E C O N O M I C S   B U L L E T I N

# The Long−Run Keynesian Multiplier

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## *Abstract*

We study the impact of investment on employment. In the short−run an increase in investment stimulates employment (this is the standard Keynesian multiplier). However increases of investment translate into increases in the capital stock. If labor and capital are substitutes (resp. complements), an increase in investment today decreases (resp. increases) employment tomorrow. We provide a formula to measure the overall effect of an increase in investment on emplyment, assuming that certain regularities hold.

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# 1. Introduction

The Keynesian multiplier says that an increase in investment increases national income in a larger proportion. An important consequence of this is that an increase in investment decreases unemployment today. Keynes was aware of the long-run consequences of investment in capital stock.[1] But given that he decided to focus on the short-run, he never incorporated in his model the fact that investment today affects employment tomorrow via increase in capital stock. In other words, the Keynesian multiplier is just one side of the story of how investment affects employment.

In this note we want to take into account all the effects of investment on employment: Today's effects -as is done by the standard multiplier-, and tomorrow´s effect via the increase in capital stock. As an illustration, suppose that the economy lasts for two periods. There is no depreciation. Aggregate output is the sum of the capital stock and labor, the latter available in a quantity of 25. Suppose that investment is exogenously given at 20 in the first period and 30 in the second period. Capital stock in the first period is 30 and, thus, in the second period is 50. If the Keynesian multiplier is 2, aggregate output is 40 in the first period and 60 in the second period. Employment is $40 - 30 = 10$ in the first period and $60 - 50 = 10$ in the second period. Now compare this situation with one in which, in the first period, investment is 25, all other magnitudes being constant. Now aggregate production is 50 in the first period and 60 in the second period. But employment is $50 - 30 = 20$ in the first period and $60 - 55 = 5$ in the second period. In other words, if capital and labor are substitutes an increase in today´s investment increases tomorrow's capital and thus, decreases tomorrow´s employment. However, if capital and labor are complements, (i.e. fixed coefficients), and in some period in the future labor demand is driven by capital stock, an increase in today´s investment increases tomorrow´s employment.

In this paper, we offer a theoretical framework to analyze this question. We show that, assuming some regularities in the dynamic behavior of investment, the long-run variation of employment can be predicted by a simple formula, see Equation 2.6, for the case of capital and labor being substitutes, and Equation 3.1 for the case of capital and labor being perfect complements.[2] In the first (resp. sec-

[1] And hence his famous sentence on pyramids, cathedrals and trains (Keynes, 1936, p. 130).

[2] These dynamic regularities are: In the first case that the economy is in a steady state with all the variables growing at a constant rate. In the second case that there are regular cycles produced by investment.

ond) case the long run effect of investment on employment is smaller (larger) than the value predicted by the multiplier since the increase in investment increases capital which, in turn, decreases (increases) demand for labor. We remark that the values of the short-run multiplier can be found by an easy calibration exercise with relative accuracy, but in order to calculate the long-run multiplier we need to know the technology and to make assumptions that determine the outcome of the exercise.[3]

An implication of our analysis is that to stimulate the economy by means of investment, as it is currently advocated by some Keynesians for some countries, is a risky business unless the social planner has a precise knowledge of the available technology or unless the current situation is so desperate that long-run effects could be neglected.[4]

## 2. Capital Substitutes Labor

In this section we assume that capital and labor are substitutes. Thus, an increase in investment today decreases tomorrow's employment because tomorrow's capital has increased.

### 2.1. The Short-Run Keynesian Model

We first recapitulate standard concepts from elementary macroeconomics. Consider the standard Keynesian model,

$$S = sY, \ S = I, \ Y = f(L), \ w = f'(L)$$

where $S$ = savings, $s$ = marginal propensity to save, $I$ = investment, $Y$ = gross national product, $f(\ )$ = short-run production function, $L$ = employment, $w$ = real wage and $f(\ )$ = derivative of $f(\ )$. It is assumed that capital stock is given and that $f(\ )$ is strictly concave.

Short-Run Equilibrium: In this model, investment is exogenously given - i.e. we do not consider financial markets- and employment is determined by gross

[3]Of course, our´s is not the first paper casting doubts on the Keynesian multiplier. But the difference of our paper with others is that we stay in a Keynesian framework where employment is determined by demand.

[4]Long-run effects can also be neglected if the economy is stimulated by means of unproductive public expenditure. This has the advantage of having a totally predictible effect on the long-run employment (zero!), but the disadvantage of not contributing at all to the building of capital.

national product -i.e. labor market is not explicitly modeled. Thus we assume implicitly that labor supply is larger than labor demand. Labor and financial markets could be considered at cost of complicating the formulae, so we decided to work with the simplest approach.

The model has a unique solution, namely

$$S = I, Y = \frac{I}{s}, L = f^{-1}(\frac{I}{s}), w = f(f^{-1}(\frac{I}{s}))$$

**Short-Run Multiplier:** From equations above we calculate the (infinitesimal) variation of employment with respect to a (infinitesimal) variation of investment,

$$\frac{dL}{dI} = \frac{1}{sf'(L)} = \frac{1}{sy\alpha},$$

where $y = \frac{Y}{L}$ is the (apparent) labor productivity and $\alpha = f'(L)\frac{L}{Y}$ is the share of labor in the gross national product. It is clear from the above that in the short-run employment increases with investment.

## 2.2. A Long-Run Keynesian Model

When considering long-run, capital stock, denoted by $K$, is no longer constant. Thus, if capital and labor are substitutes, an increase in investment today, increases capital tomorrow and thus decreases employment tomorrow. In this subsection we quantify the effect of today´s investment on tomorrow´s employment.

**Long-Run Equilibrium:** From now on, we will write all variables with a subindex indicating time. If capital depreciates at a constant rate, $\delta$,

$$K_t = K_{t-1}(1-\delta) + I_{t-1}. \tag{2.1}$$

We assume that capital is always fully utilized.[5] The production function is now written as

$$Y_t = F(A_t L_t, K_t), \tag{2.2}$$

where technical progress increases the labor productivity in time $t$ by the factor $A_t$. This factor is assumed to grow at an exogenous and constant rate $g_A$. Let $\mathcal{L}_t =$

[5] Machines have no alternative use. Thus, as long as the market for capital goods is competitive, full employment of capital occurs.

$A_tL_t$. We assume constant returns to scale in $A_tL_t$ and $K_t$. The representative firm maximizes instantaneous profits by choosing labor and capital and this yields

$$w_t = A_t \frac{\partial F(\mathcal{L}_t, K_t)}{\partial \mathcal{L}_t}. \tag{2.3}$$

From the short-run model we also have the following equations,

$$S_t = sY_t, \;\; S_t = I_t. \tag{2.4}$$

In this model, we keep the simplifications made in the short-run model, namely, that investment is exogenously given in each period and employment is determined by output. Thus, in the Long-Run equilibrium we have

$$Y_t = \frac{I_t}{s} \text{ and } L_t \text{ solves } \frac{I_t}{s} = F(A_tL_t, K_{t-1}(1-\delta) + I_{t-1}). \tag{2.5}$$

Thus the latter equation determines actual employment.

Steady State: In order to solve the model we assume that investment grows at an exogenous constant rate, say $g$, and that initial values of the variables are such that there is a steady state, i.e. a situation in which all variables grow at a constant rate. This assumption is purely simplificatory and it is sometimes used by Keynesian economists, see e.g. Robinson and Eatwell (1973), p. 189. Two more pieces of notation: All the variables that are constant in the steady state will be denoted without the time subindex and the rate of growth of a variable, say $X$ will be denoted by $g_X$.

From (2.1) we have that $g_K = \frac{I_{i-1}}{K_{i-1}} - \delta$. Thus if $g_K$ is constant, $\frac{I_{i-1}}{K_{i-1}}$ should also be constant for all $i$ and then, $g_K = g$. From (2.4) we have that $g_Y = g$. By constant returns to scale, the production function can be written as $Y_t = K_tF(\frac{A_tL_t}{K_t}, 1)$, and from this and the previous findings we see that $g_{AL} = g_K = g$. Since $g_{AL} = g_A - g_L$, we obtain that $g_L = g - g_A$. Thus, capital-output ratio, denoted by $v$, remains constant and labor productivity, $y$, grows at a rate $g_A$. Since

$$\frac{\partial F(\mathcal{L}_t, K_t)}{\partial \mathcal{L}_t} \text{ depends only on } \frac{\mathcal{L}_t}{K_t}$$

and this magnitude is constant, the wage rate grows at a rate $g_A$ and, therefore, the share of labor in the national product, denoted by $\alpha = \frac{w_tL_t}{Y_t}$, is constant. Finally, the capital-labor ratio, denoted by $k$ $(= vy)$, grows at rate $g - g_L = g_A$.

Summing up, in the Steady State,

$$g_K = g_Y = g, \; g_L = g - g_A.^{6}$$

Long-Run Multiplier: Let 0 be the initial period. The (infinitesimal) effect of an increase in investment in period 0 on employment in period $t = 1, 2..., \infty$ is

$$\frac{dL_t}{dI_0} = \frac{dL_t}{dK_t}\frac{dK_t}{dI_0}, \text{ where } \frac{dK_t}{dI_0} = (1-\delta)^{t-1}.$$

Since $Y_t$ does not depend on $I_0$, we have that

$$\frac{dL_t}{dK_t} = -\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial L}} = -\frac{(1-\alpha)}{\alpha k_t}. \text{ Thus,}$$

$$\sum_{t=1}^{\infty}\frac{dL_t}{dI_0} = -\sum_{t=1}^{\infty}\frac{(1-\alpha)(1-\delta)^{t-1}}{\alpha k_t} = -\frac{(1-\alpha)}{\alpha k_0(1+g_A)}\sum_{t=1}^{\infty}\left(\frac{1-\delta}{1+g_A}\right)^{t-1} = -\frac{(1-\alpha)}{\alpha k_0(g_A+\delta)}$$

Finally, the total effect of investment of investment on employment, or long-run multiplier, denoted by $M$ is

$$M \equiv \sum_{t=0}^{\infty}\frac{dL_t}{dI_0} = \frac{1}{sy_0\alpha} - \frac{(1-\alpha)}{\alpha k_0(g_A+\delta)} = \frac{1}{k_0\alpha}\left(\frac{v}{s} - \frac{(1-\alpha)}{(g_A+\delta)}\right) \quad (2.6)$$

From there we see that

$$M > 0 \Leftrightarrow \frac{v}{s} > \frac{(1-\alpha)}{g_A+\delta}.$$

## 3. Capital Complements Labor

In this section we assume that capital and labor are complements. Thus, an increase in investment today might increases tomorrow's employment because tomorrow's capital has increased.

[6] The equations are identical to a neoclasical growth model but here $g$ is the exogenous variable and $g_L$ is endogenous. In a neoclassical model there is full employment and the rate of growth of the population determines $g$.

## 3.1. The Short-Run Keynesian Model

We assume that production takes place under fixed coefficients. Let

$$Y = \min\{\frac{K}{v}, Ly\}$$

where $v$ -the capital-output ratio- and $y$ -the productivity of labor- are now constant in a particular period. Let $\bar{L}_t$ be the available quantity of labor. Thus, gross national product is determined by the following equation

$$Y = \min\{\frac{I}{s}, \frac{K}{v}, \bar{L}y\}$$

**Short-Run Equilibrium:** As we did in the previous section we assume that $\bar{L}y > \min\{\frac{I_t}{s}\frac{K_t}{v}\}$, i.e. supply of labor is larger than demand of labor. Thus,

$$Y = \min\{\frac{I}{s}, \frac{K}{v}\}.$$

**Short-Run Multiplier:** From the equation above we calculate the (infinitesimal) variation of employment with respect to a (infinitesimal) variation of investment,

$$\frac{dL}{dI} = \frac{1}{sy} \text{ if } \frac{I}{s} < \frac{K}{v}, \ \frac{dL}{dI} = 0 \text{ otherwise.}$$

In words, short-run employment increases with investment, as long as $\frac{I}{s} < \frac{K}{v}$, i.e. when investment is relatively low.

## 3.2. A Long-Run Keynesian Model

We now assume that $v$ is constant on time but $y$ grows at rate $g_A$. Therefore the capital/labor ratio $k$ also grows at rate $g_A$. As before, capital depreciates at a constant rate $\delta$. Since now capital and labor are complements, an increase in investment today, increases capital tomorrow and thus might increase employment tomorrow. In this subsection we will quantify the effect of today´s investment on tomorrow´s employment.

**Long-Run Equilibrium:** From the previous discussion we obtain that

$$Y_t = \min\{\frac{I_t}{s}, \frac{K_{t-1}(1-\delta) + I_{t-1}}{v}\}, L_t = \min\{\frac{I_t}{sy_t}, \frac{K_{t-1}(1-\delta) + I_{t-1}}{vy_t}\}.$$

Cycles: In this model the assumption of steady state becomes very implausible (as pointed out by Harrod (1948)) because if capital were fully utilized, we would have

$$\frac{I_t}{s} = \frac{K_{t-1}(1-\delta) + I_{t-1}}{v} \Leftrightarrow g_K = \frac{s}{v} - \delta,$$

which, generically, is impossible because $g_K, s, v$, and $\delta$ are all parameters of the model.

In order to solve the model we assume (in the tradition of Harrod [1948]) that investment produces regular cycles.. Specifically, we assume two things: During $x$ periods

$$\frac{I_t}{s} > \frac{K_{t-1}(1-\delta) + I_{t-1}.}{v} \Leftrightarrow g_{tK} > \frac{s}{v} - \delta$$

where $g_{tK}$ is the rate of growth of $K$ at time $t$. These are periods were investment is booming. During these $x$ periods national product and employment depend on the stock of capital.

After these $x$ periods, we have $\tau$ periods where

$$\frac{I_t}{s} < \frac{K_{t-1}(1-\delta) + I_{t-1}.}{v} \Leftrightarrow g_{tK} < \frac{s}{v} - \delta.$$

These are periods were investment is low and consequently, capital grows slowly. During these $\tau$ periods national product depends on current investment.

Long-Run Multiplier: As before, let 0 be the initial period. In order to simplify our calculations we assume that 0 is the last period where the national product depends on current investment. The effect of an infinitesimal increase of investment in period 0 on employment in period 0 is

$$\frac{dL_0}{dI_0} = \frac{1}{sy_0}.$$

In the Appendix we show that the long-run effect of investment is

$$\frac{T}{k_0(1+g_A)(1-(\frac{1-\delta}{1+g_A})^{x+\tau})},$$

Thus, the total effect of investment on employment is:

$$M \equiv \sum_{t=0}^{\infty} \frac{dL_t}{dI_0} = \frac{1}{sy_0} + \frac{T}{k_0(1+g_A)(1-(\frac{1-\delta}{1+g_A})^{x+\tau})} \tag{3.1}$$

## 4. Extensions and Conclusions

1- In Section 2 we assumed that labor is hired until the marginal productivity of labor equals the wage rate. It is hard to think this policy to be followed in socialist China. However if the production function is Cobb-Douglas, profit maximization is not needed. We get directly that

$$\frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial L}} = \frac{(1-\alpha)}{\alpha k_t}$$

without invoking that the marginal productivity of labor equals the wage rate.

2- The assumption that the economy is in a Steady State is just a way to solve the model. If we want to know the exact loss of employment in, say, the next five years when capital and labor are substitutes, all we have to do is to compute

$$\sum_{t=1}^{5} \frac{dL_t}{dI_0} = \sum_{t=1}^{5} \frac{(1-\alpha_t)(1-\delta)^{t-1}}{\alpha_t k_t}$$

using data of the economy under consideration. The same can be said about our assumptions on cycles in the case were capital and labor are complements.

3- Finally, it would be interesting to consider financial and labor markets, public capital, more general production functions, taxes and international trade and to derive the corresponding long-run multiplier. This is left for future work.

## 5. References


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## 6. Appendix

From Period 0 on, we have $x$ periods where capital stock determines employment. Thus $L_i = \frac{K_i}{k_i}, i = 1, ...t$ and the induced increase of employment in period $i$ is

$$\frac{dI_0(1-\delta)^{i-1}}{k_i} = \frac{dI_0(1-\delta)^{i-1}}{k_0(1+g_A)^i}. \text{ Thus,}$$

$$\sum_{i=1}^{x} \frac{dL_i}{dI_0} = \frac{1}{k_0(1+g_A)}\left[1+\frac{1-\delta}{1+g_A}+....+(\frac{1-\delta}{1+g_A})^{x-1}\right]. \qquad (6.1)$$

After these $x$ periods we have $\tau$ periods where employment is determined by current investment. In all these periods employment is independent on $dI_0$. After these periods have elapsed, we have again $x$ periods where employment is determined by capital. The induced increase of employment in period $i$ of this cycle is

$$\frac{dI_0(1-\delta)^{x+\tau+i-1}}{k_{x+\tau+i}} = \frac{dI_0(1-\delta)^{x+\tau+i-1}}{k_0(1+g_A)^{x+\tau+i}}.$$

Adding up all terms between time $x+\tau+1$ and time $2x+\tau$ we have

$$\sum_{i=x+\tau}^{2x+\tau} \frac{dL_i}{dI_0} = \frac{1}{k_0(1+g_A)}\left[\frac{1-\delta}{1+g_A}\right]^{x+\tau}\left[1+\frac{1-\delta}{1+g_A}+....+(\frac{1-\delta}{1+g_A})^{x-1}\right]. \qquad (6.2)$$

In the next cycle in which employment is determined by capital, we have that

$$\sum_{i=2x+2\tau}^{3x+2\tau} \frac{dL_i}{dI_0} = \frac{1}{k_0(1+g_A)}\left[\frac{1-\delta}{1+g_A}\right]^{2x+2\tau}\left[1+\frac{1-\delta}{1+g_A}+....+(\frac{1-\delta}{1+g_A})^{x-1}\right], \qquad (6.3)$$

so on and so forth. In order to save notation let

$$T \equiv 1+\frac{1-\delta}{1+g_A}+....+(\frac{1-\delta}{1+g_A})^{x-1} = \frac{1-(\frac{1-\delta}{1+g_A})^{x}}{1-\frac{1-\delta}{1+g_A}}.$$

With this notation in hand we can write equations 3.1, 3.2 and 3.3 as follows

$$\begin{aligned}\sum_{i=1}^{x} \frac{dL_i}{dI_0} &= \frac{T}{k_0(1+g_A)},\\ \sum_{i=x+\tau}^{2x+\tau} \frac{dL_i}{dI_0} &= \frac{T}{k_0(1+g_A)}\left[\frac{1-\delta}{1+g_A}\right]^{x+\tau},\\ \sum_{i=2x+2\tau}^{3x+2\tau} \frac{dL_i}{dI_0} &= \frac{T}{k_0(1+g_A)}\left[\frac{1-\delta}{1+g_A}\right]^{2x+2\tau},\end{aligned}$$

so on and so forth. Thus total effect of the increase of investment in period $0$ on employment in periods $1, ...., \infty$ is

$$\sum_{t=1}^{\infty} \frac{dL_t}{dI_0} = \frac{T}{k_0(1+g_A)}(1+\left[\frac{1-\delta}{1+g_A}\right]^{x+\tau}+\left[\frac{1-\delta}{1+g_A}\right]^{2x+2\tau}+.....).$$

Letting $z \equiv \frac{1-\delta}{1+g_A}$ and $p \equiv z^{x+\tau}$ the parenthesis in the equation above can be written as

$$1 + z^{x+\tau} + z^{2x+2\tau} + .... = 1 + p + p^2 + .... = \frac{1}{(1-(\frac{1-\delta}{1+g_A})^{x+\tau})}. \text{ Then,}$$

$$M \equiv \sum_{t=0} \frac{dL_t}{dI_0} = \frac{1}{sy_0} + \frac{T}{k_0(1+g_A)(1-(\frac{1-\delta}{1+g_A})^{x+\tau})}$$