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SMALL FIRMS, BORROWING CONSTRAINTS, AND REPUTATION

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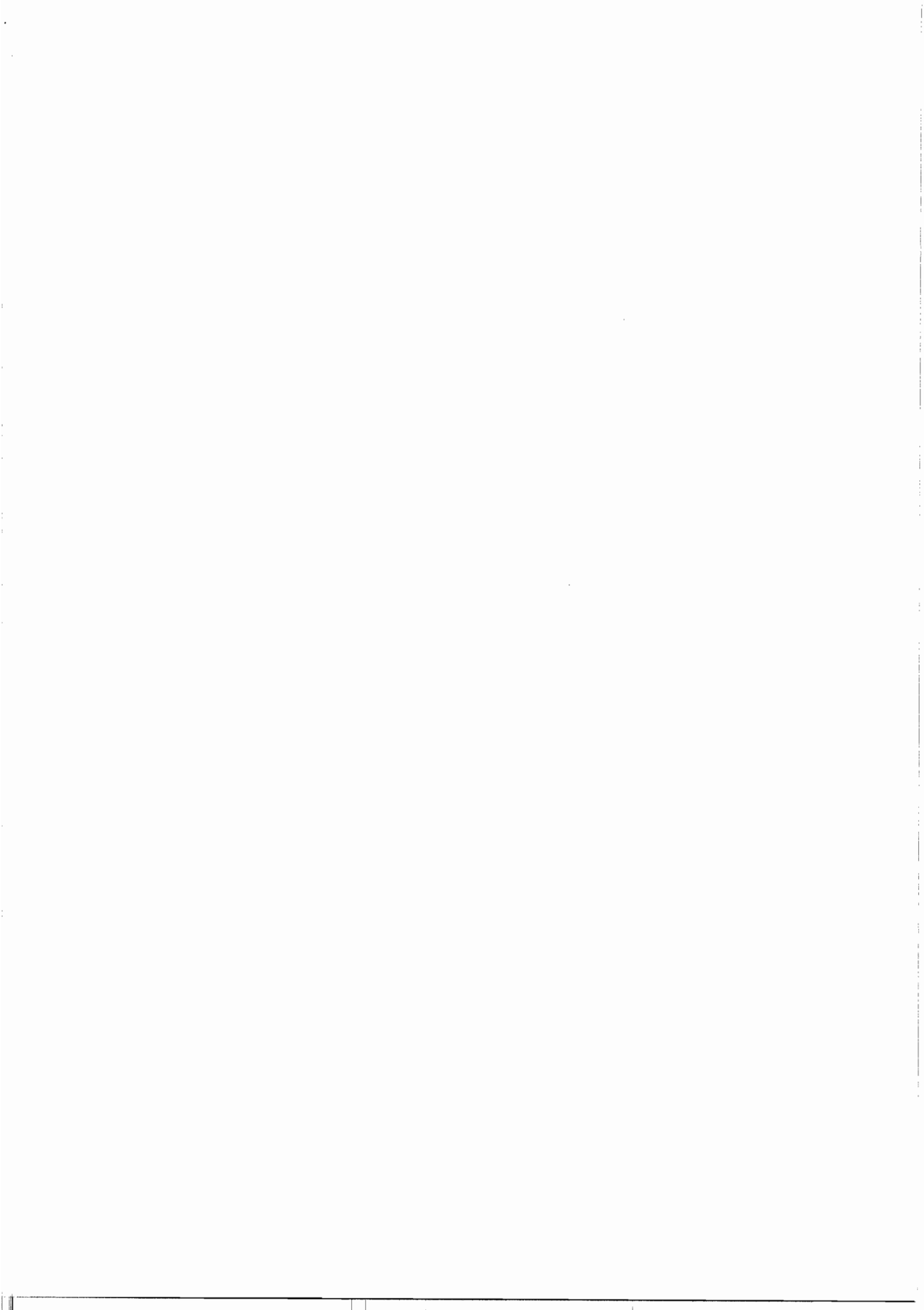
Abstract

This paper presents a simple model relating firm age with firm size and access to credit markets. Lending to new firms is risky because lenders have had no time to accumulate observations about them. As a result, interest rates are high and loans are small for entering firms. As firms need credit to operate, credit markets impose a limit on the scale of operation of new firms. Reputation building by the firms allows markets to overcome these difficulties over time. Large firms face lower interest rates than small firms, and credit markets fluctuations are shown to have different effects on firms of different size.

Keywords

Small Firms, Credit Markets, Borrowing Constraints, Repeated Games, Reputation

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1 Introduction

Small firms pay higher interest rates, are more likely to be liquidity constrained, and, as shown by recent empirical work, suffer disproportionately from reductions in credit market flows in response to economy-wide disturbances. This paper provides a simple explanation to these observations in the context of a reputational model relating firm age with firm size and access to credit markets. The model predicts that the length of the credit history of a firm will be negatively correlated with the interest rate it faces and positively correlated to the size of the loans it can get from lenders. Credit markets impose limits on the size of new firms. Macroeconomic fluctuations are amplified for new firms through the working of the credit market.

Gertler and Gilchrist (1993, 1994) review the available evidence on the cyclical behavior of small versus large firms in the U.S., and present some new evidence. They show that credit flows to small firms contract relative to credit flows to large firms after tight money episodes. After tight money, at the onset of recessions, small firms account for a disproportionate share of the decline in manufacturing output. This poses the question of why, under those circumstances, small firms seem less able to borrow than large firms. Credit market frictions seem a natural place to start looking for an answer. Gertler and Gilchrist (1993), in particular, point out that informational frictions adding to the cost of external finance apply mainly to smaller firms, due to the existence of fixed costs in evaluation and monitoring, proportionally greater bankruptcy costs for small borrowers, and proportionally more collateralizable net worth for large borrowers.

The thrust of this paper is that informational frictions may be important for small firms because they tend to be relatively new borrowers. We do not consider explicitly differential costs in evaluation, monitoring or bankruptcy procedures. We do, however, assume that small firms must rely on their reputation in order to keep access to the credit market, and focus on the risk of lending to borrowers with a short

track record.

We support our point of view by developing a model in which firms play an infinitely repeated game with adverse selection and moral hazard. Adverse selection problems are strong for entrant firms: Among firms which are observationally equivalent, there are many which only have access to excessively risky projects ("bad" firms), so that they have a high probability of defaulting. High interest rates (due to "lemons" premia over the economy-wide riskless interest rate) are required to guarantee a competitive return to lenders. If firm owners are patient enough, those of them who have access to sound investment projects ("normal" firms) will avoid undertaking excessively risky projects in order to keep their access to the loan market in the future. In equilibrium, borrowing limits to firms serve to reduce the gains of choosing excessively risky projects in the present. Borrowing limits are a substitute for patience: If the firm owners' discount rate approaches one, credit limitations become less restrictive.

As firms who default are recognized as the bad ones and excluded from future lending, firms that stay in the loan market face lower lemons premia. As interest rates decline, higher profits provide an incentive for the borrowing firms to keep their reputation and, therefore, larger loans are made to individual firms. Thus, newer firms pay higher interest rates and are more credit constrained (and therefore, smaller) than older firms.

Macroeconomic disturbances are introduced in the model as movements in the riskless interest rate. It is shown that borrowing constraints become tighter for new firms when the riskless interest rate increases. As we ignore multiperiod contracting and possibilities of using collateral, we believe our model is best suited to describe the evolution of small business' credit terms.¹

There is an extensive literature on the effects of moral hazard in loan contract-

¹We can also think of the situation of many developing nations, where the legal environment is not reliable enough to allow for multiperiod contracts or the use of collateral.

ing at least since Stiglitz and Weiss (1981). Our stage game presented is similar to their setup. In Stiglitz and Weiss' article, lenders are assumed to be scarce, and discrimination among observationally equivalent borrowers emerges to avoid increasing the interest rate, which would exacerbate moral hazard problems. In this paper, lenders are not scarce. There is no discrimination among observationally equivalent borrowers, but limits on individual borrowing are imposed to counter moral hazard.

Our model is most related to Diamond (1989) work on reputation acquisition by new firms. The adverse selection problem we consider is similar to his. In our model, as in Diamond's, the interest rate depends on adverse selection considerations (the probability of a firm being bad). Unlike what happens in Diamond's model, however, borrowing limits emerge in our model as a result of moral hazard (the need to provide incentives for the firms to repay). Other related work on reputation acquisition includes Cole, Dow, and English (1995) and Petersen and Rajan (1995). Cole, Dow, and English develop a model of sovereign debt with changing types. In their model, separation between normal and bad types occur in the first period a country becomes normal. Adverse selection stops being a problem, and the model relies in trigger strategies in later periods. Petersen and Rajan (1995) focus on small business. Unlike us, they assume that adverse selection problems are dispelled after the first period, and concentrate on the effects of creditors' market power on credit terms.

Section 2 presents the model and Section 3 derives the equilibrium. In order to build an intuition of how the model works, both sections describe the evolution of one generation of firms keeping constant the riskless interest rate. Section 4 introduces fluctuations in the riskless interest rate in a context in which there are many generations of firms. Section 5 concludes.

2 The model

There is an infinite sequence of dates indexed by $t = 0, 1, \dots$. There are two different sets of agents: lenders and borrowers (or firms). All agents are risk-neutral.

Lenders survive only one period. They are born each period with an endowment of 1 unit of input. They can lend all or part of it to firms, and invest the rest in a constant-returns-to-scale technology that yields r units of the consumption good at the end of the period (i.e., they have access to a risk-free return of r). Borrowers are relatively scarce as compared to lenders. One interpretation of this setup is that we are focusing on a particular industry, so that the economy-wide opportunity cost of funds, r , is exogenously given. Another possible interpretation is that we are focusing on a small economy that faces an internationally given, risk-free interest rate r .

Firms are born at time 0.² They are infinitely-lived, and have a discount factor $\beta \in (0, 1)$. The assumption that firms are long-lived but lenders live only one period is intended to focus the attention on borrowers' reputation as the link between the present and the future. Firms have no endowment, but have access to investment projects each period. Therefore, they must borrow from lenders in order to finance investment.

Firms can operate only one project per period. There are two types of projects: "safe" and "risky." Safe projects have a return of Gy , where $y \in [0, 1]$ is the amount invested. Risky projects obtain B_y with probability π and 0 with probability $1 - \pi$, where $\pi \in (0, 1)$.

Allowing y to take any value from 0 to 1 is the main departure of our assumptions with respect to those in Diamond (1989), where firms are constrained to invest either 0 or 1 (that is, they face fix-size projects). This departure is crucial because, as we will see below, contracts between lenders and borrowers will have to specify not only the interest rate but also the amount to be lent. Of course, our results about

²In Section 4, it will be assumed that a generation of firms is born in every period.

borrowing constraints are possible because of this feature of loan contracts.³

It will be assumed that:

$$\pi B < r < G < B \quad (\text{A1})$$

That is, the expected return of safe projects is larger than the risk-free interest rate, while the expected return of risky projects is smaller than the risk-free interest rate (It is inefficient to invest in risky projects). The return of the risky project in case of a good outcome, however, is superior to the return of a safe project, which, as we will see, might provide an incentive for firms to undertake risky projects.

There are two types of firms: “Normal” firms can choose one of the two types of project each period, while “bad” firms have access only to risky projects. Each firm’s type is private information, as is the type of project chosen by a firm and the realized return on a project.

Lenders can commit to use a liquidation technology that destroys the output of a firm if it defaults. The contracts between borrowers and lenders are assumed to be debt contracts (Townsend (1979) shows that debt contracts are optimal in single-period principal-agent environments with similar information asymmetries).

At the beginning of each period, each firm offers a debt contract to a lender. A contract will be a pair $\{R_t, L_t\}$. If the lender accepts it, he will proceed to lend L_t units of the input to the firm. Otherwise, if the lender rejects it, the firm will not be able to invest. If the loan is made, the firm has to choose an investment project. At the end of the period, the firm is expected to repay $L_t R_t$ units of output. We will call R_t the interest rate (contingent on the contract $\{R_t, L_t\}$). Firms that do not repay their debt will be subject to the liquidation technology, in which case borrowers and lenders involved will end up with 0 units of output. If a firm repays its debt, it consumes the difference between the project’s return ($L_t y_t$) and the amount repaid ($L_t R_t$). Lenders will make use of their storage technology to obtain r units of output from whatever units of input have not been lent.

³Another departure is that, to simplify matters, we use an infinite horizon.

Within-period timing is shown in Figure 1. We assume that firms are able to make take-it-or-leave-it offers to convey the idea that they are in the short side of the market.

The population of firms has a fraction $\rho \in (0, 1)$ of the normal type, drawn once at the beginning of the game. While project choices and realized returns are privately known by firms, each firm's history of contract terms and defaults becomes common knowledge. At the beginning of time t , the public history of a firm includes information about contract terms proposed by the firm, whether they were accepted or not and whether the firm repaid its debt or defaulted on it for all periods previous to t . Let h^t be the public history of a firm after it has made a proposal at the beginning of time t , and before this proposal has been accepted or rejected by a lender. This information allows lenders to update their beliefs about a firm's type.

Let us denote by $p_t(h^t)$ the probability, as perceived by lenders, that a firm is normal given its public history. The term $p_t(h^t)$ will be called the reputation of the firm. In updating this probability, lenders take into account the distribution over possible histories induced by the equilibrium strategies of normal firms and bad firms. The reputation of a firm is important because lenders are not interested in lending to bad firms: Given (A1), the expected return of lending to bad firms is smaller than r , regardless of contract terms. With complete information, lending to bad firms would be avoided. As we will see, reputation plays a crucial role in equilibrium.

3 Competitive equilibrium

We will use the notion of perfect Bayesian equilibrium. In the context of our model, this means that for every possible history of a firm (including histories that do not occur in equilibrium) we must specify lenders' beliefs over the firm's type. These beliefs must be updated according to Bayes' law, wherever possible. Given these beliefs, normal firms and bad firms choose actions that are a best response to the

lenders' strategy, and vice versa.

Since bad firms do not want to be identified as such, in equilibrium they offer the same contract terms that normal firms do in order to obtain credit (That is, there are only pooling equilibria). Moreover, for some lending to occur, in equilibrium normal firms must undertake safe projects and repay their debts. Otherwise, the expected return for a lender would be smaller than r . Barring mixed strategies, that means that if a firm ever defaults in a period in which there is lending, it is identified by lenders as a bad firm and excluded from credit thereafter. Finally, the interest rate offered by firms in equilibrium must be such that the expected return for a lender is greater than or equal to r , taking into account that normal firms undertake safe projects and bad firms risky ones.

From the possible equilibria satisfying the description above, we will concentrate on the one where contract terms maximize the utility of normal firms, and any firm offering different contract terms sees its reputation dropping to zero and is excluded from credit thereafter. We will call it a competitive equilibrium because it maximizes the utility of the agents in the short side of the market. This is by no means the only perfect Bayesian equilibrium. For instance, there is an equilibrium in which there is never lending because normal firms are expected to undertake risky projects every time they get credit. We believe the equilibrium we propose is the most reasonable one in the context of the model.⁴

The equilibrium path, then, consists of a sequence of loans $\{L_t\}_{t=0}^{\infty}$ and a sequence of interest rates $\{R_t\}_{t=0}^{\infty}$ such that the (expected discounted) utility of normal borrowers is maximized, given that lenders suspend credit to any firm that ever defaults, normal firms find it advantageous to undertake safe projects in every period, contract terms are such that the expected return for a lender is greater or equal to r in every

⁴This is a restriction on beliefs - (of sorts!). In a similar environment (but considering also a type of firms committed to undertake safe projects in every period), Diamond (1989) shows that the best equilibrium for borrowers survives a variety of refinements that put constraints on out-of-equilibrium beliefs.

period, and reputation p_t of firms that have never defaulted is updated by Bayes' rule. Only bad firms with a zero outcome from their risky project will default in equilibrium, because firms who default are subject to the liquidation technology and their output is destroyed.

It is useful to state the following assumption, whose meaning will be clear later:

$$\rho \geq \frac{r/G - \pi}{1 - \pi} \text{ and } (1 - \pi) \left(\frac{G - r}{1 - \beta} \right) > \pi (B - G) \quad (\text{A2})$$

The following Proposition characterizes the equilibrium path:

Proposition 1 *Under the assumptions (A1) and (A2) there is lending along the equilibrium path. Lenders only accept contracts from borrowers who have never defaulted. Normal firms undertake safe projects. The reputation of firms which have never defaulted is updated according to*

$$p_{t+1} = \frac{p_t}{p_t + \pi(1 - p_t)} \quad (1)$$

with $p_0 = \rho$. If a firm ever defaults, its reputation drops to zero. The interest rate offered by firms which have never defaulted is given by:

$$R_t = R(p_t) = \frac{r}{p_t + \pi(1 - p_t)} \quad (2)$$

Define \tilde{t} as the earliest time such that $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau}) \geq \pi (B - G)$. Before \tilde{t} , firms ask for loans given by:

$$L_t = L(p_t) = \frac{(\beta\pi(B - G))^{\tilde{t}-t-1}}{\prod_{\tau'=t}^{\tilde{t}-1} (\pi(B - G) - (1 - \pi)(G - R_{\tau'}))} \times \beta(1 - \pi) \sum_{\tau=\tilde{t}}^{\infty} \beta^{\tau-\tilde{t}} (G - R_{\tau}) \quad (3)$$

From \tilde{t} onwards, L_t is equal to 1.

Proofs are provided in the Appendix. The intuition for this result is discussed here. Equation (1) follows from the fact that bad firms avoid default with probability π in any given period, while normal firms never default. Equation (2) gives the

minimum interest rate such that the expected return of lending to a firm which has never defaulted is equal to r , taking into account the different probabilities of default of normal and bad firms. As time goes on, the reputation of a firm which has never defaulted increases. Interest rates, then, decline over time, approaching r from above.

Loan sizes are determined in a forward looking manner, taking into account current and future interest rates. Borrowing constraints are used to prevent normal firms from considering profitable to chose risky projects.

Let us define $V_t \equiv \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau})$. V_t is the value for a normal firm of undertaking safe projects from t onwards if there are no borrowing limits (i.e., if $L_{\tau} = 1$ for $\tau = t, t + 1, \dots$). By undertaking a risky project, a normal firm faces to lose V_t with probability $(1 - \pi)$, in case the project goes wrong, and to gain $(B - R_t) - (G - R_t)$ with probability π , in case the project goes well. Since interest rates decline over time, $(1 - \pi) V_t$ increases as time goes on. As soon as $(1 - \pi) V_t$ becomes larger or equal to $\pi (B - G)$, which happens at time \tilde{t} , borrowing limits become unnecessary.

For $\tilde{t} - 1$, equation (3) can be rewritten as

$$\pi (B - G) L_{\tilde{t}-1} = (1 - \pi) (G - R_{\tilde{t}-1}) L_{\tilde{t}-1} + (1 - \pi) \beta V_{\tilde{t}}$$

This expression tells us that the expected gain of choosing a risky project at $\tilde{t} - 1$ (the LHS) is no larger than (in fact it is equal to) the expected loss (the RHS), taking into account that $L_{\tau} = 1$ for $\tau = \tilde{t}, \tilde{t} + 1, \dots$

For $t < \tilde{t} - 1$, equation (3) can be rewritten as

$$\pi (B - G) L_t = (1 - \pi) (G - R_t) L_t + \beta \pi (B - G) L_{t+1}$$

This expression tells us that borrowing limits equalize the expected gain of choosing a risky project (the LHS) with the expected loss (the RHS). Notice that the term $\beta \pi (B - G) L_{t+1}$ represents the (discounted) expected loss due to the possibility of being excluded from the market from $t + 1$ onwards, which is equal, along the equilibrium path, to the expected gain of choosing a risky project at $t + 1$.

The next results follows from Proposition 1:

Corollary 2 *Under the assumptions (A1) and (A2), R_t declines over time and approaches r from above. L_t increases over time as long as $t < \tilde{t}$.*

New firms in need of building a reputation to gain access to loan markets are initially finance-constrained and face interest rates with high risk premia. As time goes on, the interest rate approaches asymptotically from above the default-free competitive rate r . Borrowing constraints loosen, and are no longer binding after some time \tilde{t} .

Corollary 3 *If either $\rho < \frac{r/G - \pi}{1 - \pi}$ or $(1 - \pi) \left(\frac{G - r}{1 - \beta} \right) < \pi(B - G)$, there is no lending along the equilibrium path.*

The minimum interest rate that guarantees a competitive return to lenders at time 0, under the supposition that normal firms do not default, is equal to $R_0 = \frac{r}{\rho + \pi(1 - \rho)}$. But if $\rho < \frac{r/G - \pi}{1 - \pi}$, R_0 becomes larger than G and normal firms will not be able to repay it. If $\rho < \frac{r/G - \pi}{1 - \pi}$ the market does not open because the adverse selection problem is too strong: There are simply too many bad firms.

Let us define $V_\infty \equiv \frac{G - r}{1 - \beta}$. If $(1 - \pi) V_\infty < \pi(B - G)$, the expected loss of choosing a risky project would always remain smaller than the expected gain of doing so, given that the interest rate is always larger than r . Lending at any particular time could only be sustained by an ever increasing sequence of loans, as in a bubble. But since L_t is bounded by one, this is impossible. The market does not open in this case because the moral hazard problem is too strong, even ignoring the initial adverse selection problem.⁵

Notice that, if we assume $L_t \in \{0, 1\}$, the market could only open if $(1 - \pi) V_0 \geq \pi(B - G)$ (which is a stronger constraint than $(1 - \pi) V_\infty > \pi(B - G)$). Borrowing constraints permit to overcome this additional restriction, which comes from the

⁵In the limit case where $(1 - \pi) \left(\frac{G - r}{1 - \beta} \right) = \pi(B - G)$, the market could open in the absence of adverse selection, and lending is not discarded by Corollary 2.

interaction of adverse selection and moral hazard problems. In a way, borrowing limits do this because they are a substitute for patience.

Corollary 4 *If new firms become more patient, borrowing constraints are relaxed and may stop being binding sooner.*

Increasing β increases the possible loss due to undertaking a risky project at any given time t while keeping constant the possible gain.

4 Lending to small firms and macroeconomic fluctuations

The model presented in the previous two sections describes the evolution of a generation of firms born at time 0. In this section it is modified to introduce a new generation of (infinitely-lived) firms born in every period.⁶ The public history of a firm includes now the date at which the firm was born. Let $\{L_t^{t'}, R_t^{t'}\}$ be the equilibrium contract terms offered at time t by a firm born at time t' which has never defaulted, and let $p_t^{t'}$ be the reputation at time t of a firm born at time t' which has never defaulted.

Clearly, if all parameters of the model are kept constant, each generation's history is a mirror image of the history of the generation born at time zero. That is, $p_t^{t'} = p_{t-t'}$. Similarly, $\{L_t^{t'}, R_t^{t'}\} = \{L_{t-t'}, R_{t-t'}\}$, where $\{L_{t-t'}, R_{t-t'}\}$ are the equilibrium contract terms offered by the generation born at time zero as described by Proposition 1. Moreover, a cross section of the contract terms offered by firms getting loans in the market looks like the history of the credit terms offered by the generation born at time 0. It follows that:

⁶Since firms are infinitely-lived, the number of firms grows without bound. We can remedy this unpleasant implication by assuming some exogenous, constant probability of a firm exiting each period. In terms of the model, it is necessary to redefine β as the product of the "true" discount factor with the probability of survival.

Corollary 5 *In the model with many generations of firms, firms from older generations which have never defaulted face lower interest rates and more relaxed or nonbinding borrowing constraints than younger firms.*

Younger firms are smaller in the sense that borrowing constraints force them to undertake (safe) projects at a lower scale than older firms do. It follows from Corollary 4 that smaller firms pay larger interest rates. The reason is that older firms have a better established reputation.

In the framework presented, macroeconomic fluctuations can be introduced as variations in the exogenously given riskless interest rate r . With the purpose of analyzing the effects of an unexpected, temporary variation in macroeconomic conditions we will consider in this section that the risk-free interest rate is subject at some period \hat{t} to a shock ϵ , which can be either positive or negative. The timing of events is modified in the following way: At the beginning of period \hat{t} , before firms get to offer credit terms to a lender, the value of $r_{\hat{t}} = r + \epsilon$ is revealed to all agents in the economy. For simplicity, at any other period $r_t = r$ (We do not consider, for instance, the possibility of persistence in the shock).

For the remainder of this section we will make use of the following assumptions:

$$\pi B < r + \epsilon < G < B \quad (\text{A1}')$$

$$\rho \geq \frac{(r + \epsilon)/G - \pi}{1 - \pi} \text{ and } (1 - \pi) \left(\frac{G - r}{1 - \beta} \right) > \pi (B - G) \quad (\text{A2}')$$

Assumption (A1') is a translation of Assumption (A1) to the conditions in period \hat{t} . Assumption (A2') makes sure that the market stays open in period \hat{t} for firms that have not defaulted previously.

We have:

Corollary 6 *Interest rates paid by younger (and hence, smaller) firms are more affected in absolute terms by the shock to the risk-free interest rate.*

Shocks to the risk-free interest rate raise interest rates paid by different firms in the same proportion. Since smaller firms pay higher interest rates, the rates they pay also suffer larger movements.

Corollary 7 *A bad shock to the risk-free interest rate (a positive ϵ) results in tighter borrowing limits. Borrowing limits can become binding for some firms that would not be finance constrained in the absence of the shock. Conversely, a good shock to the risk-free interest rate (a negative ϵ) results in less stringent borrowing limits. Borrowing limits can stop being binding for firms that would be finance constrained in the absence of the shock.*

Since borrowing limits depend on the interest rate paid by a firm, an increase in r will result in more restrictive borrowing limits. A noticeable difference with Proposition 1 is that it is possible for a firm that faces no borrowing constraints at time $\hat{t} - 1$ to face them at time \hat{t} . A bad shock at time \hat{t} may have raised the level of the interest rate paid for that firm to the extent that borrowing limits have to be reimposed. When fluctuations in the riskless interest rate are considered, then, borrowing limits have an additional role. In bad times, they serve to make sure the firm has no incentive to undertake risky projects.

In spite of the apparent symmetry implied by Corollary 6, good and bad shock do not necessarily have symmetric effects. Let us define $\bar{\epsilon}$ by

$$(1 - \pi) \left(G - \frac{r + \bar{\epsilon}}{\rho + \pi(1 - \rho)} + \sum_{t=\hat{t}+1}^{\infty} \beta^t (G - R_t^{\hat{t}}) \right) = \pi (B - G)$$

and $\bar{\bar{\epsilon}}$ by

$$(1 - \pi) \left(\frac{G - r}{1 - \beta} - \bar{\bar{\epsilon}} \right) = \pi (B - G).$$

Corollary 8 *If $\epsilon > \bar{\epsilon}$, some firms remain constrained in period \hat{t} . If $\epsilon > \bar{\bar{\epsilon}}$, borrowing constraints are imposed upon all firms in period \hat{t} .*

Depending on the sizes of positive and negative shocks, it is possible that during good times only some firms are freed from borrowing constraints; while during bad

times borrowing limits may be imposed upon all firms. This suggests an explanation of why the response of small firms to shocks to the interest rate might be disproportionate during bad times, if all firms in the model are considered to be “small.”

5 Final remarks

We have presented a model of reputation acquisition and borrowing constraints to new firms. Borrowing limits emerge in equilibrium as a way to provide firms with a poor reputation with incentives to repay their debts: Borrowing limits are relaxed over time, increasing the value for firms of being able to borrow in the future and thus discouraging them from undertaking projects with some risk of default in the present. As reputation acquisition requires time, newer firms have a poorer reputation and are more credit constrained. When the opportunity cost of lending increases, firms with poorer reputations suffer the most from higher interest rates. Bad shocks can lead to the imposition of borrowing limits to all firms in the model.

The model is useful to understand some observed features of lending to small firms, such as the fact that they tend to pay higher interest rates, are more likely to be finance constrained, and suffer the most from periods of tight money. Newer firms are smaller in the model in the sense that borrowing limits force them to operate at a lower scale. All firms in the model are “small” in the sense that they rely on their reputation to access the credit market. Introducing investment in the firm’s capacity could allow to consider more closely the issue of firm’s growth. After some threshold, it could become profitable for lenders to monitor firm’s project choices, so that adverse selection and moral hazard considerations would lose some relevance. We leave open the possibility of extending the model along those lines.

Although the model is highly-stylized, the results are likely to be robust to changes in some of the specifications. In particular, universal risk neutrality and linear technologies allow to obtain a neat separation between the determination of interest rates

and that of borrowing limits. Abandoning risk neutrality and linear technologies would only make more difficult to disentangle the effects of adverse selection on interest rates and of moral hazard on borrowing limits, without adding much insight. It is also interesting to ask what happens if a richer menu of investment projects is allowed; for instance, what happens when the project with the highest mean return involves some risk. In this case, it seems likely to have an equilibrium with excusable default. In such an equilibrium, the probability of each firm being a normal borrower (its credit rating) will be updated according to its past record of defaults; in turn, credit ratings will affect the interest rates that different borrowers can offer to lenders. Too low a credit rating will lead to a permanent exclusion from the credit market. The difference with the model discussed in this paper is that a normal borrower could end up being excluded from credit with positive probability.⁷

Finally, it is left open the empirical matter of the relevance of firm's reputation vis-a-vis other explanations of the peculiarities of small firms' access to credit markets.

⁷Eaton (1990) offers a similar result in a two-period model of international debt.

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APPENDIX

Proof of Proposition 1:

The equilibrium path is the result of maximizing $\sum_{t=0}^{\infty} \beta^t L_t (G - R_t)$, that is, the utility of a normal borrower that chooses safe projects every period, with respect to $\{L_t, R_t\}_{t=0}^{\infty}$ under the constraints:

(i) $R_t \leq G$, $L_t \leq 1$ (normal borrowers are able to repay when undertaking safe projects),

(ii) $r \leq R_t (p_t + \pi(1 - p_t))$ (it is advantageous for lenders to lend to firms that have never defaulted, given that bad borrowers default with probability $1 - \pi$ and normal borrowers do no default),

(iii) $\sum_{\tau=t}^{\infty} \beta^{\tau-t} L_{\tau} (G - R_{\tau}) \geq \pi (L_t (B - R_t) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} L_{\tau} (G - R_{\tau}))$ (normal borrowers prefer to select a normal project rather than a risky project in any given period under the threat of being excluded from future borrowing in case of default), and

(iv) $p_{t+1} = \frac{p_t}{p_t + \pi(1 - p_t)}$ with $p_0 = \rho$ (reputation of firms which have never defaulted is updated according to Bayes' rule, taking into account that bad borrowers default with probability $1 - \pi$ and normal borrowers do no default).

Increasing R_t does not help to relax the constraint (iii), so R_t is given by the minimum consistent with constraint (ii), as in equation (2). Notice from equations (1) and (2) that R_t approaches r from above.

With respect to L_t , if $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau}) \geq \pi (B - G)$, then the constraint (iii) is satisfied by $L_{\tau} = 1$ for $\tau = t, t + 1, \dots$ which clearly maximizes $\sum_{t=0}^{\infty} \beta^t L_t (G - R_t)$ given $L_{\tau} \leq 1$ (Constraint (i)). Given that R_t approaches r from above and $(1 - \pi) (G - r) / (1 - \beta) > \pi (B - G)$ (from assumption (A2)), there is a time \tilde{t} such that $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau}) \geq \pi (B - G)$ from \tilde{t} on. Then, L_{τ} takes the value of 1 for $\tau = \tilde{t}, \tilde{t} + 1, \dots$

For $t < \tilde{t}$, the constraint (iii) is binding. From (iii) we get:

$$L_t = \frac{1}{\pi(B - G) - (1 - \pi)(G - R_t)} \times (1 - \pi) \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} L_{\tau} (G - R_{\tau}) \quad (\text{I})$$

Using (I) to calculate L_{t+1} and substituting back in L_t we get for $t < \tilde{t} - 1$:

$$L_t = \frac{1}{\pi(B - G) - (1 - \pi)(G - R_t)} \times \beta \pi (B - G) \times L_{t+1} \quad (\text{II})$$

From (I) and $L_{\tau} = 1$ for $\tau = \tilde{t}, \tilde{t} + 1, \dots$ we get for $t = \tilde{t} - 1$:

$$L_{\tilde{t}-1} = \frac{1}{\pi(B - G) - (1 - \pi)(G - R_{\tilde{t}-1})} \times \beta (1 - \pi) \sum_{\tau=\tilde{t}}^{\infty} \beta^{\tau-\tilde{t}} (G - R_{\tau}) \quad (\text{III})$$

Using (III) and (II) recursively we get equation (3). \square

Proof of Corollary 1:

The evolution of R_t follows directly from equations (1) and (2).

For $t < \tilde{t} - 1$, from equation (3),

$$\frac{L_{t+1}}{L_t} = \frac{\pi(B - G) - (1 - \pi)(G - R_t)}{\beta \pi (B - G)}$$

But, at $t < \tilde{t} - 1$, $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau}) < \pi(B - G)$, which, since the interest rate declines over time, implies $\frac{1-\pi}{1-\beta} (G - R_t) < \pi(B - G)$. This, in turn, implies $L_{t+1}/L_t > 1$ for $t < \tilde{t} - 1$.

Similarly, at $t = \tilde{t} - 1$, $(1 - \pi) \sum_{\tau=\tilde{t}-1}^{\infty} \beta^{\tau-(\tilde{t}-1)} (G - R_{\tau}) < \pi(B - G)$, which implies $(1 - \pi)(G - R_{\tilde{t}-1}) + \beta(1 - \pi) \sum_{\tau=\tilde{t}}^{\infty} \beta^{\tau-\tilde{t}} (G - R_{\tau}) < \pi(B - G)$. Using (III), this implies $L_{\tilde{t}-1} < 1$. Since L_t takes the value of 1 from \tilde{t} onwards, $\frac{L_{\tilde{t}}}{L_{\tilde{t}-1}} > 1$. \square

Proof of Corollary 2:

There can be no lending if $\rho < (r/G - \pi)/(1 - \pi)$ because then there is no interest rate that can satisfy simultaneously constraints (i) and (ii) as defined in the Proof of Proposition 1.

Suppose that $(1 - \pi)(G - r)/(1 - \beta) < \pi(B - G)$ and there is some lending at period t . From these assumptions we have that $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_{\tau}) <$

$\pi(B - G)$ for all t so that constraint (iii) is always binding. Hence, expression (II) is valid for all t . From constraint (i) we know that $L_{t+n} \leq 1$ for all t and for n arbitrarily large. Then, using expression (II) we can get:

$$L_t < \prod_{\tau=t}^{t+n-1} \frac{\beta\pi(B - G)}{\pi(B - G) - (1 - \pi)(G - R_\tau)} \quad (\text{IV})$$

From $(1 - \pi)(G - r) / (1 - \beta) < \pi(B - G)$ we have that $(1 - \pi)(G - r) / (1 - \beta) = \pi(B - G) + k$ for some $k > 0$. Then, $(1 - \pi)(G - R_\tau) / (1 - \beta) < \pi(B - G) + k$ for all τ and so the quotient in expression (IV) is uniformly bounded away from one. Then, as n grows large, the RHS must converge to zero and L_t must be zero. \square

Proof of Corollary 3:

From equation (3), as β increases L_t increases.

Also, as β increases, $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_\tau)$ increases for all t but $\pi(B - G)$ remains constant, so it can be the case that the first period at which the expression $(1 - \pi) \sum_{\tau=t}^{\infty} \beta^{\tau-t} (G - R_\tau)$ becomes greater than $\pi(B - G)$ comes sooner. \square

Proof of Corollary 4:

From Corollary 2, we have that $t' < t''$ implies $R_{t-t'} < R_{t-t''}$. Using $R_t^{t'} = R_{t-t'}$ and $R_t^{t''} = R_{t-t''}$ we get $R_t^{t'} < R_t^{t''}$ for all $t \geq t''$. \square

Proof of Corollary 5:

After the shock to the riskless interest rate is considered, credit terms for each generation are obtained by solving a problem entirely similar to the one described in the Proof of Proposition 1, with the exception of constraint (ii) that at time \hat{t} becomes $r + \epsilon \leq R_{\hat{t}}^{t'} (p_{\hat{t}}^{t'} + \pi(1 - p_{\hat{t}}^{t'}))$.

The interest rate paid at time \hat{t} by a generation born at time $t' \leq \hat{t}$ becomes $R_{\hat{t}}^{t'} = \frac{r + \epsilon}{p_{\hat{t}}^{t'} + \pi(1 - p_{\hat{t}}^{t'})}$. Hence, $\partial R_{\hat{t}}^{t'} / \partial \epsilon = (p_{\hat{t}}^{t'} + \pi(1 - p_{\hat{t}}^{t'}))^{-1}$. We also have that $t' < t''$ implies $p_{\hat{t}}^{t'} > p_{\hat{t}}^{t''}$ and $\partial R_{\hat{t}}^{t'} / \partial \epsilon < \partial R_{\hat{t}}^{t''} / \partial \epsilon$. (Notice also that $\partial (R_{\hat{t}}^{t'} / R_{\hat{t}}^{t''}) / \partial \epsilon = 0$). \square

Proof of Corollary 6:

As mentioned in the previous Proof, after the shock to the riskless interest rate is considered, credit terms for each generation are obtained by solving a problem entirely similar to the one described in the Proof of Proposition 1, with the exception of constraint (ii) at time \hat{t} . It is still the case that, if $(1 - \pi) \sum_{\tau=\hat{t}}^{\infty} \beta^{\tau-t} (G - R_{\tau}^{t'}) \geq \pi (B - G)$, $L_{\hat{t}}^{t'} = 1$. Otherwise, firms ask for loans given by:

$$L_{\hat{t}}^{t'} = \frac{(\beta\pi (B - G))^{\hat{t}-\hat{t}-1}}{\prod_{\tau'=\hat{t}}^{\hat{t}-1} (\pi (B - G) - (1 - \pi) (G - R_{\tau'}^{t'}))} \times \beta (1 - \pi) \sum_{\tau=\hat{t}}^{\infty} \beta^{\tau-\hat{t}} (G - R_{\tau}^{t'})$$

Since $R_{\hat{t}}^{t'} = \frac{r+\epsilon}{p_{\hat{t}}^{t'} + \pi(1-p_{\hat{t}}^{t'})}$, a bad shock increases $R_{\hat{t}}^{t'}$. This means that a bad shock decreases $(1 - \pi) \sum_{\tau=\hat{t}}^{\infty} \beta^{\tau-t} (G - R_{\tau}^{t'})$ (making more likely for a firm to be credit constrained) and increases the denominator in the expression for $L_{\hat{t}}^{t'}$ (leading to more stringent borrowing constraints). A good shock has the opposite effects. \square

Proof of Corollary 7:

If $(1 - \pi) \left(\frac{G-r}{1-\beta} - \epsilon \right) < \pi (B - G)$, then $(1 - \pi) \sum_{\tau=\hat{t}}^{\infty} \beta^{\tau-t} (G - R_{\tau}^{t'}) < \pi (B - G)$ for all possible values of $R_{\tau}^{t'}$, so that borrowing constraints are imposed upon all firms after a shock of size ϵ .

Similarly, if $(1 - \pi) \left(G - \frac{r+\epsilon}{\rho+\pi(1-\rho)} + \sum_{t=\hat{t}+1}^{\infty} \beta^t (G - R_t^{\hat{t}}) \right) < \pi (B - G)$, using $R_{\hat{t}}^{\hat{t}} = \frac{r+\epsilon}{\rho+\pi(1-\rho)}$ we get that firms born at time \hat{t} remain constrained after the shock. \square

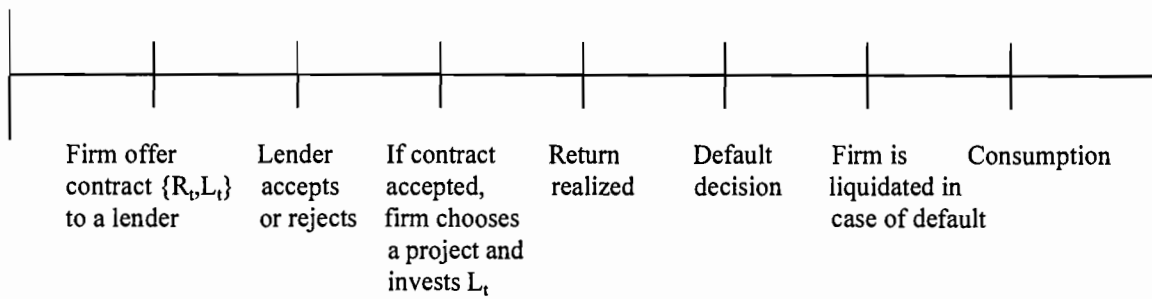


FIGURE 1
WITHIN-PERIOD TIMING