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UNEMPLOYMENT SPELLS AND INCOME DISTRIBUTION DYNAMICS

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Abstract

In the U.S., during the 1948-86 period, an approximation to the Gini Index based on the quintiles and on the top 5% of the income distribution yielded a value of 0.351. Further, during this same period, the income share earned by the first quintile was procyclical and 7% more volatile than aggregate yearly output. In this paper we quantify the role played by unemployment spells in determining these and other related issues. To this purpose, we use an extension of the general equilibrium stochastic growth model that includes an endogenous distribution of households indexed by wealth and employment status. Our main findings are the following: *i)* in a model economy where all households have the same endowments of skills and are subject to the same employment processes, uninsured unemployment spells alone account for a very small share of the concentration of income observed in the U.S., and of the income distribution dynamics —the approximated Gini Index in this model economy is 18% of the one observed in the U.S., and the income share earned by the first quintile is 58% more volatile, *ii)* this result is robust to including a technology that allows for cyclically moving factor shares, and *iii)* in a model economy where households are partitioned into different skills groups that are subject to different employment processes in accordance to U.S. data, unemployment spells account for a significantly greater share of the U.S. statistics —the approximated Gini Index in this model economy is 70% of the one observed in the U.S., and the income share earned by the first quintile is 10% more volatile.

Key Words
Quantitative General Equilibrium; Heterogeneous Agent Economies; Income Distribution Dynamics; Unemployment Spells.

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1 Introduction

In the U.S., during the 1948-86 period, an approximation to the Gini Index based on the quintiles and on the top 5% of the income distribution yielded a value of 0.351. Further, during this same period, the income share earned by the first quintile was procyclical and 7% more volatile than aggregate yearly output. In this paper we explore the role played by unemployment spells in determining these and other related issues. Specifically, we want to quantify the extent to which unemployment spells account for: *i)* the average shape of the income distribution, and *ii)* its business cycle dynamics.

a) Methods

To answer these questions, in Section 2, we start by documenting both the average and the business cycle behavior of the U.S. income distribution. Our data source is the Consumption Population Survey (CPS) March files that report the income shares earned by the quintiles and by the top five percent of the U.S. income distribution. Next, in Section 3, we construct a general equilibrium stochastic growth model, with a large number of infinitely lived households. These households face an uninsured, household-specific, stochastic disturbance to their employment opportunities. Consequently, different households face different random flows of labor income and they choose to accumulate assets at different rates partly to smooth out their consumption. As a result of these differences in individual employment histories, at any point in time there is a distribution of households that can be indexed by household wealth and employment status. Finally, the households are subject to an economy-wide disturbance that drives the business cycles.

The quantitative nature of the questions posed in this paper requires a numerical solution of the model economies, which in turn requires their calibration. To calibrate our economies we do the following: First, we use data on employment and on labor income to characterize the household-specific employment processes and the processes on wages. Then, we choose the model economy's functional forms and parameters so that the model aggregates mimic certain statistics of the U.S. economy regarding both the first and the second moments of some of its aggregate variables. Finally, we simulate the calibrated model economies and we report the average behavior and the business cycle dynamics of their income distributions.

The large size of the state of this class of model economies —recall that it includes a time varying distribution of wealth and employment status— precludes the use of standard computational methods, and presents serious computational difficulties. These computational difficulties have lead many researchers to avoid the stochastic growth model with uninsured idiosyncratic shocks as an analytical tool for quantitative theoretical purposes.[1]

In order to solve their decision problem, households need to know current period prices and they need to predict future prices.[2] These prices are a function of the first moment of the wealth distribution. Krusell and Smith (1994) have recently shown that simple affine functions of the current moments of a distribution are very good predictors of the future moments. In this paper we approximate the wealth distribution by its first moment and we exploit Krusell and Smith (1994) result to construct a predictor for future prices.[3] The computational approach that we follow is described in Appendix 1.

b) Findings

First, we study the income distribution and its business cycle dynamics in a model economy where every household faces the same employment opportunities. We call this economy the baseline model economy and our main findings, which we report in Section 4.2, are the following: *i)* uninsured unemployment spells alone generate a very flat income distribution. In the baseline model economy the value of our approximation to the Gini Index based on the quintiles and on the top 5% of the distribution of income is

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[1] See Ríos-Rull (1995) for a review of the different approaches used to study heterogenous agents economies.

[2] Strictly speaking agents need to know the entire set of future prices for every possible history.

[3] In a previous version of this paper, see Díaz-Giménez and Ríos-Rull (1991), we used Markov chains to characterize the processes for the distributions moments. This approach proved to be more cumbersome and less accurate that the one we follow now.

0.063, or 18% of the value obtained from U.S. data,[4] and *ii)* while some of the qualitative patterns of the business cycle dynamics of the model economy's income distribution resemble those observed in the U.S. —the highest volatility of income corresponds to the first quintile, and this group's income is the most highly positively correlated with output, for instance— the match between the model economy results and U.S. data is far from being satisfactory —the model economy severely overpredicts the volatility of the income share earned by the first quintile of the distribution, and it underpredicts the volatility of the income shares earned by some of the other groups, specially those of the second quintile, and of the top 5%.

Given these rather disappointing answers we then explore some variations of the baseline model economy. First, we try different parameterizations of that economy. Namely, we lower the return to the home production technology, we lengthen the duration of the unemployment spells, and we use a combination of these two features. We find that none of these changes improves the behavior of the model economy substantially. These results are reported in Section 4.3.

Next, we modify the technology to include cyclically moving factor shares that account for the countercyclical behavior observed in the U.S. labor share. If the main source of income of the poor is their labor, and if the labor share of income is countercyclical, the income share earned by the poor will tend to increase in contractions and to decrease in expansions. Consequently, including this feature in our model economy should reduce the excessive volatility of the income share earned by the first quintile. We call this model the countercyclical labor share economy, and our main findings, which we report in Section 5.1, are the following: *i)* the average income distribution changes very little when compared to the corresponding one in the baseline model economy. More specifically, the approximated Gini Index in in the countercyclical labor share model economy is 0.064, which is only 1.6% higher than the one that obtains in the baseline model economy; and *ii)* the volatility of the income share earned by the first quintile

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[4] Aiyagari (1994) reports essentially the same finding as a steady state property of the model economy analyzed in his paper.

remains too high, and that of the income share earned by the top 5% remains too low. Lower, in fact, than the one that obtains in the baseline model economy. Overall, we find that cyclically moving factor shares do very little to improve the behavior of the model economy. The fact that the distribution of wealth is very disperse in this model economy accounts for most of this behavior.

Finally, we construct a version of the model economy where households are partitioned in five different groups with different endowments of skills and, hence, with different income levels and different employment processes.[5] We call this model the economy with multiple household types and our main findings, which we report in Section 6.2, are the following: *i)* the average income distribution becomes significantly more unequal, and it starts to resemble the average income distribution observed in the U.S. The value of the approximated Gini Index in this model economy increases significantly: it is now 0.246, or 70% of the value obtained from U.S. data, *ii)* the cyclical behavior of the multiple household type model economy comes very close to reproducing some of the key statistics of the income distribution dynamics observed in U.S. data, and *iii)* in this model economy, total income is more concentrated than capital income. This result indicates that the relatively low labor income earners are relatively high wealth holders. This is not a surprise given that they face a riskier employment process and that we abstract from life-cycle considerations that would induce a high positive correlation between asset holdings and labor income.

In his seminal work, Blinder (1974), mentions the following sources of income dispersion: dispersion in wages due either to unequal abilities or to unequal education and training, dispersion in tastes, increasing rates of return to wealth, racial and sexual discrimination, uneven incidence of unemployment and the effects of monopolies and monopsonies. In this paper we focus on the role played by unemployment spells, especially as it relates to the uneven distribution of wages, and we abstract from the remaining sources of income dispersion cited by Blinder and from life-cycle considera-

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[5] See Clark and Summers (1981), Kydland (1984) and Ríos-Rull (1993) for a rationale of this type of partitions.

tions.

The rest of the paper is organized as follows: Section 2 analyzes the data and characterizes the business cycle behavior of the U.S. income distribution. Section 3 describes the model economies and defines the equilibrium. Sections 4, 5, and 6 discuss the calibration choices and report the main findings for, respectively, the baseline model economy, the countercyclical labor share model economy, and the model economy with multiple household types. Section 7 concludes. The paper also includes two appendices. Appendix 1 describes our computational methods which involve an approximation to the equilibrium defined in Section 3. Finally, Appendix 2 describes the data collection and processing, and it contains a complete version of Table 2.

2 Data Analysis

To summarize the income distribution, we partition the households into quintiles[6] and we divide the last quintile into its first 15% and the top 5% percent. This summary corresponds to the one constructed by the *U.S. Bureau of the Census* based on the answers to the total income question asked in the March files of the CPS, and published in various issues of *Money Income of Households, Families, and Persons*, as part of the *Current Population Reports*, series P60. The definition of income considers includes all monetary income earned during the previous year before payments for personal taxes. It includes items such as Social Security benefits, Unemployment Compensation, Public Assistance, Retirement Benefits, Dividends, and others but it excludes non-cash benefits such as food stamps or health benefits. It is the most comprehensive notion of income in the CPS data set. The questions used to construct the data appear in the March files of the CPS only. Furthermore, it is important to note that this survey is not a panel since each year the sample of households changes completely. The data is reported yearly.[7]

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[6] Strictly speaking the i-th quintile of a distribution F is the value in the support of that distribution that solves the equation $F(x) = 0.2i$. In this paper we report the share of total income earned by different groups: the poorest 20%, the next 20% and so on. Sometimes we abuse the language and we call these groups quintiles.

[7] The frequency with which the data is collected is important. The reason for this is that income differences across households that arise from unemployment spells should decrease with the length of

The sample period available is 1948–1986. Once the data has been collected, the Bureau of the Census reports the shares of total income earned by the five quintiles and by the top 5% of the income distribution for families and unrelated individuals.[8]

Table 1: The average income distribution in the U.S. economy (1948-86)

	Income groups (%)					
	$0-20$	$20-40$	$40-60$	$60-80$	$80-95$	$95-100$
U.S. Economy	5.05	11.95	17.56	23.91	25.56	15.97

Table 1 reports the period averages of the income shares earned by each group. These income groups can be used to construct an approximation to the Gini Index,[9] that yields a value of 0.351.

Table 2 reports the percentage standard deviations and the contemporaneous correlations with output, Y, of output, consumption, C, investment, I, aggregate employment, N, average labor productivity, Y/N, and, for reasons that will become clear later, the labor share of output, $\mathcal{L}$. To compute the second moments we log the series and we filter them using the Hodrick and Prescott filter with a smoothing parameter of 100.[10] Table 2 also reports the second moments of the different income groups. For additional details on the methods used to construct the data reported in Tables 1 and 2, see Appendix 2.[11]

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the period being considered since unemployment spells tend to average out over time.

[8] Families and unrelated individuals do not correspond exactly with households. The concept of household considers a group of unrelated individuals sharing a housing unit as one household, and live-in employees are counted as part of the household of their employers, while the concept of families and unrelated individuals considers both unrelated individuals and live-in employees as different households. The Bureau of the Census has only been publishing the data for households since 1967. This lead us to use the families and unrelated individuals series which has been collected since 1948.

[9] This is an approximation to the Gini Index since we only use six observations to approximate the Lorenz curve. The true value of the Gini Index is somewhat higher. In this paper we use exactly the same approximation to compute the concentration indicators of the model economies.

[10] For details on the properties of the Hodrick and Prescott filter see Cooley and Prescott (1995). Two other papers that analyze the business cycle properties of yearly series are Backus and Kehoe (1989) and Ríos-Rull (1994a).

[11] Note that Table 2 reports the standard business cycle facts that obtain form yearly data. Namely, that consumption and investment are strongly correlated with output; that investment is about six

Table 2: The business cycle behavior of the U.S. economy (1948-1986)
Standard Deviations other than that of output are relative to output

	Aggregate Variables					
Variables	Y	C	I	N	Y/N^a	$\mathcal{L}$
St Dev	2.63%	0.48	2.99	0.48	0.74	0.25
Corr	1.00	0.78	0.70	0.71	0.89	−0.10

	Total Income Quintiles					
	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
St Dev	1.07	0.48	0.26	0.17	0.36	0.74
Corr	0.53	0.49	0.31	−0.29	−0.64	0.00

Source: Citibank Database, and the CPS March files.

The most outstanding features of the cyclical properties of the income shares earned by the different groups are the following:

i. The income share earned by the first quintile is the most volatile. It is slightly more volatile than aggregate output, and its correlation with output is positive and large.

ii. The income share earned by the top 5% is the second most volatile. It is about 75% as volatile as output, and its correlation with output is zero.

iii. The income shares earned by the remaining groups are between 25% and 50% as volatile as aggregate output. The correlations of the shares earned by the bottom 60% of the distribution and output are positive, and those of the shares earned by the groups between the 60% and the 95% are negative.

Additional properties of the cyclical behavior of the income quintiles are reported in Table 11 in Appendix 2.

3 The Model Economies

The model economies analyzed in this paper are modified versions of the stochastic neoclassical growth model. These models can also be interpreted as extensions of Aiyagari

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times more volatile than consumption and three times more volatile than output; that average labor productivity is about 75% as volatile as output, and that employment is about half as volatile as output.

(1994) and of Huggett (1993) who analyze model worlds that are similar to ours but that do not include either aggregate uncertainty or type multiplicity. The key features of these economies are *i)* that they include a large number of heterogeneous households, *ii)* that these households face both uninsured, household-specific employment shocks, and economy-wide productivity shocks, and *iii)* that these households accumulate assets both for precautionary reasons as a substitute of insurance against these shocks, and for the standard real business cycle motive of taking advantage of higher expected future rates of return.

3.1 Description of the environment

3.1.1 Population

We assume that at each point in time the economy is inhabited by a continuum of households of different skill levels, $i \in \mathcal{I} \equiv \{1, \cdots, I\}$. The mass of households of type i is μ_i, and $\sum_{i \in \mathcal{I}} \mu_i = 1$. Household-types differ in their efficiency labor factor, denoted ϵ_i, and in the transition probabilities of their idiosyncratic employment processes that we describe below.

3.1.2 Technology

3.1.2.1 Production possibilities. We assume that aggregate output, Y_t, depends on aggregate capital, K_t, on the aggregate labor input, L_t, and on the economy-wide shock, z_t, through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t, z_t)$. The capital stock depreciates at a constant rate δ.

3.1.2.2 Employment opportunities. We assume that the household-type specific employment processes take two possible values, $s \in S = \{e, u\}$. When a household of type i draws shock e, it receives an endowment of $h_i(z_t) > 0$ productive hours which it allocates inelastically to the aggregate production technology, and we say that it is employed. Note that the efficiency labor units supplied to the market by each of these households is $\epsilon_i h_i(z_t)$ and that the hours worked, h_i, depends on the economy-wide shock, z_t. When a household draws shock u, it receives no endowment of productive time, it is driven to operate the home production technology, and we say that it is unemployed.

We denote the measure of households of type i that draw shock e by N_{it}. Consequently, aggregate employment, N_t, is the sum over household-types of the measures employed of each type, *i.e* $N_t = \sum_{i\in\mathcal{I}} \mu_i N_{it}$, and the aggregate labor input, L_t is the sum over household types of the measures employed of each type weighted by the number of efficiency labor units supplied by each type, *i.e.* $L_t = \sum_{i\in\mathcal{I}} \mu_i \epsilon_i h_i(z_t) N_{it}$.

3.1.2.3 Processes on the exogenous shocks There is an exogenous economy-wide stochastic process $\{z_t\}$. This process follows a stationary finite state Markov chain with transition probabilities given by $\Pi(z' \mid z) = Pr\{z_{t+1} = z' \mid z_t = z\}$ where $z, z' \in Z = \{1, 2, ..., n_z\}$. We assume that the Markov chain generating z is such that it has a single ergodic set, no transient states and no cyclically moving subsets.

Each household also faces an idiosyncratic random disturbance, s, to its employment opportunities. Conditional on the realizations of z_t and z_{t+1}, these idiosyncratic disturbances are assumed to be independently distributed across households and identically distributed within each household-type. The process for these household-specific employment shocks, $\{s_t\}$, is also assumed to follow a finite-state Markov chain with conditional transition probabilities given by: $\pi_i(s' \mid z, s, z') = Pr\{s_{t+1} = s' \mid z_t = z, s_t = s, z_{t+1} = z'\}$ where $s, s' \in S = \{1, 2, \dots n_s\}$ and $z, z' \in Z$.

The joint processes on (s, z), therefore, are Markov chains with $n = n_s \times n_z$ states. Their transition probabilities are:

$$\Gamma_i[(s', z') \mid (s, z)] = Pr\{s_{t+1} = s', z_{t+1} = z' \mid s_t = s, z_t = z,\} \tag{1}$$

We assume that, for every household type i, the Markov chain generating (s, z) is such that it has a single ergodic set, no transient states and no cyclically moving subsets. Households know the laws of motion of both $\{s_t\}$ and $\{z_t\}$ and they observe the realizations of both stochastic processes at the beginning of each period. Note that $\Gamma_i(.,z'|.,z) = \Gamma_j(.,z'|.,z)$ for all $i,j \in \mathcal{I}$.

3.1.2.4 Home production. We assume that every household has access to the same home production technology. In any given period this technology allows households to produce $\bar{w}$ units of that period's consumption good without using any capital.[12]

3.1.3 Preferences.

We assume that households order their random streams of consumption according to:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_{it})\right\} \tag{2}$$

where u is a continuous and strictly concave utility function, $0 < \beta < 1$ is the subjective time-discount factor and $c_{it} \geq 0$ is the household's allocation of the period t perishable consumption good.

3.1.4 Market arrangements

We assume that there are no insurance markets for the household-specific shock, s.[13] We also assume that there are no markets for contracts contingent on the realization of the economy-wide shock, z.[14] To buffer their streams of consumption against these shocks, households can accumulate assets in the form of real capital. Moreover, household asset holdings are restricted to belonging to a compact set $\mathcal{A}$. Aiyagari (1994) shows that in this class of incomplete market economies, the requirement that debt has to be repaid

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[12] Alternatively, the returns to the home production technology, $\bar{w}$, could be thought of as some form of unemployment compensation. In this case they would be possibly different for different household-types, and the model economy would have to include a public sector to levy the resources required to finance the unemployment compensation scheme.

[13] This is the key feature of this class of model worlds. When insurance markets are allowed to operate this economy collapses to a standard representative agent model, as long as the right initial conditions hold.

[14] The reasons for this assumption are computational: the equilibrium is significantly easier to compute when markets for contracts contingent on the aggregate state of the economy are precluded. Ríos-Rull (1994b) compares the equilibrium allocations of heterogeneous agents economies that differ in the market structure for aggregate shocks, and finds that the differences in the behavior of these economies are very small.

imposes a lower bound on the set of assets holdings endogenously. If the lower bound of $\mathcal{A}$ is zero, this restriction can be understood as a borrowing constraint.

Firms rent factors of production from households in competitive spot markets. Consequently, factor prices are given by the corresponding marginal productivities, *i.e.*, $r_t = f_1(K_t, L_t, z_t) + (1-\delta)$, and $w_t = f_2(K_t, L_t, z_t)$, where r_t denotes the gross real rental price of capital, and w_t denotes the real wage.[15]

3.2 Equilibrium

In this paper we consider recursive, *i.e.* stationary Markov, equilibria only. This equilibrium concept might exclude some other type of equilibria such as those that model arrangements that can implement history dependent allocations as described, for instance, in Atkeson and Lucas (1992) and (1993). In this paper we are interested in the aggregate consequences of a specific set of market arrangements. We do not attempt to account for the reasons that justify the existence of those markets.

Each period, the economy-wide state is the pair (μ, z), where μ is a measure[16] defined over $\mathcal{B}$, an appropriate family of subsets of $\{\mathcal{I} \times \mathcal{A} \times S\}$.[17]

3.2.1 The household decision problem

For each household type i, the individual state variable is the vector (a, s, μ, z) which includes the stock of assets held, a, the realization of the employment shock, s, and the economy-wide state, (μ, z). The decision problem of a household of type i can be written as:

$$v_i(a, s, \mu, z) = \max_{c \geq 0, a' \in \mathcal{A}} \left\{ u(c) + \beta \sum_{s', z'} \Gamma_i[(s', z') \mid (s, z)] \; v_i(a', s', \mu', z') \right\} \quad \text{s.t.:} \quad (3)$$

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[15] In this class of model economies firms do not play any intertemporal role for two main reasons: first, they do not make profits and, second, they cannot be used by the households who own them to substitute for insurance by choosing non-profit maximizing strategies.

[16] Note that we have abused the language and while μ_i denotes the mass of households of type i, μ denotes the measure of households.

[17] Note that we do not need to keep track of household names since the decisions of households in the same individual state are always the same.

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\begin{aligned}
c + a' &= ar + w\,\epsilon_i\, h_i(z) && \text{if} \quad s = e, \\
c + a' &= ar + \bar{w} && \text{if} \quad s = u, \\
r &= r(\mu, z), \\
w &= w(\mu, z), \\
\mu' &= g(\mu, z, z').
\end{aligned}
$$

where function v_i is household's i value function, r and w are functions that describe the factor prices, and function g describes the law of motion of the wealth distribution.

Note that because of home production, aggregate consumption is different from market consumption. To compute the amount of the period good produced at home we define function $\psi_i(a, s, \mu, z)$, where $\psi_i(a, e, \mu, z) = \bar{w}$ and $\psi_i(a, u, \mu, z) = 0$.

3.2.2 Definition of Equilibrium

Definition 1 *A Recursive Competitive Equilibrium is a set of household policies* $\{c_i(a, s, \mu, z), \psi_i(a, s, \mu, z), a_i'(a, s, \mu, z)\}_{i \in \mathcal{I}}$, *pricing processes* $r(\mu, z)$ *and* $w(\mu, z)$, *aggregate input functions* $K(\mu)$, *and* $L(\mu, z)$, *and a law of motion for the distribution of household-types* $\mu' = g(\mu, z, z')$, *such that:*

i. *Optimality: given* $g(\mu, z, z')$, $r(\mu, z)$ *and* $w(\mu, z)$, *the household decision rules solve the maximization problems described in (3), and factor prices are factor marginal productivities:*

$$r(\mu, z) = f_1\left(K(\mu), L(\mu, z), z\right) + (1-\delta) \text{ and } w(\mu, z) = f_2\left(K(\mu), L(\mu, z), z\right). \quad (4)$$

ii. *Feasibility:*

$$\int_{\mathcal{I}, \mathcal{A}, \mathcal{S}} \left(a_i'(a, s, \mu, z) + c_i(a, s, \mu, z) - \psi_i(a, s, \mu, z)\right) d\mu \leq f\left(K(\mu), L(\mu, z), z\right) + (1-\delta) K(\mu) \quad (5)$$

iii. *Aggregation: factor inputs are generated by aggregation over agents.*

$$K(\mu) = \int_{\mathcal{I}, \mathcal{A}, \mathcal{S}} a \; d\mu, \qquad \text{and,} \qquad L(\mu, z) = \int_{\mathcal{I}, \mathcal{A}, \mathcal{S}} h_i(z)\, \epsilon_i\, \xi_{s=e}\, d\mu \quad (6)$$

where ξ is the indicator function.

iv. *Consistency of individual and aggregate behavior:*

$$\mu'(\mathcal{I}_0, \mathcal{A}_0, \mathcal{S}_0) = g(\mathcal{I}_0, \mathcal{A}_0, \mathcal{S}_0)(\mu, z, z') = \int_{\mathcal{A}_0, \mathcal{S}_0} \left\{ \int_{\mathcal{I}_0, \mathcal{A}, \mathcal{S}} \xi_{a'=a'_i(a,s,\mu,z)} \, \Gamma_i(s', z'|s, z) d\mu \right\} da'ds' \tag{7}$$

for all $(\mathcal{I}_0, \mathcal{A}_0, \mathcal{S}_0) \in \mathcal{B}$, and all (μ, z, z').

In Appendix 1 we describe an approximation to this equilibrium and we provide an algorithm to compute its solution.

4 The Baseline Model Economy

To calibrate this economy we impose some targets in terms of second moments of the aggregate variables. We do this because in this paper we want to explore the behavior of the income distribution in a model economy whose aggregate business cycles resemble those observed in the U.S. We do not want to account for the model economy aggregate business cycles.

4.1 Calibration

4.1.1 Model period and Population

The CPS data on the U.S. income distribution is collected yearly. However, the appropriate length of period to model unemployment spells is much shorter: it is probably as short as one week. A weekly model period imposes very high computational costs. As a compromise we choose one eighth of a year or six and a half weeks for our model period.[18] In the baseline model economy there is only one type of households. Consequently $i = 1$, and $\mu_1 = 1$.

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[18] Note that the model period cannot last longer than the minimum duration employment or unemployment spell. Note also that the model period does not need to coincide with the data collection period.

4.1.2 Technology

We choose a Cobb-Douglas aggregate production function for reasons that we discuss below. We also choose a two-state symmetric process for the economy-wide shock, *i.e.* $z \in Z = \{z_1, z_2\}$. We are not explicit about the decomposition of $z\, h(z)^{1-\theta}$ into its two components because we are not interested in the behavior of unweighted aggregate hours. Further, we normalize $z_1\, h(z_1)^{1-\theta}$ to 1. This leaves us with a total of 14 parameters to be determined. These parameters are the following: the capital depreciation rate, δ, the capital share, θ, the aggregate employment in the two states, $N(z_1)$ and $N(z_2)$, the value of $z_2\, h(z_2)^{1-\theta}$, the conditional transition probabilities of the economy-wide shocks $\Pi(z_1|z_1) = \Pi(z_2|z_2)$, and the conditional transition probabilities on the household-specific shocks $\pi(e|z_1, e, z_1)$, $\pi(u|z_1, u, z_1)$, $\pi(e|z_2, e, z_2)$, $\pi(u|z_2, u, z_2)$, $\pi(e|z_1, e, z_2)$, $\pi(u|z_1, u, z_2)$, $\pi(e|z_2, e, z_1)$, $\pi(u|z_2, u, z_1)$. To determine the values of these parameters we impose 14 restrictions. We impose 4 of these restrictions to make aggregate employment, $N(z_t)$, dependent on the current realization of the economy-wide process only. These 4 restrictions are: $N(z') = \int_A \pi(e|z, e, z')d\mu(a, e) + \int_A \pi(e|z, u, z')d\mu(a, u)$, for $\{z, z'\} \in Z \times Z$. The remaining 10 restrictions are the following:

- 1. *Consumption-output ratio.* The model only includes consumption and investment as components of output. Therefore, the first statistic that we want to match is the ratio of consumption to the sum of consumption and investment. In the U.S. economy, this ratio is 72.8%.

- 2. *Behavior of factor shares.* After World War II in the U.S., the real wage has increased at a constant rate —at least until 1973— and factor income shares have displayed no trend. To account for these two properties we choose a Cobb-Douglas aggregate production function, $Y = zK^{\theta}L^{1-\theta}$.[19]

To construct our measure of the capital share we follow Cooley and Prescott (1995)

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[19] Note that this functional form generates factor shares that are constant at every frequency. On the other hand, in the U.S. the labor share is countercyclical (see Table 2). We address this issue in Section 5 where we construct a model economy with cyclically moving factor shares.

but we abstract from the role played by the government. Essentially, their procedure considers consumer durables as capital goods and, therefore, their measure of output has to be adjusted to include the flow of services of consumer durables.[20] When measured in this way, the value for the capital share is 0.375.

- 3. *Average employment rate.* Our model economy is too abstract to distinguish between households that are outside the labor force and households that are unemployed. Moreover, in the U.S. economy the labor force participation is strongly procyclical. To address this issue we interpret the lower labor force participation in downturns as discouraged workers, *i.e.* as people that do not have an employment opportunity. Specifically, to determine the average employment rate in our model economy, we divide the average employment rate in the U.S. during the period under consideration —which was 62%— by one of the highest values for the U.S. labor force participation rate in that same period —67%— and we obtain a value of 92% which is the value for the average employment rate that we target.[21] Of course our choice implies higher average unemployment rates that those reported by the Bureau of Labor Statistics.
- 4. *Output volatility.* We want the volatility of logged, detrended output in the model economy to match the value of 2.63% observed in U.S. data.
- 5. *Employment volatility.* We want the volatility of logged, detrended employment in the model economy to match the value of 1.26% observed in US. data.
- 6. *Persistence of business cycles.* We target the autocorrelation of logged, detrended yearly output to match the value of 0.56 observed in U.S. data.
- 7. and 8. *Expected duration of unemployment spells:* We assume that the average duration of unemployment spells is 10 weeks during expansions and 14 weeks during recessions. İmrohoroğlu (1989) and Díaz-Giménez (1990) make these same choices.
- 9. and 10. We assume that in the switches from expansions to recessions no jobs are created. and that in the switches from recessions to expansions no jobs are destroyed.

These considerations imply the following parameter values:

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[20] Details on how our measure of the labor share was constructed can be found in Appendix 2.

[21] This choice is fairly standard in the literature. See İmrohoroğlu (1989). Díaz-Giménez (1990) and Díaz-Giménez *et al* (1992), amongst others.

Table 3: Calibration of Technology Parameters Values in Yearly Terms

δ	0.1000	θ	0.3750	$N(z_1)$	0.9594
$N(z_2)$	0.8806	$z_2\,h(z_2)^{1-\theta}$	0.9130	$\Pi(z_1\vert z_1)$	0.9722
$\pi(e\vert z_1, e, z_1)$	0.9615	$\pi(u\vert z_1, u, z_1)$	0.0419	$\pi(e\vert z_2, e, z_2)$	0.9525
$\pi(u\vert z_2, u, z_2)$	0.5714	$\pi(e\vert z_1, e, z_2)$	0.9580	$\pi(u\vert z_1, u, z_2)$	1.0000
$\pi(e\vert z_2, e, z_1)$	1.0000	$\pi(u\vert z_2, u, z_1)$	0.6048		

4.1.2.1 Home production. The returns to the home production technology represent the value to the households of their endowment of time when they do not work in the market, measured in terms of current period consumption. This parameter is difficult to target. We assume that in our model economies the value of home production is time invariant, and that it is 25% of the average earnings.[22]

4.1.3 Preferences

To characterize the household decision problem described in equation (3) completely, we must choose a form for the utility function. As is customary in quantitative general equilibrium exercises, we use a constant relative risk aversion utility function. Our choice for the risk aversion coefficient is 1.5. This is in line with many other studies. Mehra and Prescott (1985) describe some of those studies.

We target a value for the net real rate of return of 4% for the deterministic version of the model economies. This value, together with our choices for δ, and θ implies a value for the capital-output ratio of 2.66. The value of the households common subjective time discount factor that implements this choice is $\beta = 0.96$. All these numbers are reported in annual terms.

4.2 Findings

Table 4 reports the average total income distribution for the 1948-1986 period in the U.S. economy and the average distributions of total income, labor income and total income plus the value of home production for 25 samples of 39 observations of the baseline model

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[22] Below we explore the implications of reducing this value significantly.

economy. When judging the results contained in that table we must keep in mind two things: first, that the partition of the population into quintiles depends on the variable that is being considered and, therefore, the composition of the quintiles is different for the different definitions of income; and, second, that the model economy eighthly data has been aggregated into years before being reported.

Table 4: The average distribution of income in the U.S and in the baseline model economies.

	Income groups (%)					
	0 – 20%	20 – 40%	40 – 60%	60 – 80%	80 – 95%	95 – 100%
U.S. Economy Total Income	5.05	11.95	17.56	23.91	25.56	15.97
Model Economy Total Income	15.23	19.81	21.12	21.57	16.56	5.71
Model Economy Labor Income Only	14.83	20.07	21.70	21.70	16.28	5.42
Model Economy Total Income Plus Value of Home Prod	16.45	19.79	20.73	21.17	16.25	5.60

Probably the most outstanding feature of Table 4 is that, in the baseline model economy, uninsured unemployment spells alone generate a very flat distribution of income. Indeed, the approximated Gini Index in the baseline model economy is 0.063, while in the U.S. economy the value of this statistic is 0.351. This finding can be justified by the following reasons: first, in the baseline model economy every household has the same expected labor income, and, second, the endogenous differences in capital holdings across households are very small. So much so that when we compare the second and third rows of Table 4, we find that the maximum difference between the shares of labor income and total income earned by the different groups is only 0.58%.

Note that labor income share earned by each of the top three quintiles is exactly the same: 21.70%. The reason for this equality at the top is that the households in the top 60% of the labor income distribution are those that were never unemployed during the sample period. Households who suffered short unemployment spells dropped to the second quintile, and those who were unemployed for longer spells dropped to the first quintile. When the value of home production is added to total income we find that the income share earned by the households in the first quintile increases significantly. This result was to be expected since the households who were unemployed for longer periods of time —and, therefore, who allocate a larger fraction of their time to home production— belong to this quintile.

Table 5: The business cycle behavior of the baseline model economy
Standard deviations other than output are relative to output
25 samples, 39 observations

	Aggregate variables					
Variables	Y	C	I	N	Y/N	$\mathcal{L}$
St Dev	2.62%	0.38	2.85	0.49	0.52	0.00
Corr	1.00	0.86	.98	0.98	0.99	0.00

	Total income quintiles					
	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
St Dev	1.69	0.13	0.44	0.42	0.37	0.29
Corr	0.98	0.24	−0.97	−0.98	−0.99	−1.00

Table 5 reports the business cycle behavior of the baseline model economy. Note that the standard deviations of output, employment and productivity in the first row of that Table have been targeted as part of our calibration choices. The relative volatilities of aggregate consumption and investment were not targeted in our calibration and, consequently, they differ from their U.S. economy counterparts: consumption in the model economy fluctuates less than in the U.S. (38% and 48% of output respectively) and investment also fluctuates a little less (2.86 and 2.99 times the volatility of output respectively).

When we compare the business cycle behavior of the income distribution of the baseline model economy and of the U.S. economy, reported in Table 2, we find that they have some qualitative patterns in common. In both cases, the income share earned by the first quintile fluctuates more than the income shares earned by the other groups, and, in both cases, the income share earned by the first two quintiles is positively correlated with output. But most of the remaining business cycle statistics differ significantly across both economies. In the baseline model economy, for instance, the fluctuations of the income share earned by the first quintile are more than one and a half times larger than those in the data, while the fluctuations of the share earned by the second quintile are four times as small. The differences between the fluctuations of the income shares earned by the remaining groups, and the differences between the correlations of the income shares and output across both economies are also large.

Overall, these results cannot be considered a success in replicating the business cycle properties of the income distribution in the U.S. The main reasons for this failure are the following: first, in the model economy every household is subject to the same employment process and, second, in the model economy the resulting distribution of household wealth, and hence of capital income is very disperse. This implies that the volatility of the income earned by employed households —essentially those that belong to the top 70% or 80% of the income distribution— is very similar to the volatility of aggregate output, while the volatility of the income earned by households that are unemployed at least during a fraction of the year is much larger. As a consequence, the volatility of the income share earned by the first quintile is very large. Another important item in the data that the model economy fails to reproduce is the large volatility of the income share of the top 5%. Most probably, these large fluctuations in the income of the rich arise from reasons other than unemployment spells.

4.3 Deviations from the baseline model economy

The results reported in Subsection 4.2 lead us to try some variations of the baseline model economy. Specifically we consider the following changes: longer average unemployment spells —19 weeks during expansions and 26 during recessions— a lower return to home production —0.5% of the average wage rather than 25%, and a combination of these two changes. We find that none of these changes improves the performance of the baseline model economy in any significant way. In all three cases the income shares earned by the bottom quintiles in the deviant economies are smaller than in the baseline model economy, either because bad shocks to the household-specific process last longer, because they impose a higher cost on unemployed households, or for both these reasons. This property increases the differences between the income shares earned by the first quintile and the shares earned by the rest of the groups. In all three cases the volatility of the income shares earned by the first quintile increases, and there is no significant improvement in the volatilities of the shares earned by the other groups.

The main difference between these three model economies lies in the behavior of the income share earned by the second quintile. In the economies with long unemployment spells, the number of households that are unemployed for a fraction of a year is smaller than in the baseline model economy. Consequently, in these economies, the behavior of the second quintile resembles those of the higher income groups —its volatility increases and it presents a strong negative correlation with output.

5 The countercyclical labor share model economy

The technology used in the previous section generates labor shares that are constant at every frequency. This is not the case in U.S. data. As we report in Tables 2 and 11, the U.S. labor share leads output and is countercyclical. Specifically the standard deviation of labor share relative to that of output is 0.25, the contemporaneous correlation with output in the U.S. economy is -0.10, and the correlation of two leads of the labor share and output is -0.42.

In this project we do not investigate the causes of the countercyclical behavior of the labor share.[23] We take its behavior as given and we explore its implications for the behavior of the income distribution. The intuition for including this feature in the model

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[23] Gomme and Greenwood (1993) document and discuss the cyclical behavior of factor shares. They ask whether this property is the outcome of risk sharing contracts between relatively risk averse, low human capital workers, and less risk averse and more talented entrepreneurs. They conclude that this may be the case. However, in their model economy the resulting countercyclical behavior of the labor share small.

economy is the following: if the main source of income of the poor who are employed is their labor, and if the labor share is countercyclical, then the income share earned by the working poor will tend to increase in contractions and to decrease in expansions. Consequently, including this feature in our model economy should reduce the excessive volatility of the income share earned by the first quintile. To this purpose, we postulate an aggregate production function that has the same long run properties as the Cobb-Douglas production function, but that generates a countercyclical labor share under competitive pricing of the factors of production. This function is:

$$f(K, L, z) = zK^{\theta+\eta(z)}L(z)^{1-\theta-\eta(z)} \tag{8}$$

where z is the productivity shock, and $\eta(z)$ is positive in expansions and negative in recessions. The expected value of $\eta(z)$ is zero, and it can be parameterized to mimic the variance of U.S. factor shares. The values of $\eta(z)$ that result in the best approximation to the cyclical behavior of the U.S. labor share are $\eta(z_1) = -\eta(z_2) = 0.007$.[24]

5.1 Findings

Table 6 reports the average income distribution in the countercyclical labor share economy. Our main finding is that this assumption leaves the first moments of the income distribution virtually unchanged. In this model economy the approximated Gini Index is 0.064. In the baseline model economy the value of this statistic is 0.063, and in the U.S. economy during the period under consideration, it was 0.351. We conclude that when differences in household wealth arise from differences in the realizations of unemployment spells alone, the quantitative importance of cyclically moving factor shares is small.

Table 7 reports the business cycle behavior of the countercyclical labor share economy.[25] Once again, the main conclusion to be drawn from this table is that includ-

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[24] This also requires changing the value of $z_2 h_2(z_2)^{1-\theta-\eta(z_2)}$. It is now equal to 1.02.

[25] Note that the absolute value of the contemporaneous correlation of the labor share and output is too large: it is -0.98 in this model economy and -0.10 in the data. Two reasons that justify this behavior of the model economy are that productivity shocks are the only source of aggregate fluctuations, and that the process on these shocks takes two values only.

Table 6: The average income distribution in the U.S. and in the countercyclical labor share economies

	Income groups (%)					
	$0-20$	$20-40$	$40-60$	$60-80$	$80-95$	$95-100$
U.S. Economy Total Income	5.05	11.95	17.56	23.91	25.56	15.97
Model Economy Total Income	15.17	19.81	21.15	21.60	16.57	5.71

ing countercyclical labor share leaves most business cycle statistics virtually unchanged. This result is more an implication of the dispersion of household wealth than of the fact that the labor share is countercyclical. When most households own very similar amounts of wealth, the sources of income for all employed households are very similar and, therefore, the quantitative importance of the cyclical behavior of factor shares is very small. Consequently, the business cycle behavior of the income distribution in the countercyclical labor share economy the baseline model economy are very close. When we compare the results reported in Tables 5 and 7 we find that this is indeed what happens, except for small variations at both tails.[26] We conclude that the countercyclical behavior of the labor share does not have quantitatively important effects on the income distribution, unless the model economy households differ significantly in the sources of their income, and hence in their wealth.

6 The model economy with multiple household types

Our findings so far suggest that if we want our model economy to mimic the business cycle dynamics of the income distribution in the U.S., we must include some feature that generates income distributions that are more concentrated. This can be done by extending our model in different ways. One of these ways is to include a more

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[26] Specifically, the countercyclical behavior of the labor share reduces the volatilities of both the first quintile and the top 5% of the distribution.

Table 7: The business cycle behavior of the countercyclical labor share economy
Standard deviations other than that of output are relative to output
25 samples of 39 observations

	Aggregate Variables					
Variables	Y	C	I	N	Y/N	$\mathcal{L}$
St Dev	2.63%	0.35	2.98	0.47	0.54	0.25
Corr	1.00	0.81	0.98	0.98	0.99	−0.98
	Total Income Quintiles					
	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
St Dev	1.67	0.14	0.47	0.41	0.33	0.20
Corr	0.98	0.01	−0.98	−0.98	−0.99	−0.99

sophisticated description of the employment processes that would account, perhaps, for periods of extraordinary success or ill-fate.[27] This feature, however, is unrelated to the role played by unemployment spells in determining the income distribution dynamics which is the main focus of this paper. Another natural way to extend this class of model worlds is to include additional dimensions of household heterogeneity. One of these dimensions is to model savings for retirement or old age. Another of these dimensions is to model households with different endowments of skills that are subject to different employment processes. The latter is the line we choose.

Some of the reasons to model households that differ in their efficiency labor factor, in their provision of hours when they are employed, and in the conditional transition probabilities of their employment processes can be found in the literature on labor economics. Clark and Summers (1981), Kydland (1984) and Rios-Rull (1993), among others, report that in the U.S. economy there is a tight inverse relationship between average wages and the volatility of individual employment. In this section, we model this relationship partitioning the population into five household types that differ in their endowments of efficiency labor units, ϵ_i, and in the transition probabilities of their employment process, $\pi_i(s'|z,s,z')$.

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[27] Murphy and Topel (1987), for instance, report the duration of unemployment spells and they find that people unemployed for 40 weeks account for more that 15% of total unemployment.

6.1 Calibration

The key issue in the calibration of this economy is how to partition households into groups. In this paper we follow Ríos-Rull (1990) and (1993) who uses PSID data on wages to partition the population into five groups of equal sizes for males, females and total population. He also reports the average hours worked and the individual standard deviation of hours worked for each of these groups. To proxy for households we use data on males. We do this because, in general, females work fewer hours and, therefore, using data on both sexes would have inflated the differences in earnings across groups of households.

6.1.1 Population

In the multiple household type economy, therefore, the number of types is $I = 5$, and the mass of each type is $\mu_i = 0.20$ for all i.

Table 8: The distribution of skills in the U.S. economy

	Skills Groups				
	First	Second	Third	Fourth	Fifth
Efficiency Factor	0.468	0.782	1.000	1.290	2.096
Average Employment	0.846	0.905	0.920	0.924	0.925
Unconditional Standard Deviation of Employment	2.28	2.21	1.92	1.74	1.37

6.1.2 Technology

As far as the employment opportunities for the different household types are concerned we impose the following properties:

- *Efficiency labor factors.* We assume that the efficiency labor factors for the different household types, ϵ_i, are the relative wages of the different income groups reported in the first row of Table 8.

- *Equal work-weeks.* We assume that when employed all household types work the same number of hours, i.e. $h_i(z) = h(z)$ for all i, and all z. We impose this restriction because we assume that the households that work fewer hours do so because they are subject to more frequent unemployment spells instead of working shorter hours when employed.
- *Average employment rates.* Ríos-Rull (1990) reports the average annual hours worked in the market by each of the five groups in which he partitions the PSID sample. Under the assumption of equal work-weeks, we use his data to compute the relative employment rates for the different household types. We normalize the average employment rate of the median household type, *i.e.* household type 3, to 92% which is the average employment rate in the baseline model economy. We report the average employment rates that result for each group in the second row of Table 8. We use these average employment rates as one of the two restrictions that we need to select the values for $N_i(z_1)$ and $N_i(z_2)$. For each group i, this restriction is that $(N_i(z_1) + N_i(z_2))/2$ equals the average employment rate for group i which is reported in the second row of Table 8.
- *Standard deviation of employment.* The standard deviation of logged, detrended aggregate employment for annual data of U.S. economy is 1.26%. Ríos-Rull (1990) reports for each of the five groups in which he partitions the PSID sample the average individual standard deviation of annual hours worked in the market. We normalize his data so that the standard deviations that we report in the third row of Table 8 match both the aggregate volatility of employment in the U.S. and the relative employment volatilities of the different skills groups. We use these unconditional standard deviations as the second restriction that we need to select the values for $N_i(z_1)$ and $N_i(z_2)$. For each group i, $N_i(z_1) - N_i(z_2)$ is proportional to the values in the third row of Table 8. The constant of proportionality is such that the aggregate volatility of detrended employment matches its counterpart in the U.S. data.

The remaining household characteristics are the same for every household type, and they coincide with the corresponding ones in the baseline model economy.

6.2 Findings

Table 9 reports the average distribution of income in the U.S. and in the multiple household type economies. We find that the multiple household type economy is significantly more successful in replicating some of the key features of the U.S. income distribution than both the baseline and the countercyclical labor share model economies. Specifically, we find that in this model economy, the income shares earned by the different income groups, and especially those earned by the three middle quintiles, are close to those observed in U.S. data. The value of the approximated Gini Index in this model economy is 0.246, which is still lower than the 0.351 observed in the U.S. economy, but which is significantly higher than the 0.063 that obtains in the baseline model economy. This result is due in part to the fact that the multiple household-type economy fails to account for the share of income earned by the top 5% of the distribution. Part of this failure could be due to sampling problems since the PSID considers a much smaller sample than the CPS, but the key reason that justifies it is that very probably the wealth of the very rich is mostly independent of the fluctuations in their employment status.

Table 9: The average distribution of income in the U.S. and in the multiple household type economies

	Income groups (%)					
	0 – 20	20 – 40	40 – 60	60 – 80	80 – 95	95 – 100
U.S. Economy Total Income	5.05	11.95	17.56	23.91	25.56	15.97
Model Economy Total Income	9.70	13.89	17.72	22.84	26.19	9.66
Model Economy Labor Income	8.88	13.42	17.60	23.08	27.09	9.93

The third row of Table 9 reports the labor income distribution in the model economy. From this table we conclude that most of the success in replicating the U.S. income distribution is accounted for by the distribution of labor income and, therefore, that it arises from the partition of households into different skills groups. In fact, in this model economy, capital income is less concentrated than labor income.

This feature of this model economy arises because, in equilibrium, its average real rate of return is smaller than the household's common subjective time discount rate, and, therefore, the model economy households have little incentive to save. Moreover, low skill types are subject to greater employment risk than high skill types and, therefore, they have stronger incentive to save. Another reason that accounts for the dispersion of capital income is that we abstract from other reasons to save such as life-cycle considerations. These reasons would most probably have induced high wage earners to save more.

Table 10: The business cycle behavior of multiple household type economy
Standard Deviations other than that of output are relative to output

15 samples of 39 observations

	Aggregate Variables					
Variables	Y	C	I	N	Y/N	$\mathcal{L}$
St Dev	2.64%	0.46	2.36	0.47	0.54	0.00
Corr	1.00	0.92	0.98	0.99	0.99	0.00
	Total Income Quintiles					
	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
St Dev	1.18	0.28	0.02	0.18	0.25	0.50
Corr	0.97	0.96	0.62	-0.98	-0.95	-0.98

The cyclical behavior of the income distribution in the multiple household type economy is reported in Table 10. We find that the cyclical behavior of the total income quintiles in this model economy is very similar to the one displayed by U.S. data. The relative volatility of consumption and investment is 19% in the model economy, 16% in U.S. data, and 13% in the baseline model economy. The orders of magnitude of the fluctuations of the shares of total income earned by the different groups are also similar to the corresponding ones for the U.S. economy reported in Table 2. More specifically, our main findings are the following:

- The relative volatility of the share of income earned by the first quintile is 1.18 times greater than that of output in this model economy, 1.67 times in the baseline model economy and 1.07 times in the U.S. data.
- The volatility of the income share earned by the second quintile is about 28% of the volatility of output in this model economy, 13% in the baseline model economy and 48% in the U.S. data.
- The relative volatility of the income share earned by the 80-95% group in this model economy is about two thirds of the corresponding value observed in the U.S., and the same is true for the income share earned by the top 5%. Compared to the baseline model economy the share earned by the top 5% in this model economy is almost two times more volatile.
- Finally, as far as the correlations between the income shares and aggregate output are concerned, we find that the correlations of the first five income groups in this model economy have the correct signs but are too large in absolute value and that the correlation of the income share earned by the top 5% and aggregate output is way off: its correlation is almost minus one in the model economy and zero in U.S. data.

6.3 The model economy with multiple household types and cyclically moving factor shares

Finally, we explore a model economy that includes both multiple household types and cyclically moving factor shares. To calibrate this model economy we simply add the stochastic labor share described in Section 5 to the economy with multiple household types described in Section 6.

We find that the average behavior of the income distribution in these two model economies is very similar, and, once again, we find that in the model economy with multiple household types and cyclically moving factor shares labor income is more concentrated than total income.

As far as the business cycle dynamics of the income distribution in this model economy are concerned, we find that the relative volatility of the share of total income earned by the first quintile is now 1.39. In the multiple household type model economy with constant factor shares, this statistic is 1.07, and in the U.S. economy, 1.18. Once again, this result arises from the fact that labor income is more concentrated than capital income. During expansions, the low skill types are the ones that fare best in relative terms since their employment processes are the most volatile. In this model economy, capital ownership increases the procyclicality of total income and, therefore, the volatility of the income shares earned by the first quintile is very high. We also find that the volatility of the income shares earned by the other groups also increases albeit by significantly smaller amounts in relative terms: approximately by 10%. This is a direct consequence of the increased volatility of the income share earned by the bottom group.

In order for the countercyclical labor share to improve the match of the volatility of the income shares earned by the different groups, the model would have to produce a better relation between the labor income and the capital of the different groups. Unemployment spells alone do not seem to be capable of doing this job.

7 Concluding comments

In this paper we quantify the role played by unemployment spells in shaping *i)* the average income distribution, and *ii)* its business cycle dynamics. As far as the average income distribution is concerned, and using an approximated Gini Index as our concentration measure, we find the following: When every household has the same skills and is subject to the same employment process, the concentration of income in our model economy is only 18% of that observed in the U.S. Moreover, we find that this result is robust to changes in the model economy technology that account for the cyclical behavior of the factor shares observed in U.S. data. When we include an additional dimension of household heterogeneity and we partition the population into five groups that differ in their skills and in their employment processes, we find that the model economy accounts for 70% of the income concentration observed in U.S. data. However in this model economy low skill households face greater employment risks than high skill households and, therefore, have stronger incentive to save. Consequently, we find that in this class of economies labor income is more concentrated than capital income.

As far as the business cycle dynamics of the income distribution are concerned we find the following: the business cycle behavior of the baseline model economy differs significantly from the one observed in the data. Specifically, the fluctuations of the income share earned by the first quintile are very large and those of the shares earned by the remaining groups are very small. On the other hand, the business cycle dynamics of the income distribution in the model economy with multiple household types come significantly closer to replicating the corresponding ones observed in the U.S. This is so in spite of the fact that we abstract from savings for retirement, or to finance down-payments for houses, college education, or the purchase of consumer durables or for as a buffer against other forms of ill-fate such as illness or accidents.

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Appendix 1: Computation

Our model economy, like most recursive models, has an associated operator, T, that maps a suitable set of laws of motion of the economy-wide state, into itself, i.e. $T : G \mapsto G$, where $G \subset \{g : \mu' = g(\mu, z)\}$. More specifically, operator T returns the law of motion of μ, $Tg(\mu, z)$, implied by the household optimal decisions when they take as given a law of motion for μ, $g(\mu, z)$, in the formulation of their decision problem. The fixed points of this class of operators are part of the equilibria of these models and, therefore, successive approximations on these operators are frequently used in the algorithms that compute the equilibria of this class of models. In our case, however, the large size of the elements of G —recall that the state space includes a time-varying distribution of assets and employment opportunities— makes it very hard to implement the associated operator in the computer.

To get around this problem we use a different operator, defined over a much smaller space, whose fixed point can be readily computed. This new operator is associated to an economy that resembles the one described in Section 3, but with boundedly rational households. The gist of its logic is to approximate the distribution, μ, by a small set of functions.[28]

Let $q_j(\mu)$ be a multivariate function that maps the space of measures into $\mathcal{R}^j$,[29] so that $\{q_j(\mu), z\}$ includes a sufficient statistic for factor prices, and let h_j be a linear function that maps $\mathcal{R}^j \times Z \mapsto \mathcal{R}^j$, where function $h_j(\cdot)$ is used to predict the values of $q_j(\mu')$. Consider the following problem where we are implicitly assuming that the maximization is subject to the budget constraint and factor pricing functions, and where, to simplify the notation we define $x \equiv q_j(\mu)$:

$$v(a, x, s, z; h_j, q_j) = \max_{c \geq 0, a' \in \mathcal{A}} \quad u(c) + \beta E\{v(a', x', s', z'; h_j, q_j)|s, z\} \quad \text{s.t.} \tag{9}$$

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[28] See Krusell and Smith (1994) for an exhaustive description of this class of approximations. In their paper essentially they show that the first moment of the wealth distribution suffices to produce a high quality approximation.

[29] Function $q_j(\cdot)$ could map the set of measures into their first $j-$moments, for instance.

$$x' = h_j(x, z).$$

Note that we have indexed the value function both with function q_j, that returns the functions that proxy for measure μ as state variables, and by the predictor function h_j that returns the future values of those functions. The households that solve this problem can be said to be boundedly rational in a variety of ways. First, when they predict the future values of prices they do not use all the information at their disposal — they approximate the distribution by a finite set of its functions and they only consider the current-period values of these functions to predict their future values. Second, in their forecasts x' they do not take into account the prediction errors.[30]

Once the expressions for the budget constraint and the factor prices have been substituted into the program defined in (9), its solution is an optimal savings rule $a'_j(a, x, s, z; h_j, q_j)$, which, together with the process on z, generates a law of motion for the economy, $g(\mu, z; h_j, q_j)$. Let $b_j(q_j, h_j)$ denote the best, linear, unbiased, forecaster of $q_j(\mu')$, which we denote $b_j(q_j, h_j)$. Note that b_j maps the set of linear j−dimensional functions into itself, and can be readily computed through long simulations. Successive approximations can be used to obtain a fixed point in the space of predictor functions, $h_j^* = b_j(q_j, h_j^*)$. Note that this fixed point is an essential part of any equilibrium in a model world with boundedly rational households: when they use linear predictor h^*, their behavior generates a law of motion whose best linear predictor is also h^*.

Since a given distribution, μ, can be approximated by a large class of functions $q_j(\mu)$, which, in general, are associated to different predictor functions $h^*(q_j)$, we are not done yet. We have to single out a q_j.

Let $\{q_j\}_{j=1}^{\infty}$, be a nested sequence of multivariate functions of dimension j, such that it contains at least every moment of that measure in their natural order. Our objective is to find a multivariate function q_j of small dimension j, with the property that economies in which the households use functions of larger dimension to approxi-

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[30] Most numerical approximations to compute equilibria impose a form of certainty equivalence that avoids this problem. We do not think that this issue is quantitatively important.

mate the distribution display a similar equilibrium behavior. To determine whether the equilibrium behavior of two economies is similar we have to choose a metric that allows us to compare that behavior. There are several candidates for this metric. Closedness of the stochastic realizations of the economies is one, and closedness of the optimal decision rules a_j' is another one. The one that we use is the following: choose a function q_j, compute its associated equilibrium predictor, $h^*(q_j)$, and compute a measure of its predictive accuracy, its $R-$squared, or the variance of the one-period ahead forecasting errors, for instance.[31] Next, choose a multivariate function of dimension $j+1$, such that $q_j \subset q_{j+1}$, compute the accuracy of the bests forecasts of $q_j(\mu')$, conditional on $\{q_{j+1}(\mu), z\}$, and compare it with those implied by using $h^*(q_j(\mu), z)$. If the difference is small, we conclude that the approximation to the equilibrium with boundedly rational households that use q_j is satisfactory. Otherwise, the multivariate function of higher dimension must be used.

In this paper we follow Krusell and Smith (1994). We chose $j = 1$, and we define $q_1(\mu)$ to be aggregate capital. This approximation turned out to work remarkably well.

A1.1 The computational algorithm

The outline of the computational algorithm used is the following:

- *Step 1:* Choose the vector of functions, q_j.
- *Step 2:* Choose an initial prediction function, $h_{j0}(x, z)$, where $x = q_j(\mu)$.
- *Step 3:* Given h_{j0}, solve the household decision problem described in (9) and obtain the vector of household decision rules, $a_j'(a, x, s, z; h_{j0}, q_j)$.
- *Step 4:* Given $a_j'(a, x, s, z; h_{j0}, q_j)$, simulate a long realization of the economy and obtain a new prediction function, $h_{j1}(x, z)$, by Ordinary Least Squares.
- *Step 5:* If $h_{j0}(x, z) = h_{j1}(x, z)$, goto Step 6. Else use a weighted average of h_{j0} and h_{j1} to update $h_{j0}(x, z)$ and goto Step 3.

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[31] Continuity of the decision rules with respect to predicted values of $q_j(\mu')$ guarantees that small improvements in prediction imply small changes in actions. This property relates the metric defined on the accuracy of the predictor to the metric defined on the decision rule space.

- *Step 6:* Add another function to the vector of functions q_j that now becomes q_{j+1}, and compute a regression of $q_j(\mu')$ on $\{q_{j+1}(\mu), z\}$. If the R−squared increases more than a certain bound, then let $j = j + 1$, and goto Step 2. Otherwise we are done.

The outline of the algorithm used to solve the household decision problem is the following:[32]

- *Step 3.1:* Impose a grid on the household state space $\{\mathcal{A} \times \mathcal{R}^j \times S \times Z\}$.
- *Step 3.2:* Initialize the savings decision rule $a'_{j0}(a, x, s, z)$ in the grid points and assume that it is piece-wise linear in the remaining points of set $\mathcal{A}$.
- *Step 3.3:* For each point in the grid find the value of a' that solves the Euler equation of the household decision problem described in equation (9), namely:

$$u'(a', a, x, s, z) = \beta \sum_{s', z'} r(x', z') u'[a'_{j0}(a', x', s', z'), a', x', s', z'] \Gamma(s', z' \mid s, z) \qquad (10)$$

Note that given the prediction function, $x' = h_{j0}(x, z)$, equation (10) is a function of a, x, s, z only, and, therefore, it can be solved for $a' = a'_{j1}(\cdot)$.

- *Step 3.4:* If $a'_{j1}(\cdot) = a'_{j0}(\cdot)$ in every grid point, we are done. Else update $a'_{j0}(\cdot)$ and goto Step 3.3.

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[32] Huggett (1993) uses a similar algorithm.

Appendix 2: Data sources, and statistical description

This appendix describes the methods that we have used to construct the data for the U.S. economy. We have considered annual data from 1948 to 1986 which is period for which income distribution data for families and unrelated individuals are available. We have logged every variable except those that report either shares or rates. To construct the business cycle statistics we have detrended the series using Hodrick and Prescott's filter with a smoothing parameter $\lambda = 100$. In the cases of rates and shares, in the column that corresponds to the standard deviations, we report the coefficients of variation of those variables. Most of the procedures used to construct the data series follow Cooley and Prescott (1995), except that we abstract from the government. The methods used to construct the different series are the following:

- We define output as *GDP* plus imputed services from the stock of consumer durables. The stock of consumer durables is taken from Musgrave (1992). We assume that the rate of return on durables is the same as the rate of return on the standard measure of the capital stock. To compute this rate of return, we use NIPA data, we assume a constant depreciation rate and we follow the procedures described in Cooley and Prescott (1995). To calculate the depreciation rate we assume that the economy is on a balanced growth path.

- The series for consumption includes non-durables, services and the imputed flow of services from the stock of consumer durables net of depreciation.

- The series for investment includes the NIPA definition of investment plus purchases of consumer durables.

- We define the capital income share as the ratio of capital income to total output. To construct the capital income series we considered Rental Income of Persons, Capital Consumption Allowances, Corporate Profits and Net Interest, as unambiguous capital income, and Compensation of Employees, as unambiguous labor income. The remaining components of the NIPA measure of income: Proprietor's Income, Indirect Business Taxes and Non-tax Liabilities, Business Transfers Payments, and Statistical Discrepancies were allocated to capital and labor in the same proportions as those of unambiguous labor income and unambiguous capital income to the sum of these two quantities.

- For the series for hours worked we used the Household Survey data.
- The data on the U.S. Income Distribution is the "Money Income of Households, Families, and Persons in the United States: 1986" from the Current Population Reports of the Bureau of the Census of the U.S. Department of Commerce for Consumer Income. We used the published series for the shares of income of the quintiles and the top five percent of the distribution for families and unrelated individuals. The data reported in those series is obtained from the March files of the Current Population Survey which reports pre-tax money income only.

Table 11: Cyclical Behavior of the U.S. Economy 1948–86
Deviations From Trend

			Cross Correlations of output with				
Variables	St Dev %	Relt to Y	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)
Output	2.63	1.00	0.02	0.56	1.00	0.56	0.02
Consumption	1.27	0.48	−0.16	0.39	0.78	0.53	0.19
Investment	7.86	2.98	0.07	0.48	0.70	−0.01	−0.33
Total Hours (Household)	1.74	0.66	−0.17	0.24	0.79	0.45	−0.10
Productivity per Hour	1.66	0.63	0.25	0.65	0.77	0.39	0.09
Employment	1.26	0.48	−0.30	0.10	0.71	0.46	−0.02
Productivity Per Person	1.96	0.74	0.22	0.68	0.89	0.47	0.05
Labor Share	0.66	0.25	−0.42	−0.41	−0.10	0.39	0.30
First Quintile (0-20%)	2.83	1.07	−0.17	0.04	0.53	0.36	0.05
Second Quintile (20-40%)	1.26	0.48	−0.19	−0.01	0.49	0.52	0.31
Third Quintile (40-60%)	0.69	0.26	−0.06	−0.04	0.31	0.33	0.41
Fourth Quintile (60-80%)	0.46	0.18	0.09	−0.19	−0.29	−0.10	0.07
Next 15% (80-95%)	0.94	0.35	0.04	−0.28	−0.64	−0.39	−0.03
Top 5% (95-100%)	1.95	0.74	0.13	0.29	0.00	−0.20	−0.37

Source: Citibank data Base, and CPS March files.