Working Paper 94-39
Economic Series 20
December 1994

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QUANTITY RESTRICTIONS AND ENDOGENOUS QUALITY CHOICE

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Abstract

In this paper we show that in an imperfectly competitive market the imposition of quantity restrictions at, or less than, the free trade equilibrium has important strategic effects on the choice of quality. In a vertical product differentiation model where foreign and domestic firms compete in quantities, both firms respond by lowering their qualities for a restriction at, or close enough, to the free trade level. If the restriction is substantially smaller than the free trade equilibrium an increase in average quality is observed only when the foreign firm produces the low quality. The change in quality depends not only on whether the recipient is a high, or low, quality foreign firm. It also depends on how restrictive the constraints are.

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I. Introduction

The issue of quality choice due to the imposition of trade restrictions has received considerable attention both in the theoretical and the applied literature. In this paper we study the effect of quantity restrictions on endogenous quality choice. A vertical product differentiation model (Motta, 1993), with a foreign and domestic firm, is analyzed for the case of imposition of quantity restrictions such as Voluntary Export Restraints, or import quotas. The government first announces an import quota, or equivalently the foreign country chooses a VER, on the foreign firm. Then the firms simultaneously choose the quality of the goods they want to produce. Following which they compete in quantities in the domestic market.

The results in the empirical literature have been on the side of quality increase due to the imposition of the restriction (Feenstra (1985), Boorstein and Feenstra (1991), and Smith and Venables (1991)). The existing theoretical literature has focussed attention on perfect competition and monopoly. Quality upgrading is typically observed in these cases[1]. The issue of quality choice in an oligopolistic framework has been analysed by Das and Donnenfeld (DD) (1989) and Ries (1993). DD look at the strategic effect of quantity restraints on quality choice in an oligopoly setting where firms decide simultaneously on output and quality. They show that quantity restraints always increase the quality of the imported good of the foreign firm. The quality of the domestic good increases if the foreign firm produces the high quality good. Contrarily, if the foreign firm produces the low quality good domestic firm lowers its quality. Ries extends this result to the case of a multiproduct

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[1] See Falvey (1979), Rodriguez (1979), Santoni and Van Cott (1980) Das and Donnenfeld (1987), Krishna (1987), Donnenfeld (1988). For a summary of results on perfect competition and monopoly, and for a very good selective survey on export restraints with imperfect competition see Krishna (1990).

monopoly. In his case if the foreign firm produces a range of low quality goods then it lacks an incentive to increase its level of qualities.

DD and Ries, however, do not explicitly take time into account. This simultaneity (in the choice of quality and output) removes the ability of the firms to manipulate its rivals quantity by commiting to a level of quality. In our model firms are able to commit as they incur a sunk cost of improving quality. Quality is chosen in the first stage of the game, given the equilibrium output choices in the second stage[2]. This is in agreement with the choice of the appropriate long-run/short-run variables as defined in Krishna (1990).

We show that the choice of quality by the foreign and domestic firm after the imposition of quantity restrictions depends not only on which of the two firms produces the low, or high, quality good. It also depends on how binding the restrictions on the foreign firm are. A low, or high, quality foreign firm *always* downgrades its quality in response to the import restriction. When the domestic firm is low quality it downgrades (its quality) for restrictions close enough to the free trade level and upgrades for a sufficiently binding quantity restriction. However, if the home firm is of high quality then it always downgrades for any level of import restriction. For very restrictive quotas the average quality of the mix of outputs increases if the foreign firm produces the low quality only. If quality is a sunk cost firms have an incentive to lower qualities in the presence of quantity restraints. Thus, the imposition of quantity restrictions facilitates collusion between the firms. This result is similar in flavor to that of Krishna (1989) and Harris (1985).

The paper is structured as follows. The model is presented in Section-II with the derivation of the benchmark results of Motta (1993). In Section-III we look at the effect of

[2]Rietzes (1992) uses a similar approach to analyse the effects of tariffs on quality choice.

quantity restrictions. In Section-IV we have the welfare analysis and Section-V is the conclusion.

II. The Model.

As in Motta (1993) we consider the simple case of 2 countries, foreign and domestic. There are two firms, one in each country producing a vertically differentiated good. The marginal cost of production is zero for both firms and does not depend on the choice of quality (s). High quality is indexed as s_1 and low quality is indexed as s_2, such that $s_1 > s_2$ always. We concentrate on the effects of the quantity restriction in the domestic market only. There is a continuum of consumers in the domestic market, each is identified by its taste parameter θ which is uniformly distributed over the interval $[0, \Theta]$ with density one. Each consumer has unit demand for the good and the utility function,

$$U = \begin{cases} \theta s - p & \textit{if it buys a unit of the good of quality s.} \\ 0 & \textit{otherwise.} \end{cases} \tag{1}$$

Quality is costly and the marginal cost of producing an additional unit of quality is increasing in s and is given by $(s^2/2)$.

The game proceeds as follows. At time '0' the government announces its decision to impose the quantity restriction on the foreign country. The government may impose an import quota, or alternatively the foreign country may choose a VER. We limit the role of the government to the announcement of the quantity restriction only. It is assumed that the government credibly precommits to a level of quantity restraint. After the government announces the quantity restriction the firms, first, simultaneously choose their qualities. At the stage of quality selection each firm bears the cost $s^2/2$. Finally, firms simultaneously choose quantities. We solve the game using sub-game perfectness.

The demand for the low and high quality good is first derived. Setting $s_1 > s_2$, the consumer indifferent between buying the high, or low, quality good has the taste parameter $\theta_{12}=[(p_1-p_2)/(s_1-s_2)]$. The consumer indifferent between buying the low quality good and not buying at all has the taste parameter $\theta_{02} = p_2/s_2$. The utility of this consumer is zero if it purchases good s_2. Now, all the consumers for whom $\Theta \geq \theta \geq \theta_{12}$ purchase good with quality s_1. All consumers for whom $\theta_{12} > \theta \geq \theta_{02}$ will purchase quality s_2. Those described by $\theta_{02} > \theta$ do not buy the good.

The demands for the high and low quality good are given by,

$$D_1(p_1,p_2) = \Theta - (p_1-p_2)/(s_1-s_2) \tag{2}$$

$$D_2(p_1,p_2) = [(p_1-p_2)/(s_1-s_2)]-(p_2/s_2). \tag{3}$$

Solving for the inverse demands,

$$p_1(x_1,x_2) = \Theta s_1-s_1x_1-s_2x_2 \tag{4}$$

$$p_2(x_1,x_2) = \Theta s_2-s_2x_1-s_2x_2 = [\Theta-x_1-x_2]\, s_2. \tag{5}$$

Note, that $p_1/s_1 > p_2/s_2$ always, that is, there exists a ´discount´ for the low quality. The cross derivative of the inverse demand with respect to own quality and own quantity is negative. This is the condition under which Krishna (1987) gets quality upgrading for the case of a foreign monopoly.

To analyze the effects of quantity restrictions we first study quality choice in an unrestricted market (following Motta, 1993). The equilibrium outcome under free trade in quantities and qualities is thus obtained. This is then compared against the outcomes observed after the quantity restrictions are imposed.

For any given pair of qualities (s_1,s_2) firm i maximizes its profits, $p_i(x_i,x_j)x_i$, over the choice of quantities x_i given the quantity of its rival x_j. This gives us the first order conditions (f.o.c),

$$x_1= (\Theta/2)-(s_2/2\, s_1\,)\, x_2. \text{ and } x_2=(1/2)\ (\Theta-x_1) \tag{6}$$

Solving for x_1^* and x_2^* we get,

$$x_1^*\ (s_1,s_2)=\Theta(2s_1-s_2)/(4s_1-s_2)\ =\ \Theta\ [1\ -\ (2s_1/(4s_1-s_2)],\ and \tag{7}$$
$$x_2^*\ (s_1,s_2)=\ \Theta\ s_1/(4s_1-s_2). \tag{8}$$

(Note that, $(dx_i^*/ds_i)>0$, $(dx_i^*/ds_j)<0$, i,j=1,2.) From the focs we have $p_i^*=s_ix_i^*$. Thus, the net profits for a given pair of qualities are as follows.

$$\pi_i^*\ (s_1,s_2)=s_ix_i^{*2}-(s_i^2/2),\ i=1,2. \tag{9}$$

It can be checked that $d^2\pi_1^*/ds_1ds_2=\ 8\Theta^2s_1s_2\ (s_1-s_2)/(4s_1-s_2)^4>0$. That is, low quality is a strategic complement for the high quality. As firm 2's quality increases, firm 1's marginal profit from increasing its quality increases. This gives firm-1 an incentive to increase its quality and differentiate its product from that of its rival. On the other hand $d^2\pi_2^*/ds_2ds_1=[-2\Theta^2s_1s_2(8s_1\ +s_2)/(4s_1\ -\ s_2)^4]<0$, that is, the high quality is a strategic substitute for the low quality. As firm 1's quality increases, firm 2's marginal benefit from increasing its quality decreases.

In the first stage firm-i chooses its quality s_i to maximize its profits given in (9) given, the quality of its rival s_j. The focs then are as follows,

$$\Theta^2\ (2\ s_1-s_2)\ (8\ s_1^2-2\ s_1s_2+s_2^2)/(4\ s_1-s_2)^3=s_1 \tag{10}$$
$$\Theta^2\ \ s_1^2\ \ (4\ s_1+s_2)/(4\ s_1-s_2)^3=s_2 \tag{11}$$

Solving for the equilibrium quality levels we get, $s^*_1=\ 0.2519\ \Theta^2$ and, $s^*_2=\ 0.0902\ \Theta^2$. Dividing, the relative equilibrium qualities are obtained, $(s^*_1/s^*_2)=2.79243$. This gives us the equilibrium expression for $x_1^*(s_1^*,s_2^*)=0.4508\Theta$, $x_2^*(s_1^*,s_2^*)=0.2746\Theta$, and the relative outputs $x_1{*}/x_2{*}=1.64166$. Finally, the market average quality, $s_{AV}=(x_1^*s_1^*+x_2^*s_2^*)/(x_1^*+x_2^*)$, equals $0.1907\Theta^2$ under free trade.

III. The effect of quantity restrictions:

We first consider the case (-1) where the quantity restriction (Q_H) is imposed on the foreign firm producing the high quality good and the domestic firm the low quality good. The second case (-2) where the quantity restriction (Q_L) is imposed on the foreign firm producing the low quality good is then considered. The restriction imposed on the foreign firm, an import quota or a VER, is such that the foreign firm cannot sell more than the amount of the restriction.

Case-1: Foreign Firm Produces High Quality Good.

Let the quantity restriction, Q_H, be such that $0.252235\Theta \leq Q_H \leq x_1^*(s_1^*, s_2^*) = 0.4508\Theta$.[3] Then the foreign firm chooses $x_{1H} = min.[Q_{H'}, (1/2)(\Theta - (s_2/s_1)x_2]$. Assuming for the moment that the restriction is binding, that is, the foreign firm always produces at the level of the quota. We then show that this is also true in equilibrium. Thus, writing $x_{1H}^* = Q_H$, $x_{2H}^* = (1/2)(\Theta - Q_H)$ (from (6)) and solving for the prices we get, $p_{1H}^* = (1/2)(2s_1 - s_2)(\Theta - Q_H)$, $p_{2H}^* = (1/2)(\Theta - Q_H)s_2$, and the net profits, $\pi_{1H}^*(s_1, s_2) = (Q_H/2)(2s_1 - s_2)(\Theta - Q_H) - (s_1^2/2)$, $\pi_{2H}^*(s_1, s_2) = (1/4)\ (\Theta - Q_H)^2\ s_2 - (s_2^2/2)$.

Taking the first order conditions,

$$(d\pi_{1H}/ds_1) = Q_H(\Theta - Q_H) - s_1 = 0 \Rightarrow s_{1H}^* = Q_H(\Theta - Q_{H'}) \qquad (12)$$

$$(d\pi_{2H}/ds_2) = (1/4)\ (\Theta - Q_H)2 - s_2 = 0 \Rightarrow s_{2H}^* = (1/4)\ (\Theta - Q_H)^2 \qquad (13)$$

Note, that given $(s_{1H}^*, s_{2H}^*, x_{2H}^*)$ it can be checked that $Q_H < (1/2)(\Theta - (s_2/s_1)x_2.$ is satisfied in the no leapfrogging region. Now, for a quantity restriction at the free trade level $(Q_H = 0.4508\ \Theta)$ the low quality domestic firm selects a quality level of $0.0754\ \Theta^2$ $(< s_2^* = 0.09022\ \Theta^2)$ producing an output of $0.2747\ \Theta$. The foreign firm chooses a quality

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[3] We restrict attention to quantity restrictions such that neither of the firms has an incentive to 'leapfrog' its rival. That is, the condition that a high (low) quality firm would not choose to produce the low (high) quality good is always satisfied. The proofs are available to the interested reader.

In a related paper (Herguera et. al, 1995) we analyse the implications on quality choice of very restrictive import quotas (VER's).

level of $0.24758\,\Theta^2$ ($< s_1^* = 0.251942\,\Theta^2$) producing an amount equal to the quantity restriction imposed upon it. It can be seen from figure-1 that the quality choice of both the firms, for restrictions close enough to free trade level, is always below their choice, (s_1^*, s_2^*), under free trade. Also, note that s_{1H}^* ($=0.1886\Theta^2$) $> s_{2H}^*$ ($=0.13979\Theta^2$) for the smallest quota ($=0.252235\Theta$) at which there are no incentives to leapfrog. It is also seen that for sufficiently restrictive quotas (to the left of point A) the domestic firm produces a higher level of quality than under free trade.

<figure-1 here>

The effect on quality choice of a restriction at the free trade level can be explained by looking at the effect on the reaction function of the foreign firm (figure-2). Producing a higher level of quality is costly for both firms. The quantity restriction on the foreign firm decreases the competitive pressure on the domestic firm given the qualities. Thus the domestic firm is unilaterally able to decrease its quality (which moves $R_1(x_2)$ outwards to $R_1'(x_2)$) maintaining its market share. It can thus increase its net profits, given that costs are quadratic and revenues are linear in its own quality. Meanwhile, the foreign firm can also lower its quality (moving $R_1'(x_2)$ inwards), without losing its market share, and increase its profit at the lower level of quality (saving on the cost of quality). Thus, we get quality downgrading from both, the foreign and the domestic firms. A similar argument applies for quantity restrictions close enough to the free trade level.

<figure-2 here>

In the region where there is no leapfrogging average quality under the restriction is always lower than under free trade. The average quality for the quota at the free trade level ($0.182403\,\Theta^2$) is less than the average under free trade ($0.190723\,\Theta^2$). Further, given that average quality (s_{AV}) equals, $Q_H(\Theta^2 - 2\Theta Q_H + 9Q_H^2)(\Theta - Q_H)(1/4(\Theta + Q_H))$, it can be checked that

Figure-1

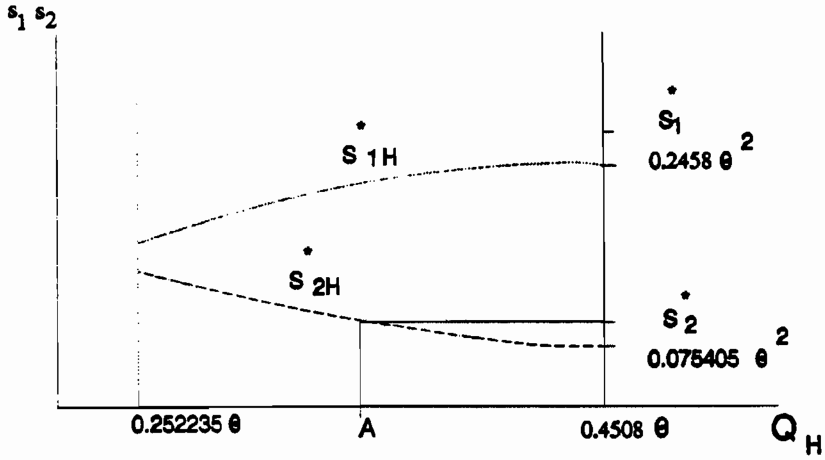
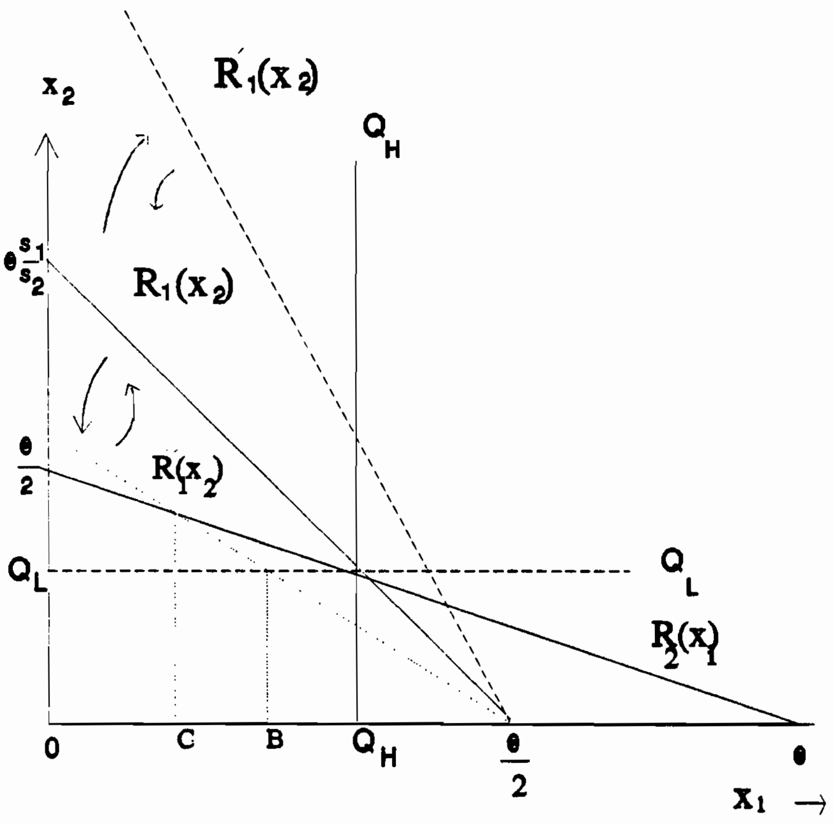


Figure-2



$R_1'(x_2)$
x_2
Q_H
$\theta \frac{s_1}{s_2}$
$R_1(x_2)$
$\frac{\theta}{2}$
$R_1'(x_2)$
Q_L
Q_L
$R_2(x_1)$
0
C
B
Q_H
$\frac{\theta}{2}$
θ
x_1

it decreases as the quota becomes more restrictive.[4]

The above results are summarized in the following proposition.

Proposition-1: *For a level of restriction at, or close enough, to the free trade level both firms downgrade qualities. As the quantity restraint becomes more restrictive average quality in the market decreases.*

Case 2: Foreign Firm Produces Low Quality Good.

Let the restriction $Q_L \leq x_2^*(s_2^*) = 0.2746\ \Theta$ be imposed on the low quality foreign firm (firm-2)[5]. Then the foreign firm chooses $x_{2L} = min.[Q_L, (1/2)(\Theta - x_1)]$. Again, we assume first that the quota is binding on the foreign firm and then we show that this is also true in equilibrium. Hence, $x_{2L}^* = Q_L$, $x_{1L}^* = (1/2)[\Theta - (s_2/s_1)Q_L]$, and solving for the respective prices we get, $p_{1L}^* = (1/2)(\Theta s_1 - Q_L\ s_2)$ and $p_{2L}^* = (s_2/2)[\Theta - Q_L(2 - s_2/s_1)]$. The net profit functions are, $\pi_{1L}^* = (1/4s_1)(\Theta s_1 - Q_L s_2)^2 - (s_1^2/2)$ and $\pi_{2L}^* = (s_2 Q_L/2)[\Theta - Q_L(2 - (s_2/s_1))] - (s_2^2/2)$. Let, $\lambda = s_2/s_1$, then the first order conditions give us the following expressions,

$$s_2 = [(\Theta/2) - Q_L]Q_L + (s_2/s_1)\ Q_L^2 = [(\Theta/2) - Q_L]Q_L + \lambda\ Q_L^2. \tag{14}$$

$$s_1 = (\Theta^2 s_1^2 - Q_L^2 s_2^2)/(4s_1^2) = (1/4)\ [\Theta^2 - Q_L^2\ \lambda^2]. \tag{15}$$

Dividing, (14) and (15) we get,

$$4[(\Theta/2) - Q_L]Q_L + \lambda Q_L^2]/(\Theta^2 - Q_L^2\ \lambda^2) = \lambda\ \Rightarrow (\lambda^3 + 4\lambda - 4)\ Q_L^2 + 2\ \Theta\ Q_L - \lambda\ \Theta^2 = 0. \tag{16}$$

Solving for Q_L we get,

$$Q_L = \Theta[-1 + (1 - 4\lambda + 4\lambda^2 + \lambda^4)^{(1/2)}]/(\lambda^3 + 4\lambda - 4)\} \tag{17}$$

From (16), $\lambda = 0$ for $Q_L = 0$ and $\lambda = 0.359523$ for $Q_L = 0.2746\Theta$. Note, that λ increases with Q_L because $dQ_L/d\lambda > 0$ for all $Q_L \leq 0.2746\Theta$ (for all $\lambda \leq 0.359523$). Thus, from (15) we know

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[4] Taking the first derivative with respect to Q_H we get, $d(s_{AV})/dQ_H = -(1/2(\Theta + Q_H)^2)[(2\Theta^3 - 11\Theta^2 Q_H + 8\Theta\ Q_H^2 + 9Q_H^3)] > 0$. Note, that the term in brackets is negative for all relevant Q_H.

[5] Firms are indexed by qualities.

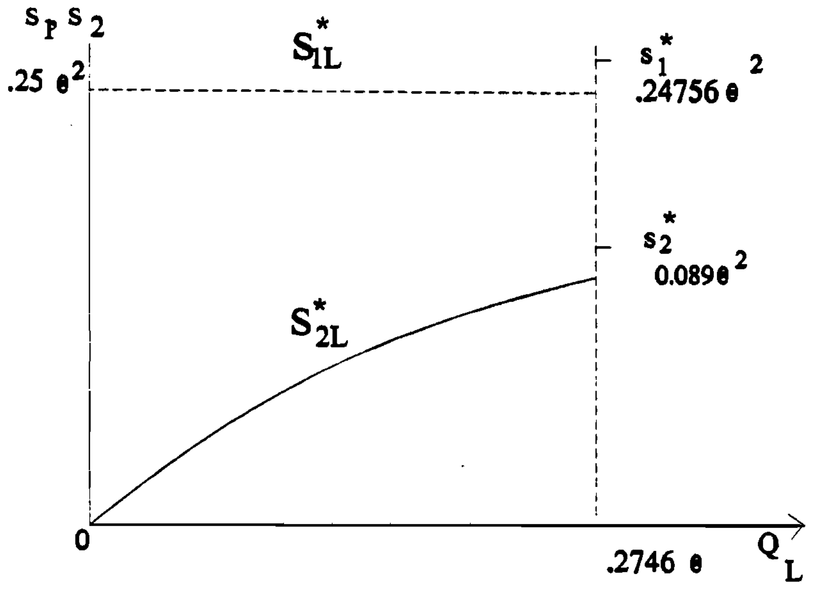
that s_1 increases when Q_L becomes more restrictive. Further, substituting (17) into (14) it can be checked that s_2 is a decreasing function of λ, and thus of Q_L. To summarise, as Q_L becomes more restrictive firm-2 lowers its quality and a slight increase in the quality of firm-1 is observed (figure-3). For a restriction imposed at a free trade level the domestic and foreign firm choose lower qualities, $s_{1L}^*=0.24756\Theta^2$ ($<s_1^*$) and $s_{2L}^*=0.089\Theta^2$ ($<s_2^*$), than under free trade.

Both the high-quality domestic and low-quality foreign firm lower their qualities. Looking at figure-2 (by our quality indexing ´2´ is now the foreign firm and ´1´ the domestic), the quantity restriction (Q_LQ_L) is now imposed at the free trade level (0.2746Θ) on the vertical axis. Note, that the domestic firm, given the quota on the foreign firm, loses a smaller amount of market share under the restriction (figure-2), Q_HB, than it would under free trade, Q_HC, for the same decrease in its quality (that shifts $R_1(x_2)$ to $R''_1(x_2)$). Given, that the costs of quality are quadratic the incentive exists for the domestic firm to save on these costs by lowering its quality in the presence of the restraint. Meanwhile, the foreign firm also responds by lowering its quality ($R''_1(x_2)$ shifts back). As it does not lose any market share and saves on the costs-of-quality. Thus, both firms optimally respond by lowering their qualities. The same argument applies for quotas close to the free trade level. Given, that for the quota at the free trade level s_{1L}^*/s_{2L}^*(=2.78146) is less than s_1^*/s_2^* (=2.79243) the restriction will be binding on the foreign firm ($R''_1(x_2)$ lies below $R_1(x_2)$). It can be checked that the restriction is always binding on the foreign firm. To verify this, note that $Q_L < (1/2)(\Theta - x_{1L}^*)$ iff $(4-\lambda)\, Q_L < \Theta$. Substituting Q_L from (17) it can be seen that the latter inequality is always true in the relevant range of λ.

<figure-3 here>

Average quality (s_{AV}) for a restriction at the free trade level is 0.187524. However,

Figure-3


s_1, s_2
S_{1L}^*
$.25\ \theta^2$
s_1^*
$.24756\ \theta^2$
s_2^*
$0.089\theta^2$
S_{2L}^*
0
$.2746\ \theta$
Q_L

as the quota becomes restrictive average quality increases and is in-fact higher for a sufficiently restrictive quota than under free trade. By substituting for x_{1L}^*, x_{2L}^*, s_{1L}^* and $\lambda = s_{2L}^*/s_{1L}^*$ into s_{AV} we get the expression, $(1/4)\ (\Theta - Q_L\lambda)\ (\Theta + Q_L\lambda)^2/(2Q_L + \Theta - \lambda Q_L)$. Further substituting Q_L from (17) it is can be seen that $ds_{AV}/d\lambda < 0$; thus, $ds_{AV}/dQ_L < 0$ in the relevant region. Finally, it can also be checked that $s_{AV}=0.1907$ (average quality under free trade) for $Q_L = 0.2441\Theta$. We summarize the results in proposition-2.

Proposition-2: *For a level of restriction at, or close, to the free trade level both firms downgrade qualities. As the quota becomes more restrictive average quality in the market increases. Specifically, for a sufficiently restrictive quota, $Q_L < 0.2441\Theta$, higher average quality (than under free trade) is offered. However, for $Q_L \in (0.2441\Theta, 0.2746\,\Theta)$, the average quality offered is lower than under free trade.*

IV. Welfare Analysis.

The welfare results are summarized in propositions 3 and 4, and are followed by a brief discussion.

Proposition-3: Regardless of whether the foreign firm produces the low, or the high, quality good:

(i) Both firms always earn a higher level of profits for a restriction at, or close to, the free trade level than under free trade. The profits of the domestic firm increase as the quota becomes restrictive. Contrarily, the profits of the foreign firm decrease.

(ii) Consumer surplus is higher under free trade than for a quota at the free trade level and decreases as the quota becomes more restrictive.

Proposition-4: When the domestic firm produces the low quality good the total welfare of the home country is always less than under free trade. Contrarily, when the domestic firm produces the high quality good total welfare of the home country increases as the quota becomes restrictive. Total welfare is highest at the point where the foreign firm is shut out of the market and is also greater than under free trade.

(i) Foreign firm high-quality:

Both firms prefer a quota at the free trade level over no restrictions as their profits are higher when the restriction is imposed at the level of free trade. Their profits[6] are

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[6] $\pi_{1H}^* = (5Q_H - \Theta)(\Theta - Q_H)^2 Q_H/8$, $\pi_{2H}^* = (\Theta - Q_H)^4/32$. As Q_H decreases, π_{1H}^* decreases and π_{2H}^* increases.

$\pi_{1H}^{*}=0.0213\Theta^{4}$ ($>\pi_{1}^{*}=0.01946\Theta^{4}$) and $\pi_{2H}^{*}=0.00284\Theta^{4}$ ($>\pi_{2}^{*}=0.002734\Theta^{4}$). The low quality domestic firm always prefers a higher level of restriction on the foreign firm as its profits increase as the quota becomes restrictive. As the profits of the foreign firm are highest for a VER at the level of free trade the foreign country would opt for such a VER. The total welfare[7] of the home country is higher under free trade than under any level of quota in the relevant region. A quota at the level of free trade results in higher prices, lower total output, lower average quality and thus a decrease in the consumer surplus. This decrease in the consumer surplus outweighs the increase in the home firm´s profits. For more restrictive quotas the rate of increase of home firm´s profits is of a similar magnitude as the rate of decrease of the consumer surplus. Thus, the home government prefers a quota at the free trade level.[8]

(ii) Foreign firm low-quality.

Both the firms prefer a quota at the free trade level as they make a higher level of profits than under free trade. Their profits[9] are $\pi_{1L}^{*}=0.01963\Theta^{4}$ ($>\pi_{1}^{*}=0.01946\Theta^{4}$) and $\pi_{2H}^{*}=0.002754\Theta^{4}$ ($>\pi_{2}^{*}=0.002734\Theta^{4}$). Once more, as the profits of the foreign firm are highest for a VER at the level of free trade the foreign country would opt for such a VER.

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[7] A word of caution is needed here. Our definition of TW gives equal weight to the producer profits and the consumer surplus. It is not clear if the home ´country´ welfare is maximised and thus this point is preferred by everyone in the home country to the one over free trade. This is a very restrictive view of a ´measure´ of welfare and should be taken with a pinch of salt.

[8] We can write consumer surplus (CS) as; $CS={}_{(1/2)(\Theta-QH)}\int^{(\Theta-QH)}(\Theta\ s_{2H}^{*}-p_{2H}^{*})d\Theta+{}_{(\Theta-QH)}\int^{\Theta}(\Theta\ s_{1H}^{*}-P_{1H}^{*})d\Theta=(\Theta-Q_{H})(\Theta^{3}+\Theta^{2}Q_{H}-5\Theta Q_{H}^{2}+19Q_{H}^{3})/32$. Defining total welfare (TW_{H}) as the $CS+\pi_{2H}$ we get, $TW=(\Theta-Q_{H})(\Theta^{3}-\Theta^{2}Q_{H}-\Theta Q_{H}^{2}+9Q_{H}^{3})/16$. Thus, consumer surplus is decreasing as Q_h decreases. The first derivative of TW is positive if $Q_{H}>0.3745\Theta$, otherwise it is negative. Total welfare is maximised at a retriction level of ($0.4508\ \Theta$).

[9] $\pi_{1L}^{*}=(\Theta-Q_{L}\lambda)^{2}(\Theta^{2}-2\Theta Q_{L}\lambda-3Q_{L}^{2}\lambda^{2})/32$, $\pi_{2L}^{*}=\lambda Q_{L}(\Theta^{2}-Q_{L}^{2}\lambda^{2})(\Theta-2Q_{L})/16$. As λ decreases (Q_L decreases) π_{1L}^{*} increases and π_{2L}^{*} decreases.

The domestic firm prefers a higher level of restriction on the foreign firm as its profit increases and acheive its maximum when the foreign firm is competely shut out of the market. At this point the total welfare of the home country is higher ($TW_L = 0.0625\,\Theta^4 > TW^* = 0.059643\,\Theta^4$) than under free trade. Thus, the home country prefers to completely shut out imports of the low quality good. From the viewpoint of the consumer prices go up and total output sold decreases as the quota becomes more restrictive. As is stated in Proposition-2 average quality increases for a level of restriction less than 0.2441Θ and decreases for levels of restrictions between ($0.2441\,\Theta$, 0.2746Θ). Note, that despite the increase in average quality ($<0.2441\Theta$) consumer surplus decreases for all levels of the restriction.[10]

VI. Conclusion.

This paper adds to the existing literature on imperfect competition (DD (1989), Ries (1993)) in that it treats the choice of quality as a long run strategic variable. Due to this a firm can pre-commit to a level of quality before it competes in the market (as it incurs a sunk cost of improving its quality). We show that in a vertically differentiated model with quantity restrictions a firms´ choice of quality not only depends on whether the restricted firm produces the low, or high, quality good but also on the restrictiveness of the quota. Contrary to the existing results we show that both firms downgrade their qualities for a restriction at, or close, to the free trade level. Only for the case where a very restrictive quota is imposed on the low quality foreign firm does average quality decrease. Further, we get quality downgrading even under the condition for which the foreign monopolist in Krishna (1987)

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[10] The consumer surplus; $CS_L = \int_{(1/2)(\Theta - QL(2-\lambda))}^{(1/2)(\Theta - QL\,.\lambda)} (\Theta\, s_{2L} - p_{2L}).\, d\Theta + \int_{(1/2)(\Theta + QL\,\lambda)}^{\Theta} (\Theta\, s_{1L} - P_{1L})\, d\Theta = (1/32)\,(\Theta^2 - Q^2_L\,\lambda^2)\,(\Theta^2 + 2\,\lambda\,\Theta\,Q_L + 4\,\lambda Q_L^2 - 3\,\lambda^2 Q_L^2)$. The consumer surplus decreases as Q_L decreases. Total welfare, $TW_L = (1/16)\,((\Theta^2 - Q^2_L\,\lambda^2)\,(\Theta - 2\lambda\Theta\,Q_L + 2\,\lambda\,Q_L^2)$, and is highest at $Q_L = 0$.

upgrades its quality, i.e., if $P_{xq} < 0$ (x is the good and q is the quality). Our results not only highlight the importance of quality choice under stragtegic interaction but, also the role of the timing of the decisions.

In our framework firms compete first in qualities and then in quantities. For a quota imposed on the foreign firm at, or close to, the free trade level, the home firm is able to decrease its quality and thus is able to save on the sunk costs of quality, while barely losing any market share. Similarly, the foreign firm also lowers its quality, maintaining its market share, and increases its profit by (also) saving on the costs of quality.

That quantity restrictions work as facilitating practices (as in Harris (1985), Krishna (1989)) is further reinforced in our framework. After the imposition of the restriction at the free trade level both firms lower qualities, raise prices and attain a higher level of profits. Our analogue of the result is even more striking because quotas work as a facilitating practice device for the case of competition in quantities. This is contrary to the well known result that quotas have no influence on market outcomes when firms compete in quantities. It will be worthwhile to further study if the role of quotas as facilitating practice devices in our framework is even stronger when the firms compete in prices.

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Appenidx-I: (The solutions in this part were obtained by using Mathematica)

We check for leapfrogging in the following four cases. (1a) and (1b) check for incentives to leapfrog in the case-1 (domestic firm low quality and foreign firm high quality) of the main text, and (2a) and (2b) (domestic firm high quality and foreign firm low quality) in the case-2. We present case (1a) in detail. Similar steps are used for the other cases, and thus are explained briefly.

(1a) Low-quality domestic firm leapfrogs the high-quality restricted foreign firm.

The low-quality domestic firm, which leapfrogs and then produces the high quality, is denoted as firm-1. Thus, $s_1 > s_{1H}^* = Q_H(\Theta - Q_H)$, s_{1H}^* being the quality choice of the foreign firm (equation (9)). The ex-ante (before leapfrogging) profits of the domestic firm are, $\pi_{2H}^* = (\Theta - Q_H)^4/32$. We need to compare these against the profits after leapfrogging when;

(i-a) Foreign firm is restricted (R) ex-post, $\pi_1^R = s_1 (\Theta - Q_H s_{1H}^*/s_1)^2/4 - s_1^2/2$.

(i-b) Foreign firm is non-restricted (U) ex-post, $\pi_1^U = \Theta^2 [1 - 2 s_1/(4 s_1 - s_{1H}^*)]^2 - s_1^2/2$.

To check under what conditions the foreign firm is restricted ex-post, we first define the quality level, $s_1^{\#}(Q_H)$, of the domestic firm which makes the quota just binding on the foreign firm. To do this, equate Q_H to $(\Theta - x_{1H})/2$ in (I-a) and take into account (I-b).

(I-a) $x_{1H} = min\ [Q_H, (\Theta - x_1)/2]$, and (I-b) $x_1 = [\Theta - (s_{1H}^*/s_1)\ x_{1H}]$.

We thus get $s_1^{\#}(Q_H) = [Q_H^2(\Theta - Q_H)/(4Q_H - \Theta)]$. Note, that the foreign firm is ex-post restricted for all $s_1 \leq s_1^{\#}$. Then $s_1^{\#}(Q_H) < s_{1H}^* = Q_H(\Theta - Q_H)$ for all $Q_H > \Theta/3$. Given that $s_1 > s_{1H}^*$, the foreign firm is ex-post non-restricted for quotas larger than $\Theta/3$. Now, to find the lower value of the quota for which the foreign firm is ex-post non-restricted, $Q_H^{\#}$, we first evaluate the derivative of (i-b), $d\pi_1^U/ds_1$ at $s_1^{\#}$. Then, by equating the result to 0 and solving for Q_H, we get $Q_H^{\#} = 0.316179\Theta$.

We check first for leapfrogging in the range of quotas where the foreign firm is ex-post non-restricted, i.e., for $Q_H \in [0.316179\Theta, 0.4508\Theta]$. Numerically, the difference of $(\pi_1^U - \pi_{2H}^*)$ was plotted and found to be < 0 for all s_1 and Q_H. Thus leapfrogging will not occur in this region. Next, we check for leapfrogging in the range of quotas where the foreign firm is restricted ex-post (i-a), i.e. for the interval $Q_H \in (0.2\Theta, Q_H^{\#})$. (Note, 0.2Θ is the quota for which the profits of the foreign firm become zero). By plotting the difference of $(\pi_1^R - \pi_{2H}^*)$ in the relevant range, we obesrve that it is positive for small enough values of Q_H. Then solving the system of equations $\{\pi_1^R - \pi_{2H}^* = 0,\ d\pi_1^R/ds_1 = 0\}$ for (Q_H, s_1) we obtain the lower bound for Q_H for which the domestic firm has no incentive to leapfrog. This is $Q_H^+ = 0.252235\Theta$. (Note, that the corresponding optimal quality $s_1^* = 0.240192\Theta$ is higher than $s_{1H}^* = 0.18861\Theta$). Thus, if the quantity restrictions are in the range $[0.2\,\Theta, 0.252235\Theta]$ the domestic firm will leapfrog the foreign firm and produce a higher quality than s_{1H}^*.

(1b) High-quality restricted foreign firm leapfrogs the low-quality domestic firm.

Denote the foreign firm that leapfrogs and produces the low quality as firm-2. Thus it produces a quality $s_2 < s_{2H}^* = (1/4)\ (\Theta - Q_H)^2$. Its ex-ante profits are given by $\pi_{1H}^* = [(5Q_H - \Theta)\ (\Theta - Q_H)^2\ Q_H/8]$. We then compare these against the profits after leapfrogging when;

(ii-a) Foreign firm restricted (R) ex-post, $\pi_2^R = (s_2\ Q_H/2)\ \{\Theta - Q_H\ [2-(s_2/s_{2H}^*)]\} - s_2^2/2$.

(ii-b) Foreign firm non-restricted (U) ex-post, $\pi_2^U = [\Theta\ s_{2H}^*/(4\ s_{2H}^* - s_2)]^2 - s_2^2/2$.

Defining, $s_2^{\#}(Q_H)$ as the quality level of the foreign firm which makes the quota just binding on itself. This is obtained from the following conditions;

(II-a) $x_{2H} = [(\Theta/2) - (s_2/2\ s_{2H}^*)\ x_2]$, and (II-b) $x_2 = min[Q_H, (1/2)\ (\Theta - x_{2H})]$

This gives us the expression for $s_2^{\#}(Q_H) = s_{2H}^*\ [4-(\Theta/Q_H)]$. Note, that the foreign firm is ex-post restricted for all $s_2 > s_2^{\#}(Q_H)$. Thus if $s_2^{\#}(Q_H) \geq s_{2H}^*$ then the foreign firm is not restricted ex-post. This is true for all $Q_H > \Theta/3$. Note, that π_2^U attains its maximum at the point $s_2^* = s_{2H}^*$. Thus the lower value of the quota for which the foreign firm is restricted is in fact $Q_H^{\#} = \Theta/3$.

Thus, checking for leapfrogging when the foreign firm is ex-post non-restricted (ii-b) we find that $\pi_2^U < \pi_{1H}^*$ for all $Q_H \in [\Theta/3,\ 0.4508\Theta]$ and for all $s_{2H}^* \geq s_2$. Now, for $Q_H < \Theta/3$ (ii-a) is the relevant expression (the foreign firm is restricted ex-post). As before, solving the system of equations $\{\pi_2^R - \pi_{1H}^* = 0,\ d\pi_2^R/ds_2 = 0\}$ we get the solutions $Q_H^{++} = 0.236592\ \Theta$ and $s_2 = 0.101201\ \Theta^2$. This implies that if the foreign firm is of high quality no leapfrogging is observed in the range of quantity restrictions $Q_H > 0.252235\Theta$.

To conclude, if the foreign firm is of high quality then no firm has incentive to leapfrog if $Q_H > max[Q_H^+,\ Q_H^{++}]$ (Note, Q_H^+ is obtained from Case-1a.). Thus, for all $Q_H \geq 0.252235\ \Theta$, (s_{1H}^*, s_{2H}^*) is an equilibrium.

(2a) Low quality restricted foreign firm leapfrogs the high-quality domestic firm.

Denote the low-quality foreign firm that leapfrogs as firm-1. Then $s_1 > s_{1L}^* = (1/4)[\Theta^2 - Q_L^2\lambda^2]$, where $Q_L(\lambda) = \Theta[-1+(1-4\lambda+4\lambda^2+\lambda^4)^{1/2}]/(\lambda^3+4\lambda-4)$. The ex-ante profits for the foreign firm are $\pi_{2L}^* = \lambda Q_L(\Theta^2 - Q_L^2\lambda^2)\ (\Theta - 2Q_L)/16]$. We compare the ex-ante profits against;

(iii-a) Foreign firm restricted (R) ex-post, $\pi_1^R = (Q_L/2)\ (\Theta - Q_L)\ (2\ s_1 - s_{1L}^*) - s_1^2/2$.

(iii-b) Foreign firm non-restricted (U) ex-post, $\pi_1^U = \Theta\ s_1[1-2\ s_1/(4\ s_1 - s_{1L}^*)] - s_1^2/2$.

Asking the question; Under what Q_L, is (iii-b) the relevant expression and solving for $s_1^{\#}(Q_L)$ from the following expressions;

(III-a) $x_{1L} = (1/2)\ (\Theta - x_1)$ and (III-b) $x_1 = min[Q_L,\ (\Theta/2) - (s_{1L}^*/2\ s_1)\ x_{1L}]$.

We obtain $s_1^{\#}(Q_L) = s_{1L}^*(\Theta - Q_L)/2(\Theta - 2Q_L)$. Now, if the $max\ s_1^{\#}(Q_L) < min.\ s_{1L}^*(Q_L)$, then (iii-a) is the only relevant expression (foreign firm is restricted ex-post). We get, $min\ s_{1L}^* = 0.24756\Theta^2$ and the $max.s_1^{\#}(Q_L) = \{max.s_{1L}^*\}[max.(1/2)(\Theta - Q_L)(\Theta - 2Q_L)] = (1/4)(0.80457) = 0.20114\Theta^2$. Thus, $max\ s_1^{\#}(Q_L)\ (=0.20114\Theta^2) < min\ s_{1L}^*\ (=0.24756\Theta^2)$. Thus, the foreign firm is always restricted ex-post. Further, the restricted profit $(\pi_1^R) <$ the ex-ante profits (π_{2L}^*) for all $(Q_L,\ s_1)$. Thus, there does not exist any incentive to leapfrog in this case.

(2b) High-quality domestic firm leapfrogs the low-quality restricted foreign firm.

Denoting the high quality domestic firm that leapfrogs as firm-2. Then, $s_2 < s_{2L}^* = (\lambda/4)[\Theta^2 - Q_L^2\lambda^2] = Q_L(\Theta/2) - Q_L) + \lambda Q_L^2$. Given, the ex-ante profits $\pi_{1L}^* = (\Theta - \lambda Q_L)^2(\Theta^2 - 2\Theta Q_L\lambda - 3Q_L^2\lambda^2)/32$, where, $Q_L(\lambda) = \Theta[-1+(1-4\lambda+4\lambda^2+\lambda^4)^{1/2}]/(\lambda^3+4\ \lambda-4)$. We compare this against;

(iv-a) Foreign firm restricted (R) ex-post, $\pi_2^R = (1/4)\ (\Theta - Q_L)^2\ s_2 - s_2^2/2$.

(iv-b) Foreign firm non-restricted (U) ex-post, $\pi_2^U = [\Theta\ s_{2L}^*/\ (4\ s_{2L}^* - s_2)]^2\ s_2 - s_2^2/2$.

Once more, asking the question; Under what Q_L, is (iv-b) the relevant expression and obtaining $s_2^{\#}(Q_L)$ from the following;

(IV-a) $x_2= (\Theta/2)-(x_{2L}/2)$ and (IV-b) $x_{2L}=min[Q_L, (\Theta/2)- (s_2/2\, s_{2L}^*)x_2.]$.

Which gives us $s_2^{\#}(Q_L)=2s_{2L}^*(\Theta-2Q_L)/(\Theta-Q_L)$. Note, that π_2^R attains its maximium at $s_2=s_{2L}^*$ Note, that $s_2^{\#}(Q_L)/s_{2L}^*>1$ for all $Q_L\in[0, 0.2746\Theta]$ (in fact for all $Q_L<\Theta/3$). Hence, the only relevant region is (iv-a). However, the $\text{Max}[\pi_2^R-\pi_{1L}^*]<0$ for all values of Q_L. Thus, the no-leapfrogging condition is always satisfied if the foreign firm is of low quality.

In conclusion, when the domestic firm is of high quality no firm has an incentive to leapfrog its rival. Thus, (s_{1L}^*, s_{2L}^*) is an equilibrium for all Q_L.