TO MERGE OR NOT TO MERGE: THAT IS THE QUESTION

Luis C. Corchón and Ramón Faulí-Oller

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Correspondence to Luis C. Corchón: Universidad Carlos III. Departamento de Economía. C/ Madrid 126. 28093 Getafe. Spain e-mail: lcorchon@eco.uc3m.es.

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** L. Corchón: Universidad Carlos III de Madrid, Ramon Fauli-Oller: Universidad de Alicante.

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A B S T R A C T

In this paper we analyze the implementation of socially optimal mergers when the regulator is not informed about the parameters that determine social and private gains from potential mergers. We find that most of the standard tools in dominant strategy implementation, like the revelation principle or the Vickrey-Clarke-Groves mechanism can not be applied in our framework. We show that implementation in dominant strategies of the optimal merger policy without budget balance is possible under an additional assumption. The same assumption makes possible the implementation in Nash equilibrium of the optimal merger policy with budget balance.

Keywords: Merge, Antitrust, Welfare.

1. Introduction.

Let's suppose two firms, say Boeing and McDonnell- Douglas, decide to merge. What effects should such a merger have on social welfare?. First, the degree of competition falls and this affects social welfare negatively. On the other hand, there might be reductions in certain costs (saving fixed costs, synergy gains, etc.) or technological improvements that enhance social welfare. The effect that finally dominates depends entirely on the specification of the problem at hand.[1]

It is clear that individual incentives to merge may lead to the wrong decisions from the social welfare point of view. That is why, in most western countries, certain mergers have had to be submitted for the approval of an independent body, such as the Justice Department or the Federal Trade Commission, in USA. Both issued Merger Guidelines that expressed the opinions of the regulators about when a merger would be likely to be contested by this department.The basic information used by the Merger Guidelines refers to market shares. However, they also ask information to the firms since they recognize that they may posses information which is not in the hands of the regulator.[2]

The use of market shares in Merger Guidelines has been rationalized in several papers (Farrell and Shapiro (1990) and Levin (1990)). However, this procedure is problematic in two counts. First it does not take into account that firms may manipulate market shares in order to fool the regulator. Second, market shares can not be used to predict postmerger equilibrium. In the present paper, we explore, using the Theory of Implementation, the potential use of the dialog between the regulator and the firms in the process of deciding whether to approve a merger.

The theory of implementation studies the outcomes that are reachable under certain game theoretical solutions when the designer is not informed about the characteristics of agents. The main application of implementation theory is the design of procedures that achieve certain social goals when agents behave strategically.

We study the implementation of socially optimal decisions on merger by means

[1] These effects have been studied by Williamson (1968), Salant, Switzer and Reynolds (1983), Davidson and Deneckere (1985), Perry and Porter (1985), Salop et alia (1987) and Salinger (1988).

[2] The policy-making implications of the game played by the antitrust authority and firms have been studied by Besanko and Spulber (1993) in the particular case of synergy gains. However they do not use the standard framework of implementation. Rather, in their model, the antitrust authority is a full-fledged player.

of dominant strategies and Nash equilibrium in certain specific environments. We first show that some of the basic results of dominant strategy implementation do not hold in our framework. For instance, the Vickrey-Clarke-Groves mechanism can be adapted to our set up, but this adaptation is not convincing since it requires consumers to announce the characteristic of all firms (Proposition 1). Moreover, the usual Revelation Principle is not true in our framework (Proposition 2). Instead, a weak form of this principle holds (see Proposition 3). We find a condition (our Assumption 1, Section 4) under which the optimal decision on mergers is implementable in dominant strategies without budget balancing (Proposition 5) and in Nash equilibrium with budget balance (Proposition 7). If this assumption is not fulfilled, the optimal decision is not implementable in dominant strategies without budget balance (Proposition 4) or in Nash equilibrium with budget balance (Proposition 6).

2. The model.

There are n players (firms). In most of the paper we will assume that n = 2. The type of firm i, denoted by θ_i, is a description of all relevant characteristics (costs, demand, price of inputs, etc.) before and after any possible merger regarding firm i. Let Θ_i be the set of all possible characteristics of firm i. Let $\Theta = X_{i=1}^{n}\Theta_i$, be the set of characteristics with typical element θ (most of our results do not need the assumption that Θ has Cartesian product structure). We now spell out two special instances of our problem that will be used in the sequel.

Rationalization: We have two firms with average cost c_1 and c_2 respectively. It is known that $c_1 \leq c_2$, but their actual values are unknown. The merger allows us to transfer production from the high cost to the low cost firm.

Synergy Gains: We have two firms producing with average cost c. If both firms merge, average cost will be $d < c$. d and c are unknown to the regulator.[3]

Let A be the set of alternatives. An alternative, denoted by a, is a description of how the n firms are merged. These merger decisions involve transfers of money among firms. Let t_i be the transfer of money to player i. Typically, if firm i is bought during the merger stage t_i will be positive.[4] We assume that once

[3]Empirical evidence about the existence of synergies in the Financial Services industry is gathered in Berger, Demsetz and Strahan (1998).

[4]We assume that all payments are controlled by the regulator. Depending on the context this may be an apropriate assumption or not. However when the firms can make side payments

the merger decision has been taken, the remaining firms engage in some form of competition (Cournot, Bertrand, etc.). We represent this by writing $\Pi_i(a, \theta)$ as the expected payoffs of i as a function of market structure (a) and characteristics of all firms (θ).[5] If firm i is bought during the merger stage $\Pi_i(a, \theta) = 0$. Thus, the payoff function of firm i is $\Pi_i(a, \theta) + t_i$ also written as $V_i(a, \theta, t_i)$. Notice that in general the payoff of firm i depends on the characteristics of all firms because the payoff function encapsulates after-merger competition (i.e. it is a reduced form). In the context of Bayesian games this situation is called common values. The case in which the payoff of firm i depends only on the characteristic of i is called private values (see e.g. Fudenberg and Tirole (1991) pp. 297-8). The latter would occur if characteristics of firms are fixed costs provided that they are such that all firms are always active. As we will see in this paper, the fact that we deal with common values has important implications for Dominant Strategy implementation.

We assume that the regulator has no power whatsoever to interfere in the nature of competition, once merger decisions have been taken.[6] In this sense we focus on *structure regulation* and not on *conduct regulation* (Vickers (1995)). However the regulator can enforce the rules under which mergers and transfers take place by means of a mechanism $\{M_i, g\}_{i=1...n}$ where M_i is the set of all possible messages sent by i, with typical element m_i. Let $m \in M \equiv X_{i=1}^{n} M_i$ be a list of messages. $g = (h(),\ t_1(), ..., t_n())$ is the outcome function where $h : M \longrightarrow A$ decides mergers as a function of messages and t_i: $M \longrightarrow R$, $i = 1, ..., n$ decides the money transferred from or to firm i. This way of writing transfers assumes that there is a clearing house in which all payments are centralized. In the case of two firms (which is the main focus of our analysis) these transfers are the payment made by the buyer and the payment received by the seller. Sometimes we will require that $\sum_{i=1}^{n} t_i(m) \leq 0$ for any $m \in M$ (feasibility) or $\sum_{i=1}^{n} t_i(m) = 0$ for any $m \in M$ (budget balance).

If the regulator had complete information, she would like to allow certain merg-

that the regulator can not control, any kind of merger regulation becomes hopeless and this is why this issue is usually ignored in the literature.

[5] If there are several equilibria we might assume that each firm has a subjective probability distribution on the occurrence of different equilibria and so $\Pi_i()$ represents expected profits.

[6] This is just another way of saying that, if truthful revelation of characteristics is achieved, this information is not used later on to regulate firms. In the implementing mechanism presented in Section 4 the messages sent by firms do not permit the complete identification of characteristics. Another implication of this assumption is that output is not contractible, something that may be appropriate for certain type of mergers (banks, airlines,...).

ers and to forbid others depending on the characteristics of firms. Let $\phi : \Theta \longrightarrow A$ represent the optimal structure of mergers as a function of the characteristics of firms. This function is called a Social Choice Rule (SCR). In what follows we will be mostly concerned with a specific SCR: Let the consumer surplus be written as $W(a, \theta)$. The social surplus, demoted by SS, is defined as $\sum_{i=1}^{n} \Pi_i(a, \theta) + W(a, \theta)$. Social surplus as a function of market structure is denoted by $SS(a_i, \theta)$. Then, ϕ^o is defined as follows; $\phi^o(\theta) = \arg\max_{a \in A} SS(a_i, \theta^j)$. We will call ϕ^o the efficient merger policy. An extended SCR $\phi : \Theta \longrightarrow A \times R^n$ maps the characteristics of firms into the decision on mergers *and* transfers.

A strategy for i is a mapping $s_i : \Theta \rightarrow M_i$.

A mechanism $\{M_i, g\}_{i=1...n}$ implements the extended SCR ϕ in dominant strategies if there are strategies $(s_1(), ..., s_n()) = s()$ such that:

a) $g(s(\theta)) = \phi(\theta)$ for all $\theta \in \Theta$.

b) $V_i(g(s_i(\theta), m_{-i}), \theta) \geq V_i(g(m_i, m_{-i}), \theta)$ for all $(m_i, m_{-i}) \in M$, and $\theta \in \Theta$.

A mechanism $\{M_i, g\}_{i=1...n}$ implements the extended SCR ϕ in Nash equilibrium if there are strategies $(s_1(), ..., s_n()) = s()$ such that:

a) $V_i(g(s_i(\theta), s_{-i}(\theta)), \theta) \geq V_i(g(m_i, s_{-i}(\theta)), \theta)$ for all $m_i \in M_i$ and $\theta \in \Theta$.

b) For all strategies $s()$ fulfilling a) above, $g(s(\theta)) = \phi(\theta)$ for all $\theta \in \Theta$.

3. Dominant Strategies: Impossibility Results

The most satisfying game-theoretical solution is dominant strategies, because if agents have dominant strategies, the strategic analysis of the game becomes akin to an individual decision problem. However, if the domain of possible characteristics is large enough, efficient and non-dictatorial allocations can not be achieved by means of dominant strategies (Hurwicz (1972), Gibbard (1973), Satterthwaite (1975)). However, in economies with public goods and quasi-linear utility functions, there is a mechanism (called Vickrey-Clarke-Groves mechanism), such that announcing the true characteristics is a dominant strategy for each agent, and the decision regarding the public good is efficient (Clarke (1971), Groves (1973), Groves and Loeb (1975)).[7] However, this mechanism does not achieve budget balance. Notice that the decision on mergers can be regarded as the decision on the level of a public good and payoffs are linear in money and thus, the domain restrictions cited above are satisfied in our problem.

[7] Moreover, in the above domain, any mechanism attaining efficient outcomes must be a Vickrey-Clarke-Groves mechanism (Green and Laffont (1979) and Laffont and Maskin (1980))

However, there are two important differences between our model and the setting where the above results are proved. First, we consider common values. Second, the welfare of consumers enters the social surplus, but it is not realistic to assume that they can participate in the process because it is hard to see how they would be informed about the characteristics of firms. We now show that this second difference is the one that explains the impossibility results regarding the implementation in dominant strategies of the efficient merger policy.

To illustrate this point, we introduce an additional player that represents consumers. Her payoff coincides with the consumer surplus and she knows the parameters defining the economy. In this case it is possible to implement the efficient merger policy in dominant strategies by using a mechanism that generalizes the Vickrey-Clarke-Groves mechanism to the case of common values.

For simplicity let us assume that there are only two firms. Subscript 0 identifies the variables that refer to consumers and subscripts 1 and 2 identify the variables that refer to firms. Let $M_1 = M_2 = M_0 = \Theta$. The outcome function is as follows:

$h(m) = \arg\max_{a \in A} W(a, m_0) + \Pi_1(a, m_1) + \Pi_2(a, m_2)$
$t_0(m) = \Pi_1(h(m), m_1) + \Pi_2(h(m), m_2)$
$t_1(m) = W(h(m), m_0) + \Pi_2(h(m), m_2)$
$t_2(m) = W(h(m), m_0) + \Pi_1(h(m), m_1)$

Proposition 1. *Let $\bar{\theta}$ be the true economy. Then $(\bar{\theta}, \bar{\theta}, \bar{\theta})$ is a dominant strategy for 0, 1 and 2.*

Proof. $W(h(\bar{\theta}, m_1, m_2), \bar{\theta}) + \Pi_1(h(\bar{\theta}, m_1, m_2), m_1) + \Pi_2(h(\bar{\theta}, m_1, m_2), m_2) \geq$
$W(h(m_0^{'}, m_1, m_2), \bar{\theta}) + \Pi_1(h(m_0^{'}, m_1, m_2), m_1) + \Pi_2(h(m_0^{'}, m_1, m_2), m_2)$ for any $m_0^{'}, m_1, m_2$. Thus, $\bar{\theta}$ is a dominant strategy for the consumer. A similar reasoning applies to firms 1 and 2. ■

The problem with the mechanism above is that consumers are assumed to know the parameters that define cost functions.[8] From now on, we will consider mechanisms in which only firms send messages.

A way of narrowing down the class of implementing mechanisms is to focus on those in which agents have incentive to tell the truth. This procedure is validated by the Revelation Principle defined below.

[8]In cases in which consumer surplus only depends on market structure, consumers need not to be considered. This includes the case where the only effect of mergers is saving in fixed costs.

REVELATION PRINCIPLE: Suppose that an extended SCR ϕ is implemented by a mechanism $\{M_i, g\}_{i=1...n}$ in dominant strategies. Then, there is a revelation mechanism in which the message space of each player is Θ_i and the outcome function is $\phi : \Theta \to A \times R^n$ for which it is a dominant strategy for each player to announce her true characteristic.

The usefulness of the revelation principle is that knowledge of the social choice rule to be implemented is sufficient to construct the revelation mechanism. Unfortunately, the revelation principle is not true in our setting as is shown by the following result.

Proposition 2. *The Revelation Principle stated above does not hold in our framework.*

Proof. Let $n = 2$, $\#\Theta_2 = 1$ (i.e. the characteristic of firm 2 is known) and $\Theta_1 = \{\theta^1, \theta^2\}$. Suppose that ϕ^o can be implemented. If the revelation Principle were true, ϕ^o may be truthfully implemented by a direct mechanism in which only firm 1 is allowed to send messages. Hence,

$\Pi_1(a_1, \theta^1) + t_1(\theta^1) \geq \Pi_1(a_2, \theta^1) + t_1(\theta^2)$ for any θ^1 for which $\mathrm{a}_1 = \phi^o(\theta^1)$ and

$\Pi_1(a_2, \theta^2) + t_1(\theta^2) \geq \Pi_1(a_1, \theta^2) + t_1(\theta^1)$ for any θ^2 and $\mathrm{a}_2 = \phi^o(\theta^2)$.

Now let a_1 = merger of 1 and 2, a_2 = no merger, θ^1 a state for which a_1 is socially optimal and θ^2 a state for which a_2 is socially optimal. Then manipulating the expressions above we get:

$$\Pi_1(a_1, \theta^1) - \Pi_1(a_1, \theta^2) \geq \Pi_1(a_2, \theta^1) - \Pi_1(a_2, \theta^2) \tag{3.1}$$

for any $\mathrm{a}_1, a_2, \theta^1$ and θ^2 such that $\mathrm{a}_1 = \phi^o(\theta^1)$ and $\mathrm{a}_2 = \phi^o(\theta^2)$.

Now we construct an example in which (3.1), above, is violated.

Suppose that demand is given by P=1-X and n=2. Firm 2's cost is known and it is denoted by c_2. It is not lower than the unknown cost of firm 1. We denote by c the cost of firm 1 if its cost is lower than $\mathrm{c}_1^* = \dfrac{-5 + 22c_2}{17}$ and by d otherwise. It can be shown that in the former case monopolization increases total surplus while in the latter case it reduces total surplus. Condition (3.1) in this case is given by:

$$(\frac{1-c}{2})^2 - (\frac{1-d}{2})^2 \geq (\frac{1-2c+c_2}{3})^2 - (\frac{1-2d+c_2}{3})^2.$$

However, this condition is never satisfied because we have that

$$(\frac{1-c}{2})^2 - (\frac{1-d}{2})^2 - (\frac{1-2c+c_2}{3})^2 + (\frac{1-2d+c_2}{3})^2 =$$

$$\frac{(d-c)(2+7c+7d-16c_2)}{36},$$

which is negative because

$$(2+7c+7d-16c_2) \leq 2+7(\frac{-5+22c_2}{17})+7d-16c_2 = \frac{c_2-1}{17} < 0.$$

Although implementation of the optimal merger policy is not possible by asking firm 1 about its costs, it becomes possible if we ask firm 2 instead. This implies that the revelation principle does not hold. Consider the following mechanism where m is the message sent by Firm 2 about c_1.

If $m < c_1^*$, then merger and firm 2 receives $\Pi_2(a_2, c_1^*)$.

If $m \geq c_1^*$, then no merger and no transfer.

Let c_1^+ be the true state.

Case 1: $c_1^+ < c_1^*$ (merger increases welfare). If $m < c_1^*$, firm 2 receives $\Pi_2(a_2, c_1^*)$. If $m \geq c_1^*$, firm 2 receives $\Pi_i(a_2, c_1^+)$. It obtains more by announcing $m < c_1^*$ because firm 2 duopoly profits are decreasing on c_1 and the merger is implemented.

Case 2: $c_1^+ \geq c_1^*$ (merger reduces welfare). If $m < c_1^*$, firm 2 receives $\Pi_2(a_2, c_1^*)$. If $m \geq c_1^*$, firm 2 receives $\Pi_2(a_2, c_1^+)$. It obtains more by announcing $m \geq c_1^*$, because firm 2 duopoly profits are decreasing on c_1 and the merger is not implemented. ■

Clearly, the fact that we deal with common values is the main culprit of the failure of the Revelation Principle. In our case, only a weak form of this principle holds. This form is stated in the next proposition and it will be used later on to prove the impossibility of implementing $\phi^o()$ with or without budget balance.

Proposition 3. *Suppose that an extended SCR ϕ is implemented by a mechanism $\{M_i, g\}_{i=1...n}$ in dominant strategies $(s_1(), ..., s_n()) = s()$. Then, there is a mechanism in which the message space of each player is Θ and the outcome function is $s \circ g \equiv f : \Theta^n \rightarrow A \times R^n$ for which it is a dominant strategy for each player to announce the true state of the world.*

Proof. From the definition of dominant strategy implementation it follows that:

$V_i(g(s_i(\theta), s_{-i}(\theta')), \theta) \geq V_i(g(m_i, s_{-i}(\theta')), \theta)$ for all $m_i \in M_i$, $\theta, \in \Theta, and\ \theta' \in \Theta^{n-1}$.

$V_i(g(s_i(\theta), s_{-i}(\theta')), \theta) \geq V_i(g(s_i(\theta''), s_{-i}(\theta')), \theta)$ for all $\theta, \theta'' \in \Theta, \theta' \in \Theta^{n-1}$.

$V_i(f(\theta, \theta'), \theta) \geq V_i(f(\theta'', \theta'), \theta)$ for all $\theta, \theta'' \in \Theta, \theta' \in \Theta^{n-1}$. ■

Note that the message space for a player in the mechanism above is such that she reveals the characteristics of *all* firms. In this sense, the link between dominant strategies and incomplete information is lost in our case. This is not surprising since the domain of a strategy for firm i is Θ and not Θ_i. Notice too that in contrast with the Revelation Principle, in the result proved above, in order to know the outcome function of the implementing mechanism, we must know the strategies used by agents.

We are now ready to establish an impossibility result:

Proposition 4. *ϕ^o can not be implemented in dominant strategies in every possible domain when only firms are allowed to send messages.*

Proof. Assume two firms. The two possible market structures are: monopoly because firm1 buys firm 2 (denoted by a_1) and duopoly (denoted by a_2). There are two possible economies, denoted by θ^1 and θ^2 such that $\phi^o(\theta^1) = a_1$ and $\phi^o(\theta^2) = a_2$.

Suppose we have a mechanism that implements the SCR ϕ^o in dominant strategies. By Proposition 3, it is a dominant strategy for each player to announce the true state of the world. If firm i announces economy θ_j this is denoted by θ_i^j. Thus, if θ^1 is true,

$$\Pi_1(h(\theta_1^1, \theta_2^2), \theta^1) + t_1(\theta_1^1, \theta_2^2) \geq \Pi_1(h(\theta_1^2, \theta_2^2), \theta^1) + t_1(\theta_1^2, \theta_2^2) \tag{3.2}$$

$$\Pi_2(h(\theta_1^1, \theta_2^1), \theta^1) + t_2(\theta_1^1, \theta_2^1) \geq \Pi_2(h(\theta_1^1, \theta_2^2), \theta^1) + t_2(\theta_1^1, \theta_2^2) \tag{3.3}$$

And if θ^2 is true,

$$\Pi_1(h(\theta_1^2, \theta_2^2), \theta^2) + t_1(\theta_1^2, \theta_2^2) \geq \Pi_1(h(\theta_1^1, \theta_2^2), \theta^2) + t_1(\theta_1^1, \theta_2^2) \tag{3.4}$$

$$\Pi_2(h(\theta_1^1, \theta_2^2), \theta^2) + t_2(\theta_1^1, \theta_2^2) \geq \Pi_2(h(\theta_1^1, \theta_2^1), \theta^2) + t_2(\theta_1^1, \theta_2^1) \tag{3.5}$$

Adding equations (3.2) and (3.4) we have:

$$\Pi_1(h(\theta_1^1, \theta_2^2), \theta^1) + \Pi_1(h(\theta_1^2, \theta_2^2), \theta^2) \geq \Pi_1(h(\theta_1^2, \theta_2^2), \theta^1) + \Pi_1(h(\theta_1^1, \theta_2^2), \theta^2).$$

Adding equations (3.3) and (3.5) we have:

$$\Pi_2(h(\theta_1^1,\theta_2^1),\theta^1)+\Pi_2(h(\theta_1^1,\theta_2^2),\theta^2)\geq \Pi_2(h(\theta_1^1,\theta_2^2),\theta^1)+\Pi_2(h(\theta_1^1,\theta_2^1),\theta^2).$$

Since the mechanism implements ϕ we have that $h(\theta_1^i,\theta_2^i)=a_i$ $i=1,2$ and $\Pi_2(a_1,\theta^1)=0$. Thus, the previous two equations can be rewritten as follows:

$$\Pi_1(h(\theta_1^1,\theta_2^2),\theta^1)+\Pi_1(a_2,\theta^2)\geq \Pi_1(a_2,\theta^1)+\Pi_1(h(\theta_1^1,\theta_2^2),\theta^2)$$

$$\Pi_2(h(\theta_1^1,\theta_2^2),\theta^2)\geq \Pi_2(h(\theta_1^1,\theta_2^2),\theta^1).$$

If $h(\theta_1^1,\theta_2^2)=a_1$, the first equation can be written as:

$$\Pi_1(a_1,\theta^1)-\Pi_1(a_2,\theta^1)\geq \Pi_1(a_1,\theta^2)-\Pi_1(a_2,\theta^2) \tag{3.6}$$

If $h(\theta_1^1,\theta_2^2)=a_2$ the second equation can be written as follows:

$$\Pi_2(a_2,\theta^2)\geq \Pi_2(a_2,\theta^1). \tag{3.7}$$

Therefore, a necessary condition for implementation is that one of the last two equations hold.

We now prove that the previous conditions do not hold in the following example. Market demand is given by $P=100-q$, where P denotes price and q quantity. The marginal costs of the firms are known. In duopoly, each firm has a marginal cost 10. In monopoly the firm produces at marginal cost 5. In economy θ^1, each firm has a capacity $k_1=\frac{95}{4}$ and in economy θ^2 each firm has a capacity $k_2=30$. The production capacity of the monopoly amounts to $2k_i$.

Profit functions are the following.

$$\Pi_1(a_1,\theta^1)=\Pi_1(a_1,\theta^2)=\left(\frac{95}{2}\right)^2$$

.

$$\Pi_i(a_2,\theta^1)=\frac{7600}{9} \text{ and } \Pi_i(a_2,\theta^2)=30^2, i=1,2.$$

Social surplus, denoted by $SS(a_i,\theta^j)$, can be easily calculated:

$$SS(a_1,\theta^1)=SS(a_1,\theta^2)=\left(\frac{3}{2}\right)\left(\frac{95}{2}\right)^2$$

$$SS(a_2, \theta^1) = \frac{25175}{8} \text{ and } SS(a_2, \theta^2) = 4\left(\frac{90}{3}\right)^2.$$

Thus:

$$SS(a_1, \theta^1) > SS(a_2, \theta^1) \text{ and } SS(a_2, \theta^2) > SS(a_1, \theta^2).$$

Implying that $\phi^o(\theta^1) = a_1$ and $\phi^o(\theta^2) = a_2$. However,

$$\Pi_1(a_1, \theta^1) - \Pi_1(a_2, \theta^1) < \Pi_1(a_1, \theta^2) - \Pi_1(a_2, \theta^2) \text{ and}$$

$$\Pi_2(a_2, \theta^2) < \Pi_2(a_2, \theta^1),$$

contradicting the necessary conditions for implementation stated above. ■

Notice that, unlike the standard impossibility theorems mentioned at the begining of the section, our impossibility result does not depend on the budget being balanced or not. Since our domain of economies includes those with private values, an adaptation of standard arguments shows the impossibility of implementing efficient decisions with a balanced budget and thus in the next section, we will concentrate on mechanisms in which the budget is not balanced.[9] We will see that in this case, the possibility of implementing the efficient merger policy hinges on the kind of environments in which the mechanism is applied.

4. Dominant Strategies: Possibility results.

In this section we demonstrate that in a suitably restricted domain, the efficient merger policy ϕ^o can be implemented in dominant strategies.

We focus on the case of two firms, 1 and 2. We consider just two possible market configurations: duopoly and monopoly because firm 1 buys firm 2. This simplification of the merger process is justified since it is the simplest possible form of merger. If in this case the efficient merger policy can not be implemented, there is no hope that this could be done in more complicated cases. If it can, we may hope to obtain insights that may be useful to treat the general case.

The following notation is needed. ΔW denotes the change in social surplus due to monopolization, P_a seller's duopoly profits and P_b the difference between

[9]If lump-sum taxes are feasible, in principle, it will be possible to balance the budget by taxing or subsidizing consumers, providing that the tax is small.

the buyer's monopoly and duopoly profits. Think of P_a as the minimum price that induces firm 2 to sell and of P_b as the maximum price that firm 1 is prepared to pay for the acquisition of firm 2. When needed, ΔW, P_a and P_b will be written as a function of the underlying characteristic θ as $\Delta W(\theta)$, $P_a(\theta)$ and $P_b(\theta)$.

Let D be the set of possible values taken by P_a and E the set of possible values taken by P_b. We are now ready to state the following assumption that later on we will show to allow the implementation in dominant strategies of the efficient merger policy.

Assumption 1: There is a strictly increasing and onto function $f : D \longrightarrow E$ such that

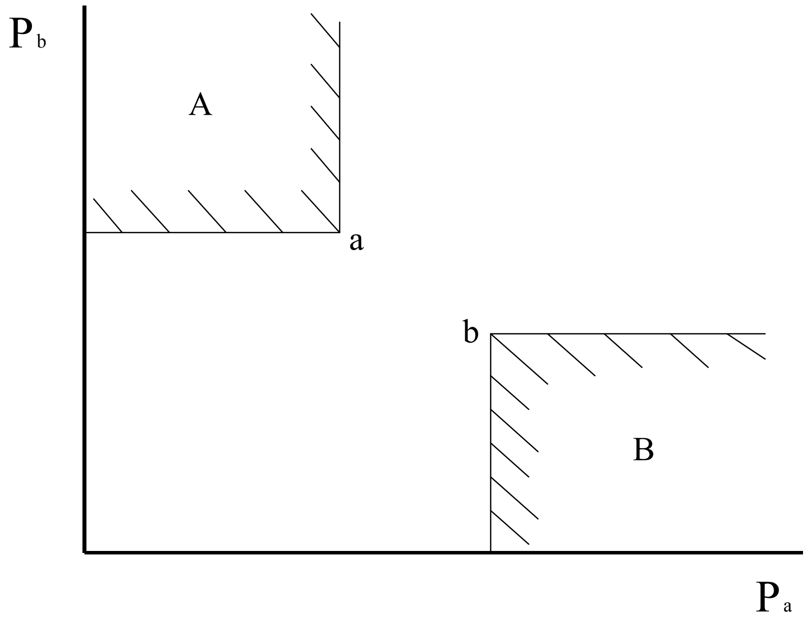
$$\Delta W > 0 \text{ iff } \mathrm{P}_b > f(\mathrm{P}_a).$$

Notice the relation between Assumption 1 and the necessary condition in (3.6) and (3.7) which can be rewritten in the following way:

$$\Delta W(\theta^1) > 0 \geq \Delta W(\theta^2) \text{ then}$$

$$\text{either } P_b(\theta^1) \geq P_b(\theta^2)$$
$$\text{or } P_a(\theta^2) \geq P_a(\theta^1)$$

This states that if a merger is socially optimal in θ, in any θ' with $P_b(\theta') > P_b(\theta)$ and $P_a(\theta') < P_a(\theta)$ merger has to be socially optimal. Conversely, if no merger is socially optimal in θ, in any θ' with $P_b(\theta') < P_b(\theta)$ and $P_a(\theta') > P_a(\theta)$ merger has to be socially optimal. In Figure 1, if at point a merger is socially optimal, merger must be socially optimal at any point into set A. Conversely, if at point b no merger is socially optimal, no merger must be optimal for any point into set B.



Notice that the necessary condition and Assumption 1 are very close. If the border between the merge and the no merger zones does not include any flat step, the necessary and Assumption 1 are identical.

We now spell out several examples in which Assumption 1 holds. We will assume that $D = E = \mathcal{R}$.

4.1. Synergy gains: Homogenous product

Let us assume that firms compete a la Cournot, and market demand, given by $P(X)$, satisfies $P'(X) < 0$ and

$$P'(X) + P\text{"}(X)X < 0. \tag{4.1}$$

Condition (4.1) guarantees existence and uniqueness of Cournot equilibrium. Define $\beta(X) \equiv \dfrac{P\text{"}(X)X}{P'(X)}$ as the degree of concavity. Then (4.1) can be rewritten as:

$$\beta(X) > -1. \tag{4.2}$$

We state the following results concerning a symmetric oligopoly with n firms and constant marginal cost denoted generically by e, where e may be either c or d. Denote respectively by $X_n(e)$, $\pi_n(e)$ and $W_n(e)$ the output, profits and social welfare in the Cournot equilibrium. Derivation of the following results is relegated to Appendix A:

$$\frac{dX_n(e)}{de} = \frac{n}{(n + \beta(X_n(e))P'(X_n(e))} < 0.$$

$$\frac{d\pi_n(e)}{de} = -\frac{(\beta(X_n(e)) + 2)X_n(e)}{n(n + \beta(X_n(e)) + 1)} < 0.$$

$$\frac{dW_n(e)}{de} = -\frac{(n + \beta(X_n(e)) + 2)X_n(e)}{(n + \beta(X_n(e)) + 1)} < 0.$$

Merger increases social welfare if:

$$\Delta W = W_1(d) - W_2(c) > 0.$$

Given that $\pi_i(e)$ is invertible we have:

$$W_1(\pi_1^{-1}(\pi_1)) > W_2(\pi_2^{-1}(\pi_2)).$$

As $\pi_1 = P_a + P_b$ and $\pi_2 = Pa$,

$$W_1(\pi_1^{-1}(P_a + P_b)) > W_2(\pi_2^{-1}(P_a)).$$

As $W_1()$ and $\pi_1()$ are strictly decreasing we have that:

$$P_a + P_b > \pi_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right).$$

$$P_b > \pi_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right) - P_a.$$

So, we take $f(P_a) = \pi_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right) - P_a$.[10]

[10] We can also consider that the buyer is a Stackelberg leader. In the case of linear demand, Assumption 1 holds by taking $f(P_a) = 3P_a$.

Finally, we check that f is strictly increasing.

$$f'(P_a) = \left(\frac{\frac{d\pi_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right)}{de}}{\frac{dW_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right)}{de}} \right) \left(\frac{\frac{dW_2\left(\pi_2^{-1}(P_a)\right)}{de}}{\frac{d\pi_2\left(\pi_2^{-1}(P_a)\right)}{de}} \right) - 1$$

$$= 2\left(1 + \frac{-1}{\left(\beta\left(X_1\left(W_1^{-1}\left(W_2(\pi_2^{-1}(P_a))\right)\right)\right) + 3\right)}\right)\left(1 + \frac{2}{\left(\beta\left(X_2(\pi_2^{-1}(P_a))\right) + 2\right)}\right) - 1 > 0.$$

4.2. Synergy gains: Differentiated products

We model the market with two differentiated products (good 1 and 2) following Singh and Vives (1984). We have a consumer endowed with a consumer surplus of the following form:

$$A\left(X_1 + X_2\right) - \frac{X_1^2}{2} - \frac{X_2^2}{2} - bX_1X_2$$

where X_i represents consumption of good i and b represents the degree of product differentiation.

We compute the profits of firms and Social Welfare in the different market configurations. The important point for the proof below is that Social Welfare can be written as a function of profits.

Profits in monopoly amount to $\pi_1 = \frac{(A-d)^2}{2(1+b)}$. Given that the monopoly sells $x = \frac{A-d}{2(1+b)}$, Social Welfare amounts to $SS = U(x,x) - 2dx = \frac{3(A-d)^2}{4(1+b)} = \frac{3}{2}\pi_1$.

Profits in duopoly and Cournot competition amount to $\pi_2 = \left(\frac{A-c}{2+b}\right)^2$ and Social Welfare to $\left(\frac{A-c}{2+b}\right)^2(3+b) = (3+b)\pi_2$. Thus, $\Delta W > 0$ iff $(\frac{3}{2})(P_a + P_b) > P_a(3+b)$. Rearranging the last expression we have $P_b > P_a(\frac{3+2b}{3})$. Thus, in this case $f(P_a) = P_a(\frac{3+2b}{3})$

Profits in duopoly and Bertrand competition amount to $\pi_2 = \left(\frac{1-b}{1+b}\right)\left(\frac{A-c}{2-b}\right)^2$ and Social Welfare to $\left(\frac{3-2b}{1+b}\right)\left(\frac{A-c}{2-b}\right)^2 = \left(\frac{3-2b}{1-b}\right)\pi_2$. Thus, $\Delta W > 0$ iff $(\frac{3}{2})(P_a + P_b) > P_a(\frac{3-2b}{1-b})$. Rearranging the last expression we have $P_b > P_a(\frac{3-b}{3(1-b)})$. Thus, in this case $f(P_a) = P_a(\frac{3-b}{3(1-b)})$.

In the examples above mergers that increase welfare if $P_b > g(b)P_a$ for some function $g()$. Mergers increase profits if $P_b > P_a$. Therefore $(g(b)-1)$ can be used as a measure of the discrepancy between social and private incentives. It is always positive except when goods are independent (in this case $b = 0$ and $g(b) = 1$ and therefore, private and social incentives coincide). Given b, $(g(b)-1)$ is greater with Bertrand competition than with Cournot competition, and given the type of competition $(g(b)-1)$, increases with b.

4.3. Rationalization.

We consider a market with two differentiated goods with the same demands as in the previous Section. To keep expressions tractable we develop the case where $b = \frac{1}{2}$. We have two firms competing with average cost c_1 and c_2 ($c_1 \leq c_2$) respectively. The planner must decide whether to approve the takeover of the inefficient firm by the efficient firm. A, c_1 and c_2 are unknown to the planner. The efficient merger policy is implementable with Cournot competition because $\Delta W > 0$ iff $P_b > P_a\left(\frac{5+2\sqrt{3}}{6}\right)$ and with Bertrand competition because $\Delta W > 0$ iff $P_b > P_a\left(\frac{2(55+8\sqrt{10})}{81}\right)$. The proof of these expressions is relegated to Appendix B.[11].

We now present our main result in this section.

Proposition 5. *Under Assumption 1, the efficient merger policy can be implemented in dominant strategies using the following mechanism: The buyer an-*

[11]In the case of homogeneous products we can generalize the previous result by allowing n firms with marginal cost c_1 and one firm with marginal cost $c_2 > c_1$ when the inefficient firm merges with any other firm. In this case $f(P_a) = (\frac{3+2n}{n+n^2})P_a$.

nounces $m_b \in E$ and the seller announces $m_a \in D$. If $m_b \leq f(m_a)$, the merger is not allowed. If $m_b > f(m_a)$, the merger takes places and the buyer pays $f(m_a)$ and the seller receives $f^{-1}(m_b)$.

Proof. The mechanism yields the efficient merger policy if players tell the truth. We show that the truth is a dominant strategy for the buyer. Denote by P_b^o the true value of P_b. If $\mathrm{P}_b^o > f(m_a)$, the buyer is better-off with the merger and this is obtained simply by telling the truth. If $\mathrm{P}_b^o \leq f(m_a)$, the buyer is better-off without the merger and this is obtained by telling the truth.

We show that the truth is a dominant strategy for the seller. Given that $f(P_a)$ is strictly increasing we have that $\Delta W > 0$ iff $f^{-1}(P_b) > P_a$. Denote by P_a^o the true value of P_a. If $f^{-1}(m_b) > P_a^o$, the seller is better-off with the merger and this is obtained simply by telling the truth. If $f^{-1}(m_b) \leq P_a^o$, the seller is better-off without the merger and this is obtained by telling the truth. ■

Our mechanism has some resemblance to the pivotal mechanism (Clarke (1971), Groves (1973)). In this mechanism (and in our's) an agent's payment is independent of her announcement unless it changes the level of the public good (the merger decision in our case). In our case we need the function f to signal if those changes are welfare enhancing or not. This is not needed in the pivotal mechanism because social welfare equals the sum of the utilities of the agents involved in the game. Notice that if a firm tells the truth, it obtains payoffs larger or equal than those that can be obtained under duopoly. In other words our mechanism is individually rational.

We remark that in order to construct the implementing mechanism the regulator has to know the function $f()$ and this implies that she has to know the kind of post-merger competition (i.e. Bertrand, Cournot, etc.)

We end this section by recording three corollaries of Proposition 5.

Corollary 1. *(Synergies) Assume that firms compete a la Cournot and the product is homogenous. The planner does not know either the premerger or the post-merger constant marginal cost, but she knows demand. Then, the efficient merger policy is implementable in Dominant Strategies.*

Corollary 2. *(Synergies) Assume that we have two goods (i = 1, 2) with demands $p_i = A - X_i - bX_j$. Symmetric duopoly with cost c and monopoly with cost d. A, c, and d are unknown to the planner. Then, the efficient merger policy is implementable with Cournot competition and with Bertrand competition.*

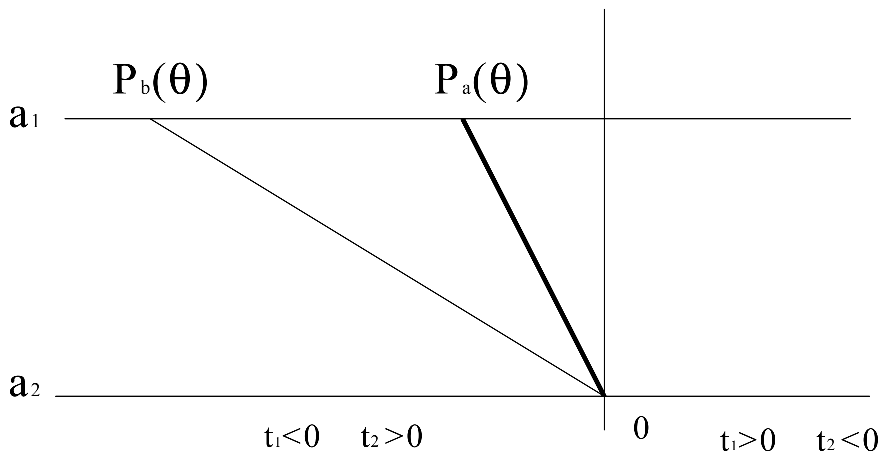
Corollary 3. *(Rationalization) Assume that we have two goods (i = 1, 2) with demands* $p_i = A - X_i - \left(\frac{1}{2}\right) X_j$*. Duopoly with costs* c_1 *and* c_2 $(c_1 \leq c_2\ )$ *and monopoly with cost* c_1*. A,* c_1 *and* c_2 *are unknown to the planner. Then, the efficient merger policy is implementable with Cournot competition and with Bertrand competition.*

5. Nash Implementation

The positive results obtained in the previous section depended crucially on the fact that the budget is not balanced. In this section we will consider an equilibrium concept weaker than dominant strategies, namely Nash equilibrium, in the hope of implementing the efficient merger policy with budget balance. We will see that if Assumption 1 does not hold, the efficient merger policy can not be implemented with budget balance in Nash equilibrium. Moreover, under Assumption 1 the efficient merger policy can be implemented in Nash Equilibrium with a balanced budget. We therefore have a trade-off regarding the implementation of the efficient merger policy: On the one hand, implementation *without* budget balance is possible in dominant strategies (a very robust equilibrium concept). On the other hand implementation *with* budget balance is possible in Nash equilibrium (a not so appealing equilibrium concept).

Since the emphasis of this section is on budget balance, it is important to specify the transfers associated with the efficient merger policy. We will assume that in the case of no merger these transfers are zero and in the case of a merger, they are any transfer that makes merger individually rational. In the case of a merger, the transfer is the acquisition price.

In this section, we will appeal to graphical arguments, assuming two firms (one potential buyer and one potential seller). In Figure 2, in the horizontal axis we measure transfers. Starting from point O (zero transfers), positive (resp. negative) transfers to firm 1 (the potential buyer) are located to the right (resp. left) of O. By budget balance the same axis can be used to measure transfers to firm 2 (the potential seller). Thus starting from point O, negative (resp. positive) transfers to firm 2 are located to the right (resp. left) of O. However, we remark that our allocation space includes points in which both firms may receive negative transfers (this was called feasibility in Section 2). Since this situation can never be optimal we will not consider it in our picture. The vertical axis measures market structure. We only have two possible values; a_2 (duopoly) and a_1 (merger). We draw indifference curves for a given characteristic, say θ. Even though there is nothing in between a_1 and a_2 we will joint points in both lines in order to indicate indifference. The indifference curves of firm 1 (depicted by a thin line) show increasing payoffs when we move to the right (since this firm gets more transfers). Similarly, the indifference curves of firm 2 (depicted by a thick line) show increasing payoffs when we move to the left for the same reason. Starting from O (no merge, no transfers) the indifference curves of both firms show that they could be better off if they merge and make the appropriate transfers. Finally, $P_a(\theta)$ and $P_b(\theta)$ can be easily located in the picture. $P_a(\theta)$ is the intersection of the indifference curve of firm 2 passing through O with the merger line and $P_b(\theta)$ is the intersection of the indifference curve of firm 1 passing through O with the merger line. Thus, the transfers associated with merger in the efficient merger policy belong to the interval $[P_a(\theta), P_b(\theta)]$.



We now invoke a result obtained by Moore and Repullo (1990, p. 1094) on the implementation of (in our terminology) extended SCR with two agents:

Theorem 5.1. *If a two agent extended SCR Φ satisfies monotonicity and restricted veto power and there is a bad outcome, then Φ can be implemented in Nash equilibrium.*

Rather than giving formal definitions of these terms (which may be found in the original paper) we will give literary (but we hope precise) descriptions of the conditions of the above theorem.

Monotonicity (M): Suppose outcome z is selected by Φ under characteristic θ. Now consider a new characteristic θ' such that z goes up (or remains constant) in the preferences of all firms. Then z should be selected by Φ under characteristic θ'.

Restricted Veto Power (RVP): Suppose outcome a is top ranked under θ by firm j and there is an outcome b in the range of Φ such that under θ, firm i -different from j- weakly prefers a to b. Then a must be selected by Φ under θ.

Bad Outcome (BO): z is a bad outcome if for any θ , z is strictly worse for both agents than any outcome in the range of Φ.

Clearly, RVP and BO hold in our framework with regard to the implementation of the efficient merger policy: If the maximum amount of negative transfers is large enough, the top ranked outcome of, say, firm 1 involves such a large transfer that the other firm will prefer any outcome in Φ to this situation. The bad outcome can be constructed by imposing very large negative transfers to both firms. Thus, if we show that M holds, Theorem 5.1 implies that the efficient merger policy can be implemented in Nash equilibrium.

First, notice that monotonicity is very easy to prove in Figure 2: Take any socially optimal outcome z under characteristic θ. Draw the upper contour set for firm i = 1, 2, i.e. the set of allocations which are weakly preferred by i to z. Consider now a new characteristic θ' such that the upper contour set shrinks for both firms. Then z should be optimal under θ'. With this preliminaries at hand we can show that monotonicity does not always hold:

Proposition 6. *: Monotonicity does not hold in every possible domain.*

Proof. Take the example presented at the end of the proof of Proposition 4. In the state of the world θ^1 merger is the socially optimal alternative. Consider now indifference curves in state θ^2. It is readily calculated that $P_a(\theta^2) = 30^2 < P_a(\theta^1) = \dfrac{8075}{8}$ and $P_b(\theta^2) = \left(\dfrac{95}{2}\right)^2 - 30^2 > P_b(\theta^1) = \left(\dfrac{95}{2}\right)^2 - \dfrac{8075}{8}$. Thus when we go from θ^1 to θ^2 the upper contour sets, evaluated at the point selected by the efficient merger policy and the corresponding transfers, become smaller. However, as we saw before, the merger is not socially optimal at θ^2. ■

The main implication of proposition 6 is that the efficient merger policy can not be implemented in Nash equilibrium in unrestricted environments. Nevertheless, if Assumption 1 holds, implementation becomes possible.

Proposition 7. *If Assumption 1 holds, then the efficient merger is implementable in Nash equilibrium with budget balance.*

Proof. We have demonstrated before that RVP and BO hold in our framework Then using Theorem 5.1. we only have to show that Monotonicity also holds to prove Nash implementation.

Given economy θ^1 we may have that the merger either increases welfare

$$\Delta W(\theta^1) > 0 \tag{5.1}$$

or that the merger reduces welfare:

$$\Delta W(\theta^1) \leq 0 \tag{5.2}$$

Suppose that (5.1) holds. That upper contour sets srink is equivalent to:

$$P_a(\theta^1) \geq P_a(\theta^2) \text{ and } P_b(\theta^1) \leq P_b(\theta^2). \tag{5.3}$$

Assumption 1 and (5.1) imply:

$$f(P_a(\theta^1)) < P_b(\theta^1).$$

Using (5.3), we have:

$$f(P_a(\theta^2)) \leq f(P_a(\theta^1)) < P_b(\theta^1) \leq P_b(\theta^2).$$

which implies, if Assumption 1 is satisfied, that

$$\Delta W(\theta^2) > 0.$$

And this is what is implied by Monotonicity.

Suppose now that (5.2) holds. That upper contour sets shrink is equivalent to:

$$P_a(\theta^1) \leq P_a(\theta^2) \text{ and } P_b(\theta^1) \geq P_b(\theta^2). \tag{5.4}$$

Assumption 1 and (5.2) imply

$$f(P_a(\theta^1)) \geq P_b(\theta^1).$$

Using (5.4) we have:

$$f(P_a(\theta^2)) \geq f(P_a(\theta^1)) \geq P_b(\theta^1) \geq P_b(\theta^2).$$

which implies if Assumption 1 is satisfied, that

$$\Delta W(\theta^2) \leq 0.$$

And this is what is implied by Monotonicity. ■

Finally we remark that the converse of Proposition 7 is not true, i.e. monotonicity does not imply Assumption 1. Suppose, for instance, that $D = E = [1, 2]$ and that merging is always the socially optimal alternative. The function required by Assumption 1 obviously does not exist, but the corresponding social choice rule is trivially implementable in Nash equilibrium by a mechanism in which any message sent by the firms yields the alternative "merge". Thus, by a theorem of Maskin (see e.g. Moore and Repullo (1990), p. 1087) this social choice rule must be monotonic, but it does not satisfy Assumption 1.

6. Conclusions.

In this paper we have studied the possibility of designing a mechanism that implements the efficient merger decision in dominant strategies and in Nash equilibrium. Remarkably, the key to implement in both cases is our Assumption 1 which allows us to implement the efficient merger policy without budget balance in dominant strategies, and with budget balance in Nash equilibrium. We have seen that this assumption is satisfied in some standard models used in Industrial Organization.

Our paper is silent on several important issues. First, we do not know how far we can go with implementation in dominant strategies if Assumption 1 does not hold. In the case of savings in fixed costs the efficient merger decision is implementable by a Vickrey-Clarke-Groves mechanism (see footnote 8), but we do not know if a more general assumption may be available. Secondly, we have proven that the efficient merger policy is implementable in Nash equilibrium but we have not provided a particular mechanism for doing the job. It would be interesting to find a simple and well-behaved (i.e. continuous, etc.) mechanism for implementation in Nash equilibrium. Third, other solution concepts must be analyzed. For instance, we know from the work of Moore and Repullo (1988) that any social choice rule is implementable in Subgame Perfect Nash equilibrium in quasi-linear environments. Four, more complex merger situations (for instance, involving the transfer of shares or the acquisition of part of a firm) should be considered. Fifth, the issue of coalition among firms must be addressed. It is easy to show that since we implement a social choice *function* in Nash equilibrium, the Nash equilibrium is also a coalition-proof equilibrium. We suspect that implementation of the efficient merger policy is not possible under other concepts like strong equilibrium. Finally, it will be interesting to consider that firms have asymmetric information, and to relate the problem of mergers to the auction literature. All of these points are left for future research.

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8. Appendix A.

In the expressions below $P(), P'(), P"()$ and $\beta()$ are evaluated at the equilibrium output $X_n(e)$.

$X_n(e)$ satisfies the equilibrium condition:

$$P - e + P'\frac{X_n(e)}{n} = 0 \tag{8.1}$$

Differentiating (8.1) with respect to e we have:

$$P'(\frac{dX_n(e)}{de}) - 1 + P"(\frac{dX_n(e)}{de})\frac{X_n(e)}{n} + P'\frac{\frac{dX_n(e)}{de}}{n} = 0$$

$$(\frac{dX_n(e)}{de})P'\left(1 + \frac{P"X_n(e)}{nP'} + \frac{1}{n}\right) = 1$$

$$\frac{dX_n(e)}{de} = \frac{n}{(n+\beta+1)P'} \tag{8.2}$$

Profits in equilibrium satisfy:

$$\pi_n(e) = -P'\left(\frac{X_n(e)}{n}\right)^2$$

$$\frac{d\pi_n(e)}{de} = -(\frac{\frac{dX_n(e)}{de}}{n^2})\left(P"(X_n(e))^2 + 2X_n(e)P'\right)$$

$$\frac{d\pi_n(e)}{de} = -\frac{(\beta+2)P'X_n'(e)X_n(e)}{n^2}$$

Using (8.2), we have

$$\frac{d\pi_n(e)}{de} = -\frac{(\beta+2)X_n(e)}{n(n+\beta+1)}$$

Social Welfare satisfies:

$$W_n(e) = \int_0^{X_n(e)} (P(x) - e)dx$$

$$\frac{dW_n(e)}{de} = (P-e)X_n'(e) - X_n(e)$$

Using(8.1) and(8.2), we have that

$$\frac{dW_n(e)}{de} = -\frac{(n+\beta+2)X_n(e)}{(n+\beta+1)}.$$

9. Appendix B.

9.1. Cournot competition.

To simplify expressions we use $a \equiv A - c_1$ and $d \equiv c_2 - c_1$. The following system gives us a and d as a function of P_a and P_b.

$$\left(\frac{a(2-b)-2d}{4-b^2}\right)^2 = P_a$$

$$\frac{a^2}{2(1+b)} - \left(\frac{a(2-b)+bd}{4-b^2}\right)^2 = P_b$$

The only solution when b=$\frac{1}{2}$ satisfying $a > 0$ and $d < \frac{3a}{4}$ is given by:

$$a = \sqrt{3(P_a + 4P_b)} - \frac{3}{2}\sqrt{P_a} \tag{9.1}$$

$$d = \frac{3}{4}\sqrt{3(P_a + 4P_b)} - 3\sqrt{P_a} \tag{9.2}$$

Using equilibrium outputs the change in welfare can be written as a function of a and d:

$$\Delta W = \frac{252ad - 27a^2 - 188d^2}{450} \tag{9.3}$$

Using (9.1) and (9.2), (9.3) can be rewritten as a function of P_a and P_b:

$$\Delta W = \frac{6P_b - 4P_a - \sqrt{3P_a(P_a + 4P_b)}}{4}$$

This function has two roots $P_b = P_a\left(\frac{5 \pm 2\sqrt{3}}{6}\right)$. As ΔW is convex in P_b, we have that $\Delta W > 0$ if $P_b < P_a\left(\frac{5-2\sqrt{3}}{6}\right)$and $P_b > P_a\left(\frac{5+2\sqrt{3}}{6}\right)$. However only the second restriction is compatible with $P_b > P_a$.

9.2. Bertrand competition.

The following system gives us a and d as a function of P_a and P_b.

$$\left(\frac{a(2-b-b^2)-(2-b^2)d}{(4-b^2)(4-5b^2+b^4)}\right)^2 = P_a$$

$$\frac{a^2}{2(1+b)} - \left(\frac{a(2-b-b^2)+bd}{(4-b^2)(4-5b^2+b^4)}\right)^2 = P_b$$

The only solution when b=$\frac{1}{2}$ satisfying $a > 0$ and $d < \frac{5a}{7}$ is given by:

$$a = \frac{7\sqrt{39P_b+4P_a} - 36\sqrt{P_a}}{13} \tag{9.4}$$

$$d = \frac{10\sqrt{39P_b+4P_a} - 45\sqrt{3P_a}}{26} \tag{9.5}$$

Using equilibrium outputs the change in welfare can be written as a function of a and d:

$$\Delta W = \frac{800ad - 125a^2 - 632d^2}{1350} \tag{9.6}$$

Using (9.4) and (9.5), (9.6) can be rewritten as a function of P_a and P_b:

$$\Delta W = \frac{351P_b - 434P_a - 48\sqrt{P_a(4P_a+13P_b)}}{338}$$

This function has two roots $P_b = P_a\left(\frac{2(55 \pm 8\sqrt{10})}{81}\right)$. As the function is convex in P_b, we have that $\Delta W > 0$ if $P_b < P_a\left(\frac{2(55 - 8\sqrt{10})}{81}\right)$ and $P_b > P_a\left(\frac{2(55 + 8\sqrt{10})}{81}\right)$. However only the second restriction is compatible with $P_b > P_a$.