firms will increase and they will find it profitable to start the OS project. This result explains the presence of OS projects in nearly every category of software.

When firms choose between OS or P, they compare the relative benefits of collaboration and secrecy. There are two elements associated with this trade-off. On one hand, free-riding and collaboration affect the equilibrium market shares, as has been analyzed in the previous sections. On the other hand, OS firms have a lower investment cost. Being P will be more profitable than OS only if free-riding is sufficiently strong as to overcome the positive effects of collaboration.

Lemma 3.5 will prove very important in characterizing the Subgame Perfect Equilibrium of the game. In order for an OS firm to find it profitable to become P ($f(n_{os}) < 0$), it has to be the case that the increase in market share from becoming P is large enough to compensate for the increase in cost. If $\gamma < \hat{\gamma}(n_{os}-1, n)$, then OS firms have a larger market share so it is not profitable for them to deviate ($f(n_{os}) > 0$). Corollaries 3.1 and 3.2 are two important implications of this lemma.

LEMMA 3.5 (Sufficient condition for positive f). *If* $\gamma < \hat{\gamma}(n_{os}-1, n)$ *then* $f(n_{os}) > 0$.

COROLLARY 3.1 (Necessary condition for an interior equilibrium). *At an interior equilibrium* n_{os} *it is necessary that* $\gamma \geq \hat{\gamma}(n_{os}, n)$.

COROLLARY 3.2 (Sufficient condition for an equilibrium with $n_{os} = n$). *If* $\gamma \leq \hat{\gamma}(n-1, n)$ *then there is an equilibrium where all firms decide to be OS.*

Corollary 3.1 states that in any interior equilibrium it has to be the case that the P firms have a larger market share than OS firms, and therefore a higher quality product. This is because the market share of P firms has to be large enough in order to compensate for the larger cost of investment. For P firms to have a larger market share than OS firms, in turn, it is necessary that the public good component of the investment is high enough ($\gamma > \hat{\gamma}(n-1, n)$).

Corollary 3.2 complements the previous corollary. If the degree of public good of the investment is low enough, OS firms have a larger market share for any n_{os}, and therefore all firms decide to collaborate in the OS project.

Proposition 3.3 and the previous two corollaries completely characterize the equilibrium.

PROPOSITION 3.3 (Necessary and Sufficient condition for equilibrium with $n_{os} = n$). *Given* $n > 3$ *and* δ*, there exists* $\bar{\gamma} \in (\hat{\gamma}, 1)$ *such that* $f(n) \geq 0$ *if and only if* $\gamma \leq \bar{\gamma}$.

Corollaries 3.1 and 3.2 and Proposition 3.3 imply that there are three kinds of equilibria. If $\gamma > \bar{\gamma}$, then there is an interior equilibrium with both kinds of firms, where the quality of P goods is higher than that of OS goods. If $\hat{\gamma}(n-1, n) < \gamma < \bar{\gamma}$, then all firms decide to be OS. However, if one of the firms was to become P, then it would produce a good of higher quality than the OS firms. This means that OS prevents the entry of a product of better quality. Finally, if $\gamma \leq \hat{\gamma}(n-1, n)$, then the equilibrium has only OS firms, but OS quality is higher than that of a potential deviator.

Figure 2 shows the regions corresponding to the three equilibria for different values of n and γ, and for δ equal to 1. We can see that $\bar{\gamma} < 1$ for $n \geq 4$. The area corresponding to interior equilibria first increases but then decreases as n increases. This means that large numbers favor cooperation, even without coordination of individual investments.

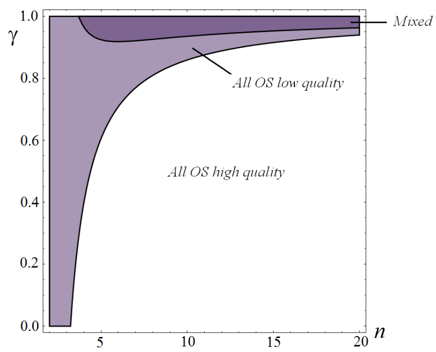


FIGURE 2. Equilibrium regions.

3.4. OS Licenses, Free-Entry and Profits. In this section I show there is an economic reason for the provision in OS Licenses (such as the General Public License) of free-entry into OS projects. Specifically, the General Public License establishes that any developer is free to join the OS project, without requiring a minimum contribution to the project. The only restriction is that whenever modifications to the program are distributed, they have to be made available to the rest of developers in the project.

The reason for this kind of provision is simply that OS firms prefer to compete against other OS firms, rather than competing against P firms. When a P firm becomes OS, the investment effort is shared between more firms. Also, if the P product is of better quality, the firm joining the OS project changes a good of high quality for a good of low quality, which benefits the rest of OS firms. On the other hand, if the OS product is initially of better quality, the saving in the investment cost compensates the competition from a higher quality substitute.

This is interesting because other forms of cooperation in R&D, like research joint ventures, cross-licenses and patent pools, generally limit the access of competitors to the agreement, or have explicit rules (cross-payments, license fees, etc.) according to which the benefit of a contributor from joining the agreement depends on her individual contributions.

Figure 3 shows the profit schedules of P and OS firms for $\gamma = 1$, $\delta = 1$ and $n = 10$. As n_{os} changes, the profits of both types of firms increase. For lower values of the parameters γ and δ, the increase in the profits of OS firms is even larger.

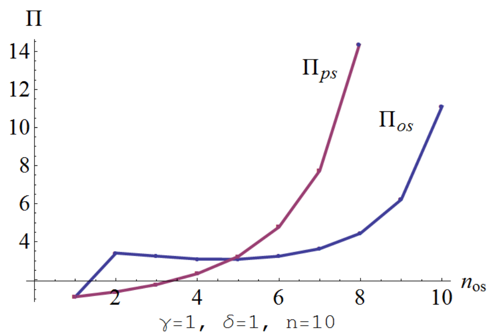


FIGURE 3. Firm profits as functions of n_{os}.

Notice that the result holds even being the case that OS firms are direct competitors in the market of software and support. If the firms were not direct competitors but were benefiting from the development of the OS program (e.g. HP and Red Hat) the result would be even stronger. However, the result also depends on the fact that the number of firms in the industry is fixed. If free-entry into the project would stimulate the entry of new firms in the industry the result could be reversed.

Finally, it is interesting to note that for γ large, entry in the OS project also benefits P firms. This result changes when γ is smaller. In this case, entry into the OS project diminishes the profits of P firms.

4. Welfare Analysis

One of the advantages of the logit model is that it can be used to construct a representative consumer whose utility embodies the aggregate behavior of the continuum of users.

Let s_i be the quantities of each variety consumed by the representative consumer, and let $\sum s_i = 1$. Total income is y and s_0 represents consumption of the numeraire. The utility of the representative consumer is:

$$U = \sum (\alpha\, a_i + \beta\, b_i) s_i - \mu \sum s_i \ln(s_i) + s_0$$

This utility embodies two different effects. The first term represents the direct effect from consumption of the n varieties, in the absence of interactions. The second term introduces an entropy-effect, which expresses the preference for variety of the representative consumer.

The social welfare function corresponding to this utility function is:

$$W = \sum (\alpha\, a_i + \beta\, b_i) s_i - \mu \sum s_i \ln(s_i) + y - \sum c\, x_i, \tag{3.11}$$

and the Social Planner's problem is to maximize (3.11) subject to $\sum s_i = 1$. It is obvious that the Social Planner would have all the firms sharing their improvements to the primary good. Also, given the concavity and symmetry of the utility function, the social planner will set $s_i = 1/n$ for all i. To determine the optimal investment, the Social Planner maximizes:

$$W = \alpha\, \ln(n\, x^*) + \beta\, \ln(x^*) + \mu \ln(n) + y - n\, c\, x^*,$$

which leads to an optimal investment equal to $x^* = (\alpha + \beta)/c\, n$.

Not surprisingly, product quality is suboptimal regardless of the number of OS and P firms. The reasons have been previously exposed. In OS free-riding leads to a lower investment in R&D. P firms, on the other hand, do not share their improvements on the primary good, generating a duplication of effort.

We know that product quality is suboptimal. However, it would be interesting to rank the different equilibria in terms of welfare. Specifically, it is interesting to analyze what is the effect of changes in n_{os} on welfare to see if the equilibrium number of firms in OS is too low or too high from a social point of view.

Simulations show that when γ is high, social welfare increases, reaches a maximum and then decreases with n_{os}, but this welfare maximum depends on the value of δ. As δ increases, the number of firms in OS which maximizes welfare also increases. When γ is low, on the other hand, welfare increases monotonically and is maximized when all firms are OS.

Figure 4 shows W and $f(n_{os})$ for $\gamma = 1$, $\delta = 1$ and $n = 10$. The equilibrium number of firms in OS is $n_{os} = 6$, but maximum welfare is not reached until $n_{os} = 8$ (simulations show that in general the maximum for $\gamma = 1$ and $\delta = 1$ is $n_{os} = n - 2$ for any n). The effect driving the increase and later decrease in welfare is the change in average quality. Even though OS firms invest less in quality than P firms when γ is high, an increase in the number of OS firms may increase average quality because it stimulates the investment of P firms (quality investments are strategic substitutes for P firms).

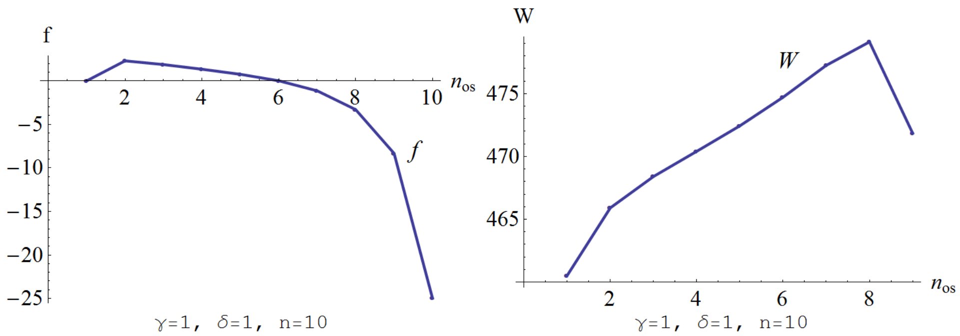


FIGURE 4. Welfare analysis of the industry structure.

As δ decreases (for high γ), the value of n_{os} at which maximum welfare is attained decreases. Figure 5 shows the welfare schedule for $\delta = 0.8$. The equilibrium number of firms in OS increases, but the welfare maximizing number of firms decreases. The reason is that when δ increases, vertical differentiation has a smaller impact on demand and firms tend to be more similar. Consequently, as n_{os} increases the increase in quality is smaller than in the previous case.

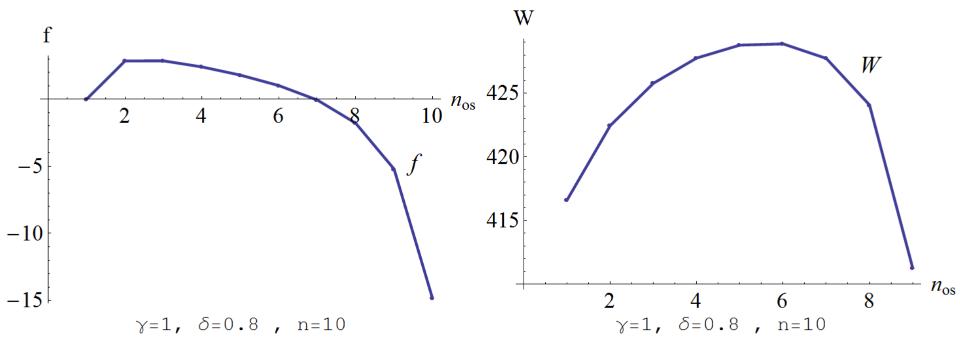


FIGURE 5. Welfare analysis of the industry structure.

When γ is low, the welfare schedule increases monotonically. Figure 6 shows the equilibrium and welfare maximum for $\gamma = 0.8$, $\delta = 0.8$ and $n = 10$. Welfare is maximized when $n_{os} = n$. In this case, the factor driving the large increase in welfare is the reduction in the investment cost.

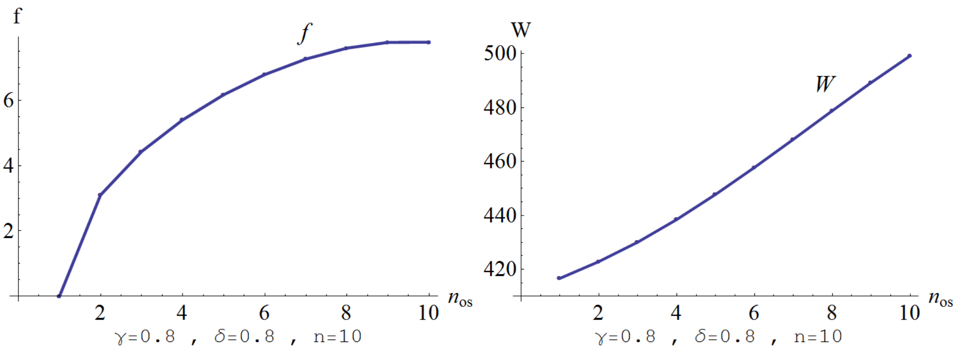


FIGURE 6. Welfare analysis of the industry structure.

The above analysis implies that when γ is high, it is socially optimal to keep both kinds of firms in the market. In this case, the equilibrium number of firms in OS may higher or lower than optimal. This will depend on the strength of horizontal product differentiation in comparison with vertical differentiation. When γ is low, on the other hand, the equilibrium has only OS firms, which is optimal from a welfare point of view.

5. Higher Substitutability Between OS Products

Given that OS packages share the same primary good, they are likely to be more similar than P packages. To introduce this difference in the degree of substitutability, I use a nested logit model (Ben-Akiva 1973). This adds an element of endogenous horizontal differentiation to the trade-off between collaboration and secrecy. By becoming P, firms get their product more differentiated in comparison with OS firms.

The main consequences are that (i) the equilibrium number of firms in OS will be smaller than in the previous model, (ii) there are equilibria with only P firms, and (iii) there are parameter values leading to multiple equilibria.

Consumers are heterogeneous in two different dimensions: they have idiosyncratic tastes for the primary good and idiosyncratic tastes for the complementary good. Differences in substitutability will be driven by the relative strength of these two forces. Following the nested logit representation of Cardell (1997), consumer j's indirect utility from consuming package i, based on primary good k is:

$$v_{ij} = \alpha\, a_k + \beta\, b_i + y - p_i + \sigma\, \eta_{kj} + (1-\sigma)\, \varepsilon_{ij},$$

where η_{kj} is a primary good idiosyncratic component, and $\sigma \in [0,1]$ weighs the different idiosyncratic components. Assumption 3.2 replaces Assumption 3.1 for the standard logit case.

Assumption 3.2. *The idiosyncratic support components ε_{ij} are i.i.d. according to the double exponential distribution with scale parameter μ. The idiosyncratic software components η_{kj} are i.i.d. according to a distribution such that $\sigma\, \eta_{kj} + (1-\sigma)\, \varepsilon_{ij}$ is distributed double exponential with scale parameter μ.*

Assumption 3.2 implies that the horizontal differentiation term $\sigma\, \eta_{kj} + (1-\sigma)\, \varepsilon_{ij}$ has the same distribution than ε_{ij} in the previous model. Cardell shows there is a unique distribution for η_{kj} such that Assumption 3.2 holds.

The parameter σ determines the relative strength of the horizontal differentiation forces. As σ increases, consumers get more differentiated in their tastes for the primary good, and less differentiated in their preferences for the complementary good. When $\sigma = 0$ consumers only have idiosyncratic preferences for the complementary good, and the model becomes the standard logit model of previous sections. When $\sigma = 1$ consumers only have idiosyncratic preferences for the primary good, and all OS firms sell a homogeneous good.

The market share of a P firm is:

(3.12)

$$s_p = \frac{\exp\left(\frac{\alpha\, a_i + \beta\, b_i - p_i}{\mu}\right)}{\exp\left(\frac{\alpha\, a_{os}}{\mu}\right)\left[\sum_{j\in OS}\exp\left(\frac{\beta\, b_j - p_j}{(1-\sigma)\mu}\right)\right]^{1-\sigma} + \sum_{j\in P}\exp\left(\frac{\alpha\, a_j + \beta\, b_j - p_j}{\mu}\right)}.$$

This market share is increasing in σ. The market share of an OS firm can be decomposed in the following way:

$$s_{os} = s_{i|os}\, S_{os}, \tag{3.13}$$

where $s_{i|os}$ is the proportion of consumers who buy the complementary good from firm i given that they buy the OS primary good, and S_{os} is the aggregate market share of OS firms. The expression for $s_{i|os}$ is:

$$s_{i|os} = \frac{\exp\left[\,(\beta\, b_i - p_i)\,/\,(1-\sigma)\mu\,\right]}{\sum_{j\in OS}\exp\left[\,(\beta\, b_i - p_i)\,/\,(1-\sigma)\mu\,\right]}.$$

Once consumers decide to buy an OS package, differences in the quality of the primary good do not play any role in the decision of which complementary good to buy. The aggregate market share of OS firms is:

$$S_{os} = \frac{\exp\left(\frac{\alpha\, a_{os}}{\mu}\right)\left[\sum_{j\in OS}\exp\left(\frac{\beta\, b_j - p_j}{(1-\sigma)\mu}\right)\right]^{1-\sigma}}{\exp\left(\frac{\alpha\, a_{os}}{\mu}\right)\left[\sum_{j\in OS}\exp\left(\frac{\beta\, b_j - p_j}{(1-\sigma)\mu}\right)\right]^{1-\sigma} + \sum_{j\in P}\exp\left(\frac{\alpha\, a_j + \beta\, b_j - p_j}{\mu}\right)},$$

which is decreasing in σ.

The optimal price and investment of P firms have the same functional forms as before. The optimal price and investment effort of OS firms become:

$$p_{os} = \mu\left(1 - s_{os} + \frac{\sigma}{1-\sigma}\,\frac{n_{os}-1}{n_{os}}\right)^{-1}, \tag{3.14}$$

$$x_{os} = \frac{\alpha+\beta}{c}\, s_{os}\left(1 - \frac{\gamma}{1-\sigma}\,\frac{n_{os}-1}{n_{os}\left(1 - s_{os} + \frac{\sigma}{1-\sigma}\frac{n_{os}-1}{n_{os}}\right)}\right). \tag{3.15}$$

From (3.12) and (3.13), we can get the ratio of market shares s_{os}/s_p. Introducing prices and investments, taking logs and rearranging terms we get:

$$\begin{aligned}(1-\delta)\ln\left(\frac{s_{os}}{s_p}\right) + \frac{1}{1 - s_{os} + \frac{\sigma}{1-\sigma}\frac{n_{os}-1}{n_{os}}} - \frac{1}{1-s_p} \\ = \delta\,\ln\left(1 - \frac{\gamma}{1-\sigma}\,\frac{(n_{os}-1)/n_{os}}{\left(1 - s_{os} + \frac{\sigma}{1-\sigma}\frac{n_{os}-1}{n_{os}}\right)}\right) + (\delta\,\gamma - \sigma)\,\ln(n_{os}).\end{aligned} \tag{3.16}$$

Proposition 3.4 shows that the equilibrium depends in δ and γ, as before, but also on the parameter σ, which represents the difference in heterogeneity between OS and P firms.

PROPOSITION 3.4. *A second-stage equilibrium for the nested model exists and is unique. Given n_{os}, the equilibrium market shares solve (3.16) and (3.7).*

Comparing equations (3.6) and (3.16), we can see that the higher substitutability between OS varieties has three effects on equilibrium market shares. First, there is a lower investment due to the lower return to investment (first term on the right hand side of 3.16). Second, there is a direct negative effect on the average value of the complementary good (second term on the right hand side of 3.16). Consumers care for variety, and therefore the value of choosing an OS package decreases when the complementary good becomes less differentiated. Third, OS firms will set a lower price in equilibrium because of higher substitutability (second term on the left hand side of 3.16). The first two effects tend to reduce the market share of OS relative to P, and the third effect tends to increase it.

Figure 7 shows the effects of the nesting on the second stage equilibrium for $\gamma = 0.5$, $\delta = 0.5$, $n = 10$ and $n_{os} = 5$. Not surprisingly, we can see that s_{os} decreases and s_p increases as σ increases, because of the higher substitutability between OS varieties.

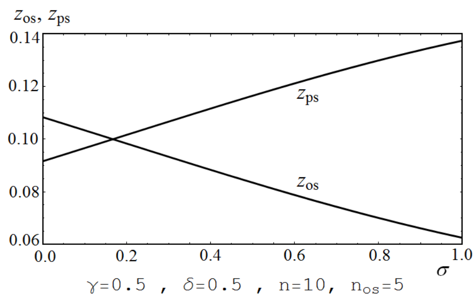
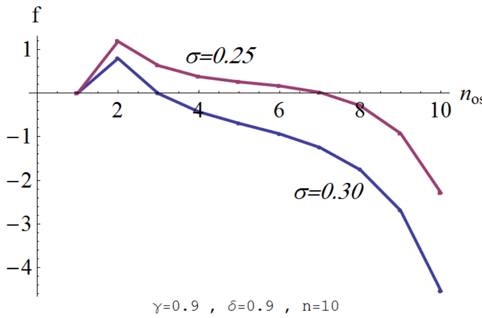
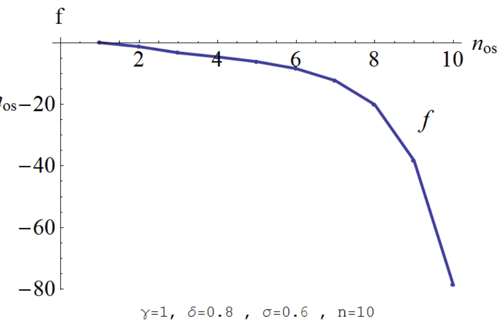


FIGURE 7. Effect of a change in σ on market shares.

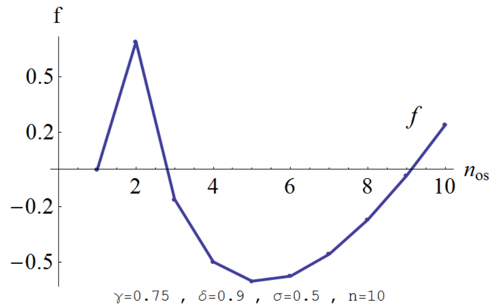
With respect to the equilibrium of the first stage of the game, there are three interesting observations to be made. First, as σ increases for given γ (OS varieties become more similar), the equilibrium number of firms in OS decreases (Figure 8a). Second, if σ is very high with respect to γ, it can even be the case of an equilibrium with all P firms (Figure 8b). Third, there are some values of the parameters for which the model exhibits multiple equilibria (in Figure 8c there is an equilibrium with $n_{os} = 2$ and another equilibrium with $n_{os} = 10$). In this case, there may be a coordination problem if the OS project fails to attract a large number of contributors.



(A) Effect of a change in σ.



(B) All P Equilibrium.



(C) Multiple Equilibria.

FIGURE 8. Nested logit equilibrium.

6. Conclusion

I have presented a model of industry equilibrium with endogenous sharing of innovation. Firms decide whether to become OS or P and how much to invest in R&D. OS firms share improvements to the source code and P firms develop the software on their own. Both kinds of firms sell a complementary good the quality of which depends on individual R&D investment. Consumers value the quality of software and support (vertical differentiation) but also have idiosyncratic tastes for the different varieties (horizontal differentiation).

The paper shows that the decision to develop technologies as OS can arise endogenously in an industry in which both OS and P firms co-exist. My main contribution is to prove the existence of an equilibrium with an asymmetric market structure, even though all firms are ex-ante symmetric. When both firms co-exist, the investment of a single P firm is larger than the sum of investments of all the OS firms. OS firms compensate the lower market share and price with a lower cost of developing the software.

The results depend on the existence of enough horizontal differentiation relative to vertical differentiation. When this assumption is relaxed to allow for a higher substitutability between OS goods, the equilibrium number of firms in OS decreases. When OS goods become nearly homogeneous the equilibrium has all firms deciding to be P. Multiple equilibria are also possible under this alternative specification.

Appendix A: Proofs of Theorems in Text

PROPOSITION 3.1. *A second-stage equilibrium exists and is unique. Given n_{os}, the equilibrium market shares solve (3.6) and (3.7).*

PROOF. The first order conditions with respect to p_i and x_i are:

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial s_i}{\partial p_i} p_i + s_i \leq 0 \qquad \text{with equality if } p_i > 0, \tag{3.17}$$

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\partial s_i}{\partial x_i} p_i - c \leq 0 \qquad \text{with equality if } x_i > 0. \tag{3.18}$$

Assume an interior equilibrium exists so that the first order conditions hold with equality. Working with equation (3.17) we get the optimal price:

$$p_i = \mu/(1 - s_i). \tag{3.19}$$

Equation (3.19) holds for both kinds of firms (OS and P). In order to find the optimal number of programming hours we need to calculate $\partial s_i / \partial x_i$, which in the case of OS firms is:

$$\frac{\partial s_i}{\partial x_i} = \frac{s_i(1 - s_i)}{\mu} \left(\alpha \frac{\partial a}{\partial x_i} + \beta \frac{\partial b}{\partial x_i} \right) - \sum_{j \in OS_{-i}} \alpha \frac{s_i s_j}{\mu} \frac{\partial a}{\partial x_i},$$

and in the case of P firms becomes:

$$\frac{\partial s_i}{\partial x_i} = \frac{s_i(1 - s_i)}{\mu} \left(\alpha \frac{\partial a}{\partial x_i} + \beta \frac{\partial b}{\partial x_i} \right).$$

Here we can see that the difference between both kinds of firms is the public good nature of a for OS firms. The improvement in the quality of the primary good due to firm i's investment benefits the rest of OS firms and therefore the increase in market share is less than what it would be if the firm was P.

Imposing symmetry and introducing these expressions in 3.18 we get:

$$x_{os} = \frac{1}{c} s_{os} \left(\alpha + \beta - \alpha \frac{n_{os} - 1}{n_{os}} \frac{1}{1 - s_{os}} \right), \tag{3.20}$$

$$x_p = \frac{1}{c} s_p (\alpha + \beta), \tag{3.21}$$

and the relation of optimal investments in equilibrium:

$$\frac{x_{os}}{x_p} = \frac{s_{os}}{s_p}\left(1 - \frac{\alpha}{\alpha+\beta}\,\frac{n_{os}-1}{n_{os}}\,\frac{1}{1-s_{os}}\right). \tag{3.22}$$

From 3.2 we get that the ratio of market shares between OS and P firms is:

$$\frac{s_{os}}{s_p} = \exp\left[\frac{\alpha\, a_{os} + \beta\, b_{os} - \alpha\, a_p - \beta\, b_p + p_p - p_{os}}{\mu}\right], \tag{3.23}$$

$$\ln\left(\frac{s_{os}}{s_p}\right) = \frac{1}{\mu}\,\Delta + \frac{1}{1-s_p} - \frac{1}{1-s_{os}}, \tag{3.24}$$

where $\Delta = \alpha\, a_{os} + \beta\, b_{os} - \alpha\, a_p - \beta\, b_p$ represents quality differences. From the definitions of a and b:

$$\Delta = (\alpha+\beta)\,\ln\left(\frac{x_{os}}{x_p}\right) + \alpha\,\ln(n_{os}). \tag{3.25}$$

From equations (3.22), (3.24) and (3.25), we get equation (3.6), which is an implicit equation determining the relation of market shares between OS and P firms in equilibrium. This equation, together with the equation establishing that the sum of the market shares is equal to 1, completely characterizes the equilibrium.

To show existence and uniqueness, we need to prove two things: (1) there is only one fixed point of the system of equations in Proposition 3.1, and (2) the profit function is quasiconcave.

Let's first show there is only one fixed point in term of equilibrium market shares. Define the function $g(s_{os})$ by plugging equation (3.7) in equation (3.6).

(3.26)

$$\begin{aligned} g(s_{os}) = (1-\delta)\ln\left(\frac{(n-n_{os})s_{os}}{1-n_{os}s_{os}}\right) - \delta\,\ln\left((1-\gamma)+\gamma\frac{1-n_{os}\,s_{os}}{(1-s_{os})n_{os}}\right) + \\ -\,\delta\,\gamma\,\ln(n_{os}) - \frac{n-n_{os}}{1+n_{os}(1-s_{os})-n} + \frac{1}{1-s_{os}}. \end{aligned}$$

By construction, s_{os} solves equations (3.6) and (3.7) if and only if $g(s_{os}) = 0$. Existence follows from a standard application of the mean value theorem. First, $\lim_{s_{os}\to 0} g(s_{os}) = -\infty$ and $\lim_{s_{os}\to \frac{1}{n_{os}}} g(s_{os}) = \infty$. Then, continuity of g implies there exists at least one s_{os} such that $g(s_{os}) = 0$. Next, I will show there exists only one such s_{os}. For this, it is sufficient to show that g

is strictly increasing, for which I will calculate its derivative:

$$\frac{\partial g}{\partial s_{os}} = \frac{1-\delta}{s_{os}(1-n_{os}\,s_{os})} + \frac{\delta\,\gamma\,(n_{os}-1)/(1-s_{os})}{(1-\gamma)(n_{os}-1)+1-s_{os}\,n_{os}} + \\ + \frac{(n-n_{os})\,n_{os}}{(1+n_{os}(1-s_{os})-n)^2} + \frac{1}{(1-s_{os})^2}.$$

All terms are positive because $s_{os}n_{os} \leq 1$. It follows there exists a unique (s_{os}, s_p) solving the system of equations.

To prove that the profit function is concave at the equilibrium candidate, I will evaluate the determinant of the Hessian of the profit function at the equilibrium price and market share, and show that it is positive definite. The determinants of the Hessian of both kinds of firms are:

$$|H_p| = \frac{(\alpha+\beta)\,s_p^2}{\mu\,x_p^2}\left(1 - \frac{(\alpha+\beta)\,(1-s_p)^2}{\mu}\right),$$

$$|H_{os}| = \frac{s_{os}^2}{\mu\,x_{os}^2}\left(\left(\frac{1-n_{os}\,s_{os}}{(1-s_{os})n_{os}^2}\alpha+\beta\right) - \frac{(1-s_p)^2}{\mu}\left(\frac{1-n_{os}\,s_{os}}{(1-s_{os})n_{os}}\alpha+\beta\right)^2\right).$$

A sufficient condition for both determinants to be positive is $\mu \geq \alpha+\beta$, which has been assumed throughout the paper, which means that the concavity of the profit function at the equilibrium is guaranteed for both kinds of firms. ■

LEMMA 3.1. *$s_p > s_{os}$ if and only if $\gamma > \hat{\gamma}(n_{os}, n)$, and $s_p < s_{os}$ in the opposite case, where $\hat{\gamma}(n_{os}, n)$ is increasing in n_{os} and n and solves:*

$$\gamma\,\frac{n_{os}^{\gamma}}{n_{os}^{\gamma}-1}\,\frac{n_{os}-1}{n_{os}} = \frac{n-1}{n}.$$

PROOF. In order to prove the first part of the lemma we only have to check the sign of $g\left(\frac{1}{n}\right)$, where g is defined in (3.26). If $g\left(\frac{1}{n}\right) < 0$, then $s_{os} > 1/n$ and therefore $s_{os} > s_p$.

$$g\left(\frac{1}{n}\right) = -\delta\left(\ln\left(1-\gamma\frac{n}{n-1}\,\frac{n_{os}-1}{n_{os}}\right) + \gamma\ln(n_{os})\right).$$

$g\left(\frac{1}{n}\right) < 0$ if and only if:

$$\ln\left(1-\gamma\frac{n}{n-1}\,\frac{n_{os}-1}{n_{os}}\right) > -\gamma\ln(n_{os}),$$

$$1-\gamma\frac{n}{n-1}\,\frac{n_{os}-1}{n_{os}} > n_{os}^{-\gamma}.$$

Rearranging this expression we get the desired result.

In order to show that $\hat{\gamma}(n_{os}, n)$ is increasing in n and n_{os}, let $h(\gamma, n_{os}) = \gamma \frac{n_{os}^{\gamma}}{n_{os}^{\gamma}-1} \frac{n_{os}-1}{n_{os}}$. Computing the derivatives:

$$\frac{\partial h}{\partial \gamma} = \frac{n_{os}^{\gamma}}{(n_{os}^{\gamma}-1)^2} \frac{n_{os}-1}{n_{os}} \left(n_{os}^{\gamma} - 1 - \gamma \ln(n_{os})\right) \tag{3.27}$$

$$\frac{\partial h}{\partial n_{os}} = \frac{\gamma}{n_{os}^{2-\gamma}(n_{os}^{\gamma}-1)^2} \left(n_{os}^{\gamma} - 1 - \gamma(n_{os}-1)\right) \tag{3.28}$$

First, I will show that $\frac{\partial h}{\partial \gamma} \geq 0$, which is enough to determine that $\hat{\gamma}$ is increasing in n. $\frac{\partial h}{\partial \gamma} \geq 0$ if and only if $n_{os}^{\gamma} - 1 \geq \ln(n_{os}^{\gamma})$. Let $x = n_{os}^{\gamma}$, $f_1(x) = x - 1$ and $f_2(x) = \ln(x)$. x ranges from 1 to n_{os}. When $x = 1$, $f_1 = f_2$, but then f_1 grows faster than f_2 for any x. This means that $n_{os}^{\gamma} - 1 \geq \ln(n_{os}^{\gamma})$ and $\frac{\partial h}{\partial \gamma} \geq 0$.

Next, I will show that $\frac{\partial h}{\partial n_{os}} \leq 0$, which implies that $\hat{\gamma}$ is increasing in n_{os} following a simple application of the implicit function theorem. $\frac{\partial h}{\partial n_{os}} \leq 0$ if and only if $\gamma\,(n_{os}-1) \geq n_{os}^{\gamma}-1$. Let $g_1(\gamma) = \gamma\,(n_{os}-1)$ and $g_2(\gamma) = n_{os}^{\gamma}-1$. It is easy to check that $g_1(0) = g_2(0)$, $g_1(1) = g_2(1)$, and that both functions are increasing but g_2 is strictly convex and g_1 is linear. Therefore, $g_1(\gamma) \geq g_2(\gamma)$ and $\frac{\partial h}{\partial n_{os}} \leq 0$. ∎

LEMMA 3.2. *s_{os} is increasing in δ if $\gamma < \hat{\gamma}(n_{os}, n)$, and decreasing in δ in the opposite case.*

PROOF. Suppose $\gamma < \hat{\gamma}$. Then, by lemma 3.1, $h(n_{os}, \gamma) < (n-1)/n$ and $s_{os}^* > 1/n$ in equilibrium. Let partial derivatives of g be denoted by subscripts, where g is defined in (3.26). By the implicit function theorem, $\partial s_{os}/\partial \delta = -g_{\delta}/g_{s_{os}}$. In the proof of proposition 3.1 it has been shown that $g_{s_{os}} > 0$. Next, I will determine the sign of g_{δ}. It can be shown that g_{δ} is decreasing in s_{os}, then if $g_{\delta}(1/n) \leq 0$, and given that $s_{os}^* > 1/n$, we can deduce that $g_{\delta}(s_{os}^*) \leq 0$. Let us compute $g_{\delta}(1/n)$:

$$g_{\delta}(1/n) = -\ln(n_{os}^{\gamma}) - \ln\left(\frac{(n-1)n_{os} - n(n_{os}-1)\gamma}{(n-1)n_{os}}\right).$$

This expression in negative if and only if,

$$\frac{(n-1)n_{os}}{n_{os}^{\gamma}\left((n-1)n_{os} - n(n_{os}-1)\gamma\right)} < 1.$$

Rearranging terms, this expression is equivalent to $h(n_{os}, \gamma) < (n-1)/n$ which holds by assumption. Thus $g_{\delta}(s_{os}^*) \leq 0$ and $\frac{\partial s_{os}}{\partial \delta} \geq 0$. The proof for $\gamma > \hat{\gamma}$ is analogous, but reversing the inequalities. ∎

LEMMA 3.3. *There exists $\gamma_d \in (0, \hat{\gamma})$ such that s_{os} is increasing in γ for $\gamma < \gamma_d$, and decreasing in γ for $\gamma > \gamma_d$.*

PROOF. By the implicit function theorem, $\partial s_{os}/\partial\gamma = -g_\gamma/g_{s_{os}}$, where g is defined in (3.26). We know $g_{s_{os}} > 0$. With respect to g_γ:

$$g_\gamma = \ln(n_{os}) - \frac{n_{os}-1}{\gamma+(1-s_{os}-\gamma)n_{os}}$$

Therefore, $\partial s_{os}/\partial\gamma = 0$ when $g_\gamma = 0$. Solving for the value $\hat{s}_{os}$ that makes $g_\gamma = 0$ we get:

$$\hat{s}_{os} = \frac{\ln(n_{os})(n_{os}(1-\gamma)+\gamma)+1-n_{os}}{n_{os}\ln(n_{os})}$$

Introducing this in $g = 0$ we get an equation determining the value γ_d that makes the derivative equal to zero.

To prove that to the right of γ_d the graph of $s_{os}(\gamma)$ is decreasing, assume this is not the case, so $g_\gamma > 0$. Then, for $\gamma > \gamma_d$ it has to be the case that $s_{os} > \hat{s}_{os}$, but this implies that $g_\gamma < 0$, which is a contradiction. This means that $\partial s_{os}/\partial\gamma < 0$ for $\gamma > \gamma_d$. A similar reasoning implies that $\partial s_{os}/\partial\gamma > 0$ for $\gamma < \gamma_d$. ■

LEMMA 3.4. *If $s_p(n_{os}) < s_{os}(n_{os})$, then $s_p(n_{os}-1) > s_p(n_{os}) > s_p(n_{os}+1)$ (s_p is decreasing in n_{os}).*

PROOF. Restating equilibrium equation (3.26) in terms of s_p:

$$\begin{aligned}\tilde{g}(s_p) =& (1-\delta)\ln\left(\frac{1-(n-n_{os})s_p}{n_{os}s_p}\right) + \frac{n_{os}}{n_{os}-1+(n-n_{os})s_p} \\ &- \frac{1}{1-s_p} - \delta\ln\left(1+\frac{(n_{os}-1)\gamma}{n_{os}-1+(n-n_{os})s_p}\right) - \gamma\delta\ln(n_{os})\end{aligned}$$

$\tilde{g}$ is continuous and differentiable in s_p and n_{os}, and continuously decreasing in s_p, with $\tilde{g}(0) = \infty$ and $\tilde{g}(1/(n-n_{os})) = -\infty$. Therefore, there is some point s_p^* where $\tilde{g}$ crosses the axis, and this determines the equilibrium level of s_p as a function of n_{os}.

Assume for the moment that n_{os} can take non-integer values. I will prove that $s_p^* < 1/n$ implies $\partial s_p/\partial n_{os} < 0$. To do this I will proceed in three steps.

First, by the implicit function theorem, $\partial s_p/\partial n_{os} = -\partial\tilde{g}_{n_os}/\tilde{g}_{s_p}$ but $\tilde{g}_{s_p} < 0$, so the sign of the derivative is equal to the sign of $\tilde{g}_{n_os}$. Second, we know that $\tilde{g}$ is continuously decreasing in s_p. It can be shown that $\tilde{g}_{n_os}$ is continuously increasing in s_p. Therefore, if $\tilde{g}(1/n) \le 0$ and $\tilde{g}_{n_os}(1/n) \le 0$, then $\tilde{g}_{n_os}(s_p^*) < 0$. Finally, it is straightforward to calculate $\tilde{g}_n(1/n)$:

$$\tilde{g}_n(1/n) = -\frac{\delta\gamma}{n_{os}}\left(1+\frac{n}{n_{os}-n(n_{os}(1-\gamma)+\gamma)}\right).$$

It is easy to show that $\tilde{g}_n(1/n) \leq 0$ if and only if $k(\gamma, n_{os}) < (n-1)/n$, where:

$$k(\gamma, n_{os}) = \frac{1 + (n_{os} - 1)\gamma}{n_{os}}.$$

However, $k(\gamma, n_{os}) \leq h(\gamma, n_{os})$, so $h(\gamma, n_{os}) < (n-1)/n$ implies $k(\gamma, n_{os}) < (n-1)/n$. Therefore, if $s_p^* < 1/n$ then $\partial s_p / \partial n_{os} < 0$.

Let us now treat n_{os} as an integer and prove that $s_p(n_{os}) > s_p(n_{os}+1)$. Given that $\partial s_p/\partial n_{os}$ is negative at $s_p(n_{os})$, then $s_p(n_{os}) > s_p(n_{os} + \varepsilon)$ for ε positive and arbitrarily small. But then $s_p(n_{os}+\varepsilon) < 1/n$ and so $\partial s_p/\partial n_{os} < 0$ at $s_p(n_{os} + \varepsilon)$. Repeating this argument for all values between n_{os} and $n_{os} + 1$, we get that s_p is continuously decreasing in all the interval and therefore $s_p(n_{os}) > s_p(n_{os}+1)$.

To show that $s_p(n_{os}-1) > s_p(n_{os})$, assume this inequality does not hold. This means that $s_p(n_{os}-1) < s_p(n_{os}) < 1/n$, and so $\partial s_p/\partial n_{os} \leq 0$ at $s_p(n_{os}-1)$. But the previous argument implies that $s_p(n_{os}-1) > s_p(n_{os})$, which is a contradiction. ∎

Proposition 3.2 (Existence and uniqueness of equilibrium). *An equilibrium for the complete game exists and is unique.*

Proof. For $n_{os} = 1$ to be an equilibrium we only need $f(2) \leq 0$. Likewise, for $n_{os} = n$ to be an equilibrium we only need $f(n) \geq 0$. In order to have an equilibrium with both kinds of firms ($1 < n_{os} < n$), we need that $f(n_{os}) \geq 0$ and $f(n_{os}+1) \leq 0$ at the equilibrium n_{os}. If $f(2) \geq 0$ and $f(n) \leq 0$ then it is guaranteed that there is at least one such equilibrium. Therefore, existence of an equilibrium with $1 \leq n_{os} \leq n$ is guaranteed. Simulations show that the equilibrium is unique for any value of the parameters. ∎

Lemma 3.5 (Sufficient condition for positive f). *If* $\gamma < \hat{\gamma}(n_{os}-1, n)$ *then* $f(n_{os}) > 0$.

Proof. Rearranging $f(n_{os})$ and dividing by μ we get:

$$\frac{f(n_{os})}{\mu} = \frac{s_{os}}{1-s_{os}}(1-\delta(1-s_{os})) - \frac{\tilde{s}_p}{1-\tilde{s}_p}(1-\delta(1-\tilde{s}_p)) + \delta\gamma\frac{s_{os}}{1-s_{os}}\frac{n_{os}-1}{n_{os}}$$

where $s_{os} = s_{os}(n_{os})$ and $\tilde{s}_p = s_p(n_{os}-1)$. The value of μ does not influence the sign of f. The first two terms have the same functional form and are increasing in s. The last term is always positive. Therefore, if $s_{os}(n_{os}) \geq s_p(n_{os}-1)$, then $f(n_{os}) > 0$. A sufficient condition is that $s_{os}(n_{os}-1) \geq 1/n$ and $s_{os}(n_{os}) \geq 1/n$, which is equivalent to $\gamma < \hat{\gamma}(n_{os}-1, n)$ and $\gamma < \hat{\gamma}(n_{os}, n)$. However, $\hat{\gamma}(n_{os}, n)$ is decreasing in n_{os}, so $\gamma < \hat{\gamma}(n_{os}-1, n)$ implies $f(n_{os}) > 0$. ∎

Corollary 3.1 (Necessary condition for an interior equilibrium). *At an interior equilibrium n_{os} it is necessary that $\gamma \geq \hat{\gamma}(n_{os}, n)$.*

Proof. For an interior equilibrium at n_{os} we need that $f(n_{os}) \geq 0$ and $f(n_{os}+1) \leq 0$, but Proposition 3.5 implies that for $f(n_{os}+1) \leq 0$ we need $\gamma \geq \hat{\gamma}(n_{os}-1, n)$. ■

Corollary 3.2 (Sufficient condition for an equilibrium with $n_{os} = n$). *If $\gamma \leq \hat{\gamma}(n-1, n)$ then there is an equilibrium where all firms decide to be OS.*

Proof. If $\gamma \leq \hat{\gamma}(n-1, n)$ then $f(n) \geq 0$, so if $n_{os} = n$ then no firm would gain by becoming a P firm. ■

Proposition 3.3 (Necessary and Sufficient condition for equilibrium with $n_{os} = n$). *Given $n > 3$ and δ, there exists $\bar{\gamma} \in (\hat{\gamma}, 1)$ such that $f(n) \geq 0$ if and only if $\gamma \leq \bar{\gamma}$.*

Proof. μ does not influence the sign of $f(n_{os})$, so I will assume $\mu = 1$ for the rest of this proof. We know that $f(n) > 0$ for $\gamma < \hat{\gamma}(n-1, n)$. We need to determine the sign of $f(n)$ for the rest of values of γ. When $n_{os} = n$, $s_{os} = 1/n$. Therefore,

$$f(n) = \frac{1}{n-1} - \frac{\delta(1-\gamma)}{n} - \frac{\tilde{s}_p}{1-\tilde{s}_p}\left(1 - \delta(1-\tilde{s}_p)\right), \tag{3.29}$$

where $\tilde{s}_p = s_p(n-1)$. We need to find the value of $\tilde{s}_p$ that makes $f(n) = 0$. There are two roots of this equation. The only positive root is:

$$\tilde{s}_p = \frac{-n^2(1-\delta) - (1-\gamma)\delta - n\gamma\delta + \sqrt{n^4 - 2\,n^2\,z + z^2}}{2\delta\,n\,(n-1)}$$

where $z = \delta\,(n-1)\,(n-1+\gamma)$. The corresponding value for $s_{os}(n-1)$ is:

$$\tilde{s}_{os} = \frac{n^2 + s - \sqrt{n^4 - 2\,n^2\,z + z^2}}{2\delta\,n\,(n-1)^2}$$

Plugging this value in the equilibrium condition (3.26) and solving for γ we get the value $\bar{\gamma}$ where $f(n) = 0$. Lemma 3.5 implies that $\bar{\gamma} \geq \hat{\gamma}(n-1, n)$. Lemma 3.3 implies that $\partial\tilde{s}_p/\partial\gamma > 0$ in the relevant area. This means that $\bar{\gamma}$ is the unique value of γ such that $f(n) = 0$.

To finish the proof we need to show that $f(n) > 0$ for $\gamma < \bar{\gamma}$ and $f(n) < 0$ for $\gamma > \bar{\gamma}$. Given the continuity and monotonicity of s_p, it suffices to show there is some value to the right or to the left of $\bar{\gamma}$ such that these inequalities hold.

Consider first the case of $\gamma < \bar{\gamma}$. We know that at $\gamma = \hat{\gamma}(n-1, n)$, $f(n) > 0$. This proves that $f(n) > 0$ for $\gamma < \bar{\gamma}$. For $\gamma > \bar{\gamma}$, consider $\gamma = 1$.

When $\gamma = 1$, the investment of OS firms is very low, and P firms have the largest advantage. In this case, $f(n) < 0$, which proves that this inequality holds for any $\gamma > \bar{\gamma}$. ∎

PROPOSITION 3.4. *A second-stage equilibrium for the nested model exists and is unique. Given n_{os}, the equilibrium market shares solve (3.16) and (3.7).*

PROOF. The first order conditions are (3.17) (3.18). Assume that an interior equilibrium exists so that the first order conditions hold with equality. Equilibrium prices and effort for P firms are identical to the logit model so I will focus on the OS firms. Let's first work with the equation corresponding to prices. In the case of OS firms the partial derivative of prices with respect to market shares is

$$\frac{\partial s_i}{\partial p_i} = -\frac{1}{(1-\sigma)\mu} s_i (1 - \sigma\, s_{i|os} - (1-\sigma)\, s_i).$$

Then from the equation (3.17) and imposing symmetry we get equation (3.14). In order to find $x_o s$ we need to calculate $\partial s_i / \partial x_i$ for OS firms:

$$\frac{\partial s_i}{\partial x_i} = \frac{\alpha}{\mu} \frac{s_i(1-s_i)}{\sum_{j \in OS} x_i} + \frac{\beta}{(1-\sigma)\mu} \frac{s_i(1 - \sigma\, s_{i|os} - (1-\sigma)\, s_i)}{x_i}.$$

From equation (3.18) and imposing symmetry we get equation (3.15), and the relation of optimal investments in equilibrium:

$$\frac{x_{os}}{x_p} = \frac{s_{os}}{s_p} \left(1 - \frac{\gamma}{1-\sigma} \frac{(n_{os}-1)/n_{os}}{\left(1 - s_{os} + \frac{\sigma}{1-\sigma} \frac{n_{os}-1}{n_{os}}\right)} \right) \tag{3.30}$$

The ratio of market shares between OS and P firms is:

$$\frac{s_{os}}{s_p} = n_{os}^{-\sigma} \exp\left[\frac{\alpha\, a_{os} + \beta\, b_{os} - \alpha\, a_p - \beta\, b_p + p_p - p_{os}}{\mu} \right] \tag{3.31}$$

$$\ln\left(\frac{s_{os}}{s_p}\right) = -\sigma \ln n_{os} + \frac{1}{\mu}\Delta + \frac{1}{1-s_p} - \frac{1}{1 - s_{os} + \frac{\sigma}{1-\sigma}\frac{n_{os}-1}{n_{os}}} \tag{3.32}$$

where $\Delta = \alpha\, a_{os} + \beta\, b_{os} - \alpha\, a_p - \beta\, b_p$ represents quality differences. From the definitions of a and b:

$$\Delta = (\alpha + \beta)\, \ln\left(\frac{x_{os}}{x_p}\right) + \alpha\, \ln(n_{os}) \tag{3.33}$$

From equations (3.30), (3.32) and (3.33), we get equation (3.16), which is an implicit equation determining the relation of market shares between OS and P firms in equilibrium. This equation, together with the equation establishing that the sum of the market shares is equal to 1, completely characterizes the equilibrium. ∎

Bibliography

ALLEN, R. C. (1983): "Collective invention," *Journal of Economic Behavior & Organization*, 4(1), 1–24.

ANDERSON, S., A. DE PALMA, AND J. THISSE (1992): *Discrete Choice Theory of Product Differentiation*. MIT Press.

BEN-AKIVA, M. (1973): "Structure of Passenger Travel Demand Models," Ph.D. thesis, Massachusetts Institute of Technology.

BESSEN, J. (2006): "Open Source Software: Free Provision of Complex Public Goods," in *The Economics of Open Source Software Development*. Elsevier.

BITZER, J. (2004): "Commercial versus open source software: the role of product heterogeneity in competition," *Economic Systems*, 28(4), 369–381.

BLOCH, F. (1995): "Endogenous Structures of Association in Oligopolies," *The RAND Journal of Economics*, 26(3), 537–556.

BONACCORSI, A., AND C. ROSSI (2004): "Altruistic individuals, selfish firms? The structure of motivation in Open Source software," *First Monday*, 9(1).

CARDELL, N. (1997): "Variance Components Structures for the Extreme-Value and Logistic Distributions with Application to Models of Heterogeneity," *Econometric Theory*, 13(2), 185–213.

CASADESUS-MASANELL, R., AND P. GHEMAWAT (2006): "Dynamic Mixed Duopoly: A Model Motivated by Linux vs. Windows," *Management Science*, 52(7), 1072–1084.

DAHLANDER, L. (2004): "Appropriating Returns From Open Innovation Processes: A Multiple Case Study of Small Firms in Open Source Software," Department of Industrial Dynamics. Chalmers University of Technology.

ECONOMIDES, N., AND E. KATSAMAKAS (2006): "Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry," *Management Science*, 52(7), 1057–1071.

GAUDEUL, A. (2005): "Competition between open-source and proprietary software: the (La)TeX case study," Discussion paper, University of East Anglia.

Henkel, J., and M. Tins (2004): "Munich/MIT Survey: Development of Embedded Linux," Institute for Innovation, Technology Management and Entrepreneurship. University of Munich.

Kamien, M. I., E. Muller, and I. Zang (1992): "Research Joint Ventures and R&D Cartels," *The American Economic Review*, 82(5), 1293–1306.

Kuan, J. (2001): "Open Source Software as Consumer Integration Into Production," Stanford University.

Lakhani, K., and R. Wolf (2005): "Why Hackers Do What They Do: Understanding Motivation and Effort in Free/Open Source Software Projects," in *Perspectives on Free and Open Source Software*, ed. by J. Feller, B. Fitzgerald, S. Hissam, and K. R. Lakhani, pp. 3–22. MIT Press.

Lerner, J., P. Pathak, and J. Tirole (2006): "The Dynamics of Open Source Contributors," *The American Economic Review*, 96(2), 114–118.

Lerner, J., and J. Tirole (2005): "The Scope of Open Source Licensing," *Journal of Law, Economics, and Organization*, 21(1), 20–56.

McFadden, D. (1974): "Conditional logit analysis of qualitative choice behavior," in *Frontiers in Econometrics*, ed. by P. Zarembka. Academic Press, New York.

Mustonen, M. (2003): "Copyleft - the economics of Linux and other open source software," *Information Economics and Policy*, 15(1), 99–121.

Nuvolari, A. (2005): "Open Source Software Development: Some Historical Perspectives," *First Monday*, 10(10).

Rossi, M. A. (2004): "Decoding the "Free/Open Source(F/OSS) Software Puzzle" a survey of theoretical and empirical contributions," Working Paper 424, Department of Economics, University of Siena.

Schmidtke, R. (2006): "Private Provision of a Complementary Public Good," Discussion Papers 134, Governance and the Efficiency of Economic Systems.

Seppä, A. (2006): "Open Source in Finnish Software Companies," Discussion Papers 1002, The Research Institute of the Finnish Economy.