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Contingent Commodities and Implementation

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Contingent Commodities and Implementation*

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Abstract

In this note we consider the problem whether contingent commodity allocations can be used when the states are not directly contractible. In such a setting a contingent commodity allocation takes the form of a social choice function, and the question is whether this function is implementable (in the sense of full implementation). Using only very mild assumptions on the rule for selecting contingent commodity allocations, we derive a strong negative result which also proves to be robust with respect to different solution concepts employed for implementation. These findings have interesting implications for the interpretation of Arrow–Debreu economies.

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1 Introduction

The Arrow–Debreu model, i. e., a general equilibrium model with location, date, and state contingent commodities, is often considered to be a benchmark for the realization of efficient resource allocation; in recent years its usage has become pervasive, particularly in macroeconomics. The cautionary tales that are told regarding the suitability of the model are usually framed in terms of imperfections like nonconvexities, thin markets, transactions costs, and asymmetries of information among agents. But it is accepted that if the economic situation to be modeled is immune to these imperfections, and the states of nature are "macroeconomic," then the Arrow–Debreu model is an acceptable approximation to reality. Our concern here is a precise formulation of the idea that a state is macroeconomic. We make the point that for the model to "work," the states have to be contractible so that an outside observer (the auctioneer in the Arrow–Debreu model) can independently verify the occurrence of a state.

When we say that the model "works" we mean that there is a mechanism which gives incentives in the right way so as to generate, or implement, the outcomes of the Arrow–Debreu model as the equilibria of a game without introducing any new outcomes (technically speaking, full implementation). As is well known, the Walrasian correspondence can be fully implemented in Nash equilibrium[1] by a number of mechanisms so it would appear that, by a standard extension of the commodity space using (state) contingent commodities, so can Arrow–Debreu equilibrium allocations. We show that such is not the case. We follow up on an early example due to Palfrey and Srivastava (1989) and show that the basic intuition given by them holds much more generally and for all the solution concepts that have been considered in the literature on implementation. Essentially, given preferences over contingent commodities, the planner comes up with a rule to assign commodities to agents as a function of the Arrow–Debreu state of nature; if the rule gives an Arrow–Debreu equilibrium allocation then it is *ex-ante* Pareto optimal. However, once nature has moved, and all the agents are told what nature's move has been, and this is common knowledge, for the planner to be able to assign commodities in the state that nature has chosen, the planner needs to know what nature did. The implementation model does not *a priori* give the planner this information; consequently, the planner must elicit this information regarding nature's choice of the Arrow–Debreu state from the symmetrically informed agents. But at this stage agents have very different incentives relative to the *ex-ante* stage with the end result that under very mild assumptions on the class of economies considered, the allocation rule is not implementable. Hence, unless the planner is able to directly verify the Arrow–Debreu state, an Arrow–Debreu equilibrium allocation cannot be realized.

We emphasize that the strong negative result goes through for all the standard game theoretical equilibrium concepts, so the consideration of stage games and subgame perfection does not help, and it goes through for any *ex-ante* Pareto optimal

[1] Subject to certain technicalities having to do with allocations which are on the boundary of some agent's consumption set.

rule that satisfies an interiority assumption.

One can ask what we learn from such a result. One always pauses and gives some thought to the appropriateness of using the Arrow–Debreu model in a particular situation; our result allows one to have a tighter feeling for the appropriateness since it says that the Arrow–Debreu model can be used provided the analyst is convinced that the uncertainty in his/her model gives rise to states which are verifiable by outside observers (earthquakes and other natural calamities, etc.). The relevant asymmetry of information is between agents and courts and not just between agents as emphasized in the earlier literature (see, e. g., Radner (1968)).

2 The Model

We consider the standard model of an Arrow–Debreu economy with uncertainty. We have a finite set $\mathcal{I} = \{1, \ldots I\}$ of agents and a finite set $\mathcal{L} = \{1, \ldots, L\}$ of physically different goods. The economy exists for two periods. At date 0 there is uncertainty about which of the possible states of the world from the finite set $\mathcal{S} = \{1, \ldots, S\}$ will be realized at date 1, while we assume that at this date the realized state of the world will become common knowledge among the agents. The commodity space is $\mathbb{R}^{L \cdot S}$, a typical element of which is a contingent commodity vector $x = (x_{1,1}, \ldots, x_{L,1}, x_{1,S}, \ldots, x_{L,S})$, where the component $x_{l,s}$ is to be interpreted as $x_{l,s}$ units of good l if at date 1 state s has occurred. An allocation in this economy is $\boldsymbol{x} \in \mathbb{R}^{L \cdot S \cdot I}$. We shall consistently use superscripts to refer to agents in the economy. Hence we write $\boldsymbol{x} = \left(x^1, \ldots, x^I\right)$, where $x^i \in \mathbb{R}^{L \cdot S}$, for all $i \in \mathcal{I}$. Sometimes it will be convenient to have the following shorthand notation available. Let $x \in \mathbb{R}^{L \cdot S}$ and $s \in \mathcal{S}$, then we write $x_s = x_{\cdot,s} = (x_{1,s}, \ldots, x_{L,s})$. Analogously, we write x_l, $\boldsymbol{x}^i$, $\boldsymbol{x}^i_s$, etc.

Each agent i is characterized by her consumption set $X^i = \mathbb{R}^{L \cdot S}_+$, her initial endowments $\omega^i \in \mathbb{R}^{L \cdot S}_+$ and her preferences $\succeq^i$ on the consumption set. We will only consider preferences which can be represented by a utility function and hence will characterize agent i by her utility function $u^i : X^i \to \mathbb{R}$. Indeed, for this note we will assume that agents maximize expected utility, i. e., that each agent has a subjective probability distribution q^i over the states and a state dependent utility function $v^i : \mathbb{R}^L_+ \times \mathcal{S} \to \mathbb{R}$ such that for all $x \in \mathbb{R}^{L \cdot S}$ we have $u^i(x) = \sum_{s \in \mathcal{S}} q^i(s)\, v^i(x_s, s)$; the conceptual importance of this assumption for our approach will become clear at the beginning of the next section. Moreover, we assume that agents' utility functions are monotone and smooth, i. e., at least once continuously differentiable (C^1), where the latter assumption should be viewed as merely technical.

Given agents' consumption sets and initial endowments, we will restrict attention to contingent commodity allocations $\boldsymbol{x}$ for which we have $\boldsymbol{x}^i \in X^i$, for all $i \in \mathcal{I}$, and $\sum_{i \in \mathcal{I}} \boldsymbol{x}^i = \sum_{i \in \mathcal{I}} \omega^i$. Such allocations will be called feasible and the set of all feasible allocations will be denoted by $\boldsymbol{A}$. Clearly, a contingent commodity allocation $\boldsymbol{x} \in \boldsymbol{A}$ can be equivalently written as a function $\boldsymbol{x} : \mathcal{S} \to \mathbb{R}^{L \cdot I}$, $s \mapsto \boldsymbol{x}(s) = \boldsymbol{x}_s$.

The usual way of interpreting a contingent commodity allocation $\boldsymbol{x} \in \boldsymbol{A}$ is as a contract signed at date 0, i. e., when agents know their preferences but the state of the world to prevail at date 1 has not been determined yet. At date 1, every agent receives the quantity specified by such a contract. So, in this interpretation, states are assumed to be "contractible". In this note we explore the consequences of dispensing with that assumption. Given that we do not assume the states of the world to be contractible, and that contingent commodity allocations can be interpreted as social choice functions $\boldsymbol{x} : \mathcal{S} \to \mathbb{R}^{L \cdot I}$, we have to ask whether these social choice functions can be implemented. So the question that interests us is, given a contingent commodity allocation, is it possible to design a mechanism in such a way that for every state s the equilibrium messages of the agents lead to the outcome prescribed by the contingent commodity allocation for that state?

Notice that, contrary to the usual implementation problem, we will assume that ex ante preferences are fixed and given; what changes, the underlying "economy", is the Arrow-Debreu state of the world which determines ex post preferences.

In formalizing the implementation problem sketched above, we will follow the general practice in the implementation literature and assume that endowments are constant. In our context this means that agents' endowments do not vary with the state of the world, i.e., $\boldsymbol{\omega}$ is such that for all pairs $(s,t) \in \mathcal{S} \times \mathcal{S}$ we have $\boldsymbol{\omega}_s = \boldsymbol{\omega}_t$. A mechanism takes the form $g = (M, f)$, where $M = \prod_{i \in \mathcal{I}} M^i$ is the message space and $f : M \to \mathbb{R}^{L \cdot I}$ the outcome function. The messages are meant to convey information concerning the realized state of the world, the outcome function then determining an allocation for that state. Given agents' preferences, and a state s, the mechanism g induces a non-cooperative game Γ_s in which M^i is player i's strategy set and her payoff function is given by $v^i(\cdot, s) \circ f$. It is at this point that the assumption of expected utility maximization becomes important. It allows us to consider separately the situation in any given state ex post, because preferences in a state do not depend on allocations in other states.

Definition 2.1

Let $E(s)$ denote the set of equilibria of Γ_s for a given equilibrium concept.[2] *We say that a mechanism g implements the social choice function $\boldsymbol{x}$ in that equilibrium if, for all states $s \in \mathcal{S}$, we have*

$$f(E(s)) = \boldsymbol{x}(s) .$$

In general, we are not interested in implementing just any contingent commodity allocation but rather want to focus on those that are suggested by some solution concept or social choice correspondence for the type of economies we consider (think, for example, of Arrow-Debreu equilibria).

[2]For a good discussion of implementation in Nash Equilibrium and different refinements, see the survey articles of Maskin (1985) or Moore (1992); also, confer Matsushima (1988) and Abreu and Sen (1991) for the alternative concept of virtual implementation.

3 An Impossibility Result

In this section we will show that there exist economies such that no solution F satisfying some very mild conditions is implementable on a set of economies containing them. Essentially, what we demonstrate is that there exists a general conflict between ex ante considerations and implementability. That this is possible was first noticed by Palfrey and Srivastava (1989) for Arrow-Debreu equilibria.[3]

In fact the following proposition states that if agents utility functions on spot markets do not depend on the state (a common assumption in the literature), the only contingent commodity allocations implementable by any equilibrium concept are the constant ones, that is, they give the same commodity bundles to an agent in every state of the world.

Proposition 3.1
Assume that agents have state independent spot market utility functions, i. e., their utility functions have the form

$$u^i(x) = \sum_{s \in \mathcal{S}} q^i(s)\, v^i(x_s)\,.$$

Let $\Phi : \mathcal{S} \rightrightarrows \mathbb{R}^{L \cdot I}$ be a social choice correspondence. If Φ is implementable (in any equilibrium concept) it has to be constant, that is,

$$\Phi(s) = \Phi(t)\,, \qquad \forall\, s, t \in \mathcal{S}\,.$$

Proof:

Let $g = (M, f)$ be a mechanism implementing Φ so that we have

$$f(\mathrm{E}(s)) = \Phi(s) \qquad \forall\, s \in \mathcal{S}\,. \tag{1}$$

Given the assumption on agents' preferences, for any two states s and t, the non-cooperative games Γ_s and Γ_t induced by the mechanism g in that state coincide; in particular, the outcome function for agent $i \in \mathcal{I}$ is $v^i \circ f$. Therefore, the sets of equilibria of these games also have to coincide. Hence

$$\mathrm{E}(s) = \mathrm{E}(t) \qquad \forall\, s, t \in \mathcal{S}\,,$$

and thus from equation (1)

$$\Phi(s) = \Phi(t)\,, \qquad \forall\, s, t \in \mathcal{S}\,.$$

$\diamond$

[3] In their paper implementation in Bayesian equilibrium is considered, which does not fit into our model. Examining the example they present (cf. Palfrey and Srivastava (1989, Example 1, p. 129) it turns out, however, that the example treats the case of Nash implementation.

While Proposition 3.1 tells us that for certain preferences the only implementable social choice correspondences are constant ones, the example of Palfrey and Srivastava (1989) is an economy of the kind considered in the proposition[4] but such that what we would like to implement, namely the unique Arrow–Debreu equilibrium allocation, is not constant.

In fact we can show that there are economies with state independent spot market utility functions for which no interior ex ante Pareto optimal contingent commodity allocation is constant; hence, there exists a conflict between implementability and ex ante Pareto optimality (for interior allocations).

Proposition 3.2

There exist economies $\mathcal{E} = \big((X^i, u^i, \omega^i)_{i\in\mathcal{I}}\big)$ such that no ex ante Pareto optimal, interior contingent commodity allocation $\boldsymbol{x} \in \mathbb{R}^{L\cdot S\cdot I}_{++}$ is implementable.

Proof:

We will construct such an economy. For $i \in \mathcal{I}$, let $X^i = \mathbb{R}^{L\cdot S}$ and let the utility function $u : \mathbb{R}^{L\cdot S} \to \mathbb{R}$ be defined by

$$u^i(x) = \sum_{s\in\mathcal{S}} q^i(s)\, v\big(x_s\big)\,,$$

where $v : \mathbb{R}^L_+ \to \mathbb{R}$ is a monotone, strictly concave, and differentiable spot market utility function that is independent of the state. Consider two states of the world $s, s' \in S$ with $s \neq s'$ and let the probabilities in agents' utility functions be such that for all $i, j \in \mathcal{I}$, $i \neq j$, we have $q^i(s)q^j(s') \neq q^i(s')q^j(s)$. For all other states the probabilities can be chosen arbitrarily.

Let $\bar{\boldsymbol{x}} \in \mathbb{R}^{L\cdot S\cdot I}$ be an interior Pareto optimal allocation. Then $\bar{\boldsymbol{x}}$ solves the following maximization problem for some $\alpha = \big(\alpha^1, \ldots, \alpha^I\big)$, with $\alpha \gg 0$ and $\sum_{i\in\mathcal{I}} \alpha^i = 1$.

$$\begin{aligned} \max_{\boldsymbol{x}\in\mathbb{R}^{L\cdot S\cdot I}_+} \quad & \sum_{i\in\mathcal{I}} \alpha^i \sum_{s\in\mathcal{S}} q^i(s)\, v\left(\boldsymbol{x}^i_s\right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{I}} \left(\boldsymbol{\omega}^i - \boldsymbol{x}^i\right) \;\geq\; 0 \end{aligned}$$

Monotonicity of the utility functions allows us to write the constraint as an equality.

The first order conditions for this problem are

$$\frac{\frac{\partial v}{\partial x_l}\left(\boldsymbol{x}^i_s\right)}{\frac{\partial v}{\partial x_l}\left(\boldsymbol{x}^j_s\right)} \;=\; \frac{\alpha^j}{\alpha^i}\,\frac{q^i(s)}{q^j(s)}\,.$$

Now consider the two states $s, s' \in \mathcal{S}$ identified above. If the allocation $\bar{\boldsymbol{x}}$ assigned the same spot market allocation in both these states of the world, we would have in particular, for all $i, j \in \mathcal{I}$, $i \neq j$, and for all $l \in \mathcal{L}$,

$$\frac{\frac{\partial v}{\partial x_l}\left(\bar{\boldsymbol{x}}^i_s\right)}{\frac{\partial v}{\partial x_l}\left(\bar{\boldsymbol{x}}^j_s\right)} \;=\; \frac{\frac{\partial v}{\partial x_l}\left(\bar{\boldsymbol{x}}^i_{s'}\right)}{\frac{\partial v}{\partial x_l}\left(\bar{\boldsymbol{x}}^j_{s'}\right)}\,,$$

[4]Notice that, in the example in Palfrey and Srivastava (1989), agent 1's utility function can equivalently be written as $\frac{1}{3}\log(x_1) + \frac{2}{3}\log(x_2)$.

while by assumption

$$\frac{\alpha^j}{\alpha^i}\,\frac{q^i(s)}{q^j(s)} \neq \frac{\alpha^j}{\alpha^i}\,\frac{q^i(s')}{q^j(s')}\,.$$

This would lead to a contradiction.

Therefore, it must be the case that $\bar{\boldsymbol{x}}_s \neq \bar{\boldsymbol{x}}_{s'}$. But then it follows from Proposition 3.1 that $\bar{\boldsymbol{x}}$ is not implementable.

$\diamond$

As it stands, Proposition 3.2 does not state that no ex ante Pareto optimal solution is partially (ex post) implementable, since it assumes interiority of the allocations.

4 A Positive Result

We have seen in Proposition 3.1 that state independent spot market utility functions impose severe conditions on implementable contingent commodity allocations. So, obviously, if we look for positive results, we have to allow for state dependence of the spot market utility functions.

In what follows, we will focus on implementability in Nash equilibrium for which Maskin monotonicity introduced by Maskin (1977) is a necessary condition, which together with no veto power proves to be also sufficient for implementability with three or more agents. If we assume agents' preferences to be monotonic, no veto power is vacuously fulfilled, so that Maskin monotonicity becomes both a necessary and sufficient condition. Defining the lower contour set of a spot market commodity bundle $x \in \mathbb{R}^L$ in state $s \in \mathcal{S}$ for the spot market utility function v^i as $\mathfrak{L}\left(x, s, v^i\right) = \left\{y \in \mathbb{R}^L \;:\; v^i\left(y, s\right) \leq v^i\left(x, s\right)\right\}$, Maskin monotonicity in our context reads as follows.

Definition 4.1
A contingent commodity allocation $\boldsymbol{x}$ is Maskin monotonic *if for all states $s, s' \in \mathcal{S}$ we have that $\mathfrak{L}\left(\boldsymbol{x}^i_s, s, v^i\right) \subseteq \mathfrak{L}\left(\boldsymbol{x}^i_s, s', v^i\right)$, for all $i \in \mathcal{I}$, implies $\boldsymbol{x}_{s'} = \boldsymbol{x}_s$.*

For the special case of $L = 2$ the well known strict Spence–Mirrlees single-crossing property provides a condition which, if satisfied by all agents' utility spot market functions $v^i \;:\; \mathbb{R}^L \times \mathcal{S} \to \mathbb{R}$, ensures that Maskin monotonicity is trivially satisfied because the situation for which the property bites never occurs.

Definition 4.2
An agent's spot market utility function $v^i \;: \mathbb{R}^2_+ \times \mathcal{S}$ satisfies the strict Spence-Mirrlees single-crossing property *if there is an ordering of $\mathcal{S}$ such that $s > s'$ implies*

$$\frac{\partial v^i(s,x)}{\partial x_1}\Big/\frac{\partial v^i(s,x)}{\partial x_2} > \frac{\partial v^i(s',x)}{\partial x_1}\Big/\frac{\partial v^i(s'x)}{\partial x_2}$$

for all $x \in \mathbb{R}^2_{++}$ and for all $s, s' \in \mathcal{S}$.

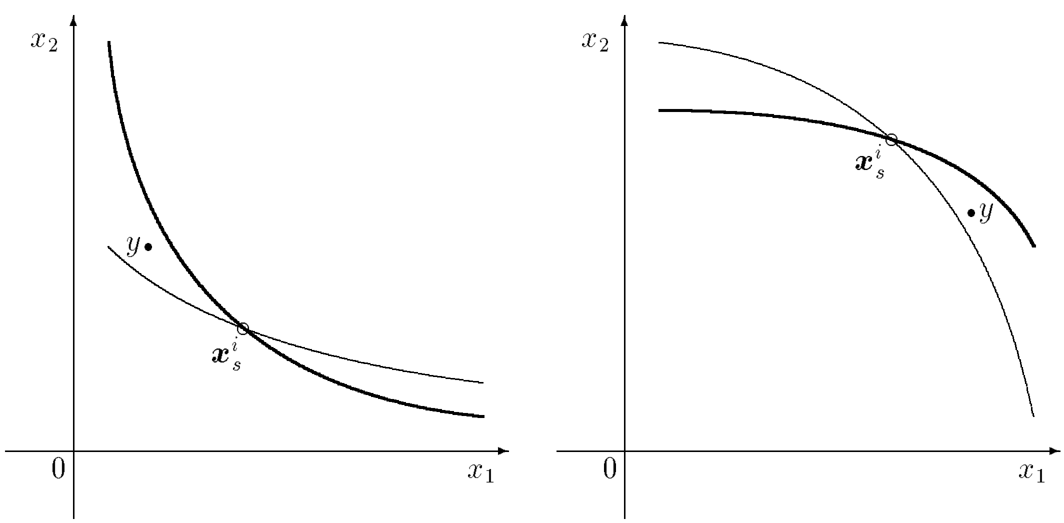
The strict single-crossing property readily admits an economic interpretation. As an example, let the two goods be umbrellas and sun-lotion and let there be two states of the world, rain and sunshine. It seems more than reasonable to assume that agents' marginal rates of substitution between umbrellas and sun-lotion change unambiguously with the weather, everybody being willing to exchange more sun-lotion for an umbrella in case it rains than on a hot and sunny summer day; that is, in this example, strict single-crossing is satisfied with the same order on the states of the world for all agents.

With the above definition in place we have the following result.

Proposition 4.3
Let $L = 2$, $I \geq 3$, and $\mathcal{E} \in \boldsymbol{\mathcal{E}}$ be an economy in which there is at least one agent whose spot market utility function satisfies the strict single-crossing property.[5] *Then any social choice correspondence F yielding interior allocations is partially Nash implementable on $\mathcal{E}$.*

Proof:
The proof follows immediately from the inspection of Figure 1 which depicts the indifference curves of agent $i \in \mathcal{I}$ in two states s and $s' \in \mathcal{S}$ through $\boldsymbol{x}_s^i$. The left

Figure 1: Single Crossing Property and Lower Contour Sets



Case 1: $s > s'$ Case 2: $s < s'$

picture, Case 1, shows the situation if $s > s'$ in the order on $\mathcal{S}$ corresponding to agent i. The bold line is the indifference curve in state s, which is steeper than the thin line depicting the indifference curve in state s'. Case 2, $s < s'$, is drawn on

[4]It is intuitively clear, that the strict single-crossing property is robust, i.e., that a slight perturbation of the preferences does not destroy the property. Indeed, this can be made precise using the techniques presented in Mas-Colell (1985).

the right, the bold line again depicting the indifference curve in state s. The lower contour sets are the areas below the indifference curves.

In both cases the point y lies in $\mathfrak{L}(\boldsymbol{x}_s^i, s, v^i)$ but not in $\mathfrak{L}(\boldsymbol{x}_s^i, s', v^i)$ which illustrates, that the condition in the definition of Maskin monotonicity cannot be satisfied for spot market utility function satisfying the strict Spence-Mirrlees single-crossing condition. Thus Maskin monotonicity is trivially satisfied for any contingent commodity allocation.

$\diamond$

Remark 4.4

Proposition 4.3 can be easily generalized to cover the case in which there are more than two goods. Also, instead of having one agent whose preferences fulfill the strict single crossing property for all states and all points in (spot market) commodity space, it would be enough to have, for any pair of states and any point in (spot market) commodity space, at least one consumer whose preferences satisfy the strict single crossing property locally. This consumer can vary among states and across commodity bundles.

5 Concluding Remarks

How should the results we have obtained be interpreted? In the case in which utilities are state independent and separable across states (a case usually assumed in the literature), they tell us that for the Arrow–Debreu model of contingent commodities to make sense, one needs to assume that the states of the world are directly contractible. In other words, the use of contingent commodities presupposes the existence of some institution which is able to verify the state of the world and to enforce transactions agreed on ex ante.

This general message is qualified by the positive result of Section 4 which shows, that insisting on contractibility of the states of the world can become unnecessary, if utilities are state dependent and satisfy the strict single crossing property.

Given the observation that, in general, use of the Arrow-Debreu model assumes the existence of an institution able to enforce contingent commodity allocations, it may be worthwhile to reconsider the model of Radner (1968) in which agents are endowed with information structures, i. e. partitions of the set of possible states of the world, where each element of the partition of an agent is to be interpreted as a set of states of the world among which this agent is unable to distinguish at the time of delivery. Radner argues that in this model the possible trades among the agents have to be measurable with respect to the join of their information structures. Our result seems to suggest that this condition is not as convincing as it may seem. If states of the world have to be directly contractible, i. e. verifiable by an institution which enforces contingent contracts, measurability with respect to agents' information structures is neither necessary nor sufficient, the only relevant thing being that all transactions be measurable with respect to the information structure of the institution.

We briefly remark that combining our findings about ex post implementation, i. e. implementation of sets of contingent commodities, with the classical results on ex ante implementation, e. g. of the Walrasian correspondence seems an interesting endeavour. But formalizing this by simply considering a two stage implementation process, by combining the results on ex ante and ex post implementation, does not suffice since in the resulting extensive form mechanism new strategic possibilities arise that were not present when considering each of the two stages separately. We hope to deal with this line of research in the future.

We end this note by mentioning that additional problems arise if we consider implementing a social choice *correspondence (SCC)*. In this case the SCC must satisfy a closedness requirement (cf. Palfrey and Srivastava (1989)); however, it can be shown by means of simple examples that the Pareto and the Arrow-Debreu correspondences are not closed and, hence, not implementable.[6]

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[6]The examples refered to are available from the authors upon request.