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OPTIMAL PROCUREMENT MECHANISM WITH OBSERVABLE QUALITY

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Abstract

In a procurement contract the Administration usually has some prior information about the quality of the bidding firms. The goal of this article is to characterize the optimal mechanism in such a situation, when firms have private information about their costs. The optimal mechanism selects low-quality firms more often than it would be efficient with perfect information. We also compare this mechanism with others frequently used by the Spanish Administration such as the first price sealed bid auction and the previous admission auction.

Key Words
Procurement; Quality; Auctions; Optimal Mechanism.

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1 Introduction

In December 1994 the Spanish Ministry of Public Works awarded the second franchise in mobile telephone service to the Airtel group[1]. The Airtel bid (85 billion ptas.) was 4 billion ptas. lower than rival Cometa-SMR's offer. The Ministry argued that the difference in price was compensated by the difference in quality between the two bidders.[2].

This kind of procurement contract where bidding firms are of different levels of quality is awarded by a mechanism we will in the following call *contest*[3]. In this mechanism the contracting board jointly evaluates both price and quality. The goal of this article is to define the features that a *contest* should have to be the optimal mechanism in procurement contract where the bidding firms have different levels of quality. In others words, we want to study how a contracting board should jointly evaluate the price and quality .

The State Contract Law regulates the procurement contract awarded by the Spanish Administration. According to the Law there are three admissible procurement mechanisms: the *auction*, the *negotiation procedure*[4] and the *contest.*

- The auction ranks the bids only according to the price and the law recommends its use whenever possible.
- The negotiation procedure is the most subjective of the three mechanisms: The Administration can award the contract to bids with the highest prices and even avoid making the decision public. Since the negotiation procedure is the most manipulable mechanism , the law limits its use to urgent projects, small projects, projects related to the national defense,

[1] El Pais (December 29,1994)

[2] There are many dimensions in this procurement contract: the technical capacity, comercial strategy, economic and finalcial solvency, employment, etc.

[3] In Spanish, *concurso.*

[4] called "adjudicación directa" by the previous law

etc.

- Finally, the *contest* mechanism is used when other features from the bids along with price have to be taken into account.

The law allows that the use of both the auction and the *contest* be restricted, thus they are previous admission mechanisms. In this article, besides studying the features of the *contest*, we analyze how the auction and the previous admission auction work, since these mechanisms are frequently used by the Administration.

As an important deregulation of the telecommunication sector is going to take place in Spain, it seems also important to study contract procurement mechanism when the bidding firms have different levels of observable quality. This process will involve many procurement auctions similar to the one mentioned above, like, for example, the one that will take place for cable television franchises.

Other important applications include the regulation of the electric sector: the last reform of the electric sector established that new power stations will be awarded by *contest*. Although electricity is a homogeneous good, power stations can produce it with different technologies, and the impact on the environment need not be the same. Thus, in this kind of procurement, quality could be measured by the impact of the technology on the environment as well as other important feature like the availability of power at different times of the day.

The literature on the relationship between procurement mechanism when goods have different qualities is relatively scarce. Thiel (1988) studies the design of the optimal auctions when bidding firms can produce goods of different quality at different costs. To simplify matters Thiel (1988) assumes that a previous decision about the price is made. With this assumption the problem is analogous to designing a unidimensional auction. Branco (1992) and Che (1993) analyze the design of multidimensional auctions in which the firms compete in price and quality. Rogerson (1990) investigates the decision-making process of government agencies with respect to procurement and its consequences on the quality of the purchased good. Manelli and Vicent (1994, 1995) seek the optimal mechanism when information on quality is private and there is a direct correlation between cost and quaity but quality is not verifiable in a court of law. Cripps and Ireland (1994) consider a model in which quality is an unknown condition in order to approve the project and analyze how three specific mechanisms perform in this situation, while not characterizing the optimal mechanism. Laffont and Tirole (1991) assume like the present paper that firms have different quality levels and no relationship exists between quality and cost. They first study the optimum auction when the information on quality is public and subsequently analyze the problem of collusion between the firms and the agent who awards the project when the information on quality is private (it is known only to the agent and the firm). Their result is that in the optimal auction the principal (Administration) has to *undervalue* quality. Since Laffont and Tirole (1991) consider a very stylized situation with two firms that can only have two different possible cost and two quality levels, this paper aims at providing a more general result that will support reliable normative conclusions.

McAfee and McMillan (1989) and Branco (1994) study auctions with foreign bidders, but their structures and their results are related with our problem. In their models, firms have a public feature –nationality– that either has value *per se* or is associated with information about the firm's technology. This allows the possibility to design an auction with favoritism for foreign or domestic firms. McAfee and McMillan (1989) show how to discriminate in favor of the firm with the worst technology can stimulate competition, while Branco (1994) justifies discrimination in favor of domestic firms since their profits have a positive effect on the national welfare (consumption, employment, etc.). In the present paper the Administration will use different observable quality levels to discriminate among firms. It will be shown that while it is natural that firms with higher quality should have a higher probability to be awarded the contract, the probability with which low quality firms will be awarded the contract has very important implications in order to maintain competition. The ultimate goal of the paper is to provide an optimal compromise between quality and price.

The paper is organized as follows. In Section 2 the model is introduced. Section 3 analyzes the auction and the previous admission auction which are often used. Section 4 identifies the optimal mechanism and discusses its features: the main result is that the optimal mechanism urdervalues quality, and therefore low quality firms are awarded the contract more often that it would be efficient. Section 5 concludes the paper.

2 The Model

An Administration wants to undertake a single indivisible project (for example, the construction of a bridge) that generates social value $V(Q)$, an increasing function of Q, the quality of the project, $V'(Q) > 0$. The welfare function of the Administration is the difference between the social value of the project and its price, $W = V(Q) - P$. The Administration is interested in finding the mechanism that maximizes its expected welfare function.

Suppose there are N firms that are capable of undertaking the project and that each firm has an observable quality level Q_i. The Administration knows the identity of each individual firm and their quality levels; let firms be indexed according to decreasing quality levels, $Q_1 > Q_2 > \ldots > Q_N$.

Firm i has private cost c_i of completing the project. The cost c_i is independently distributed according to the distribution function $F(c)$ on $[c_*, c^*]$. The distribution function $F(c)$ satisfies the monotone hazard rate assumption $\frac{d}{dc}\left[\frac{F(c)}{f(c)}\right] > 0$. Each firm knows its own cost c_i, the number of bidders N, the quality of each firm, and $F(c)$. The Administration knows the number of bidders N, the quality of each firm, and $F(c)$ but does not know individual costs c_i, i.e., there is asymmetric information about costs. We will assume that firms are risk neutral, that there is *no relation* between the cost c_i and the level of quality Q_i and that the Administration always wants to undertake the project as its social value with the lowest quality is larger than the highest cost $V(Q_N) > c^*$.

Procurement proceeds in three steps. The Administration first announces the mechanism that it is going to use; in the second stage each firm submits a bid; finally the Administration awards the project[5].

3 Auction and Previous Admission Auction

3.1 Auction

In a regular auction bids only specify a price. We will now consider first price sealed-bid auctions (FPSB)[6], where the firm with the lowest bid wins the contract and pays its bid (the contract price is therefore the lowest bid price). The next Proposition characterizes the bidding equilibrium and the expected contract price.

Proposition 1 *In a first price sealed-bid auction (FPSB), with N risk neutral firms, with private cost c_i distributed according to cumulative distribution function $F(c)$ and with each firm knowing its cost c_i, N, and $F(c)$:*

1. *The equilibrium bid for a firm of cost c_i is $P_i = c_i + \frac{\int_{c_i}^{c^*}[1-F(c)]^{N-1}dc}{[1-F(c_i)]^{N-1}}$.*
2. *The firm with lowest cost who wins the auction.*
3. *The expected contract price is $P(N) = \int_{c_*}^{c^*}(c + F(c))N[1 - F(c)]^{N-1}f(c)dc$.*

Proof: See McAfee and McMillan(1987)

[5]The three stages are called in Spanish respectively, *licitación, puja* and *adjudicación.*

[6]With the assumptions of the model the contract price does not depend on the class of auction.

Asa consequence the expected contract price is decreasing in the number of firms $P'(N) < 0$;[7] since all firms have the same probability of winning the auction, irrespective of their quality levels, the expected quality of the project is $Q_s = \sum_{i=1}^{N} Q_i/N$ which is simply the average of quality of all firms participating in the auction; finally, the Administration's expected welfare is $W_s = \sum_{i=1}^{N} V(Q_i)/N - P(N)$.

3.2 Previous Admission Auction

The Spanish Administration often uses the previous admission auction (PAA) to award projects. In PAA's firms can only take part if they fulfil some requirements, such as solvency, experience, etc. This kind of requirements will be expressed in our model by a minimum quality level $\bar{Q}$. The Administration sets a minimum quality level and the firms can take part in the auction only if they a quality level no lower than the preset standard. The PAA is obviously a more general mechanism than the FPSB auction, as the FPSB auction is a special case of previous admission auction in which the minimum quality requirement does not prevent the entry of any firm (in our model $\bar{Q} \leq Q_N$).

Let $N^*(\bar{Q})$ be the number of the firms with a quality level higher that the minimum $\bar{Q}$ ($N^*(\bar{Q})$ is obviously non-increasing in $\bar{Q}$). The contract price that follows from Proposition 1 is $P(N^*(\bar{Q}))$ and the expected quality is $Q_{PA}(\bar{Q}) = \sum_{i=1}^{N^*(\bar{Q})} Q_i/N^*(\bar{Q})$. An increase in the minimum quality requirement to enter the auction implies an expected quality increase but also an increase in the expected contract price as a consequence of the lower number of firms participating in the auction. The Administration's expected welfare will be $W_{PA} = \frac{\sum_{i=1}^{N^*(\bar{Q})} V(Q_i)}{N^*(\bar{Q})} - P(N^*(\bar{Q}))$. Expected quality and price of the PAA are no lower than those of the FPSB auction, but expected welfare of the PAA *with an optimum minimum quality requirement* is never lower than the FPSB auction's as the

[7] Since the FPSB auction is equivalent in expected terms to a second price sealed bid auction, the expected contract price will be the second lowest value of cost in a sample of size N from $F(c)$. It is then simple to check that the expected contract price is decreasing in the number of firms $P'(N) < 0$.

former is a more general mechanism than the latter (it would be equal if $\bar{Q}_* \leq Q_N$, and it cannot be lower with an optimum minimum quality requirement, $\bar{Q}_*$).

The optimum minimum quality is found as the solution of a simple maximization problem:

$$\bar{Q}_* \in \arg\max \left\{ \frac{\sum_{i=1}^{N^*(\bar{Q})} V(Q_i)}{N^*(\bar{Q})} - P(N^*(\bar{Q})) \right\}$$

The Administration often uses both PAA and FPSB auction, although they are not optimum mechanisms. Due explanation stems from the fact that the mechanisms are simple to apply and they are more difficult to manipulate than the contest thereby avoiding corruption problems[8].

4 The Optimal Mechanism

As we said in the introduction the *contest* is a procurement mechanism that takes into account the price as well as other features of the bid that in our model will be summarized by a quality component. Our objective is to characterize the *contest* as an optimal mechanism and with this goal in mind we address the problem using the methods developed by Myerson (1981) on the optimal design of auctions.

Our problem is similar to Branco (1994). This paper analyzes how a project is awarded to one of two firms, one of them domestic and the other foreign. The main feature of the model is that the regulator is interested in maximizing the home country expected welfare. This objective function generates an asymmetry between the firms, because if the foreign firm gets the contract, its profit is not relevant for domestic welfare. If the domestic firm gets the contract, on the other hand, the regulator adds to the project's value the domestic firm's profit. A clear relationship between this work and our study exists as the domestic firm can be thought of as a firm of higher quality, which if awarded the project will provide higher value. In Branco's optimal mechanism, the regulator

[8]See, e.g. Laffont and Tirole (1991).

discriminates in favor of the domestic firm and we can therefore expect the solution to our problem to share have the same features, though in our case the Administration is going to discriminate according to the quality level of the firms.

We design a mechanism with two variables $\{p_i(c_i, Q_i), x_i(c, Q)\}$, Q and c being the quality and cost vectors, respectively, and p_i is the expected payment made to the firm i; as is clear, the expected payment is conditional only on the own cost and quality, while $x_i \in [0, 1]$, the probability of awarding the project to firm i depends on the whole vector of costs and quality levels. The type of each firm is given by its cost and its quality level, but as the latter is public information, we simplify the notation by omitting it $\{p_i(c_i, Q_i), x_i(c, Q)\} = \{p_i(c_i), x_i(c)\}$.

We are going to use a revelation mechanism in three steps. First the Administration announces the mechanism that it is going to use $\{p_i(c_i), x_i(c)\}$. The second step is the bidding stage, in which the use of a revelation mechanism ensures that firms report their own true costs. Finally, the Administration awards the project and makes payments.

We want to find a revelation mechanism that maximizes the ex-ante welfare of the Administration. Following the revelation principle (Myerson (1979)), there is no loss of generality if we concentrate on revelation mechanisms. Moreover, we restrict ourselves to mechanisms that satisfy incentive compatibility. This condition in our setting is the following:

$$E_{c_{-i}}\{\pi(c_i, c)\} \geq E_{c_{-i}}\{\pi(c', c)\} \quad \forall c_i, c' \in [c_*, c^*] \; \forall i \in N$$

where, $\pi(c', c) = p_i(c') - c_i x_i(c', c_{-i})$. According to the above condition a revelation mechanism is incentive compatible if all firms, no matter what their cost and quality, are willing to report their cost truthfully, i.e., all firms maximize their profits by reporting their true cost parameter. The mechanism must also satisfy other constraints on the probability of awarding the project, because the Administration only awards one project.

$$\sum_{i=1}^{N} x_i \leq 1, x_i \geq 0 \ \ \forall c_i \in [c_*, c^*], \ \forall i \in N$$

Finally, we also want to guarantee that all firms taking part in the mechanism get greater or equal profits than simply staying out. This restriction is called the individual rationality constraint.

$$E_{c_{-i}}\{\pi(c_i, c)\} \geq 0 \ \ \forall c_i \in [c_*, c^*], \ \forall i \in N$$

If a mechanisms satisfies the three conditions above (incentive compatibility, individual rationality and the constraint on the probabilities) we call it a *feasible mechanism.* Then our problem is to find the feasible mechanism $\{p_i(c_i), x_i(c)\}$ that maximizes the expected welfare of the Administration.

$$\begin{array}{lll} & \max_{p(c),x(c)} E\left[\sum_{i=1}^{N} V(Q_i)x_i - \sum_{i=1}^{N} p_i\right] & \\ \text{s.t.} & E_{c_{-i}}\{\pi(c_i, c)\} \geq 0, & \forall c_i \in [c_*, c^*], \forall i \in N \\ & E_{c_{-i}}\{\pi(c_i, c)\} \geq E_{c_{-i}}\{\pi(c', c)\}, & \forall c_i, c' \in [c_*, c^*], \ \forall i \in N \\ & \sum_{i=1}^{N} x_i \leq 1, x_i \geq 0, & \forall c_i \in [c_*, c^*], \ \forall i \in N \end{array}$$

The solution to this problem is characterized by the following result:

Proposition 2 *The firm that wins is the one whose cost and quality level maximize the function* $\psi(Q, c) = V(Q) - c - \frac{F(c)}{f(c)}$. *Then* $x_i = 1$ *if and only if* $\psi(Q_i, c_i) = \max_j \psi(Q_j, c_j)$. *The payment to the winning firm (contract price) will be the maximum cost, with which the winner firm would have obtained the project. If* i *is the winner and* i^* *is the second best firm* i, $\psi(Q_{i^*}, c_{i^*}) \geq \psi(Q_j, c_j)$, $\forall j \neq i$, *the contract price is:*

$$p_i = \begin{cases} c^* & \text{if } \ \psi(Q_i, c^*) \geq \psi(Q_{i^*}, c_{i^*}) \\ c_i^M : \psi(Q_i, c_i^M) = \psi(Q_{i^*}, c_{i^*}) & \text{if } \ \psi(Q_i, c^*) < \psi(Q_{i^*}, c_{i^*}) \end{cases}$$

Proof: See Appendix A.

As we said at the beginning of this section, the optimal mechanism is similar to Branco's. The optimal mechanism is an auction with favoritism: even though all firms can take part in the procurement process, the greater quality of the firm has, the higher probability that it will be awarded the contract. Therefore, the expected quality of the project is higher when the project is awarded by contest than when it is awarded by a FPSB auction. The contest's expected quality is an average of participating firms' qualities with weights decreasing with quality:

$$Q_c = \sum_1^N \alpha_i Q_i \text{ with } i > j \Rightarrow \alpha_i \leq \alpha_j \quad \text{and} \sum_1^N \alpha_i = 1$$

The contest's expected price is higher than the auction's expected price. This can be better understood thinking that if the Administration did not care about quality and would only be interested in minimizing the price, then the optimal mechanism is equivalent to a FPSB auction [9].

The comparison between the contest and the PAA is ambiguous. We know the PAA is weakly dominated by the contest, but we can not say anything about the quality and price. The problem is that expected price and quality of the PAA depends on the minimum quality $\bar{Q}$ [10].

The next proposition summarizes the main features of the optimal mechanism.

Proposition 3 *The optimal mechanism undervalues the quality with respect to what would be efficient with perfect information.*

Proof : Consider two firms with different levels of quality $Q_j < Q_i$, but the same evaluation according to the optimal mechanism $\psi(Q_i, c_i) = \psi(Q_j, c_j)$. Using the monotone hazard rate as-

[9] Even though the contest's expected price is higher than auction's expected price, sometimes the contest's price is lower than the auction price (e.g., if the winner of the contest is the lowest quality firm, the contest's price is lower than the auction price).

[10] If the minimun quality $\bar{Q}$ is low, the outcome is similar to the auction outcome. But if the minimun quality $\bar{Q}$ is high, then expected quality and price of the PAA are higher than thecontest.

sumption, $\frac{d}{dc}\left[\frac{F(c)}{f(c)}\right] > 0$, we can show that the higher quality firm provides more welfare to the Administration than the lower quality one: $V(Q_j) - c_j < V(Q_i) - c_i$. Therefore, the optimal mechanism undervalues quality.□

In terms of regulation policy this result, the above Proposition shows that when the Administration makes the rules for evaluating bids in the procurement process, it should assign less weight to quality than the optimal weight with perfect information. This implies that low quality firms will win the contests more often than it would be efficient. In brief, the optimal mechanism discriminates in favor of the low quality firms.[11] The above result provides the same prescriptions as the one of Laffont and Tirole (1991).[12]

The idea of Proposition 3 is straightforward: There is a trade-off between increasing expected quality of the project and limiting the expected profits of high quality firms. If the optimal mechanism did not undervalue the quality in the bids, it would result in increasing profits of high quality firms, because they have more chances of winning, and this would lead to an increase in the expected price of the project[13].

The State Contract Law also considers another procurement mechanism: the *previous admis-*

[11] An other interesting question is: When is discrimination in favour of a low firm too high? The answer is complex. If the social value of quality is high and the firms are very heterogeneus, the discrimination should be low. But in other cases, the discrimination depends on $F(c)$. Though we can not give a general result for this case, we can give a simple but interesting example. Consider the family of distribution functions $F(x) = x^n$ on $(0, 1)$. The value function of the optimal mechanism is in this case $\psi(Q, c) = V(Q) - c - \frac{c}{n}$,and it follows that the discrimination is decreasing in n. In this case the relationship between variance and optimal discrimination can be made explicit. When there is low variance (low cost uncertainty and low competition's profit) little discrimination should be employed. It is also interesting to notice that discrimination does not depend on the number of biders N.

[12] This result, also goes in the way of Branco (1992) who assumes that firms also choose their quality levels. His result is that the optimal mechanism produces less expected quality than it would be efficient.

[13] We suppose, however, that there is no relationship between a firm's cost and its quality. If on the other hand individual firms had different cost distributions associated with different quality levels, $F_i(c)$, the value function of the optimal mechanism would be $\psi(Q, c) = V(Q_i) - c_i - \frac{F_i(c_i)}{f_i(c_i)}$ and this would imply a change of Proposition 3.

sion contest. This kind of contest can be necessary, for example, if for specific quality levels the Administration is not interested in awarding the project. But with our assumptions ($V(Q_N) > c^*$) and our definition of the contest, the Administration never wants to limit entry: if it did so, expected price would increase and expected welfare would decrease. The justifications for the use of this mechanism could be reducing the transaction cost (to collect information from the firms and study the bids) and creating a reputation mechanism.

Reputation mechanisms become important when auctions are repeated. For this reason the Administration can take into the account the behavior firms had in previous projects to make decisions about new ones. For example, since there are high transaction cost to prepare a complete contract for large infrastructure projects, costly conflict situations are likely to arise and a great deal of bargaining can be expected to take place between the Administration and firms during the realization of a project. In such a situation it could be worthwhile for the Administration to use a previous admission mechanism to maintain high profits for the firms and to therefore provide them with incentives not to enter in conflict with itself.[14]

The conclusion is that to limit the competition through the previous admission mechanism could give firms incentives for "good behavior".

5 Implementing the optimal mechanism

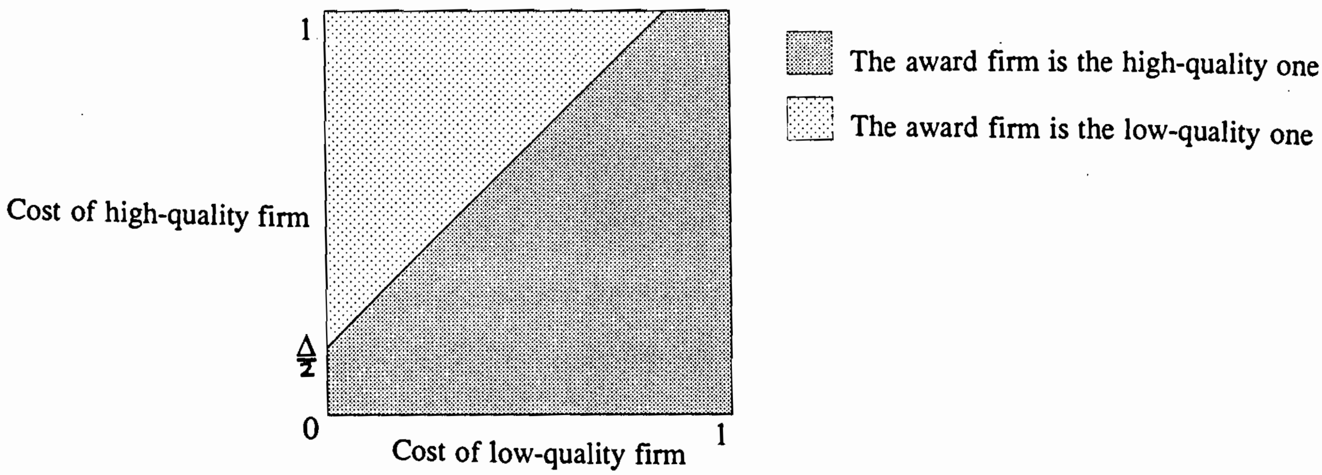
Although it is simple to characterize the optimal mechanism, its practical implementation could be complex. In fact, the regular procurement mechanism used by the Administration in this context, is

[14]The Wall Street Journal Europe (March 7,1995) reports how the German Administration limits competition in the Electric Turbine Market. General Electric complains that the German Electric Turbine Market is closed, because they have not sold a turbine since before World War II. Disputing a simple interpretation in terms of German protectionism, a quoted source claimed that "a German purchaser must think about his long-term relationship with the supplier".

a first price sealed-bid auction with specific award rules. Firms submit their bids, and a contracting board evaluates both price and quality of the firm. The contracting board decision is based on the previously stated award's rules, and the awarded firm receives only its own bid.

The aim of this section is to implement the optimal mechanism as a first price sealed-bid auction with specific award rules in an simplified environment. We only consider two firms, the high-quality firm and the low-quality firm. The social value of the quality is just the quality $V(Q) = Q$. The quality of the high-quality firm is $1 + \Delta$ and the quality of the low- quality firm is 1. The cost distribution function is a uniform function on the $[0, 1]$. The difference between the two quality values, is at most 1, $\Delta \in (0, 1)$.

We look for a favoritism auction equivalent to the optimal mechanism. [15]. The award rule of the optimal mechanism in this environment is $\psi(Q, c) = Q - 2c$. We are indifferent between the high-quality firm and the low-quality firm, when their cost are: $c_h + \frac{\Delta}{2} = c_l$. If the difference between costs $c_h - c_l$ is greater than $\frac{\Delta}{2}$, the low-quality firm will be the awarded firm, otherwise the high-quality firm will be the awarded firm.



Relation between the award firms and the costs firms.

The application of the auction that we are implementing is quite simple. The firms submit

[15]We will do this in a similar way to Branco(1994).

their bid, b_h and b_l. The Administration computes the costs associated with the bids; and also the difference between these costs $\sigma = b_b^{-1}(c_h) - b_b^{-1}(c_l)$. Later on, the Administration applies the awarding rule. If σ is higher than $\frac{\Delta}{2}$, the low-quality firm will be the awarded firm, otherwise the high-quality firm will be the awarded firm.

The equilibrium bidding function for the high-quality firm is $b_h(c_h) = \frac{1+\Delta-c_h^2}{2+\Delta-2c_h}$, and the bidding function for the low-quality firm is $b_l(c_l) = \max\{c_l, \frac{1-\Delta-c_l^2}{2-\Delta-2c_l}\}$ [16]. We can check that the high-quality bidding function is greater or equal than the low-quality bidding function for every cost $b_h(c) \geq b_l(c)$.

The probability that the awarded firm is the high-quality one is $\delta^* = 1 - \frac{(1-\frac{\Delta}{2})^2}{2}$, then, the high-quality firm has a higher probability to be awarded firm than the low-quality one. Therefore, the expected quality will be: $Q_c = (1-\delta^*) + \delta^*(1+\Delta)$.

The high-quality bidding function is increasing in the difference of quality Δ, $\frac{\partial b_a}{\partial \Delta} \geq 0$, but the low-quality bidding function is decreasing in the difference of quality Δ, $\frac{\partial b_a}{\partial \Delta} \leq 0$. Moreover, the expected quality and the award probability of the high-quality firm , are increasing in the difference of quality Δ $\frac{\partial \delta^*}{\partial \Delta} \geq 0$.

6 The previous admission auction

In this section we want to show the functioning of the previous admission auction. In the environment of the previous section we have only two choices; we can not set any requirement, and both firms can take part in the auction, or we can exclude to the low-quality firm.

If we do not set requirement (or the requirement, the minimum quality level is low $\bar{Q} \leq 1$), the previous admission auction is equivalent to the auction. The bidding function of an auction in this setup is $\frac{c+1}{2}$. The expected price is then $P_s = \frac{2}{3}$. The probability that the awarded firm is the

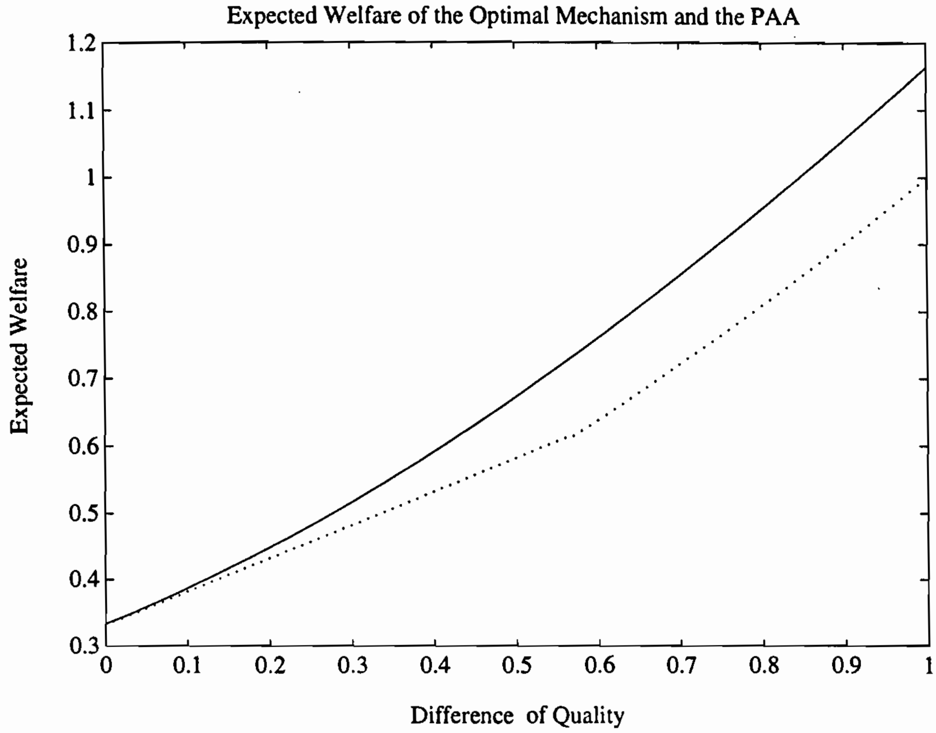
[16] See appendix B.

high-quality one is $\frac{1}{2}$, and therefore, the expected quality is $Q_s = \frac{1}{2} + \frac{1}{2}(1+\Delta)$. The expected profits of the firms are $\pi_s^a = \pi_s^b = \frac{1}{6}$. Finally, the Administration's expected welfare is $W_s = \frac{1}{3} + \frac{1}{2}\Delta$.

If we set a requirement preventing the entry of the low quality firm $1 < \bar{Q} \leq 1 + \Delta$, the Administration can only award the project to the high-quality one. In this setup , the Administration's optimal strategy is to offer a take-leave offer to the high-quality firm. The behavior of the firm in presence of the offer is simple: if the project's cost is higher than the offer, the firm reject the offer, in other case, the firm accept the offer. With that firm behavior, the Administration's expected profit is $W_{sa} = \int_0^{P_{sa}}(1 + \Delta - P_{sa})dc_a = P_{sa}(1 + \Delta - P_{sa})$. Therefore, the optimal Administration's offer is $P_{sa} = \frac{1+\Delta}{2}$. Given this offer, the expected Administration's profit is $W_{sa} = \frac{(1+\Delta)^2}{4}$. Finally, the expected profit of the high-quality firm is $\pi_{sa}^a = \frac{(1+\Delta)^2}{8}$.

The next step is to specify when it is better to prevent the entry of the low quality firm difference quality. The Administration's profit implies that if the difference in quality between firms is $\Delta \geq \frac{1}{\sqrt{3}}$, the optimal Administration's strategy is not to prevent entry, but if the difference in quality between the firms is $\Delta < \frac{1}{\sqrt{3}}$, the optimal Administration's strategy is to prevent the entry of the low-quality firm. When the Administration prevents the entry, the expected price, quality and profit of the high-quality firm rise.

Now, we can compare the different mechanisms that we have analyzed in terms of the expected welfare of the Administration. The main conclusion is that the optimal mechanism is always better, and its advantage is higher when the difference in quality is high.



In the figure, the optimal mechanism is the continuous line (-), and the previous admission auction is the point line (.).

7 Concluding remarks

If firms competing for a procurement contract are not homogeneous and have different levels of quality the optimal mechanism is an auction with "favoritism". The probability of being awarded a project is higher when a firm has high quality than when it has low quality. In the optimal mechanism's value function, however, quality is undervalued. Thus, while higher quality firms are awarded the contract with higher probability, the optimal mechanism *discriminates in favor of low quality firms*, and these firms get the contract more often than it would be efficient with perfect information.

Appendix

Proof of Proposition 2:

We first want to guarantee that the mechanism is incentive compatible. If the firm's expected profit is $\pi = E_{c_{-i}}\left[p(c', c_{-i}) - c_i x_i(c', c_{-i})\right]$, the envelope theorem implies that:

$$\frac{d\pi}{dc_i} = -E_{c_{-i}} x_i(c', c_{-i})$$

We want to replace a global incentive compatibility constraint for a local constraint. Following Guesnerie and Laffont (1984), we must prove that the mechanism satisfies the Spence-Mirrlees condition $\frac{\partial}{\partial c}\left(\frac{\partial \pi}{\partial x} / \frac{\partial \pi}{\partial p}\right) = -1$ and that the probability of awarding is monotonically decreasing in cost $\frac{\partial}{\partial c_i} E_{c_{-i}}\left[x_i(c_i, c_{-i})\right] \leq 0$. We will omit this second condition until the end of the proof.

The incentive constraint implies that the firm's profit is decreasing in c_i, Thus we must show only that the highest cost firm c^* gets positive profit. Replacing p_i for $\pi + x_i c_i$ in the objective function we get:

$$\begin{aligned} W &= E\left[\sum_{i=1}^{N} V(Q_i)x_i - \sum_{i=1}^{N} x_i c_i - \sum_{i=1}^{N} \pi_i\right] \\ &= E\left[\sum_{i=1}^{N} V(Q_i)x_i - \sum_{i=1}^{N} x_i c_i\right] - \sum_{i=1}^{N} E_{c_{-i}}\left[\int_{c_*}^{c^*} \pi_i f(c_i) dc_i\right] \end{aligned}$$

Integrating by parts and using $\frac{d\pi}{dc_i} = -E_{c_{-i}}\left[x_i(c', c_{-i})\right]$ we obtain:

$$W = E\left[\sum_{i=1}^{N} V(Q_i)x_i - \sum_{i=1}^{N} x_i c_i\right] - \sum_{i=1}^{N} E_{c_{-i}}\left[\left[\pi_i F(c)\right]_{c_*}^{c^*} + \int_{c_*}^{c^*} x_i(c_i, c_{-i}) F(c_i) dc_i\right].$$

Since the objective function is decreasing in π_i, and the individual rationality condition implies $\pi_i(c^*) = 0$, the highest cost firm gets 0 profits independent of its quality, and therefore $\left[\pi_i F(c)\right]_{c_1}^{c_2} = 0$. Given this we can now simplify the objective function above. Multiplying and dividing by the density function $f(c)$, we obtain:

$$W = E\left[\sum_{i=1}^{N} V(Q_i)x_i - \sum_{i=1}^{N} x_i c_i - \sum_{i=1}^{N} x_i \frac{F(c_i)}{f(c_i)}\right]$$

Defining the value function $\psi(Q,c)$ as $\psi(Q_i,c_i) = V(Q_i) - c_i - \frac{F(c_i)}{f(c_i)}$, the objective function is:

$$W = E\left[\sum_{i=1}^{N} \psi(Q_i,c_i)x_i\right]$$

and it follows than the Administration must choose the lowest $\psi(Q_i,c_i)$.

$$x_i(c_i,c_{-i}) = \begin{cases} 1 & \text{if } \psi(Q_i,c_i) = \max_j \psi(Q_j,c_j) \\ 0 & \text{otherwise} \end{cases}$$

Given this awarding rule and using the monotone hazard rate assumption $\frac{d}{dc}\left[\frac{F(c)}{f(c)}\right] > 0$, we can show that the monotonicity condition $\frac{\partial}{\partial ci} E_{c_{-i}} x\left[{}_i(c_i,c_{-i})\right] \leq 0$ is satisfied.

We must now define the expected payment. Given $p_i = \pi + c_i x_i$ and using $\frac{d\pi}{dc_i} = -E_{c_{-i}}\left[x_i(c',c_{-i})\right]$,we obtain:

$$p_i = c_i x_i(c_i,c_{-i}) + \int_{c_i}^{c^*} x_i(s,c_{-i})ds.$$

Let i^* be the best firm different from i, $\psi(Q_{i^*},c_{i^*}) \geq \psi(Q_j,c_j), \forall j \neq i$. If $\psi(Q_{i^*},c_{i^*}) \leq \psi(Q_i,c_i)$ and $\psi(Q_i,c^*) < \psi(Q_{i^*},c_{i^*})$, we define the cost with which firm i has the same valuation as i^*, c_i^M : $\psi(Q_i,c_i^M) = \max_j \psi(Q_j,c_j)\ \forall j \neq i$. Now using the definition of x_i, the payment is:

$$p_i = \begin{cases} c^* & \text{if } \psi(Q_i,c^*) \geq \psi(Q_{i^*},c_{i^*}) \\ c_i^M : \psi(Q_i,c_i^M) = \psi(Q_{i^*},c_{i^*}) & \text{if } \psi(Q_i,c^*) < \psi(Q_{i^*},c_{i^*}) \end{cases}$$

The payment is 0 if the firm does not get the contract while if it does, the payment is equal to the maximum cost with which the firm would have been awarded the project.□

Appendix B

The goal of this appendix is characterize the equilibrium bids of the first sealed bid auctions equivalent to the optimal mechanism.

The bid of the high-quality firm is found as the solution of this simple maximization problem:

$$\max_{b(c_a)}(1 - b^{-1}(b(c_a)) + \frac{\Delta}{2})(b(c_a) - c_a)$$

The first order condition is:

$$[1 - c_a + \frac{\Delta}{2}]b'(c_a) = b(c_a) - c_a$$

The solution of this differential equation with the initial condition $b(1) = 1$, is:

$$b(c_a) = \frac{1 + \Delta - c_a{}^2}{2(1 + \frac{\Delta}{2} - c_a)}$$

The problem of the low-quality firm is the same. Thus, the bid of the low-quality firm is found as the solution of this maximization problem:

$$\max_{b(c_b)}(1 - b^{-1}(b(c_b)) - \frac{\Delta}{2})(b(c_b) - c_b)$$

The first order condition is:

$$[1 - c_b - \frac{\Delta}{2}]b'(c_b) = b(c_b) - c_b$$

The solution of this differential equation with the initial condition $b(1) = 1$, is:

$$b(c_b) = \frac{1 - \Delta - c_b{}^2}{2(1 - \frac{\Delta}{2} - c_b)}$$

This bid is wrong if $c_b > 1 - \frac{\Delta}{2}$, because the firm can not win the auction, and with this bid, the firm will not get positive profits. We must guarantee that the low quality firm do not obtain negative profits. Taking this into account, the bid is:

$$b(c_b) = \max\{c_b, \frac{1 - \Delta - {c_b}^2}{2(1 - \frac{\Delta}{2} - c_b)}\}$$

□

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