Working Paper 98-34
Statistics and Econometrics Series 17
May 1998

Departamento de Estadística y Econometría
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-9849

A NONLINEAR MODEL FOR THE INVESTMENT FUNCTION IN SPAIN.

Eva Senra and Antoni Espasa*

Abstract

This paper developpes a nonlinear single equation econometric model for the investment function in Spain, taking as starting point the equation estimated by Andrés et al. (1990). This original model, linear in its structure, incorporates oscillant dynamic relationships between the dependent and the explanatory variables. In the nonlinear model estimated in this paper, the response of the investment to production depends at any moment on the relative prices of energy, as an indicator of uncertainty into the future. This allows the investment to response with big oscillations to movements in production only in moments of great uncertainty. This alternative model introduces a nonlinear error-correction scheme, in which the adjustments to the long-run equilibrium path are affected by an exogenous variable. The model also improves the original adjustment, by reducing the residual variance in more than 30%.

Keywords:
Transfer function, dynamic response function, time-varying parameter models, nonlinear error correction models, relative prices of energy.

*Senra, Departamento de Estadística y Econometría, Universidad Carlos III de Madrid. C/ Madrid, 126 28903 Madrid. Spain. Ph: 34-1-624.98.89; Fax: 34-1-624.98.49, e-mail: esenra@est-econ. uc3m.es; Espasa, Departamento de Estadística y Econometría, Universidad Carlos III de Madrid. e-mail: espasa@est-econ.uc3m.es. We acknowledge the finantial support given by the project PB93-0236 from the Spanish DGICYT.

1 Introduction

The specification of a linear single equation dynamic econometric model in rational distributed lags or transfer function (TF) formulation specifies the different types of dynamic dependence between the explanatory and the dependent variables. The general formulation of any of these models, supposing, for simplicity of the exposition, that it only has one explanatory variable, is given by

$$Y_t = \frac{\omega_s(L)}{\delta_r(L)} L^b X_t + \frac{\Theta_q(L)}{\Phi_{p+d}(L)} a_t \tag{1.1}$$

where

$$\omega_s(L) = \omega_0 + w_1 L + \ldots + w_s L^s, \tag{1.2}$$

$$\delta_r(L) = 1 - \delta_1 L - \ldots - \delta_r L^r, \tag{1.3}$$

$$\Theta_q(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_q L^q \tag{1.4}$$

$$\Phi_{p+d}(L) = (1-L)^d (1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p), \tag{1.5}$$

a_t is a white noise variable and the roots of $\omega_s(L)$, $\delta_r(L)$ and $\theta_q(L)$ are outside the unit circle.

The rational stationary filter $\frac{\omega_s(L)}{\delta_r(L)}$ picks the specific dynamic effect of the variable X on the variable Y, a_t is a white noise variable and $\frac{\Theta_q(L)}{\Phi_{p+d}(L)} a_t$ approximates the residual dynamic structure. If the model has more than one explanatory variable, there will be in general, a different rational polynomial $\frac{\omega_{sj}}{\delta_{rj}}$ for each one. Along this paper the polynomials $\omega_{sj}(L)$ and $\Theta_q(L)$ will be called moving average (MA) polynomials and $\delta_{rj}(L)$ and $\Phi_{p+d}(L)$ autoregressive (AR) polynomials.

When X is a strongly exogenous variable, the dynamic structure $\frac{\omega_s(L)}{\delta_r(L)}$ represents the response function of Y to impulse changes in the variable X and is given by

$$v_\infty(L) = \frac{\omega_s(L)}{\delta_r(L)} L^b = v_0 + v_1 L + v_2 L^2 + \ldots. \tag{1.6}$$

In (1.6) it must be observed that the polynomial in the numerator extends s periods with no functional structure the effect of an impulse, and the one in the denominator extends, in theory till infinity, the impulse effect imposing an exponential or oscillant behaviour to the coefficients of v_∞.

If X is not strongly exogenous there is feedback in the dynamic response of Y to changes in X and in that case (1.6) is not a real response function. However, in such situation $\frac{\omega_s(L)}{\delta_r(L)}$ represents the dynamic structural relationship between X and Y and can be represented by the right side in (1.6), though the v_j coefficients are not neccessarily, response coefficients to an impulse in X.

Since model (1.1) must be balanced, one has that the oscilations in Y which are not in X will be incorporated in the dynamic filters of the model. If those oscilations are due to an omitted variable correlated with the set of explanatory variables the estimation of the dynamic relationships in (1.1) will be inconsistent and could be spurious. In the case of not correlation with the included variables, the effect of the omitted variable will be in the residual dynamics. All this means that before accepting a model, a careful analysis of its dynamics must be done. In particular, if the dynamics show important oscillations it will be useful to test the model against omitted variables before accepting that the estimated dynamics represent a reasonable approximation of the real world.

Any dynamic model can be formulated in two ways: (a) in rational distributed lags or transfer function terms[2] and (b) in autoregressive distributed lags (AD) terms as

$$\alpha(L)Y_t = \beta(L)X_t + a_t. \tag{1.7}$$

The change from the TF, given in (1.6), to the AD formulation is made approximating the ARIMA residual structure in (1.1) by an AR polynomial of finite order and multiplying (1.1) by this polynomial. Then, the transfer function for X_t is aprox-

[2]The term transfer function is usually reserved for the case in which the explanatory variable is strongly exogenous. In this article we use the transfer function term, though the explanatory variable could not fulfill the exogeneity condition.

imated by polynomials $v(L)$ of finite orders[3]. The change from the AD to the TF formulation is made directly, multiplying the AD model by $[\alpha(L)]^{-1}$. In this case all the explanatory variables and the residual term have the same denominator in their filters. If for a particular model one of the mentioned formulations is known or an estimation is available, it can be inmediately obtained the other.

The starting point of this article is the single equation econometric model for private productive investment presented in Andrés et al. (1990). This model in the rational autoregressive distributed lags formulation (AD) is the following[4]:

$$\begin{aligned}(1-0.46L+0.25L^2)I_t &= -2.73+(2.72-1.85L^2)\text{GDP}_{t-1}+\\ &\quad +2.29\Delta\text{CU}_t+(-1.54+0.43L)(C/P)_{t-1}-\\ &\quad -0.74\Delta^2\pi_t+a_t, \end{aligned} \tag{1.8}$$

where I is the logarithm of investment, Y is the logarithm of Gross Domestic Product, CU is the logarithm of the degree of usefulness of production capacity, CP is the logarithm of the cost of capital, π is the inflation rate of the GDP deflator and a is the residual supposed to be generated by a white noise, Δ is the difference operator and L is the lag operator. The model has been estimated using annual data for the period 1964–1987 and the residual standard deviation is $\hat{\sigma}=0.0258$.

In this model the transfer functions of all the explanatory variables have a common denominator which is a second order autoregressive process

$$\delta_2(L)=1-0.46L+0.25L^2. \tag{1.9}$$

This polynomial has a pair of complex conjugated roots

$$0.23\pm 0.44i, \tag{1.10}$$

with module 0.5 and periodicity 5.74 years.

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[3]On this point, see Espasa y Cancelo (1993), section 3.10.

[4]See Andrés et al. (1990), page 153.

There is no theoretical requirements about oscillant responses between the investment and all their explanatory variables and with the same kind of oscillation amongst them, and also equal to the residual component. Such a restrictive formulation is the reason that originally motivated this article.

Another interesting consideration about this model, is that the estimated long-run elasticity between investment and GDP is greater than unity. The characteristic of elasticity equal to one between investment and GDP, can certainly be true in consolidated economies. In equation (1.8) the ratio between investment and GDP is

$$I - \text{GDP} = 0.10\text{GDP}. \qquad (1.11)$$

The authors recognize the existence of arguments based in the characteristics of the sample period used which allow to justify this elasticity to be slightly greater than unity. The main reason is that in the Spanish economy, in the sample period considered, a big accumulation of capital took place, and in those particular circunstances we would not expect to estimate a long-run unit elasticity.

The model used from now on, is the same as the one presented in equation (1.8), but reestimated with the data (rounded) such as they appear published in the appendix of Andrés et al. (1990). So the results obtained are:

$$\begin{aligned}(1 - 0.56L + 0.27L^2)I_t &= -2.39 + (2.44 - 1.66L^2)\text{GDP}_{t-1} + \\ &\quad +2.24\Delta\text{CU}_t + (-1.52 + 0.57L)(C/P)_{t-1} + \\ &\quad -0.85\Delta^2\pi_t + a_t. \qquad (1.12)\end{aligned}$$

The t-values of each coefficient are found in Table 1 and the standard residual deviation obtained is $\hat{\sigma} = 0.02109$.

The dynamic functions basically present the same characteristics than the original estimation. The second order autoregressive filter affecting the functions of all the explanatory variables is

$$\delta_2(L) = 1 - 0.56L + 0.27L^2. \tag{1.13}$$

Again this polynomial has a pair of conjugated complex variables

$$0.26 \pm 0.58i, \tag{1.14}$$

with module 0.52, which produce oscillations of periodicity 6.26 years. In graph 1, it can be seen the oscillant behaviour of the dynamic functions of all the explanatory variables in the model. Also it should be noted that the lon-run elasticity between investment and GDP is 1.09 instead of the published value of 1.10.

The rest of this paper has been organized as follows. Section 2, analyzes several features of the model, and it is detected that the oscillations developped by the autoregressive filter are required by the data basically in the period 1974–1980. In section 3, it is argued that, mostly because of the two energy crises, this is a period of special uncertainty into the future, what must affect the proccess of taking investment decissions. In such a situation it makes sense to consider, a scheme of alternative models in which the elasticity between investment and the explanatory variables are not fixed parameters, but functions of the degree of uncertainty into the future. It is proposed to approximate the uncertainty level by the relative price of energy, in a way such that if they rise abruptly, the uncertainty into the future also rises, while if they decline, also does the uncertainty into the future. The discussion of this model is made in section 4. In section 5, the alternative model is written in its error-correction mechanism form and it is placed in the context of non-linear error correction models existing in literature. Last, section 6 concludes.

2 Initial Considerations

Initially the following aspects of the model in equation (1.12) are studied:

1. the residuals of the model
2. the effects of the autoregressive filter on each variable, and
3. the contributions of the variables.

2.1 Study of the Residuals

The residuals of equation (1.12) show a mean which is not significantly different from zero. Neither are the corresponding autocorrelations and there isn't any residual bigger than $1.96\hat{\sigma} = 0.04133$ in absolute value. However, if the residuals of equation (1.12) are observed (see graph 2), two residuals a bit outstanding respect to the rest can be detected: the residuals corresponding to years 1975 and 1978.

Intervention analysis has been applied to see if the adjustment could be improved, but the results of the estimation of the coefficients relative to the dummy variables introduced happens to be non-significant.

2.2 Effects of the Autoregressive Filter on each Variable

Model (1.12), can be rewritten in terms of transfer functions in the following way

$$\begin{aligned} I_t &= -3.36 + \frac{2.44 - 1.66L^2}{1 - 0.56L + 0.27L^2}\mathrm{GDP}_{t-1} + \\ &+ \frac{2.24}{1 - 0.56L + 0.27L^2}\Delta CU_t + \\ &+ \frac{-1.52 + 0.57L}{1 - 0.56L + 0.27L^2}(C/P)_{t-1} + \\ &+ \frac{-0.85}{1 - 0.56L + 0.27L^2}\Delta^2\pi_t + \\ &+ \frac{1}{1 - 0.56L + 0.27L^2}a_t. \end{aligned} \tag{2.1}$$

The model is not balanced[5] in the variables, and equilibrium in estimation is

[5]On balanced models see Granger (1990), pages 12 and 13 and Espasa y Cancelo (1993), pages

achieved by a dynamic oscillant polynomial, which in the specification chosen by the Andrés et al. (1990) is imposed in a common form to all the variables and the residual term.

As it has been said before, the presence of the oscilating polynomial $(1-0.56L+ +0.27L^2)$ can be due to an omitted relevant variable. This will be the solution to which data will lead, when looking for a model specification guided by economic considerations. Before searching for possible omitted variables, and to outline the nature of the problem, it should be taken into consideration models in which the oscillant effect appears only in the residual element or just in one or several transfer functions but not in all of them.

The first step has been to apply the autoregressive component only to the residuals and to eliminate it from the explanatory variables. The result has been that the adjustment worsens, and the standard residual deviation takes the value $\hat{\sigma} = 0.0026$.

The same proofs have also been made changing the moving average (MA) polynomial specification -$\omega(L)$- which affects the variable GDP delayed one period, keeping in all the cases the autoregressive structure applied only to the residual.

This original MA polynomial for GDP_{t-1} was

$$\omega(L) = \omega_0 + \omega_2 L^2 \tag{2.2}$$

and the following cases were considered:

$$\omega_0 + (\omega_1 + \omega_2 L)(1-L)$$
$$(\omega_0 + \omega_1 L) + (\omega_2 + \omega_3 L)(1-L)$$
$$\omega_0 + \omega_1(1-L) + \omega_2(1-L)^2$$
$$\omega_0 + \omega_1(1-L)^2$$
$$\omega_0 + (\omega_1 + \omega_2 L)(1-L)^2.$$

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The best adjustment is around $\hat{\sigma} = 0.03$, so it is clearly worse than in the original model. This result indicates that if the problem is caused by omitted variables, it is possible that those variables are correlated with some of the included ones and also that their effect in the model could be nonlinear.

Keeping the original moving average formulation in all the variables, the next step was to incorporate the autoregressive part to a certain explanatory variable as well as to the residual component. The objective was to check if the cyclical oscillations are due only to just one explanatory variable instead to all of them, as the original model proposes. In all the cases, the adjustment is worse, being $\hat{\sigma}$ greater than 0.0239.

2.3 Variable Contributions

The contribution of an explanatory variable X_t is defined as the influence it has on the dependent variable. This contribution denoted by X_t^* is given by

$$X_t^* \;=\; \frac{w_s(L)}{\delta_r(L)} L^b X_t. \tag{2.3}$$

Graphs 3 shows the original explanatory variables and their contributions.

The most relevant effect is observed in the GDP contribution,

$$\text{GDP}_t^* = \frac{2.44 - 1.64L^2}{1 - 0.56L + 0.27L^2}\text{GDP}_{t-1}. \tag{2.4}$$

It can be clearly seen how the associated dynamic filter causes fluctuations that original GDP did not have. This means that in the specified model the best fit is obtained with a filter given in (2.4) which must provoke fluctuations which are absent in the original variable GDP. This suggests that if there were any relevant omitted variable which had this fluctuations, its inclusion in the model could make unnecessary the second order autoregressive polynomial in the filter of GDP. The consideration of a possible omitted variable with these characteristics is the objective of the next section.

3 A Nonlinear Relationship between Investment and Production motivated by Strong Changes in Expectations

In the analyzed model for investment what could be missing is a variable reflecting important changes on future expectations, which possibly happen with the occasion of the energy crises in 1974 and 1979. A potencial variable for this purpose is the relative prices of energy, because important changes in these prices reflect greater uncertainty when investing. In this paper, this variable has been built as the log of the ratio between the prices of energetic imports and the deflator of production and it is denominated RPE.

The first step has been to study carefully this variable which is represented in graph 8. Given the scarce number of observations available, it is difficult to distinguish wether this series has a unit root or if, on the other hand, on the line of Perron (1989) and Espasa (1989), the series could be generated by a model with abrupt changes in mean. In this paper, it has been considered as I(1).

The linear contribution of this variable is not significative by the usual standards. However, the impact of this variable can be bigger in a nonlinear formulation. Thus, it seems interesting to consider models in which the uncertainty into the future generated by the abrupt changes in energy prices is reflected in changes of the elasticities of the explanatory variables with respect to the investment. To this purpose, the variable RPE will act as an uncertainty indicator.

Granger and Lee (1991) extended the idea of cointegration allowing the coefficients in the model to vary over time. Concretely they suggested a relationship in which the parameters changed in function of another variable ξ_t. A general type of time–varying nonlinear model which could pick up the interactions between the explanatory and the dependent variables as a function of another variable ξ to the model is

$$I_t = \Pi(L,\xi)X_t + a_t, \tag{3.1}$$

where X_t the set of explanatory variables of the model (including delays of the dependent variable I), a_t an stationary residual and $\Pi(L,\xi)$ a polynomial in the lag operator and the variable ξ. In what follows it will be assumed that ξ is strongly exogenous and that $\Pi(L,\xi)$ can be decomposed in the following way:

$$\Pi(L,\xi) = \Pi_1(L) + \Pi_2(L,\xi) \tag{3.2}$$

and therefore (3.1) can be rewritten as

$$I_t = \left(\Pi_1(L) + \Pi_2(L,\xi)\right) X_t + a_t. \tag{3.3}$$

This formulation allows us to distinguish clearly how the indicator of uncertainty ξ affects the elasticities of the different explanatory variables, and to know which would have been these elasticities in the cases when there is stability in the variable ξ. Also, supposing that $\Pi_2(L,\xi) = 0$, the linear case would be nested in the nonlinear formulation.

Given the scarce number of observations available, it is not possible to estimate model (3.1) in a reliable way. This is the reason why in this case we have used a model less general in which only the elasticity of one of the variables, say Z, is affected by the exogenous variable ξ. Using $X_t = (Z_t, W_t)$, the kind of model considered is

$$I_t \;=\; \Pi^*(L,\xi)Z_t + \Theta(L)W_t + a_t, \tag{3.4}$$

where $\Pi^*(L,\xi) = \Pi_1^*(L) + \Pi_2^*(L,\xi)$, and $\Theta(L)$ a polynomial only on L. In our case the exogenous variable ξ is the relative prices of energy RPE defined above.

As the result of this analysis it is obtained that the only elasticity significatively affected by RPE is the one corresponding to GDP, obtaining the following alternative model

$$\begin{aligned} I_t &= -5.81 + \Pi^*(L,\mathrm{RPE})\mathrm{GDP}_t + \\ &\quad +1.03\Delta\mathrm{CU}_t - 0.83L(C/P)_t + \\ &\quad -0.27\Delta^2\pi_t + \frac{1}{1+0.79L^2}\hat{e}_t, \end{aligned} \tag{3.5}$$

being

$$\begin{aligned} \Pi^*(L,\mathrm{RPE}) &= [(2.32 - 1.05L^2)L - \\ &- 0.004(\mathrm{RPE}_{t-1}L + \mathrm{RPE}_{t-2}L^2 + \mathrm{RPE}_{t-3}L^3) \end{aligned} \tag{3.6}$$

In this case $\Pi_1^*(L) = (2.32 - 1.05L^2)L$ and $\Pi_2^*(L,\mathrm{RPE}) = -0.004(\mathrm{RPE}_{t-1}L + \mathrm{RPE}_{t-2}L^2 + \mathrm{RPE}_{t-3}L^3)$. The adjustment is $\hat{\sigma}^2 = 0.01737$. The residual autoregressive process has a pair of complex variables

$$\pm 0.8879i$$

of module 0.79 and period 4 years. The t-values for each coefficient are found in Table 1.

In this model, the contribution of GDP on investment, say (GDP_t^*), depends on RPE in the following way:

$$\begin{aligned} \mathrm{GDP}_t^* &= [(2.32 - 1.05L^2)L - \\ &\quad -0.004(\mathrm{RPE}_{t-1}L + \mathrm{RPE}_{t-2}L^2 + \mathrm{RPE}_{t-3}L^3)]\mathrm{GDP}_t = \end{aligned} \tag{3.7}$$

$$= [1.27L - 0.004(\mathrm{RPE}_{t-1}L + \mathrm{RPE}_{t-2}L^2 + \mathrm{RPE}_{t-3}L^3)]\mathrm{GDP}_t +$$
$$+ [1.05L(1+L) - 0.004(\mathrm{RPE}_{t-1} + \mathrm{RPE}_{t-2}L)L)]\Delta\mathrm{GDP}_t, \qquad (3.8)$$

where Δ is the first difference operator. Graph 4 shows the the contributions of each of the variables. Graph 6 allows to know exactly how the nonlinear term affects in the model. It can be seen how its contribution changes along the model, picking the effect of the two energy crises. The dynamic relationship between investment and GDP in presence of stability in energy relative prices in a value RPE^s is

$$I_t = [1.27 - 0.012\mathrm{RPE}^s]\mathrm{GDP}_{t-1} + \qquad (3.9)$$
$$+[1.05(1+L) - 0.004\mathrm{RPE}^s(1+L)]\Delta\mathrm{GDP}_{t-1}. \qquad (3.10)$$

When the relative prices of energy change, the previous dynamic relation is altered as a function of them.

It's concluded that, when including the relative prices of energy in the model, we are supposing that, as far as abrupt changes on these prices refer to bigger uncertainty into the future, the elasticities are sensible to variations in future expectations.

4 Considerations on the Alternative Model

Let's compare first the original model given by equation (1.12) and the alternative shown in equation (3.5). The residuals in both cases do not show sample autocorrelations significatively different from zero. But it is model (3.5) which has better results: it's residual standard deviation is $\hat{\sigma} = 0.01737$ instead of $\hat{\sigma} = 0.02109$ originally. Also, it can be checked in the graph of the residuals (graph 2) that the residuals of model (3.5) (dotted line), are not only smaller in absolute value, but also eliminate the oscillations in the years 1975–1980 detected in the original model commented in previous sections. The alternative model improves the original adjustment.

Second, looking at the transfer functions, it's obtained that the autoregressive filter common to the denominator of all of them in the original model affects only the residual part of the alternative model. This model was built pretending to solve the problem of oscillant responses that the original model had. So, in graph 4, it can be observed how the contribution of GDP has a similar effect than the one obtained by the complex roots autorgressive process showed in graph 3, and how the the oscillant behaviour has disappeared from the contributions of the rest of the explanatory variables. Also graph 5, compares the contribution of GDP in both models and it's remarkable how they both obtain the same effect.

Another point to comment is that the number of parameters is not greater with the inclusion of the new variable in the alternative model. Just the oposite as can be seen in Table 1, this alternative model has one parameter less than the original model.

In conclusion, with the alternative model proposed, the original one is rejected in favour of the new one, and also, the characteristics of the alternative model are easier to asume from a theoretical point of view than those of the original one. This model signaled that all the response functions, including the residual term, were oscillant with the same structure amongst them. With the new model, the only sistematically oscillant structure is the one corresponding to the residual element. The rest of the explanatory varialbes, except GDP, do not have oscillant response.

In the case of GDP the oscillant response is not sistematic in time, independently of specific economic circumstances, but it appears in periods of great uncertainty about the future. Such uncertainty in the model is approximated by the relative prices of energy.

In the new model, the estimated long–run elasticity of investment with respect to GDP is greater than the original one, and this seems coherent with the strong process of capital accumulation registered in the Spanish economy during the sample period considered. However, once certain development of the Spanish economy has been consolidated, this elasticity should not be greater than untiy. In an extended sample with enough number of years in which observations of an hypothetical situation of consolidated development are included, the model could be specified to allow the elasticity in (3.10) to be greater than one at the period with big capital acumulation and restricted to be equal to unity once the process of capital acumulation were consolidated.

5 Error Correction Mechanism

Now, it is studied a bit more in detail the long-run properties of the original and the alternative models. Let us comment first the integration orders discussed by Andrés et al (1990). They argue that the variables do not follow very erratic paths and some of them show a clear growth. In general, they are characterized by being integrated of order one and having tendency in mean, except GDP, which could be represented by and I(2) process. Andrés et al. (1990) argue that this is probably just a sample phenomena, because in the considered period there are some very high rates of growth and also a big depression. So they conclude that all the variables are integrated of order one.

The hypothesis that the variables in the model I, GDP, C/P, CU are integrated of order one, implies their stochastic growths have stable means in time. This hypothesis isn't realistic for variables with sistematic growth, like I and GDP, because it's difficult to accept that their growth rate tend in every moment in time to the same fixed value as can be observed in graph 7. The hypothesis of a second unit root is probably most adequate than the original one. Unfortunately the tests to determine the integration order are only valid for big samples. They have little power against alternative hypothesis and the right test to be applied depends on the the hypothesis made on the possible breaks in the trend of the corresponding series as it is our case. Again graph 7 shows how the first difference transformations of I and GDP are both characterized by having significantly different means before and after 1974 (dotted lines represent the confidence bounds at 95%). The conclusion is that the integration and cointegration orders can not often be determined with precision. In any case, it's not very relevant, what really matters is to conclude with a balanced model, in which the variables and dynamic factors which balance the right term of a model with its corresponding dependent variable do make sense and have economic interpretation. In other words, what is important is that the dynamic factors determined in the estimation process were not spurious due to the omission of relevant variables or to restrictions in the dynamic factors of the model.

None of the above mentioned hypothesis on the long-run nature of I and GDP growing is completely convincing. A better alternative perhaps would be to postulate for them an univariate ARMA model with random level shifts, as proposed by Chen and Tiao (1990). But the estimation of these level shifts also has problems. Given the difficulty of characterizing the long-run of the variables considered in this article, it is going to be taken the proposal defended by Andrés et al. (1990) of just one unit root.

So let's consider first the model given by equation (1.12) and obtain its error correction model form. In this model all the variables appear preceeded by a polynomial on L. Let's call $\beta(L)$ the polynomial affecting the dependent variable and formulate it as:

$$\beta(L) = \beta_0 L + \beta_1(L)(1-L), \tag{5.1}$$

where β_0 is the previously mentioned long-run elasticity or gain of the filter $\beta(L)$. This decomposition allows to know at the same time the effect of the variable in the long-run of the model, given by β_0 and the effect in the short-run given by

$\beta_1(L)(1-L)$. Rewriting (1.12) in this way

$$\begin{aligned}\Delta I_t &= 2.29\Delta CU_t - 0.85\Delta^2\pi_t + \\ &+1.66(1+L)\Delta \text{GDP}_{t-1} - 0.57\Delta\left(\frac{C}{P}\right)_{t-1} + \\ &+0.27\Delta I_{t-1} + \\ &-0.71\left[I_{t-1} + 3.37 - 1.09\text{GDP}_{t-1} + 1.34\left(\frac{C}{P}\right)_{t-1}\right] + \hat{e}_t \qquad (5.2)\end{aligned}$$

From here one can characterize the long-run relationship given by the original model for investment

$$I_t = -3.37 + 1.09\text{GDP}_t - 1.34\left(\frac{C}{P}\right) + \hat{u}_t. \qquad (5.3)$$

The same decomposition given by (5.2) can be applied to the alternative model given by equation (3.5).

The model obtained is

$$\begin{aligned}\Delta I_t &= 1.03\Delta CU_t - 0.27\Delta^2\pi_t + \Phi(L,\text{RPE})\Delta\text{GDP}_{t-1} + \\ &-\left[I_{t-1} + 5.81 + 0.83\left(\frac{C}{P}\right)_{t-1} + \psi(L,\text{RPE})\text{GDP}_{t-1}\right] + \\ &+\frac{1}{1+0.79L^2}a_t, \qquad (5.4)\end{aligned}$$

being

$$\Phi(L,\text{RPE}) = [1.05 - 0.004(\text{RPE}_{t-1} + \text{RPE}_{t-2}L)](1+L)\Delta\text{GDP}_{t-1} \qquad (5.5)$$

and

$$\psi(L,\text{RPE}) = 1.27 - 0.004(\text{RPE}_{t-1} + \text{RPE}_{t-2} + \text{RPE}_{t-3}). \qquad (5.6)$$

So the long-run dynamic relationship of investment in Spain is characterized by

$$I_t = -5.81 - 0.83\left(\frac{C}{P}\right)_t - \psi(L,\text{RPE})\text{GDP}_t. \qquad (5.7)$$

Equation (5.4) is an specification of a non-linear error correction mechanism. The literature on this kind of models is not very wide, but several formulations have been realized and applied to data, Burgess et al. (1993) survey and apply them to labour demand in U.K.. Granger and Lee (1989) proposed an asymmetric error correction model characterized by different adjustments depending on the sign of the error correction term

$$\begin{aligned}(1-L)I_t = & -\lambda[\alpha_1(I_{t-1}-AP_{t-1})^+ + \alpha_2(I_{t-1}-AP_{t-1})^-] + \\ & +a(L)\Delta X_t + u_t, \end{aligned} \tag{5.8}$$

where I_t is the dependent variable, P_t is the set of variables influeting the long-run, X_t is the set of all the explanatory variables involved in the short-run specification, $a(L) = a_0 + a_1 L + \ldots + pL^p$ and

$$(I_t - AP_t)^+ = \max\{(I_t - AP_t), 0\} \tag{5.9}$$

$$(I_t - AP_t)^- = \min\{-(I_t - AP_t), 0\} \tag{5.10}$$

A more general formulation is in Escribano (1986) who developped the cubic polinomial error correction representation given by

$$\begin{aligned}(1-L)I_t = & -\lambda[\alpha_1(I_{t-1}-AP_{t-1}) + \alpha_2(I_{t-1}-AP_{t-1})^2 + \alpha_3(I_{t-1}-AP_{t-1})^3] + \\ & +a_1(1-L)X_{t-1} + \ldots + a_p(1-L)X_{t-p} + u_t, \end{aligned} \tag{5.11}$$

The cubic expansion of the errror correction term allows smooth adjustments to approximate the asymmetric reactions.

Another approach was the error correction representation with time varying parameters studied by Burgess (1988, 1992) of the form

$$(1-L)I_t = -\lambda(\xi_t)(I_{t-1} - AP_{t-1}) + a_1(1-L)X_{t-1} + \ldots a_p(1-L)X_{t-p} + u_t. \tag{5.12}$$

This formulation allows the lon-run term of the equation affect in a different way as the variable ξ_t changes in time.

The model developped in this article gives a new kind of error correction formulation of the form

$$(1-L)I_t = \lambda(I_{t-1} - A(\xi_t)P_{t-1}) + a_1(\xi_t)(1-L)X_{t-1} + \dots a_p(\xi_t)(1-L)X_{t-p} + u_t \quad (5.13)$$

The difference of equation (5.13) related to the models given by equations (5.8) to (5.11) is that although they all refer to non-linearities and asymmetries, the causes are exogenous in equation (5.13) and endogenous in the rest. Model (5.13) is basically linear, leaving linearity only in moments of changes in the exogenous variable ξ_t. Another possibility appointed by Granger and Lee (1991) once one knows there are structural changes at known times (as it would be the case) could be to include them in the model using appropiate zero-one dummy variables. The approach taken in this article allows us to represent the structural break situation by means of changes in RPE giving us a plausible interpretation for the behaviour of investment in those years. The approach taken by Burgess (1989, 1990) represented by equation (5.12) picks this idea. It does not affect the long-run relationship of the variables, but how ΔI_t reacts to deviation from the long run. In fact, the factor $\lambda(\xi_t)$ in (5.12) is a restricted version of our equation (5.13) when $A(\xi_t)$ also affects in I_{t-1}.

6 Concluding Remarks

This paper evaluates the empirical model for investment in Spain developed by Andrés et al. (1990) and designes an improved specification. Two potential problems were detected, the first was the long–run elasticity between investment and production, which happened to be greater than unity. The second had to do with the oscillant behaviour of the dynamic filter of the explanatory variables. The first consideration has a potential explanation consistent with the characteristics of the sample period considered because in that time in Spain a strong process of capital accumulation was ocurring. But the second could show a problem of an unbalanced equation and sugested that a potential omitted variable should be considered in the model.

This omitted variable seems related to changes on future expectations and can be approximated by the relative price of energy. The influence of this variable is not significative when introduced in a linear way with constant parameters during the whole sample period, but it is important in the moments of the energy crises, modifying the elasticity of GDP with the investment. The alternative equation modelizes the relationship between investment and GDP as a function of relative prices of energy, producing a nonconstant parameter model only in the moment of the energy crises. So it is obtained that, as abrupt changes on these prices refer to bigger uncertainty into the future, the elasticities are sensible to variations in future expectations.

Also the long–run relationship of the investment and GDP is affected by the changes on the relative prices of energy. The model proposed introduces a nonlinear error–correction scheme, which shows how the linearity of the equation can be affected by exogenous variables to the model influencing strongly only in specific moments of time.

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Table 1

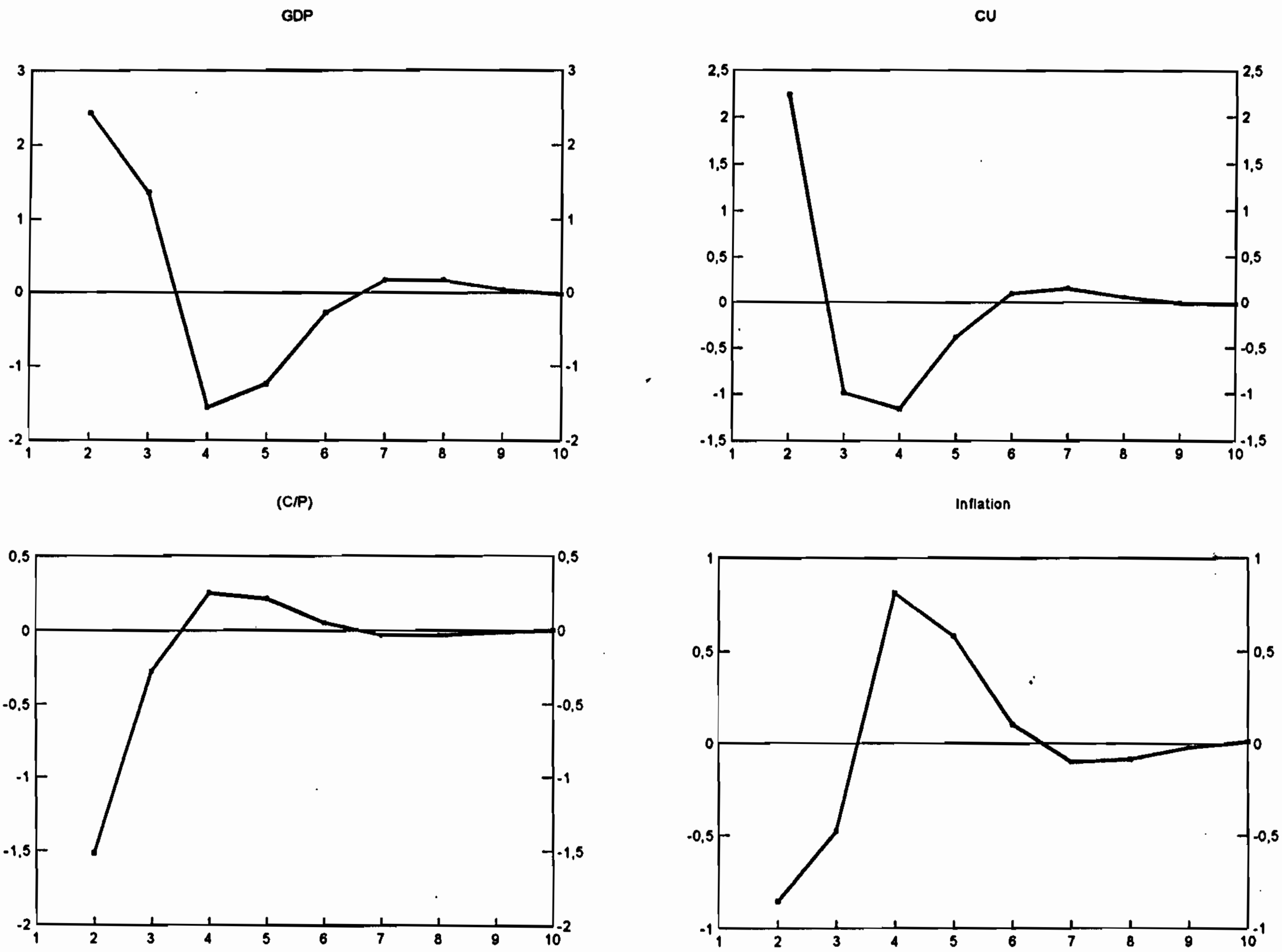
PARAMETERS BOTH MODELS

Variable	Equation (1.12)	Equation (3.5)
cte	-2.39 (-4.2)	-5.81 (-9.0)
I(-1)	0.56 (5.7)	-- --
I(-2)	-0.27 (-0.4)	-- --
GDP(-1)	2.44 (11.1)	2.32 (23.0)
	-- --	$-0.004RPE_{t-1}$ (-10.3)
GDP(-2)	-- --	-- --
	-- --	$-0.004RPE_{t-2}$ (-10.3)
GDP(-3)	-1.65 (-10.2)	-1.05 (-10.0)
	-- --	$-0.004RPE_{t-3}$ (-10.3)
ΔCU	2.24 (9.7)	1.02 (3.8)
(C/P)(-1)	-1.52 (-9.8)	-0.83 (-8.4)
(C/P)(-2)	0.57 (3.1)	-- --
$\Delta^2\pi$	-0.85 (-5.2)	-0.27 (-2.0)
Residual	White Noise	$1/(1+0.78L^2)$ (-4.8)

Graph 1

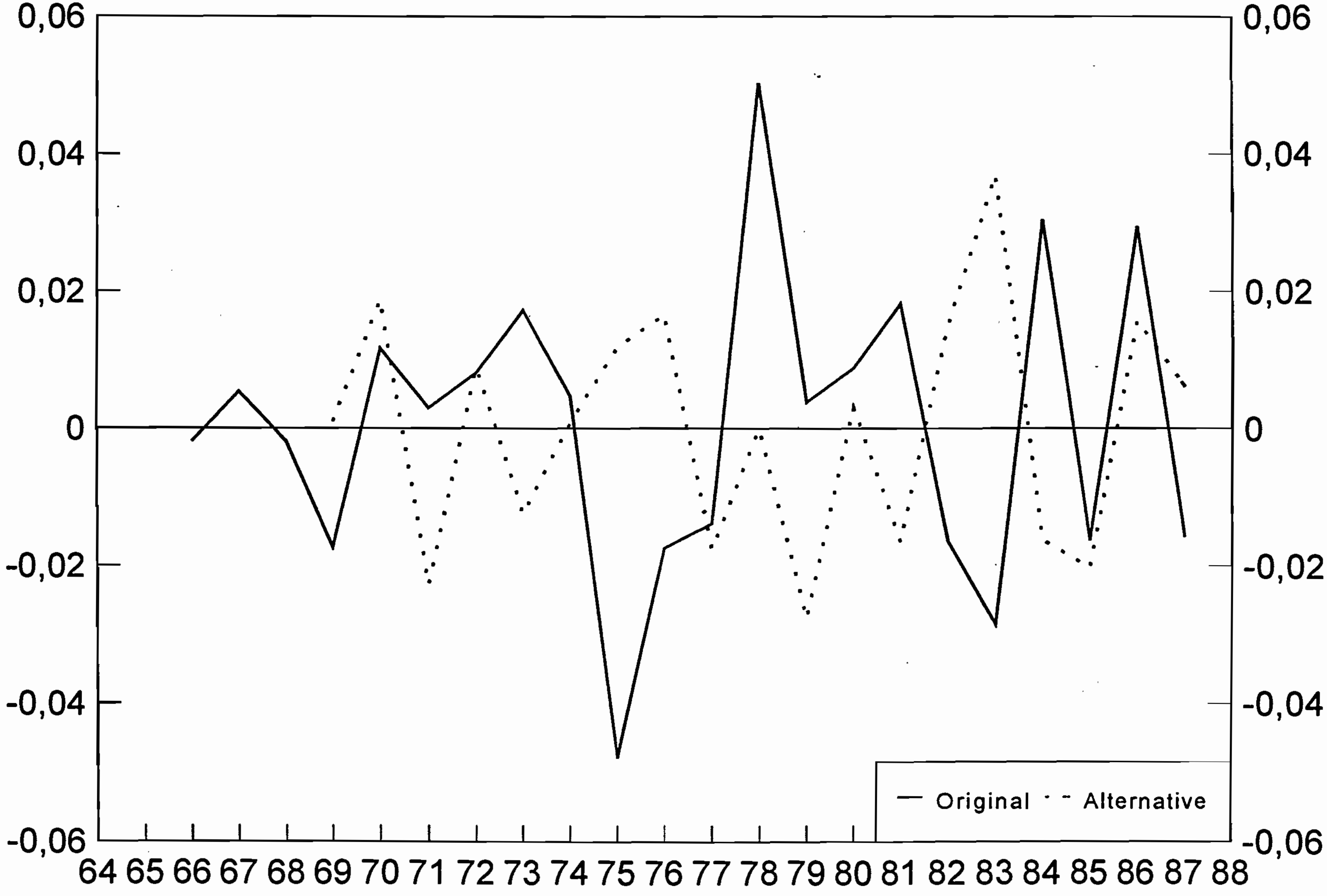
Dynamic relationship between Investment and the rest of explanatory variables in the model (1.12)

v(L) function



Graph 2

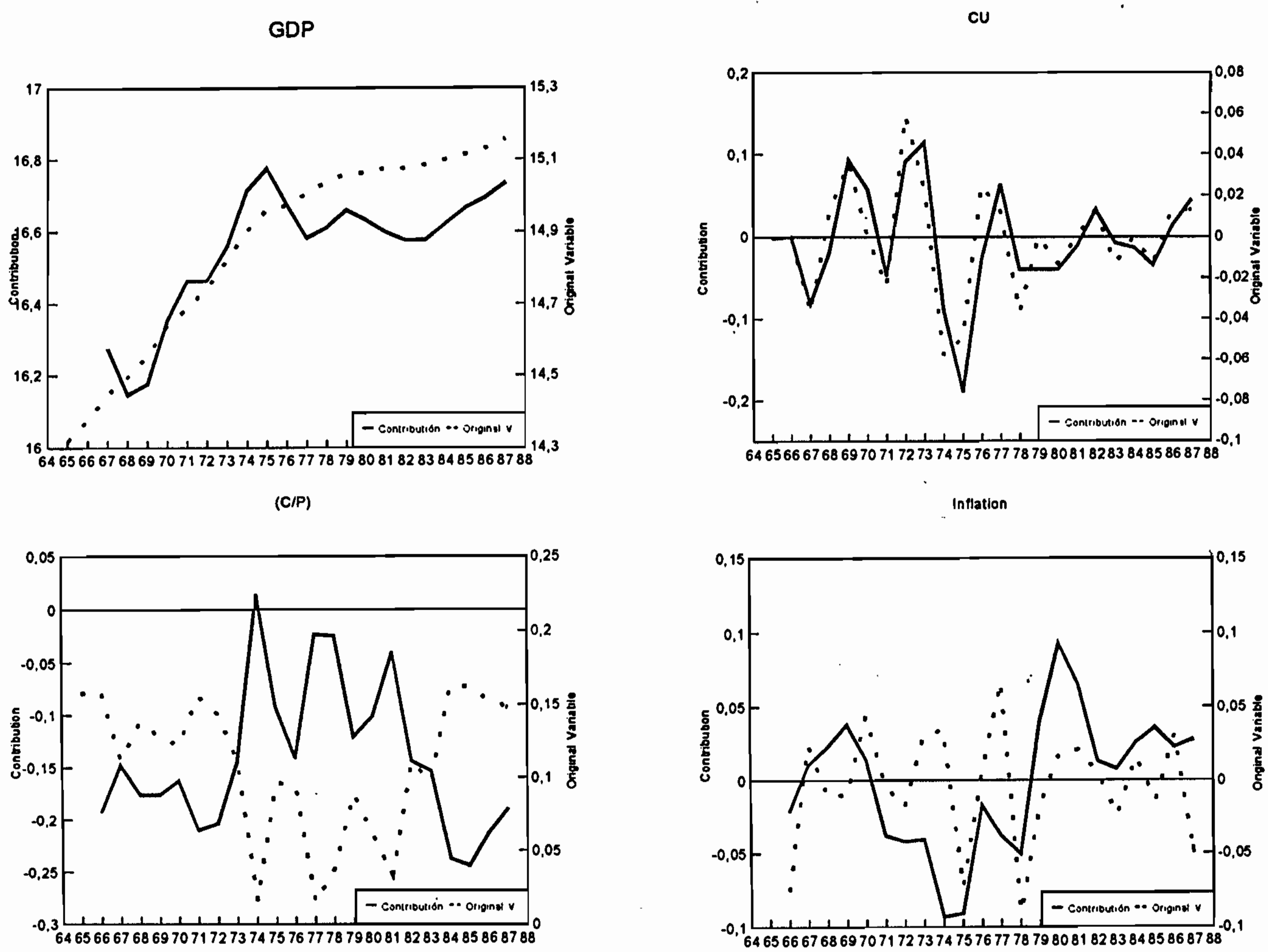
Residuals from the original model of Andrés et al. (1990) and the alternative one



Graph 3

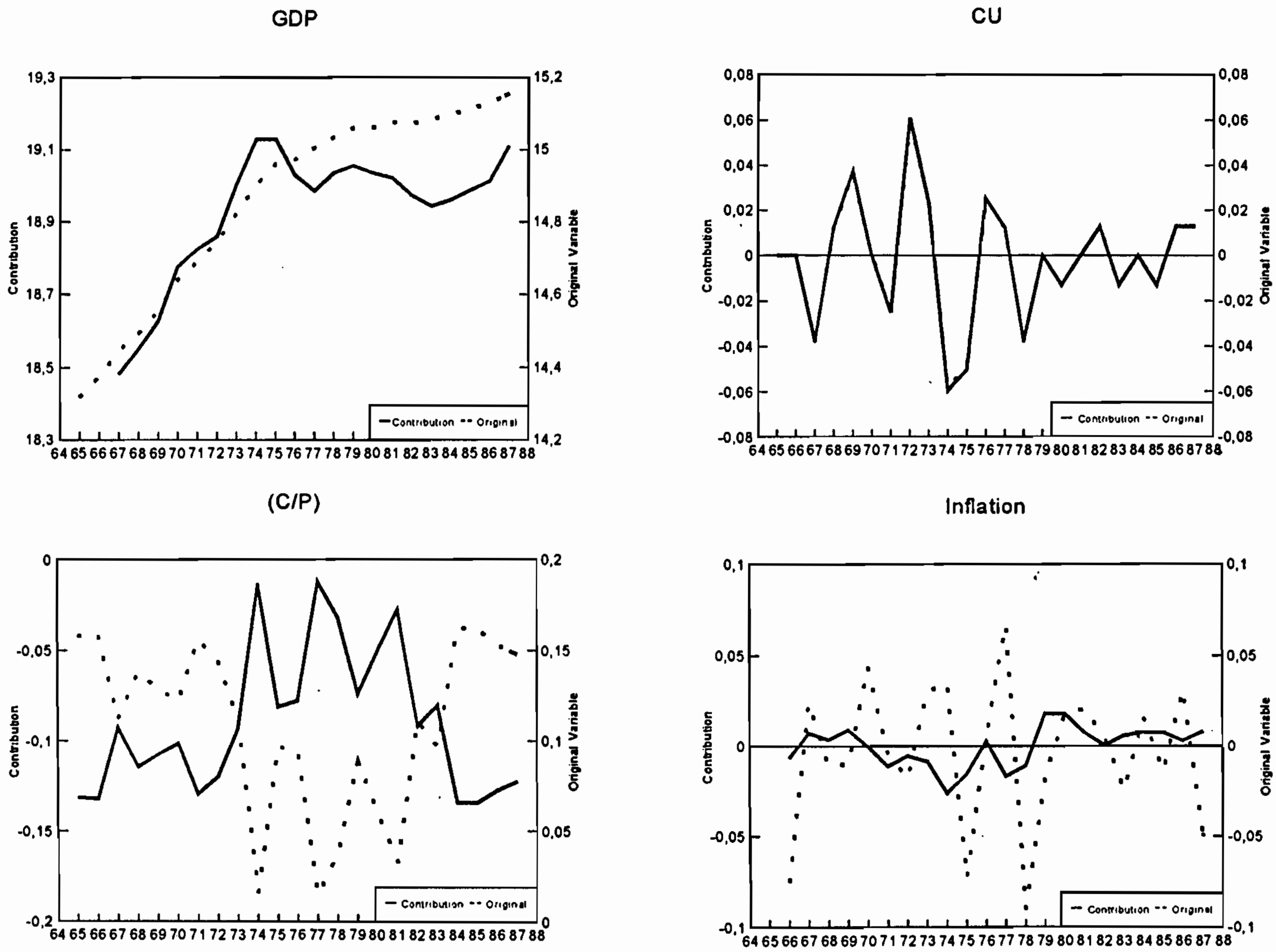
Contribution on Investment of

the explanatory variables in model (1.12)



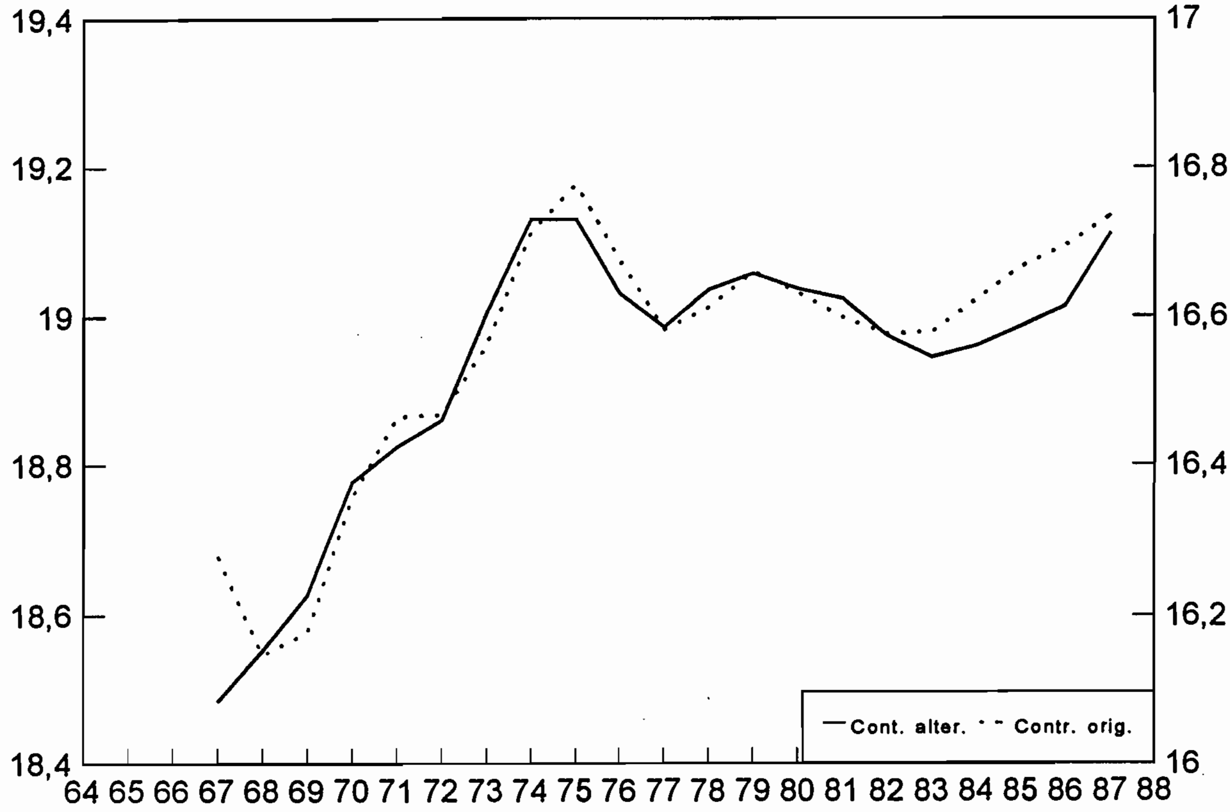
Graph 4

Contribution on Investment of the explanatory variables in model (3.5)

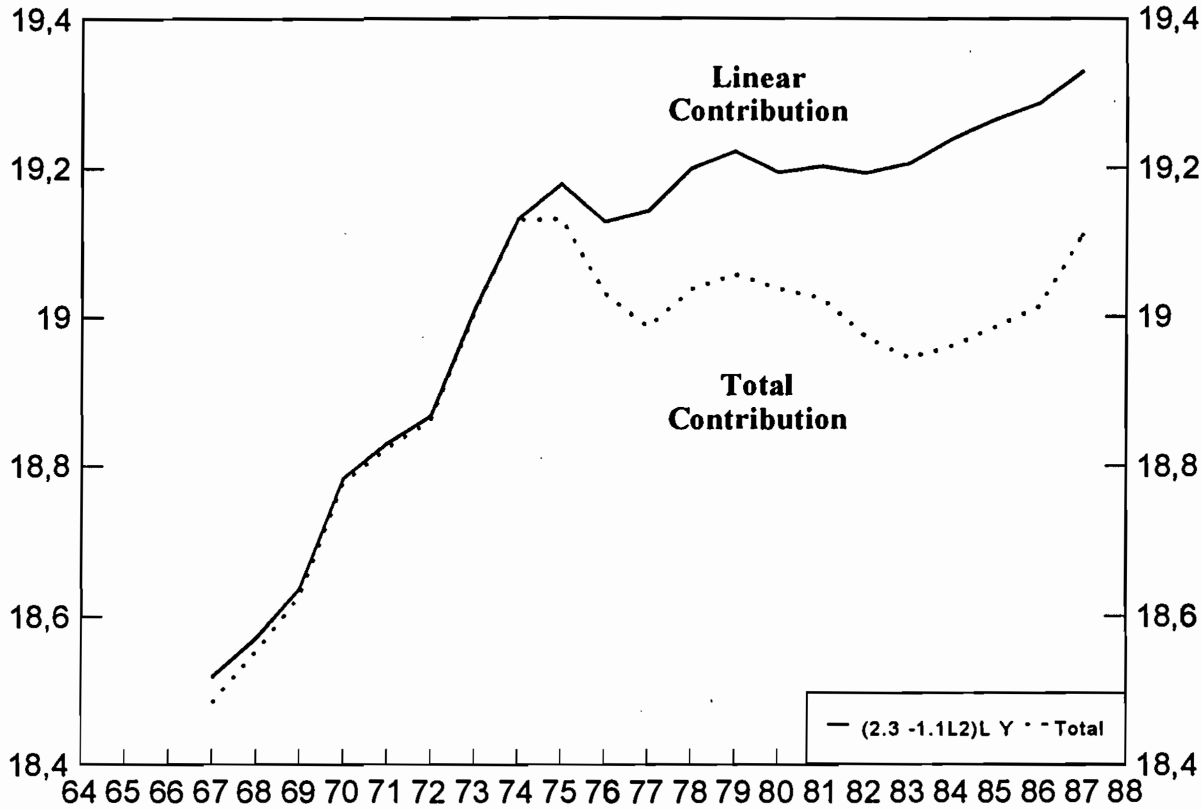


GDP Contributions on Investment in the original and alternative models

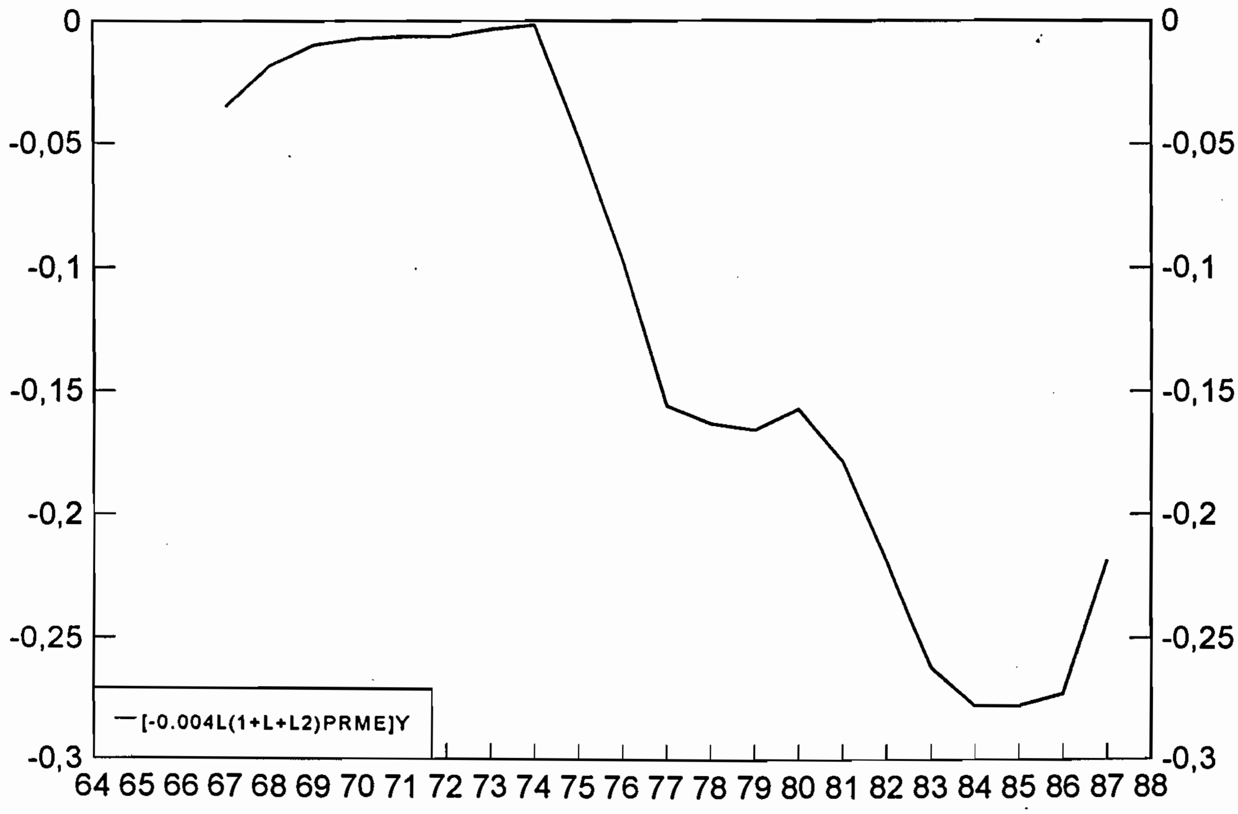
Graph 5



Decomposition of GDP Contribution on Investment in the alternative model



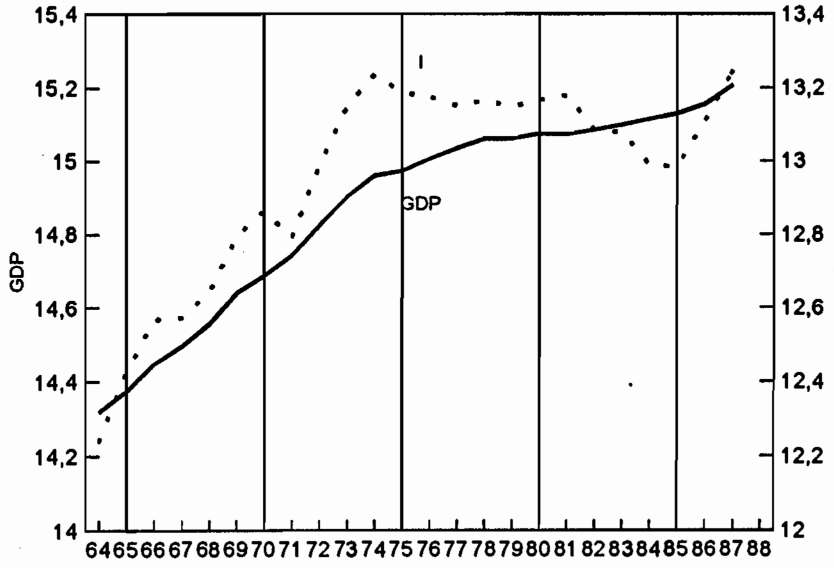
Nonlinear Part



Log of GDP and Log of I

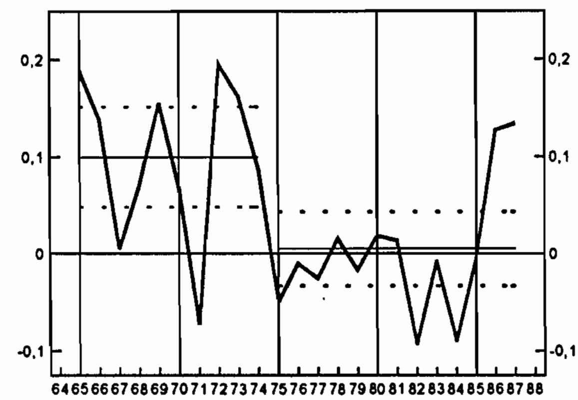
Original Series

Graph 7



Log of I

First difference

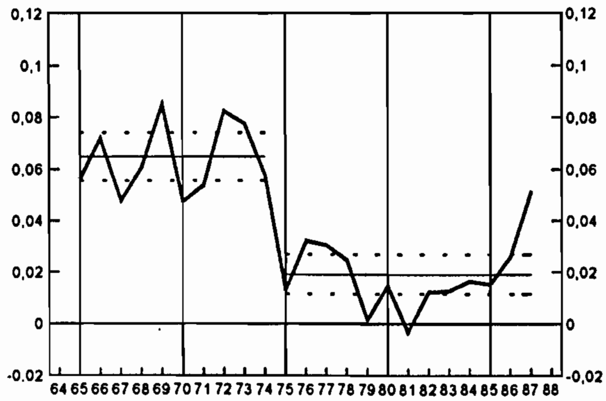


Log of I

Second difference

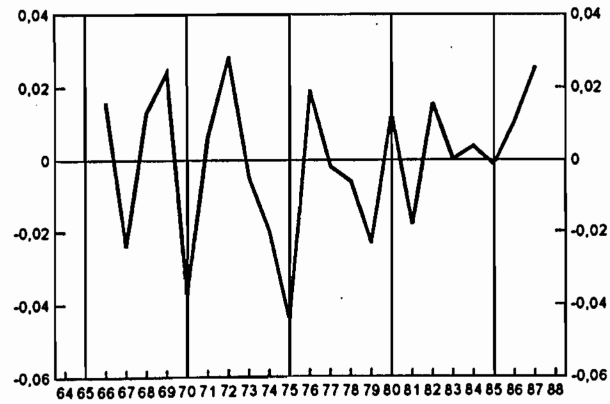
Log of GDP

First difference



Log of GDP

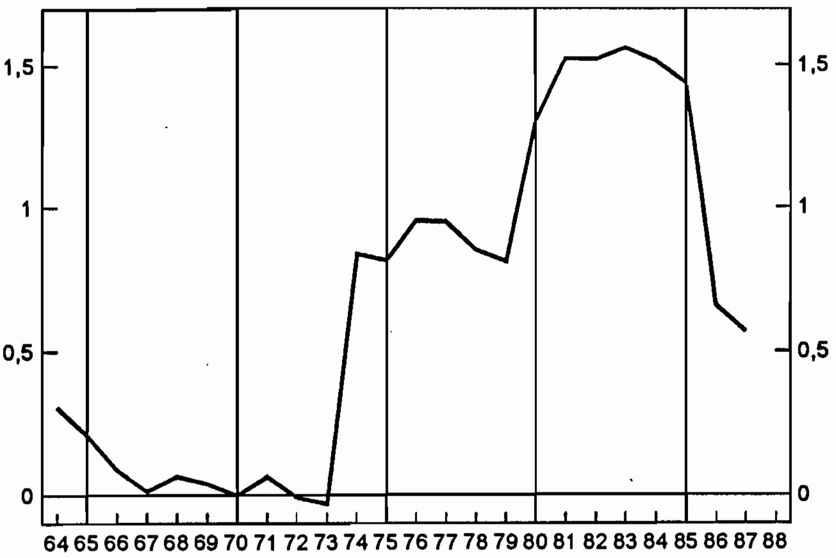
Second difference



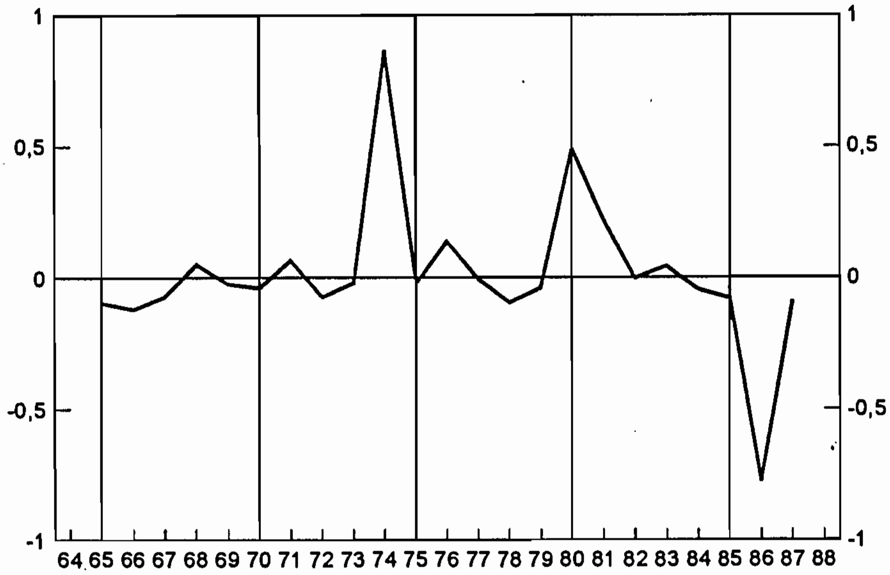
Log of RPE

Graph 8

Original Series



First difference



Second difference