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CAPACITY UTILIZATION DYNAMICS AND MARKET POWER

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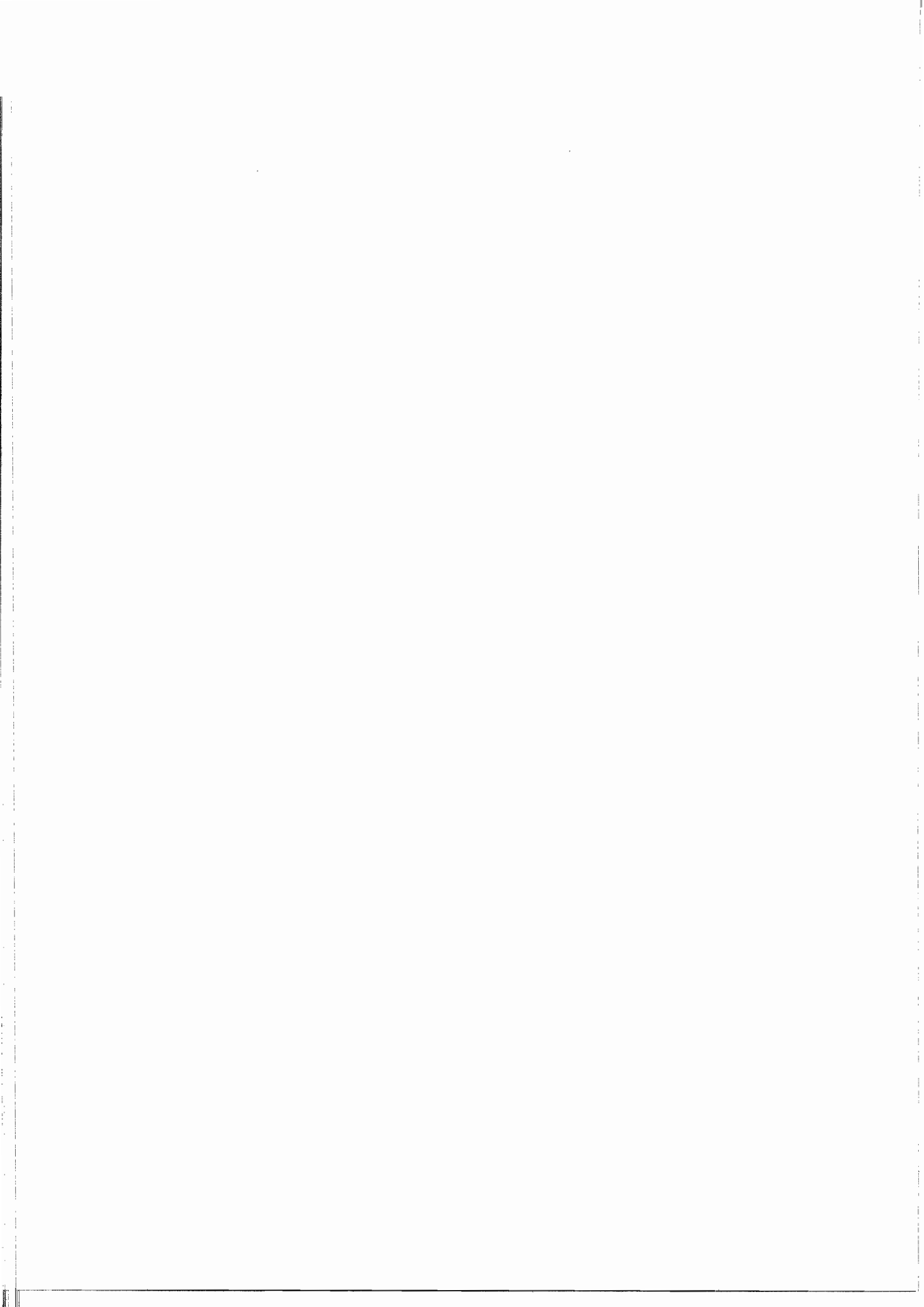
Abstract

In an intertemporal general equilibrium model with imperfect competition, we settle a relationship between factor utilization and markups, via the effect of capacity utilization rate changes on firms' market power when the demand for goods is uncertain. When competition is imperfect, the existence of capacity constraints introduces a distinction between demand and sales price elasticities. At given demand price elasticity, the price elasticity of sales will be smaller the larger the aggregate capacity utilization rate. In such a framework, capacity utilization affects the propagation mechanism of exogenous disturbances in two ways. The first effect is similar to the effect that bottlenecks and stockouts would have in a perfectly competitive setup; the second effect is related to imperfect competition and works through market power and optimal markup changes. We study these interactions and their implications for the dynamic behavior of some key macro variables in response to various "structural" changes. We show that the same shock can have quite different short run effects depending on the characteristics of the initial stationary state (low or high capacity utilization rate).

Key Words

Business Cycles, Capacity Utilization, Market Power, Markup Rate

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1. Introduction

The analysis of the determinants of factor utilization and markup rate variations is currently at the heart of several research programs in macroeconomics. It is now recognized that factor utilization and markup rate variations may play a crucial role in explaining business cycle fluctuations, either by contributing to generate endogenous fluctuations¹ or by complexifying the economic mechanisms that propagate through the economy the effects of exogenous shocks. Our contribution is based on this latter point of view. The literature on propagation mechanisms has usually discussed the effects of factor utilization and of markup rate changes separately. Factor utilization rate changes have received much attention in purely competitive real business cycle models, where they contaminate the so-called Solow residual and make it an inadequate measure of exogenous technological shocks². Business cycle models with imperfect competition have instead emphasized the effects of markup rate variations, especially counter-cyclical variations generated by exogenous demand shocks³. We emphasize in this paper the relationship between the two phenomena (factor utilization and markups), via the effect of capacity utilization rate changes on firms' market power when the demand for goods is uncertain.

In most existing intratemporal models with variable capacity utilization, the utilization rate variable is introduced in the firm's intertemporal decision problem via its impact on capital depreciation⁴. In this setup, the equilibrium capacity utilization rate is such that the marginal revenue that could be obtained from a more intensive capacity utilization and additional sales is exactly compensated by the marginal cost coming from faster capital depreciation. Although there may be such a link between capital utilization intensity and physical depreciation, we believe that the latter is unlikely to be the main driving force behind actual capacity utilization rate fluctuations, as reported in business surveys. For this reason, we chose to rely on a different representation, wherein capacity utilization rate changes follow from microeconomic demand uncertainty and (temporary) technological rigidities (which imply that firms prefer capacity underutilization to costly capital-labour ratio changes). Firms may then be either sales- or capacity-constrained, the proportion of firms in each case influencing the aggregate capacity utilization rate. In an imperfectly competitive setup, capacity constraints introduce a distinction between demand and sales price elasticities and for this reason influence optimal markup values. At given demand price elasticity, the price elasticity of sales will be smaller the larger the aggregate capacity utilization rate. In other words, the fact that some firms are producing at full capacity and unable to serve any extra demand increases the market power of firms with spare productive capacities. In such a setup, capacity constraints affect the propagation mechanism in two

¹See for instance the model developed in d'Aspremont, Dos Santos, Gerard-Varet (1994a), where the variability of markup rates leads to the possibility of endogenous business cycles. In de la Croix-Licandro (1994), factor underutilization may be a source of endogenous fluctuations.

²On labour hoarding, see e.g. Burnside, Eichenbaum and Rebelo, (1993)). On capacity utilization, see e.g. Greenwood, Hercowitz and Huffman (1988), Burnside and Eichenbaum (1994)) or Cooley, Hansen and Prescott (1994)).

³See a.o. Rotemberg-Woodford (1991), (1992) and (1993), Gali (1994).

⁴An exception is Cooley et alii (1994) who propose a modelisation close to ours.

ways. The first effect is similar to the effect that bottlenecks and stockouts would have in a perfectly competitive setup; the second effect is related to imperfect competition and works through market power and optimal markup changes.

Constructing a model with such features in an intertemporal setup raises different difficulties. First, we do want a model wherein the diversity of situations across firms is recognized and wherein the underutilization of capital at the macro level does not imply identical utilization rates at the micro level. Second, if capacity constraints can be binding, we must specify a rationing scheme, obtain effective demands and supplies and take into account spillover effects. It is of course impossible to tackle all these issues at once within a full-fledged stochastic model. Our primary objective is to build a model which remains tractable and at the same time flexible enough to be later extended. Our formulation is based on earlier work on macroeconomic models with capacity constraints and imperfectly competitive price setting (see Sneessens (1987), and subsequent work by a.o. Licandro (1992), de la Croix and Fagnart (1993))⁵.

The key elements of our stylized intertemporal model can be briefly summarized as follows. The model relies on a particular market organisation with two sectors, the intermediate and the final goods sectors respectively. In the intermediate goods sector, the firm uses the primary inputs to produce the intermediate goods. These intermediate goods are then sold to firms in the final goods sector, which combine them to produce a final good that can be either consumed or invested⁶. We assume perfect competition on the final goods market, and monopolistic competition on the intermediate goods one where product differentiation gives some monopoly power to the individual firm. The latter announces her selling price on the basis of (rational) expectations, before knowing the exact value of the demand for her production. This structure implies that an intermediate goods firm can be either sales- or capacity-constrained; it also allows different firms to face different quantity constraints. At the aggregate level, the productive capacity of the economy will be systematically underutilised. It is worth noting that the price pre-setting assumption is not necessary for aggregate capacity underutilization. As we will show, in an imperfect competition setup, it suffices that the productive capacity be pre-determined. However, the price pre-setting assumption is not unrealistic and has the advantage of giving us a symmetric equilibrium in prices.

The rest of the paper is organized as follows. Section 2 is devoted to the description of individual behaviours. Section 3 characterizes the general equilibrium of this economy and analyzes the properties of its stationary state. Section 4 illustrates the dynamic properties of the model with numerical examples. Section 5 concludes.

⁵These models have been used extensively to study the causes of unemployment persistence in Europe (see a.o. Sneessens-Drèze (1986) and Drèze-Bean (1990)). Our objective in this paper is to show that they also provide an interesting framework for a better understanding of the determinants of business cycle fluctuations.

⁶As in Romer (1987), we interpret an aggregator index à la "Dixit-Stiglitz" as the production function used by the firms in the final sector. In the present model, it may also be interesting to consider the final good producing firms as a distribution network that makes the (differentiated) goods available to final consumers under the form of a basket of goods, the precise composition -and price- of which may depend on the availability of the basic ingredients. As a consequence, consumers do not feel quantity constrained since the distribution firms keep on proposing a basket of goods which satisfies their needs.

2. Behaviours

2.1 Consumers

We suppose an infinitely lived consumer whose intertemporal preferences with respect to consumption and labour are represented by a time separable utility function. The optimal time profiles of consumptions $\{C_t\}_{t=1,\dots,\infty}$ and labour supplies $\{L_t\}_{t=1,\dots,\infty}$ are given by the solution of the following problem:

$$\max_{\{C_t, L_t\}} \sum_{t=1}^{\infty} \beta^t \mathcal{U}(C_t, L_t)$$

subject to the wealth constraint

$$A_t = A_{t-1} (1 + r_t) + w_t L_t - C_t - T_t \quad \forall t. \quad (1)$$

where β is a constant subjective discount rate; A_t and T_t denote respectively the real financial wealth -with the price of consumption as numeraire- and a lump-sum tax. Function \mathcal{U} is strictly increasing in C_t , decreasing in L_t , twice differentiable and concave. There is no aggregate uncertainty in the model and the sequence of equilibrium real wages and rates of return $\{w_t, r_t\}_{t=1,\dots,\infty}$ is perfectly forecast by the consumer. A_0 is given.

Optimal consumption demand and labour supply profiles must satisfy the following two first-order conditions at any time $t = 1, \dots, \infty$

$$U_1(C_t, L_t) = U_1(C_{t+1}, L_{t+1}) \beta (1 + r_{t+1}) \quad (2)$$

$$w_t = - \frac{U_2(C_t, L_t)}{U_1(C_t, L_t)}. \quad (3)$$

The transversality condition on wealth must also be satisfied:

$$\lim_{t \rightarrow \infty} \beta^t U_1(C_t, L_t) A_t = 0$$

The interpretation of these standard conditions is fairly straightforward.

2.2 Product firms

The homogeneous final good is produced by the so-called product firms. It is sold on a competitive market, and can be used either as a consumption or an investment good. All firms use the same production technology. There is no fixed input, which implies that the optimization programme of firms remains purely static. We have however to take into account the effect of input supply quantity constraints.

The production technology of each product firm is represented by a constant return to scale CES production function defined over a continuum of variable inputs, each one denoted by y and indexed by j , with j belonging to the interval $[0, 1]$. More formally, the representative product firm's output (denoted Y) is obtained from the following production function:

$$Y_t = \left[\int_0^1 v_t^j \frac{1}{\theta} y_t^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{with } \theta > 1 \quad (4)$$

Each parameter $v_t^j \geq 0$ is drawn from an i.i.d. stochastic process with unit mean and distribution function $F(v)$. The total supply of input j is limited to an amount $b_t k_{t-1}^j$ (see below), equal to the predetermined productive capacity of the corresponding input supplying firm, i.e., the available capital stock times its (here non stochastic) productivity. If we assume a uniform non-stochastic rationing scheme and use the final good as numeraire, we can write the optimization programme of the representative firm as:

$$\max_{\{y_t^j\}} Y_t - \int_0^1 p_t^j y_t^j dj$$

subject to the supply constraints

$$y_t^j \leq b_t k_{t-1}^j \quad \forall j,$$

with Y_t defined in (4). It is well-known⁷ that with deterministic quantity constraints and a uniform rationing scheme, effective demands are not well-defined, although realized transactions are. The latter can easily be shown to be

$$y_t^j = \begin{cases} (p_t^j)^{-\theta} Y_t v_t^j & \text{if } v_t^j \leq \bar{v}_t^j \\ b_t k_{t-1}^j & \text{if } v_t^j > \bar{v}_t^j \end{cases} \quad (5)$$

where the critical value of the productivity parameter, \bar{v}_t^j , is such that the demand for input j at price p_t^j is equal to the productive capacity of supplier j , i.e.,

$$\bar{v}_t^j = \frac{b_t k_{t-1}^j}{(p_t^j)^{-\theta} Y_t}, \quad \forall j. \quad (6)$$

For notational convenience, let us define d_t^j as follows:

$$d_t^j = (p_t^j)^{-\theta} Y_t v_t^j,$$

that is, d_t^j represents the demand for input j that would be expressed to firm j if there were no shortage of that input⁸.

Because all the product firms are identical and face the same technological and quantity constraints, we shall in the sequel use the same symbol Y to represent both the individual and the aggregate production of final goods. The aggregate production of final goods can be obtained by substituting (5) into (4). Note that the marginal productivity of a supply-constrained input remains larger than its marginal cost. With constant returns to scale, this implies positive profits despite the perfect competition assumption⁹.

⁷See for instance Green (1980), Svensson (1980).

⁸The variable d_t^j actually corresponds to the so-called Benassy (1975) effective demand concept, as opposed to the Drèze (1975) effective demand concept. The choice between the two concepts is in our case unsequential. It has no impact whatsoever on the determination of input prices nor of any other variable.

⁹Indeed, by using Euler theorem, one obtains $Y_t = \int_0^1 \frac{\partial Y_t}{\partial v_t^j} y_t^j dj > \int_0^1 p_t^j y_t^j dj$.

2.3 Input firms

2.3.1 Individual Firm's Problem

Each input j is produced by a single firm (henceforth input firm). In order to obtain a simple productive capacity concept, we assume that the production of each intermediate good combines labour and capital through a Leontief technology with technical coefficients equal to a_t and b_t respectively. The input firm's productive capacity is predetermined and equal to the capital stock k_{t-1}^j times its productivity b_t . Labour is supposed to be a purely variable input. It is bought on a competitive market and can be adjusted instantaneously to its optimal level. From equation (5), the latter is given by

$$l_t^j = \frac{y_t^j}{a_t} = \frac{1}{a_t} \min \{ d_t^j, b_t k_{t-1}^j \}$$

In other words, as long as the productive capacity is not fully utilised, the firm adjusts her production plan to her customers' orders.

We assume that the price decision has to be taken before the realized values of the stochastic terms v_t^j are known. The exact position of the demand function is thus not known with certainty and the price and investment decisions have to be taken with respect to expected sales, which are given by

$$E(y_t^j) = E(d_t^j) \int_0^{\bar{v}_t^j} v dF(v) + b_t k_{t-1}^j \int_{\bar{v}_t^j}^{\infty} dF(v) \quad (7)$$

$$\text{where } E(d_t^j) = (p_t^j)^{-\theta} Y_t$$

The expression $E(X_t)$ denotes the expected value of the variable X_t before observing the value of the parameter v_t^j . The ex ante optimization problem can then be written as:

$$\max_{\{i_t^j, p_t^j\}} \sum_{t=1}^{\infty} R_t \left\{ p_t^j E(y_t^j) - \frac{w_t}{a_t} E(y_t^j) - i_t^j \right\}$$

subject to

$$k_t^j = (1 - \delta) k_{t-1}^j + i_t^j,$$

where R_t is a real discount factor:

$$R_t = \prod_{i=1}^t (1 + r_i)^{-1}, \quad \forall t > 0.$$

δ is the depreciation rate of capital and \bar{v}_t^j represents that particular value of the input demand shock which makes demand equal to productive capacity (see (6)) with k_0 given.

In every period t , the optimality condition for price¹⁰ can be written as

$$p_t^j = \frac{\theta \pi_{dt}^j}{\theta \pi_{dt}^j - 1} \frac{w_t}{a_t} \quad (8)$$

where the variable π_{dt}^j is defined as

$$\pi_{dt}^j = \frac{E(d_t^j)}{E(y_t^j)} \int_0^{\bar{v}_t^j} v \, dF(v) \quad (9)$$

and can be interpreted the weighted probability of a demand constraint.

The optimality condition for investment is at each period t

$$b_t \frac{p_{t+1}^j}{\theta \pi_{dt+1}^j} \int_{\bar{v}_{t+1}^j}^{\infty} dF(v) = r_{t+1} + \delta. \quad (10)$$

The following transversality condition must also hold:

$$\lim_{t \rightarrow \infty} R_t k_t = 0$$

The presence of π_{dt}^j in equations (8) and (10) makes their interpretation slightly unusual. The optimal pricing rule (8) implies a markup over marginal costs, which is negatively related to the (absolute value of the) price elasticity of expected sales. The latter is equal to the elasticity of expected sales to expected demand (denoted π_{dt}^j) times the price elasticity of expected demand (θ)¹¹. In other words, when the probability of a sales constraint is large, the firm's actual market power is reduced (i.e., the price elasticity of sales is higher), which implies a smaller markup rate, and conversely when the probability of a sales constraint is small. π_{dt}^j is of course endogenous and depends on the firm's price decision (as well as on other variables). The higher the price, the larger the probability of a sales constraint *ceteris paribus*. It is easily seen from (8) that the profit margin can be written as $p_t^j - w_t^j/a_t = p_t^j/(\theta \pi_{dt}^j)$.

The optimal investment decision (10) imposes that investment in every period be such that the marginal revenue expected from an additional unit of capital (left-hand side) be equal to its marginal cost (right-hand side). The expected marginal revenue is equal to the productivity of capital b_t times the profit margin per unit of output, times the probability of using this extra unit of capital.

¹⁰The derivation of this condition supposes that each monopolistic firm only considers the direct effect of her price decision on demand and neglects all the indirect effects (e.g. the effect through Y_t). This approximation is certainly perfectly reasonable in a framework where there is a continuum of firms. For a thorough analysis of this question, see d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1994b) who compare this approximation with the solution obtained when all the indirect effects are taken into account.

¹¹As prices are pre-set, the equilibrium wage rate has not been observed yet at the time prices have to be announced. Since there is no aggregate uncertainty in the model, the equilibrium wage rate can however be perfectly anticipated. See the labour market equilibrium condition below.

2.3.2 Symmetric Equilibrium and Aggregation

At the time they have to take their investment and price decisions, all the input firms j have the same information about individual demand and face exactly the same uncertainty. At a symmetric equilibrium, they will thus all choose the same capital stock and price level:

$$\forall j \in [0, 1], \quad k_t^j = k_t \quad \text{and} \quad p_t^j = p_t < 1$$

Note that p is always lower than 1. Because the marginal productivity of the constrained inputs is larger than their price, p_t is always lower than the price of final output P_t (normalized to 1) which is also the shadow price index for intermediate inputs¹².

With all prices identical, aggregate employment, denoted L_t , is equal (up to a scaling factor) to individual expected employment levels. It is determined by:

$$L_t = \frac{1}{a_t} \left\{ (p_t)^{-\theta} Y_t \int_0^{\bar{v}_t} v \, dF(v) + b_t K_{t-1} \int_{\bar{v}_t}^{\infty} dF(v) \right\} \quad (11)$$

where K_{t-1} stands for the aggregate capital stock installed at time $t-1$ and available at time t , and

$$\bar{v}_t = \frac{b_t K_{t-1}}{(p_t)^{-\theta} Y_t} \quad (12)$$

represents the ratio of productive capacity to expected demand for intermediate inputs¹³.

3. General Equilibrium

3.1 Intertemporal Equilibrium

For each period t , we may characterize the equilibrium as follows.

¹²Indeed, the shadow price index for intermediate inputs must be computed by using the marginal productivities of inputs in the production of final output. Without binding supply constraints, marginal productivities are equal to the observed prices at equilibrium. In this model, the marginal productivity of each supply constrained inputs remains larger than its price. At a symmetric equilibrium in prices, the shadow price index is therefore

$$1 = P_t = \left[\int_0^1 \left(\frac{\partial Y_t}{\partial y_t^j} \right)^{1-\theta} v_t^j \, dj \right]^{\frac{1}{1-\theta}} > \left[\int_0^1 (p_t)^{1-\theta} v_t^j \, dj \right]^{\frac{1}{1-\theta}} = p_t$$

The difference between shadow and market prices explains why the term $p^{-\theta} (> 1)$ remains in the unconstrained demand function for input j ($d_t^j = p_t^{-\theta} Y_t v_t^j$) even though the equilibrium is symmetric in prices. This term measures in fact the spillover effects coming from the constraints on the intermediate inputs produced by the input firms that are at full capacity. Obviously, the larger the number of constrained inputs, the more p_t departs from its upper bound 1 and the larger the spillover effect $p^{-\theta}$. In the limit case where all input firms would underuse their productive capacity at equilibrium ($P_t = p_t = 1$), there would be no spillover effect. At a symmetric equilibrium in prices, each d_t^j would then be equal to $Y_t v_t^j$.

¹³The definition of \bar{v} gives us another opportunity to outline the effect of binding capacity constraints. Each firm that underuses its productive capacity satisfies a demand increased by the spillover effect $p^{-\theta}$. It is obvious that the critical value \bar{v} above which firms use fully their capacity has to take such an effect into account.

a) The **labour market** equilibrium is characterized by equations (3) for supply and (11) and (12) for demand.

b) The **intermediate input market** equilibrium is, from equation (8), characterized by

$$p_t = \frac{\theta \pi_{dt}}{\theta \pi_{dt} - 1} \frac{w_t}{a_t} \quad \text{with} \quad \pi_{dt} = p^{-\theta} \frac{Y_t}{a_t L_t} \int_0^{\bar{v}_t} v \, dF(v) \quad (13)$$

The aggregate value of π_{dt} is a weighted measure of the proportion of firms for which demand is smaller than the productive capacity¹⁴.

c) The **product market** equilibrium is characterized by:

$$Y_t = C_t + K_t - (1 - \delta) K_{t-1} + G_t \quad (14)$$

where public consumption G_t is exogenous and non-stochastic. The optimal consumption level is derived from equation (2); the optimal investment rule is, from equation (10), given by

$$r_{t+1} + \delta = \frac{p_{t+1}}{\theta \pi_{dt+1}} b_{t+1} \int_{\bar{v}_{t+1}}^{\infty} dF(v). \quad (15)$$

By substituting (5) into (4) and taking into account the equality of all input prices, the aggregate production of final goods can be written as:

$$Y_t = \left\{ \left[(p_t)^{-\theta} Y_t \right]^{\frac{\theta-1}{\theta}} \left[\int_0^{\bar{v}_t} v_t \, dF(v) \right] + (b_t K_{t-1})^{\frac{\theta-1}{\theta}} \left[\int_{\bar{v}_t}^{\infty} (v_t)^{\frac{1}{\theta}} \, dF(v) \right] \right\}^{\frac{\theta}{\theta-1}}. \quad (16)$$

d) **Financial markets**

Product and input firms are owned by consumers. Both types of firms make positive profits at equilibrium; the corresponding shares have thus positive market values. At equilibrium, these shares prices must satisfy the arbitrage condition (expected rates of return equal to the riskless market interest rate). Because share prices do not appear in the previous expressions, the financial market equilibrium conditions will not be presented here and are presented in appendix 1.

e) A **dynamic general equilibrium** for this economy is a sequence of prices $\{p_t, w_t, r_t\}_t$ and quantities $\{C_t, L_t, Y_t, K_t\}_t$ such that the above equilibrium conditions are satisfied at each time period t ($t = 1, \dots, \infty$), with K_0 given.

f) **Capacity utilization and markup**

As $\bar{v}_t > 0$, the aggregate productive capacity is underutilized at equilibrium. Let D_t be the capacity utilization rate at the aggregate economy level:

$$D_t = \frac{Y_t}{b_t K_t} \leq 1.$$

The underutilization of productive capacities at the aggregate level cumulates two effects, i.e., the underutilization of capital in some input firms and the productive

¹⁴At equilibrium, $F(\bar{v}_t)$ represents the proportion of firms that underuse their productive capacities (i.e., those for which $v_t^j \in [0, \bar{v}_t]$). The variable π_{dt} weights $F(\bar{v}_t)$ by the relative importance of the production of the latter firms in total production.

inefficiency in product firms (the marginal productivity of supply-constrained inputs remains larger than their price¹⁵).

For a given distribution function of $F(v)$, there exists of course a decreasing relationship between the capacity utilization rate D_t and the weighted proportion π_{dt} , which determines the markup rate. Indeed, the aggregate capacity utilization rate is linked directly to the proportion of firms which produce at full capacity. As the latter proportion is equal to $(1 - \pi_{dt})$, the larger D_t , the lower π_{dt} . At given price elasticity of demand, this implies a positive relationship between the capacity utilization and markup rates.

It is also worth stressing that the underutilization of the productive capacity depends more crucially on the assumption of a predetermined capacity and uncertainty than on the assumption of pre-set prices. If prices were set ex-post, each firm's optimal capacity would still imply a positive probability of underutilization. When the demand for its good turns out to be too low, the input firm would never reduce its markup rate below $\theta/(\theta - 1)$; it would instead prefer underutilize its productive capacity. There would however be no input demand rationing and the equilibrium in prices would no longer be symmetric in prices. The crucial assumption is uncertainty. Without uncertainty, the installed productive capacity would always perfectly match the demand for intermediate inputs. With constant returns to scale, all p_t^i would then again be identical.

3.2 Stationary State Equilibrium

At the stationary state, equation (2) determines the stationary interest rate as a function of the subjective time discount rate:

$$r = \frac{1 - \beta}{\beta}. \quad (17)$$

Let us denote by X the following ratio:

$$X = \frac{aL}{bK},$$

We can next rearrange the three equations (11), (15) and (16) (without time subscripts) as follows:

$$X = \frac{1}{\bar{v}} \left[\int_0^{\bar{v}} v dF(v) \right] + \left[\int_{\bar{v}}^{\infty} dF(v) \right] \quad (18)$$

$$X = \frac{\theta(r + \delta)}{bp} \frac{\bar{v}^{-1} \int_0^{\bar{v}} v dF(v)}{\int_{\bar{v}}^{\infty} dF(v)} \quad (19)$$

$$p = \left\{ \left[\int_0^{\bar{v}} v dF(v) \right] + (\bar{v})^{\frac{\theta-1}{\theta}} \left[\int_{\bar{v}}^{\infty} v^{\frac{1}{\theta}} dF(v) \right] \right\}^{\frac{1}{\theta-1}} < 1 \quad (20)$$

Equation (19) is obtained from the optimal investment rule (15), after eliminating π_d with (13). From (20), it is easy to show that p is increasing in \bar{v} . The larger the number

¹⁵Without quantity rationing, the latter inefficiency would vanish.

of constrained inputs (i.e. the lower \bar{v}), the more p departs from its upper bound 1 (see also footnote 11).

With the solutions X , \bar{v} and p of this system, the capacity utilization rate D and the proportion π_d are determined easily by using equations (12) and the definition of π_{dt} in equation (13). From the equation of prices (13), the stationary wage level w becomes:

$$w = p \left(1 - \frac{1}{\theta \pi_d} \right) a \quad (21)$$

The stationary real wage rate is thus lower than the marginal productivity of labour a . The gap between the two is increasing in the proportion of firms working at full capacity. If all the firms underused their productive capacity, π_d and p would be equal to 1 and the real wage rate would be simply equal to $(1 - 1/\theta) a$.

Note that, because of the constant return to scale assumption, the stationary state values of the variables \bar{v} , D , π_d , w do not depend on the consumer's intratemporal preferences \mathcal{U} . Defining the latter next allows the computation of the stationary levels of output Y , employment L , consumption C , the capital stock K . Of course, the fact that the stationary values of D , X , π_d , p and w do not depend on the intratemporal preferences does not prevent the specification of \mathcal{U} to have substantial effects on the dynamics of these variables.

In appendix 2, a table details the effects of the parameters of the model on the stationary values of D , π_d , p and w .

4. Transitional Dynamics and Local Stability

This section has two objectives. The first one is simply to check the local stability of the model¹⁶. The second one is to illustrate the interactions between capacity utilization and markup rate changes by analysing numerically the dynamic behavior of some key macro variables in response to various structural changes in the environment of firms. This will allow us to show that a same shock can have quite different short run effects depending on the characteristics of the initial steady state ("low" or "high" capacity utilization rate).

For this numerical part, we consider two economies which are identical except for the variance σ_v^2 of the idiosyncratic shocks v_t^j . Because of that, the main difference between the stationary equilibria of these two economies is the value of the equilibrium capacity utilisation rate, which is higher the lower¹⁷ the variance σ_v^2 . $F(v)$ is assumed a lognormal distribution function with a variance σ_v^2 equal to 0.01 in the first economy (called hereafter the High D economy) and equal to 0.25 in the second one (called hereafter the Low D economy).

¹⁶To solve this nonlinear model, we use a Newton-Raphson relaxation method proposed by Laffargue (1990) and further developed by Boucekkine (1994). As Boucekkine shows, Laffargue's algorithm allows ones to check very easily the existence of a unique stable saddle-point solution. Indeed, when the latter algorithm is used, non saddlepoint models are ill-conditioned and induce an explosive behavior of the algorithm.

¹⁷Indeed, the larger σ_v^2 , the larger the mismatch between demands and supplies, and consequently the lower D_t at any productive capacity level (see also appendix 2).

For the set of the other parameters of the model, we choose the following calibration. We assume a utility function of the CRRA variety like

$$U(C_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\tau}}{1+\tau} \quad (22)$$

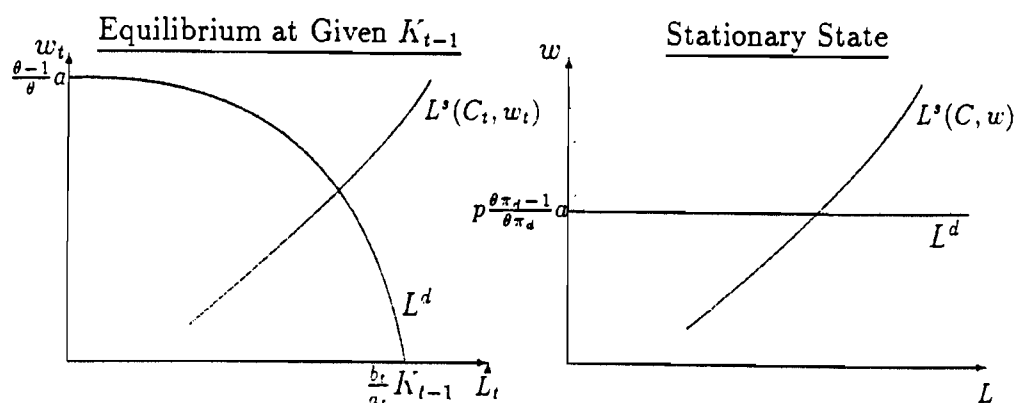
and choose the following values for the other parameters of the model: $a = b = 1$, $\theta = 7$ (which implies a mark-up rate of about 40% if π_d is about 0.5), $\beta = 0.971$ (so that the real interest rate is 3%), $G = 0$, $\delta = 0.07$, $\gamma = 1.5$ and $\tau = 1$.

The values of the most important variables at the stationary state in these two economies are given in the following table:

	D	π_d	w	p	Y	C
High D economy ($\sigma_v^2 = 0.01$)	0.952	57.5%	0.748	0.995	0.932	0.86
Low D economy ($\sigma_v^2 = 0.25$)	0.795	52.6%	0.708	0.972	0.919	0.84

Before presenting the numerical exercises, we propose a diagrammatical representation of the labour market equilibrium at given capital stock and at the stationary state. This diagrammatical apparatus will provide good intuitions on the interactions between capacity utilization and markup variations in the short run. It will thus be very useful to understand why the short run effects of a same shock depends crucially on the value of the capacity utilisation rate when the shock occurs. It will also provide a better understanding of the role played by the parameters γ and τ of the utility function.

Figure 1: Instantaneous and stationary equilibria in the labour market



4.1 A Representation of the Labour Market Equilibrium

In the two panels of figure 1, the upward-sloping curve represents the labour supply curve (see equation (3) with \mathcal{U} given by (22)). The other curve (sloping downwards in figure 1.a. horizontal in figure 1.b) represents the macroeconomic labour demand curve

given by equation (11) (combined of course with equations (12), (13), (16)). In the previous section, the stationary state wage level has been shown to be independent of the employment level, so that the long-term labour demand curve is horizontal (figure 1.b). In the very short run, i.e. at given capital stock, the labour demand curve is sloping downwards and intersects the two axes. It necessarily intersects the horizontal axis because, even at zero wage, the short run demand for labour is bounded above by the number of available work-stations corresponding to the full employment of installed capacities (i.e., from equation (11) in that case, $L_t = b_t K_{t-1} / a_t$). As labour and capital are complementary, higher employment levels imply a higher capacity utilization rate. The capacity utilization rate and other related variables (the proportion π_{dt} , markups) thus vary along the labour demand curve. Conversely, when L goes to zero, all firms underutilise their productive capacity and $\pi_{dt} = 1$. The input price p is then equal to 1 and the feasible real wage is equal to $(1 - 1/\theta) a$ (see (21)). Along the short run labour demand curve, there is thus a negative relationship between the demand elasticity of sales (π_{dt}) and employment: a downwards shift along the short run labour demand curve corresponds to an increase in the proportion of firms that produce at full capacity and therefore to an increase in the markup rate.

4.2 Transitional Dynamics: Numerical Simulations

In this section, we analyse successively the effects of four structural changes: an increase in government spendings G , a labour and capital augmenting technical progress (increase in the productivity coefficients a and b), a labour augmenting technical progress (increase in a), an increase in the elasticity of substitution between goods θ .

In each case, we first present a qualitative analysis of the instantaneous and long run impacts on the basis of the diagrammatical apparatus presented in the previous section. For the two economies (High \bar{D} and Low \bar{D}), we next report the time profiles of the capacity utilization rate (fig. 2-3-4.a), the markup rate (fig. 2-3-4.c), the proportion of firms facing a sales constraint (fig. 2-3-4.e), the real wage rate (fig. 2-3-4.b), the employment (fig. 2-3-4.d) and output (fig. 2-3-4.f) levels. In each case, the vertical axis measures the gap (in %) between the value of the variable at time t and its value at the initial stationary state. The size of the four shocks has been calibrated so as to produce a 1% increase in final output Y at the new stationary state.

a) Permanent Increase in Government Purchases

i) Qualitative Analysis (figure a in Appendix 3)

A (permanent) increase in G leaves the labour demand curves unaffected and only shifts the labour supply curve to the right (negative wealth effect). Both the instantaneous and the steady state effect on output are unambiguously positive.

The instantaneous output increase is achieved thanks to a more intensive use of existing capacities (downwards move along the short run labour demand curve). The instantaneous effect thus depends crucially on the capacity utilization rate at the time of the public spending increase. For instance, if the economy lies in the flat part of the labour demand curve (low utilization rates), firms will respond to the increase in

G by raising production with little change in markup and prices. If, to the contrary, the utilization rate is high when the change occurs (in the steeper part of the curve), firms will raise their markup and there will be little output effect in the short run.

ii) Numerical Simulation (see figure 2)

As expected, the short run effects of the government spending increase are much more expansionary in the Low D economy than in the High D one. The instantaneous output change in the former is even twice larger than in the latter where firms have initially more market power and raise much more their markup.

Following the initial increase in the capacity utilization rate (with a drop in π_d), the transitional dynamics are mainly driven by increased investments. During this phase, the output increase is smaller than the capacity extensions; there is thus a progressive fall in the capacity utilization and in the markup rates. The correlation between output and D_t (and thus between output and markups) is thus negative during this phase.

b) Labour and Capital Augmenting Technical Change

We consider here an increase in the average productivities a and b with a/b constant.

i) Qualitative Analysis (figure b in Appendix 3)

On the left-hand part of figure 1, the productivity gain shifts the labour demand curve upwards (as labour is more productive, any level of employment is now compatible with a higher real wage). However, the maximum employment level is unchanged since, at given capital stock, the simultaneous increase in the productivity of both factors has not changed the number of available work-stations in the economy. The labour supply curve shifts upwards. As a result of these two shifts, the instantaneous effect of the global productivity increase on the capacity utilization rate and the employment level is ambiguous, although final output is clearly stimulated. The net effect on employment can be positive or negative, depending on the value of the intertemporal elasticity of substitution. There are two counteracting forces at work: the increase in productive capacities produced by the change in b (at given K_{t-1}) and the positive wealth effect on the supply of labour. Note however that, in the short run, the expansionary effects of the productivity gain is likely to be larger when there are initially few idle capacities (equilibrium in the steeper part of the curve) than when capital is widely in excess.

In the right-hand part of figure 1, both the long run feasible wage and labour supply curves shift upwards. Consequently, the effects on L and K are again ambiguous even though the stationary level of output is increased. As the increase in b reduces the cost of capital per unit of output, the input firm can choose a more "risky" investment policy, i.e., a policy with a lower expected capacity utilization rate. D is thus slightly lower at the new stationary state.

ii) Numerical Simulation (see figure 3)

With our calibration, we need a productivity gain of more than 1% to produce a 1% increase in the stationary level of final output. This means a lower level of employment and capital at the new stationary equilibrium. In the present case, the capacity utilization variable seems finally to play little role in the transmission of the global productivity gain. In fact, D_t falls. This is because the productivity gain affects

installed capital as well as labour, and increases the productive capacity of firms at once. Hence, the fact that the output increase is initially larger than 1% does not follow from a more intensive use of installed capital but from a reduction in market power associated to a lower capacity utilisation rate. As expected, this effect is larger in the High D economy than in the Low D one.

c) Labour Augmenting Technical Progress *a.*

We now consider an increase in the productivity of labour, at unchanged capital productivity.

i) Qualitative Analysis (figure c in Appendix 3)

In the short run, the increase in the productivity of labour reduces the maximum employment level (less workers are now necessary for a given productive capacity) but increases the maximum wage level. The short run labour demand curve thus moves upwards at low levels of employment and inwards near the maximum employment level. As in b), the labour supply curve shifts upwards. The instantaneous effects of the productivity gain depend crucially on whether the initial employment level is to the left or to the right of the intersection point between the old and the new demand curves, i.e., on whether the initial equilibrium capacity utilization rate is small (flat part of the labour demand curve) or large (steep part of the labour demand curve). In the first case, the productivity gain induces an instantaneous rise in real wages while the effect on employment remains ambiguous and depends on the labour supply behavior. In the second case, whatever \mathcal{U} , the instantaneous effect on employment is negative; the size of the employment contraction determines whether the short run real wage falls or increases.

The stationary levels of output and capital are higher; the stationary effect on employment remains ambiguous.

ii) Numerical Simulation (see figure 3)

The two numerical exercises illustrate the "second" case referred above. The short run employment level falls below its new stationary value in both economies. At given capital productivity, the labour productivity gain leads in the short run to an increase in capacity utilization and in markup rates. But the increase in D_t is larger in the Low D economy than in the High D one whereas markups increase more in the latter than in the former. The employment contraction is thus much larger in the High D economy than in the Low D one. In the short run, the equilibrium real wage has to decrease in the former but can increase immediately in the latter.

It is also worth stressing that a labour productivity gain generates strikingly different dynamic paths and short run effects than a global (labour and capital) productivity gain, although the long run effects are similar. As it appears clearly in figure 3, the instantaneous effects on the capacity utilization rate, the proportion π_{dt} , the markup rate and the real wage go in opposite directions and the time profiles of output and employment are very different.

Figure 2: Government Spending Increase

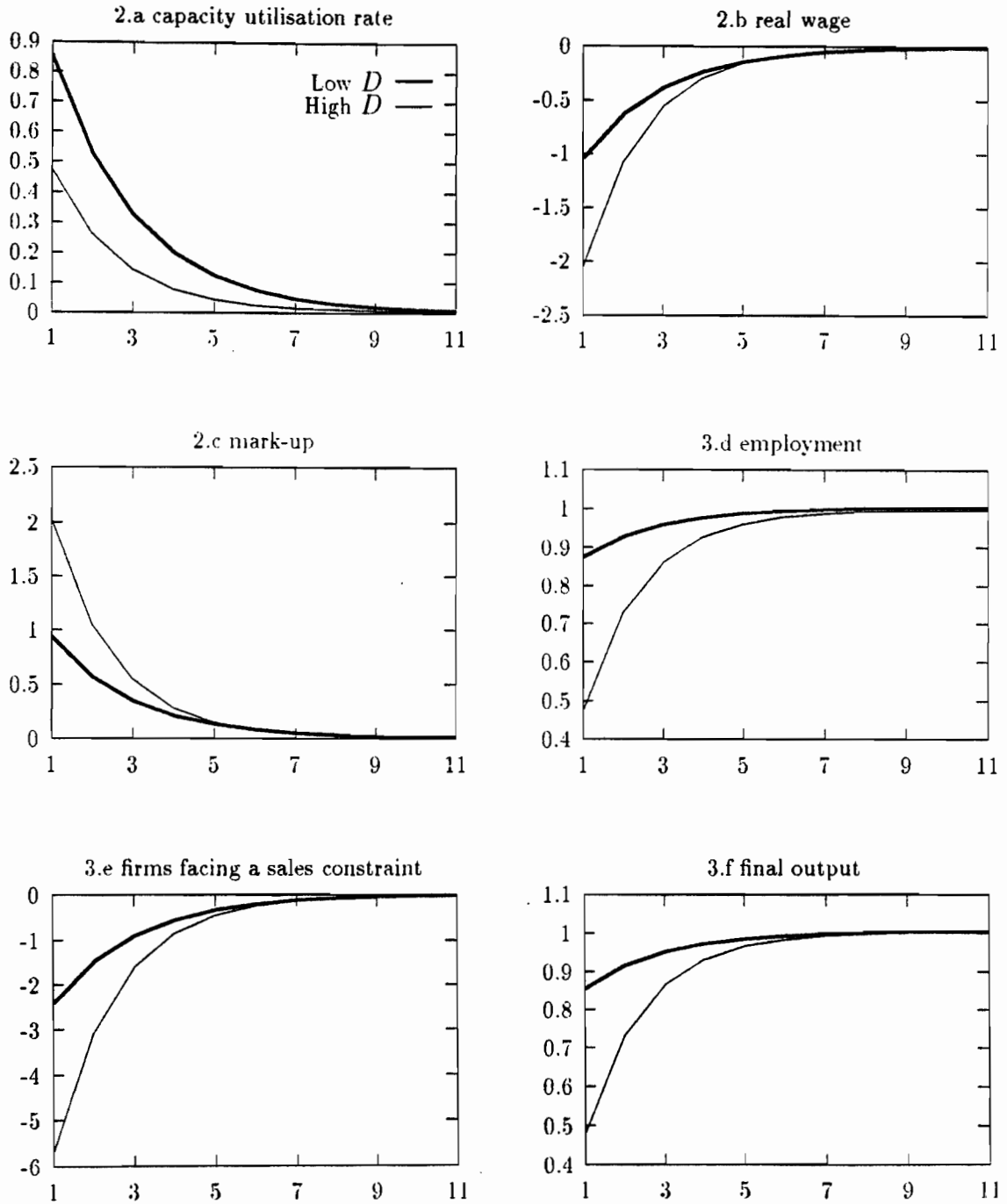
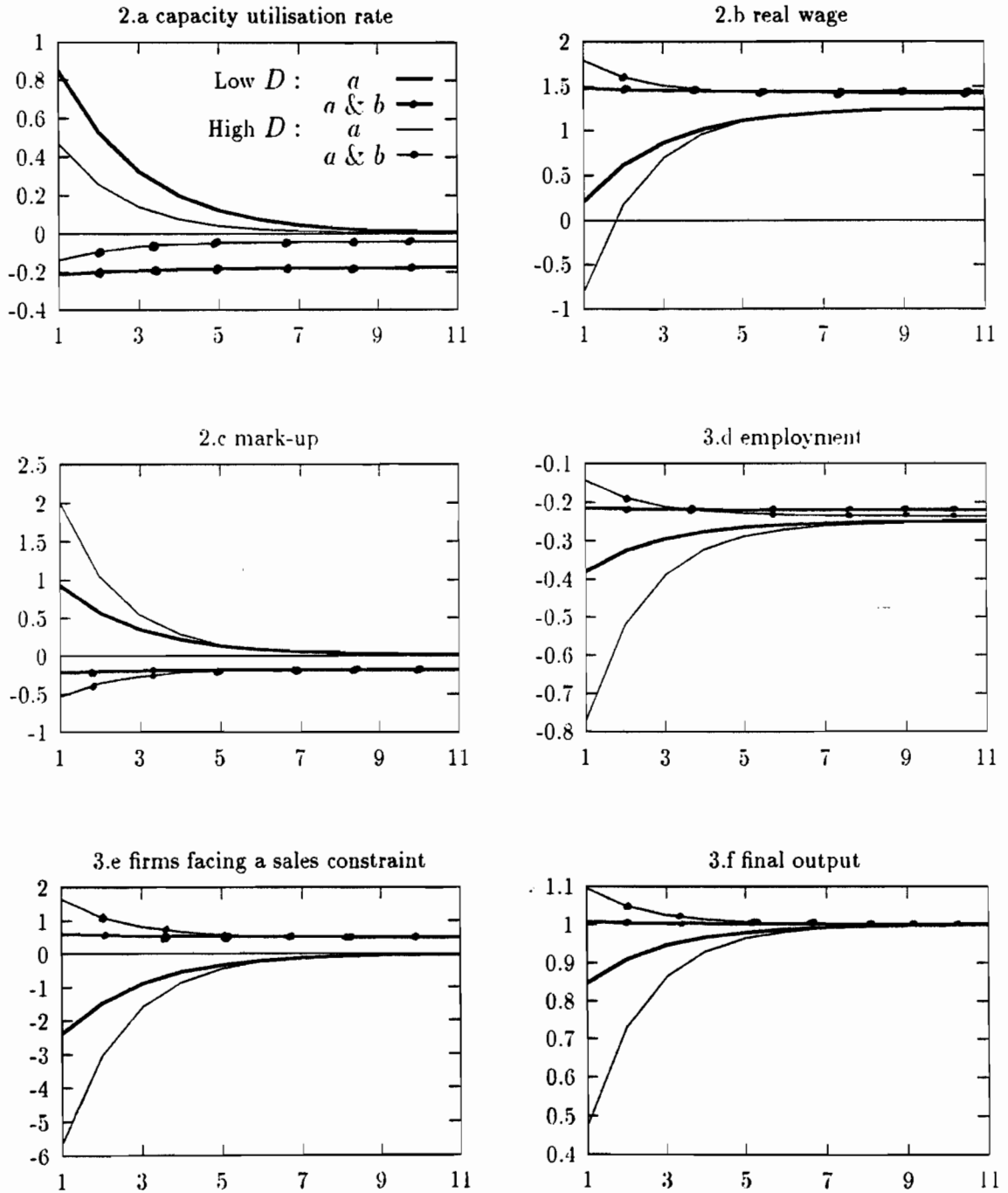


Figure 3: Productivity Increases



d) Increase in the Elasticity of Substitution θ

i) *Qualitative Analysis* (figure d in Appendix 3)

Although the price elasticity of demand is constant in this model, it would not be too difficult to endogenize it¹⁸. Here, we simply assume an exogenous change in θ . This will allow us to assess qualitatively the combined effects of changes in the price elasticity of demand θ and in the demand elasticity of sales π_d , which determine together the markup rate. Changes in the latter simply corresponds to movements along the short run labour demand curve; changes in θ shift the labour demand curve itself.

The increase in θ leads the intermediate firms to lower their margins. Both the short run and the long run labour demand curve now shift upwards, and the real wage comes closer to the marginal productivity of labour a . A positive wealth effect shifts the labour supply schedule upwards. These two shifts are responsible for the same ambiguities as in the global productivity change. Although output is stimulated, the net effect on employment can be positive or negative, depending on the value of the intertemporal elasticity of substitution.

ii) *Numerical Simulation* (see figure 4)

In both economies, the effect on employment and capital is positive. The increase in the price elasticity of demand has two immediate and opposite effects on markups. The first effect (linked directly to the elasticity change) dominates clearly the second one (linked to the higher capacity utilisation rate¹⁹). The markup thus falls below its initial value.

However, when comparing the time profiles of the variables in the two economies, it appears clearly that the increase in D_t interacts with the change in θ . It implies that the initial reduction in market power is lower in the High D economy than in the Low D one. The initial output increase is thus much larger in the latter. In both economies, however, the correlation between markup and output is negative during the adjustment phase.

5. Conclusions

Our objective was to develop a dynamic intertemporal model with imperfect competition, wherein the combined effects of capacity utilization and markup rate changes could be discussed. By introducing microeconomic demand uncertainty and aggregating over firms, we have been able to obtain a model wherein capacity underutilization is a macroeconomic equilibrium feature. In this setup, capacity utilization and markup rate changes are related via the effect of the former on the firm's actual market power. We studied these interactions and their implications for the dynamic behavior of some key macro variables in response to various "structural" shocks. Capacity constraints introduce potentially important non-linearities in the dynamic adjustment path. The same shock can have quite different effects depending on the characteristics of the ini-

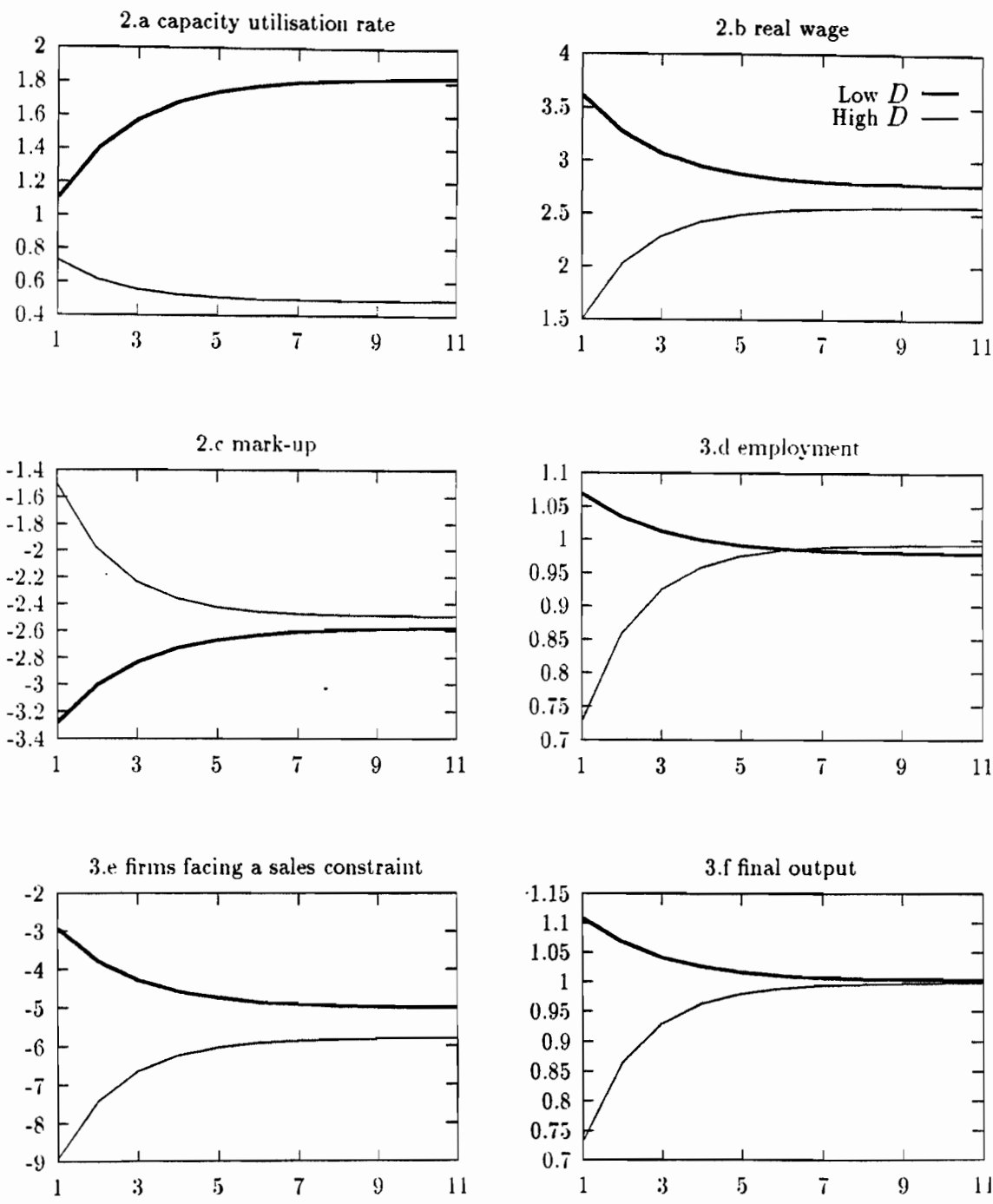
¹⁸The easiest solution would be to introduce a relationship between the price elasticity of demand and the composition of aggregate demand (see e.g. d'Aspremont et alii (op citum), Gali (op citum)).

¹⁹At any capacity level, the increase in degree of substitutability between goods eases the spillover effects between firms. There are thus less firms facing a sales constraint than before.

tial stationary state (low or high capacity utilization rate). We should emphasize in this context that the positive relationship between capacity utilization and markup rates does not imply per se a positive correlation between output and markup rates over the cycle. The latter correlation can be positive or negative, and depends on the correlation between productive capacities and output. It was furthermore shown that changes in the price elasticity of demand have a large positive effect on output and capacity utilization, and still a negative effect on markup rates. In other words, by extending the model so as to allow the price elasticity of demand to vary with output, one can generate complex and varied correlation patterns between capacity utilization, markups, real wages and output.

The model presented in this paper can be used as a starting point for further developments. The next logical step is of course to complete the model with stochastic processes and to examine the characteristics of the cyclical fluctuations so generated. We leave this topic for future research.

Figure 4: Elasticity of Substitution Increase



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Appendix 1: Financial markets equilibrium

Although there is uncertainty at the individual firm level (in the intermediate good sector), there is no aggregate uncertainty. At a symmetric equilibrium, all the intermediate good (input) firms have the same expected rate of profit at a given point in time. Consumers thus choose to diversify and eliminate risk by holding shares of all the intermediate good firms. The number of shares is assumed to be fixed and investment in fixed capital is entirely financed by borrowing at the market rate of interest. Because risk can be eliminated by diversification, bonds and (the market portfolio of) shares must, by arbitrage, bear the same sure rate of return.

Let q_t^f and q_t^i represent the market value of final good firms and of intermediate good

firms respectively. The corresponding share prices must satisfy for all $t \geq 1$ the following dynamic relationships:

$$q_{t+1}^i + \Pi_{t+1}^i = (1 + \tau_{t+1}) q_t^i \quad \text{with} \quad \Pi_{t+1}^i = \frac{p_{t+1}}{\theta \pi_{dt+1}} Y_{t+1} - (\tau_{t+1} + \delta) K_t$$

$$q_{t+1}^f + \Pi_{t+1}^f = (1 + \tau_{t+1}) q_t^f \quad \text{with} \quad \Pi_{t+1}^f = Y_{t+1} - p_{t+1} Y_{t+1}$$

where Π_{t+1}^f and Π_{t+1}^i represent the aggregate profit levels of the two types of firms. In every period t , the demand for assets by households is then given by:

$$A_t = [q_t^f + q_t^i + (1 + \tau_t) K_{t-1}] + [\Pi_t^f + \Pi_t^i + w_t L_t - C_t - T_t].$$

The financial markets equilibrium condition requires:

$$A_t = q_t^f + q_t^i + K_t.$$

Appendix 2: Influences of Parameters on the Stationary Equilibrium

The table below gives the signs of the partial derivatives (at the stationary equilibrium) of the capacity utilization rate, D , the proportion π_d , $(\theta \pi_d)^{-1}$ (i.e., the markup in percentage points), the spillover effect $p^{-\theta}$ and the real wage w with respect to the parameters σ_v^2 , θ , b , a , β and G .

	a and b	a	σ_v^2	θ	β	G
D	-	0	-	+	-	0
π_d	+	0	-	-	+	0
$(\theta \pi_d)^{-1}$	-	0	+	-	-	0
$p^{-\theta}$	-	0	+	-	-	0
w	+	+	-	+	+	0

In the case of the global productivity increase, the increase in b reduces the cost of capital per unit of output. The input firms can choose a more "risky" investment policy, i.e., a policy with a lower expected degree of capacity utilization. At macroeconomic equilibrium, D is thus lower. In the case of a labour productivity increase at unchanged capital productivity, the real cost of capital remains unaffected, hence the capacity utilization rate and the proportion π_d are unchanged.

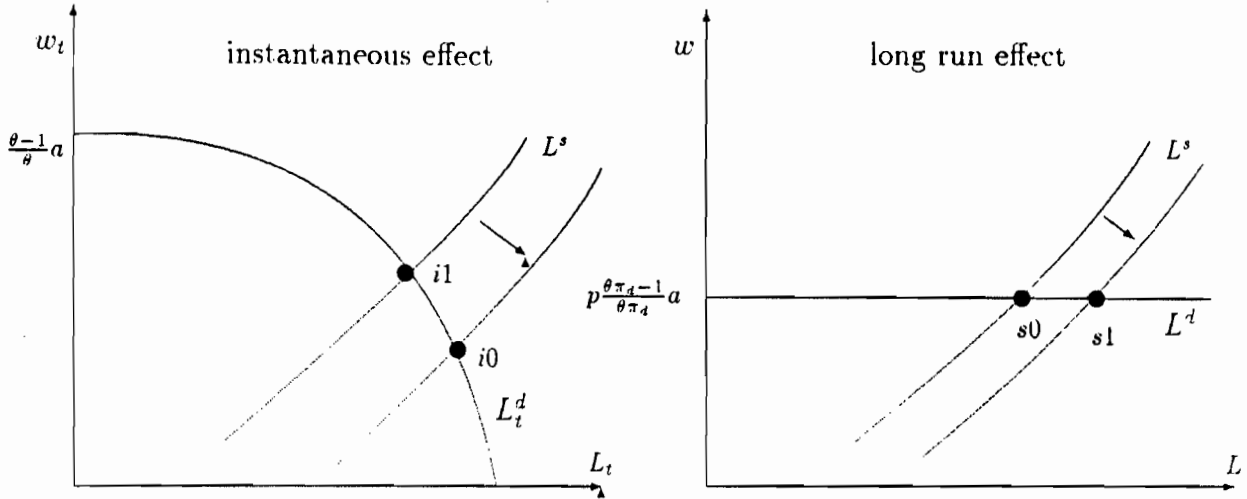
A mean-preserving spread of the distribution of the parameters v reduces both D and π_d . The larger σ_v^2 , the larger the mismatch between demands and supplies. A mean-preserving spread thus reduces the capacity utilization rate at any capacity level. In order to compensate the resulting loss of profitability, firms increase their markup rate (lower π_d).

The larger θ , the lower margin and the higher the relative cost of overcapacity (equipments are less profitable). Firms thus restore their profitability by decreasing the probability of a sale shortage, thereby choosing to increase their average capacity utilization rate.

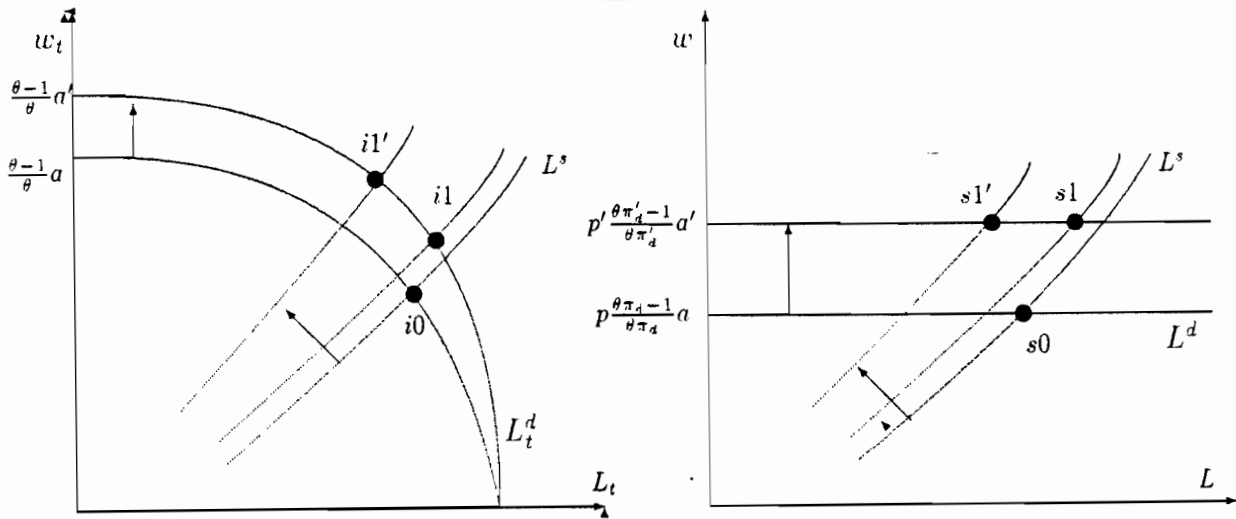
An increase in β reduces the real capital cost and, at given $F(v)$, allows the firms to reduce their optimal capacity utilization rate.

Appendix 3: Short- and Long-runs Effects of Structural Changes

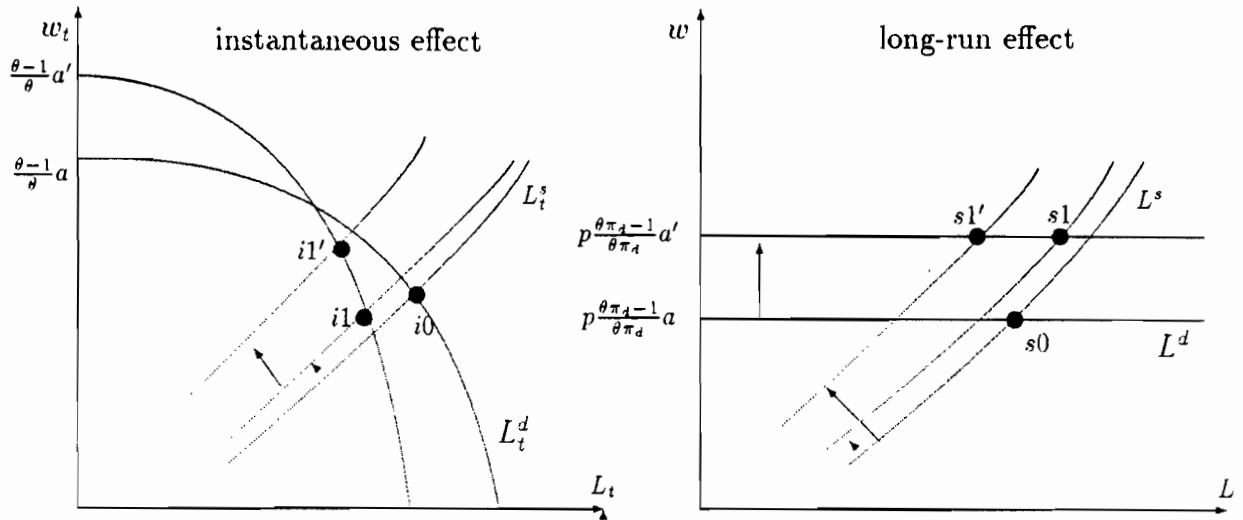
a. Government Purchase Increase



b. Productivity Increase (both factors)



c. Labour Productivity Increase



d. Demand Elasticity Increase

