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## REAL RETURNS ON GOVERNMENT DEBT: A GENERAL EQUILIBRIUM QUANTITATIVE EXPLORATION

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### Abstract

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We extend and apply computable general equilibrium methods to the study of economies with both aggregate uncertainty and uninsured household-specific uncertainty. In our economies the government issues two types of assets: a small denomination, non-interest bearing asset, which we call currency, and a large denomination, interest bearing asset, which we call T-bills. We find that a real interest rate behavior similar to that observed in the U.S. can be sustained as equilibrium behavior in our class of economies. We also find that policy induced real interest rate changes that are perceived as being permanent have significant real effects and that these effects take a few years to be fully realized.

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### Key Words

Real Returns; Heterogeneous Agents; Liquidity Constraints; Quantitative General Equilibrium.

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## 0. Introduction

Recently economists have been exploring ways to introduce money into applied general equilibrium analyses. Virtually all these efforts have followed the quantity theory tradition of Irving Fisher and Alfred Marshall. This tradition focuses on the transactions role of money. To model this role of money, most of these studies have used the Lucas and Stokey (1983, 1987) cash-credit good extension of the Lucas (1982) and Svensson (1985) cash-in-advance model. Examples of such studies are Cooley and Hansen (1989), Altig and Carlstrom (1991) and Christiano and Eichenbaum (1993), amongst others.

The behavior of real returns on government debt in these representative household economies is grossly at variance with the data. In these model economies, the average real return on government debt is close to 5 percent while in the U.S, the average rate of return on 3 month T-bills was only 0.6 percent during the 1950-1986 period. Further, in these model economies the expected real interest rate varies little, and this result is essentially independent of the monetary policy rule followed. However, in the U.S. the average real interest rate on government debt over periods as long as 5 years has varied significantly: In the 1974-78 period it averaged  $-1.6$  percent, while in 1981-85 it averaged  $4.7$ . These disparities present a very serious problem for the representative household approach, if it is to be used in evaluating the implications of short-term monetary policy.

These large disparities between the model predictions for real interest rate behavior and U.S. data have lead us to abandon the representative household abstraction for monetary policy analysis and to introduce household heterogeneity. In this exploratory study households are heterogeneous with respect to their nominal asset holdings and their production opportunities. In the permanent income tradition of Bewley (1980), these households vary their holdings of nominal assets in order to buffer their flows of consumption against uninsured idiosyncratic variations in the market value of their time endowment.<sup>1</sup>

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<sup>1</sup>Other analyses of economies with uninsured idiosyncratic uncertainty are Lucas (1980) and İmrohoroğlu (1989, 1991).

Any model economy that explores the behavior of real returns on interest bearing government debt must include at least two government issued assets: one that bears interest and another one that does not. If both of these assets are to be held in equilibrium the non-interest bearing asset must play some role which the interest bearing asset cannot play. The exact nature of this role does not seem to be all that important for the issues that we are addressing here.<sup>2</sup> In this study we follow Bryant and Wallace (1979) and we assume that interest-bearing government debt, which we call T-Bills, is issued only in large denominations. On the other hand, the denomination of the non interest-bearing asset, which we call currency, is small.

Given that our economy has heterogeneous households, its state at each point in time must specify the distribution of households as indexed by their asset holdings and by their production opportunities. When there is only idiosyncratic household uncertainty and no aggregate uncertainty, for policies that result in a constant inflation rate and a constant nominal interest rate, the equilibrium path of this distribution of households converges to a steady state distribution. We find that steady state distributions exist for policies with both low —zero— and high —three percent— real returns to T-bills. The zero percent return is significantly smaller than the infinitely lived household's subjective time discount rate, which together with the average growth rate of consumption ties down the average real interest rate in the neoclassical growth model.<sup>3</sup>

We also compute the equilibrium process for a special class of government policy rules that allow for random changes in both nominal and real interest rates. We find that policy rules with persistent changes in the real interest rate have significant effects on output and employment. Further, these effects increase over time and they take a few years to be fully realized. On the other hand, policy rules with transitory changes in the real interest rate have effects on output and employment that are negligible.

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<sup>2</sup>Marimon and Wallace (1987) assume that breaking up the large denomination T-bills is costly. Another way to model this additional role for the non-interest bearing asset is to introduce a banking system and to impose a legal requirement that banks must hold non interest-bearing reserves on deposits. Yet another way is to follow Lucas (1982) and to include a cash-in-advance constraint.

<sup>3</sup>Real rates of return have to be adjusted for inflation risk, but quantitatively, these adjustments are small.

The rest of this paper is organized as follows: In Section 1 we formally describe our class of monetary economies, we define the equilibrium processes, and we discuss calibration issues. In Section 2, we describe the computational experiments and we report our findings. Finally, in Section 3 we present our concluding comments and we provide some suggestions for future research.

## 1. Description of the Class of Monetary Economies

### 1.1 Information

There is an exogenous economy-wide stochastic process  $\{z_t\}$ . This process is a Markov chain and its transition probabilities are:

$$\pi_z(z' | z) = Pr\{z_{t+1} = z' | z_t = z\}. \quad (1)$$

for  $z, z' \in Z = \{1, 2, \dots, n_z\}$ . We assume that the Markov chain generating  $z$  is such that it has a single ergodic set, no transient states and no cyclically moving subsets.

Each household also faces an idiosyncratic random disturbance,  $s$ , that affects its individual production possibilities. Conditional on the realization of the economy-wide shock one period ahead, these idiosyncratic disturbances are assumed to be independent and identically distributed across households. The process for this household-specific production shocks,  $\{s_t\}$ , is also assumed to follow a finite-state Markov chain with conditional transition probabilities given by:

$$\pi_s(s' | s, z') = Pr\{s_{t+1} = s' | s_t = s, z_{t+1} = z'\}. \quad (2)$$

where  $s, s' \in S = \{1, 2, \dots, n_s\}$  and  $z' \in Z$ .

The joint processes on  $(s, z)$  are therefore Markov chains with  $n = n_s \times n_z$  states. Their transition probabilities are:

$$\pi[(s', z') | (s, z)] = \pi_s(s' | s, z')\pi_z(z' | z) \quad (3)$$

Households know the laws of motion of both  $\{s_t\}$  and  $\{z_t\}$ . At the beginning of each period they observe the realizations of both stochastic processes. Trade ensues.

## 1.2 The government sector

The government in this economy taxes labor income at a rate  $\theta$ . This is a proportional tax and is restricted to being a function of the current value of the economy-wide shock,  $z_t$ , only. The tax rate at date  $t$  is  $\theta(z_t)$ . The government also issues two assets. The first asset determines the unit of account and bears no interest. We denote it by  $M$ , and we call it currency. The second asset is a large denomination risk-free promise to deliver  $\gamma$  units of currency at the beginning of the period immediately after its date of issue. This asset may sell at a discount. We denote it by  $B$ , and we call it a T-bill.<sup>4</sup>

Variable  $p_t$  is the price of one unit of the date  $t$  composite good. Government policy determines the pricing process on currency,  $\varepsilon(z_t) = p_t/p_{t-1}$ , and the discounted price of government debt,  $\varrho(z_t) = q(z_t)$ , where  $q(z_t)$  denotes the price of a sure claim to one unit of currency one period ahead.<sup>5</sup> To implement these policies, the government exchanges goods and currency at a price  $p_t$  and sells and buys promises to deliver  $\gamma$  units of nominal value next period at price  $\gamma q(z_t)$ . We only consider economies with a positive nominal interest rate policy, that is, where  $q(z) \leq 1$  for all  $z \in Z$ . An additional restriction is that  $q(z)\varepsilon(z') < 1/\beta$  for all  $z$  and  $z'$ . With this restriction the real rate of return on interest-bearing government debt is always less than the households' subjective time discount rate. Finally, we make the additional assumption that households may not pool their savings and share the proceeds of T-bills.

A government policy rule is, therefore, a specification of  $\{\theta(z), \varepsilon(z), \varrho(z)\}$  and the associated processes on public consumption,  $g$ , on the government supply of T-bills,  $B^g$ , and on the government supply of currency,  $M^g$ . Under this specification for the government policy, the nominal version of the government budget constraint is the following:

$$p_t g_t + M_t^g + \gamma B_t^g = \theta y_t p_t + M_{t+1}^g + q_t \gamma B_{t+1}^g \quad (4)$$

<sup>4</sup>Note that throughout this paper we follow the convention that capital letters denote nominal quantities and, except where otherwise indicated, lowercase letters denote the real values of the corresponding variables in terms of current-period prices.

<sup>5</sup>Note that the pricing policies are also restricted to being a function of the current value of the economy-wide shock,  $z_t$ , only.

where  $y_t$  denotes the aggregate output of period  $t$ .

### 1.3 The household sector

#### *Preferences*

We assume that at each point in time the economy is inhabited by a large number, actually a measure one continuum, of households. These households order their random streams of consumption and leisure according to:

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, \tau - n_t) \quad (5)$$

where  $u$  is a continuous and strictly concave utility function,  $\beta$  is the time-discount factor,  $c_t$  is the perishable household consumption good which is restricted to being non-negative,  $\tau$  is the household endowment of productive time and  $n_t$  is time allocated to market activities. Hence,  $\tau - n_t$  is time allocated by the household to non-market activities which we call leisure.

#### *Productive opportunities*

The household's date  $t$  production of the composite good is:

$$w(s, z)n_t \quad (6)$$

where  $w(s, z)$  is that household's technology parameter. When households choose to work, they are paid their marginal product. Therefore  $w(s, z)$  equals the household's real wage. Following Rogerson (1988) and Hansen (1985), we assume a labor indivisibility. Labor services,  $n_t$ , are constrained to belonging to the set  $\{0, 1\}$  where zero corresponds to not being employed and one corresponds to being employed.

#### *Monetary Arrangements*

Households can hold integer amounts of small denomination currency  $M \in \{0, 1, 2, \dots\}$ . They can also hold integer numbers of large denomination T-bills  $B \in \{0, 1, \dots, n_b\}$ . The denomination of T-bills in terms of currency is a large integer  $\gamma$ . Since there are no insurance technologies available, agents hold these assets for consumption smoothing

and for protection against variations in their marginal productivities and therefore in their labor income.

*The households' decision problem*

Let  $M_{t+1}$  denote the nominal end-of-period household currency holdings,  $B_t$ , the nominal end-of-period household holdings of T-bills, and  $A_t = M_t + \gamma B_t$ , the beginning-of-period nominal asset holdings. Then, the nominal version of the household competitive decision problem is the following:

$$\max_{c_t, n_t, B_{t+1}, M_{t+1}} E \sum_{t=0}^{\infty} \beta^t u(c_t, \tau - n_t) \quad (7)$$

subject to the budget constraint

$$p_t c_t + M_{t+1} + q_t \gamma B_{t+1} \leq A_t + (1 - \theta(z_t)) p_t w(s_t, z_t) n_t \quad (8)$$

The maximization is also subject to  $M_{t+1}$  and  $B_{t+1}$  belonging, respectively, to sets  $M$  and  $B$  and  $n_t$  belonging to  $\{0, 1\}$ . Finally,  $M_0$  and  $B_0$  are taken as given.

Let  $m = M_{t+1}/p_t$  denote household real holdings of currency,  $b = B_{t+1}/p_t$ , household real holdings of T-Bills, and  $a = m_{t-1} + \gamma b_{t-1}$ , household real holdings of beginning-of-period assets, all three valued in terms of the current period's consumption good. Then, the functional equation for the dynamic program solved by an  $(a, s)$ -type household is the following:

$$v(a, s, z) = \max_{b, c, m, n} \left\{ u(c, \tau - n) + \beta \sum_{s', z'} v(a', s', z') \pi_s(s' | s, z') \pi_z(z' | z) \right\} \quad (9)$$

subject to the budget constraint

$$c + m + \gamma q(z) b \leq a/e(z) + n w(s, z)(1 - \theta) \quad (10)$$

and to  $n \in \{0, 1\}$ ,  $c \geq 0$ ,  $m \in \{0, \rho, 2\rho, \dots, \gamma\}$ <sup>6</sup> where  $\rho$  denotes the real value of one unit of currency, and  $b \in \{0, 1, \dots, n_b\}$  and where  $a' = \gamma b + m$ . Given that the agent's problem is a finite state discounted dynamic program, an optimal stationary Markov

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<sup>6</sup>Note that the nominal interest rate on T-bills is always positive for the policies that we consider. As a result, T-bills dominate currency in rate of return. Therefore, it is never optimal to hold more than  $\gamma$  units of  $m$  and the restriction that  $m \leq \gamma$  is never binding. This constraint is imposed, none the less, so that the problem becomes a finite dynamic program.

plan always exists. This optimal plan and the stochastic processes on  $(s, z)$  define an equilibrium state transition probability matrix on  $A \times S \times Z$ . The ergodicity of this matrix is established in Appendix 2.

#### 1.4 Definition of equilibrium

In the goods and asset market the government is not a small agent so treating it as just another price-taking agent is not reasonable. Instead, part of the specification of the economy must be the policy arrangement employed and the resulting government excess demand correspondences for the consumption good and for both assets. Features of our explicit arrangement include the properties of the assets issued by the government, the liquidity constraints and the legal restrictions. Other features of our policy arrangement are that the government taxes labor income at a rate  $\theta(z)$ ; that each date the government exchanges goods for currency at price  $p_t$ ; that this price satisfies  $p_{t+1} = p_t \varepsilon(z_t)$ ; that it exchanges T-bills for currency at a price of  $q(z_t)$  per unit of currency to be delivered the following period, and that this price satisfies  $q(z_t) = \rho(z_t)$ . For such an arrangement there is a well defined government excess-demand correspondence. We now define a recursive equilibrium.

The *state of a household* is the triple  $(a, s, z)$ . The measure of agents of type  $(a, s)$  is  $x(a, s)$ . We let  $x$  denote the corresponding measure. The *economy-wide state* is the pair  $(x, z)$ .

An *equilibrium* for a policy arrangement  $\{\theta(z), \varepsilon(z), \rho(z)\}$ , given  $x_0$ , consists of four basic parts: a government policy  $\{g(x, z), m^g(x, z), b^g(x, z)\}$ , a household policy  $\{c(a, s, z), n(a, s, z), m(a, s, z), b(a, s, z)\}$ , pricing processes  $\{e(z), q(z)\}$ , and a law of motion for the measures of agent types,  $x'_{a', s'} = f_{a', s'}(x, z, z')$ , such that:

i.) Given the processes on,  $\theta(z)$ ,  $e(z) = p_t/p_{t-1}$ , and  $q(z)$ , the household policy solves the household's optimization program described in equations. (9)–(10) above.

ii.) The goods market clears:

$$\sum_{a, s} x(a, s)[c(a, s, z) - n(a, s, z)w(s, z)(1 - \theta)] + g(x, z) = 0 \quad (11)$$



for all  $(x, z)$  in the support of the distribution of  $(x_t, z_t)$  for some  $t$ .

iii.) The currency market clears:

$$m^g(x, z) = \sum_{a,s} x(a, s)m(a, s, z) \quad (12)$$

iv.) The T-bills market clears:

$$b^g(x, z) = \sum_{a,s} x(a, s)b(a, s, z) \quad (13)$$

v.) Household and aggregate behavior are consistent:

$$f_{a',s'}(x, z, z') = \sum_{a,s \in \Omega(a',z)} x(a, s)\pi[(s', z') | (s, z)] \quad (14)$$

for all  $(a', s', x, z, z')$ , where  $\Omega(a', z) = \{(a, s) : a' = m(a, s, z) + \gamma b(a, s, z)\}$ . Note that  $f_{a',s'} \equiv x'(a', s')$  for all  $(a', s') \in A \times S$ .

vi.) The behavior of endogenous variables is consistent with the policy arrangement.

For our class of policy arrangements, this requires that  $e(z) = \varepsilon(z)$ ,  $q(z) = \varrho(z)$  and  $g(y, z) \geq 0$  for all  $(x, z)$  in the support of the distribution of  $(x_t, z_t)$  for some  $t$ .<sup>7</sup>

For the set of policy arrangements that we consider, there is at most one equilibrium. The computational procedure we use to find the equilibrium is the following: first we solve the household problem which is a finite-state discounted dynamic program. Then we use the household optimal decision rules and the initial distribution of households to obtain a stochastic realization of  $g(x, z)$  from (11). If  $g_t = g(x_t, z_t)$  turns out to be a positive stochastic process, we have found the unique equilibrium given the policy arrangement. Otherwise, we have established that no equilibrium exists for that policy arrangement. A fully documented version of the FORTRAN program used to solve this economy is available from the first author upon request.

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<sup>7</sup>Note that the households' budget constraints and the market clearing conditions imply that the government budget constraint is also satisfied. In real terms an expression for the government budget constraint is the following:  $g_t = \theta y_t + m^g - m_{-1}^g/e + \gamma(qb^g - b_{-1}^g/e)$

## 1.5 Calibration

The transitions on the economy-wide process and the parameters that specify the government policy are different for different experiments and we discuss them in the following section. The remaining calibration choices are the following:

### *Time period .*

Most U.S. time series are reported quarterly. Wages, however, are paid more frequently. Our model period, therefore, should be shorter than a quarter of a year. We chose the model period to be an eighth of a year. This choice enables us to have some temporal aggregation while keeping the computation costs within reasonable bounds.<sup>8</sup>

### *The exogenous individual-specific process*

In the three model economies considered, we assume that the individual-specific productivity process,  $\{s\}$ , can take two possible values,  $s \in S = \{1, 2\}$ . State 1 represents high productivity draws and state 2 represents low productivity draws, *e.g.* a qualified electrician who can only find a job as a janitor.

The transition probabilities are chosen so that, on average, 92 percent of the time on average, agents experience the high productivity shock and the remaining 8 percent of the time they experience the low productivity shock. We also require that the expected duration of the low productivity shock be of two model periods, or a quarter of a year. These values roughly match the average U.S. employment rate and the expected duration of unemployment in U.S. business cycles. Given that in this paper we are not specifically concerned with shocks to the aggregate technology, in Experiment 3 we also assume that the individual productivity processes are independent of  $z$ . The transition probabilities on  $s$  that satisfy these requirements are the following:<sup>9</sup>

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<sup>8</sup>During the calibration stage of this project we experimented with shorter model periods and we found that they did not result in significant changes in the aggregate properties of the model.

<sup>9</sup>These transition probabilities for the individual-specific processes are the same as those considered in İmrohoroğlu (1989).

	$s' = 1$	$s' = 2$
$s = 1$	0.9565	0.0435
$s = 2$	0.5000	0.5000

### *Preferences*

Following the applied general equilibrium tradition we choose a utility function with constant elasticity of substitution in consumption and leisure. During the last 50 years, in the U.S., per capita leisure has remained virtually constant, per capita consumption has grown at an average rate of nearly 2 percent and real wages have increased by a factor of two. To match these observations we assume a unit elasticity of substitution between consumption and leisure. The utility function for our model economies is, therefore, the following:

$$U(c_t, \tau - n_t) = (1 - \sigma)^{-1} \{ [c_t^\alpha (\tau - n_t)^{1-\alpha}]^{1-\sigma} \} \quad (15)$$

where  $\tau - n$  is leisure.

We select preference parameters  $\beta = 0.995$  and  $\alpha = 0.33$ . These parameter values imply an annual subjective time discount rate of 4 percent and a share of leisure of approximately two-thirds. These values for the time discount rate and for the share of leisure are in line with observations from national income and product accounts on the net real rate of return on capital and on the average fraction of productive time that households allocate to the market. We choose  $\sigma = 1.5$ . This value is commonly used in applied general equilibrium exercises in public finance and business cycle theory. Our choice of  $\tau$  reflects the fact that the average workweek including commuting time is roughly 45 hours or approximately 45 percent of people's weekly endowment of productive time, given that we consider the productive part of a day to be 14 hours. Parameter  $\tau$  is, therefore,  $1/0.45 = 2.22$ .

### *Technology parameters*

The model economy technology parameters are denoted  $w(s, z)$ . The values of those parameters are normalized so that the productivity of highly productive types is 1.0.

The relative size of the marginal productivities of households in their high and low productivity times is three. This number is chosen to roughly match the ratio between the average hourly wage in U.S. manufacturing and the minimum hourly wage in the U.S. With these choices we are implicitly assuming that there are always minimum wage openings for anyone who wants them. Finally in Experiment 3 the technology parameters are chosen to be independent of  $z$ . This additional restriction facilitates the comparison between Experiments 1 and 2 and the long-run asymptotic behavior of Experiment 3 when there are no policy switches. These parameter choices, together with the denomination of T-bills and the transition probability parameters, result in average holdings of both assets that are reasonably close to U.S. aggregates.<sup>10</sup>

The resulting productivity parameters for each type of agent are the following:

	$s = 1$	$s = 2$
$w(s)$	1.00	0.33

#### *Units and bounds*

In addition to the parameters already discussed, in order for the program described in equations (9) and (10) to be well defined, we must choose the real value of one unit of currency,  $\rho$ , the denomination T-Bills,  $\gamma$ , and the maximum number of T-Bills,  $n_b$ .

In every experiment we choose  $\rho = 1/100$  and  $\gamma = 300$ . These choices imply that the real value of one T-bill is 3, which approximately corresponds to 50 percent of the average yearly model economy per capita income and roughly matches the value of the ratio of the denomination of T-bills to per capita yearly income in the U.S. The choice for  $\rho$  implies that the real value of one unit of currency corresponds to approximately

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<sup>10</sup>Note, however, that there is one dimension in which our calibrated economy fails to mimic the data: in Experiment 3 the percentage variations in household annual incomes are nearly twice as large as those found in panel studies of the U.S. economy. The reason for this divergence is that in the model economies, households hold liquid assets only as a substitute for insurance against idiosyncratic income variations. It goes without saying that people hold liquid assets for many other reasons. Liquid assets are held, for instance, as a substitute for insurance against sickness and accidents, or to make large payments for consumer durables, college education, and down-payments on houses. Given that our model economies abstract from these reasons, greater income variability is needed if the average aggregate asset holdings are to come close to those observed in the data. This property of Experiment 3 could be changed by allowing the productivity parameters to vary with  $z$ . This modification, however would cloud the comparisons with the two other experiments.

0.148 percent of the model economy average per capita yearly income. If we take U.S. average per capita yearly income to be \$20,000, the smallest currency unit of the model economies would be worth, approximately, \$30. We find this unit to be sufficiently small for the purposes of this paper. Making this unit smaller raises computational costs significantly and has virtually no effect on the aggregate properties of the model. Finally, the maximum number of T-bills that a household can hold is  $n_b = 3$ . We find that this value is never binding in equilibrium.

## 2. The Experiments

In the U. S. in the 1926-80 period real returns to short-term interest bearing government debt averaged about zero percent. This low real interest rate regime changed during the 80's when interest rates increased to about 3 percent. Throughout the 1926-90 period inflation rates averaged about 4 percent. In this paper we explore the behavior of the model economy under real interest regimes that mimic this behavior: a low real interest rate regime defined by a nominal interest rate of 4%, an inflation rate of 4% and, consequently a zero real return to interest-bearing debt, and a high real interest rate regime with a nominal interest rate of 7%, an inflation rate of 4% and, consequently a 3% real return to interest-bearing debt. These policy choices are reported in Table 1.

Table 1: The Real Return Regimes

Policy parameters	Low Real Returns	High Real Returns
Nominal interest rate	7%	4%
Inflation	4%	4%
Implied real interest rate	0%	3%

In Experiments 1 and 2 described below we explore the steady state behavior of the model economy under, respectively, the low and the high real return regimes. Then, in Experiments 3 and 4 we allow for the possibility of switches between both regimes. In Experiment 3 the policy regimes are permanent with an average duration of, respectively, 50 and 10 years and in Experiment 4 the policy regimes are transitory with an average

duration of, respectively, 1 year and 1 quarter of a year. In the subsections that follow we describe the purpose of the experiments and the calibration choices that are specific to each experiment, and we report the experimental findings.

## 2.1 The steady state experiments: Experiments 1 and 2

### *Purpose*

Experiments 1 and 2 have been designed to find out whether the two policy regimes described above can be sustained as equilibrium processes in our class of model economies under reasonable specifications of the remaining components of government policy. Given the role of the government in our model worlds three outcomes are possible: first, that there exists no specification of government policy that can sustain those interest rate regimes as part of the equilibrium. Second, that the processes on government consumption and on the supplies of currency and T-Bills implied by a policy that includes those interest rates as part of the equilibrium are outrageously absurd. And, third, that the processes on government consumption and on the supplies of currency and T-Bills implied by such a policy are reasonable.

Once this first question is settled, the second purpose of these two experiments is to evaluate the steady state effects of switching from the low real interest rate regime to the high real interest rate regime.

### *Calibration Choices*

#### *i.) Transition probabilities on the exogenous economy-wide process*

Given that Experiments 1 and 2 model two steady state economies, their economy-wide processes take only one value,  $z = 1$ , with a degenerate transition probability matrix given by  $Pr\{z' = 1 \mid z = 1\} = 1$ .

#### *ii.) Government Policy*

Experiment 1 explores the steady state behavior of the model economy under the low interest rate regime. The policy parameter choices for that regime are  $\theta = 0.20$ ,  $\varepsilon = 1.05$  and  $q = 0.995012$ . These parameter choices imply an average tax rate of 20%, an

inflation rate of 4% and a nominal interest rate of 4%. Consequently, the real interest rate in this economy is zero.

Experiment 2 explores the steady state behavior of the model economy under the high interest rate regime. The monetary policy parameter choices for that regime are  $\epsilon = 1.05$  and  $q = 0.991288$ . These parameter choices imply an inflation rate of 4% and a nominal interest rate of 7%. Consequently, the real interest rate in this economy is 3%. Given those components of government policy, the labor income tax rate is then calibrated so that the steady state value of public consumption is approximately 0.305: the same value as the one obtained in Experiment 1. The tax rate that renders this value is  $\theta = 0.21682$ . Therefore, the two steady state experiments have the same inflation rates and the same levels of public consumption and they differ in their nominal interest rates and labor income tax rates, and in the steady state government supplies of currency and T-Bills.

### *Findings*

The findings from Experiments 1 and 2 are reported in Table 1<sup>11</sup>. The most significant of those findings are the following:

i) In Experiment 1, steady state government consumption is 20.5% of output and T-Bills and currency holdings are, respectively, 28.8% and 12.3%. We therefore conclude that when households are liquidity constrained and they hold nominal assets as a substitute for insurance against income risks, low real interest rate regimes are feasible under reasonable specifications of government policy.

ii) In Experiment 2, steady state government debt is also 0.205<sup>12</sup> (this value now corresponds to, approximately, 21.1% of output) and T-Bill and currency holdings are, respectively, 0.333 (34.2% of output) and 0.101 (10.4%). We therefore also conclude that high real rates of return regimes are also feasible in this class of model economies

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<sup>11</sup>The results reported in Table 1 have been renormalized using Experiment 1's output as the normalization factor. Hence in the results reported for Experiment 1, variable levels and variable shares of output coincide.

<sup>12</sup>In this case by construction.

Table 2: Steady State Model Aggregates

	Exp. 1 $r = 0$	Exp. 2 $r = 3$
<b>PRODUCT ACCOUNT</b>		
Output (y)	1.000	0.972
Private consumption (c)	0.795	0.767
Public consumption (g)	0.205	0.205
<b>LABOR INPUT</b>		
Hours (h)	3.079	2.992
Productivity (y/h)	0.325	0.325
<b>ASSET HOLDINGS</b>		
Total Real Assets (a)	0.411	0.434
T-Bills (b)	0.288	0.333
Currency (m)	0.123	0.101
<b>GVT. ACCOUNT</b>		
Tax receipts	0.200	0.211
Int. payments	0.000	0.010
Gov. Deficit <sup>a</sup>	0.005	0.004

<sup>a</sup> Note that the government deficit equals seignorage revenues.

under reasonable specifications of government policy.

*iii)* When households are liquidity constrained and hold nominal assets as a substitute for insurance against income risks, government policies that imply a lower real rate of return on those assets are expansionary. From Table 2 we see that reducing real returns to government debt while keeping public consumption constant implies an increase in output and hours of approximately 3% (2.77 and 2.93% to be precise), and a reduction in total asset holdings of about 5% —this reduction in total assets is brought about by a 14% reduction in T-Bill holdings and a 21% increase in currency holdings. Moreover, in this class of model worlds, reducing the real return to government debt while keeping public consumption constant increases the government deficit. This result arises from the fact that the reduction in income tax revenues resulting from both the lower tax



base and the lower income tax rate is greater than the reduced interest payments on government debt.

## 2.2 Persistent and transitory regime changes: Experiments 3 and 4

### *Purpose*

Experiments 3 and 4 have been designed to explore the behavior of our model economy when aggregate uncertainty is considered. In these two experiments stochastic switches between the low and the high policy regimes are possible. In Experiment 3 both regimes are relatively persistent. To mimic the U.S. experience the expected duration of the low interest rate regime is 50 years and the expected duration of the high interest rate regime is 10 years. In Experiment 4 both policy regimes are relatively transitory. To keep the duration ratios invariant, the expected duration of the low interest rate regime is 1 year and the expected duration of the high interest rate regime is 1 quarter of a year. The purpose of these two experiments is to explore the aggregate effects of permanent policy switches that occur after long periods of policy stability. To that purpose we simulate a 50 year realization of the low interest rate regime after which there is a policy switch to the high interest rate regime. In Figures 1, 2 and 3 we represent the responses of respectively aggregate output, consumption and asset holdings after the policy switch takes place.

### *Calibration Choices*

#### *i.) Transition probabilities on the exogenous economy wide process*

In Experiments 3 and 4 we model policy switches between low and high real interest regimes. Consequently, the aggregate process,  $z$ , can, therefore, take two values,  $z \in \{1, 2\}$ , where state  $z = 1$  represents the low real rate of return regime, and state  $z = 2$  represents the high real rate of return regime. In Experiment 3 the transition probabilities on  $z$  are chosen to imply expected durations of the high and low real return regimes of, respectively, 50 and 10 years which correspond to 400 and 80 model periods. Given that the expected duration of a state in a Markov chain is the reciprocal

of  $1 - \pi(z, z)$ , where  $\pi(z, z)$  is the conditional probability of state  $z$  occurring again the following period, the transition probability matrix for the economy-wide process that satisfies these properties is the following:

	$z' = 1$	$z' = 2$
$z = 1$	0.9975	0.0025
$z = 2$	0.0125	0.9875

In Experiment 4 the transition probabilities on  $z$  are chosen to imply expected durations of the high and low real return regimes of, respectively, 1 year and 1 quarter of a year, which correspond to 8 and 2 model periods. The transition probability matrix for the economy-wide process that satisfies these properties is the following:

	$z' = 1$	$z' = 2$
$z = 1$	0.875	0.125
$z = 2$	0.5000	0.5000

#### ii.) Government Policy

In Experiment 3 the policy parameter choices for, respectively, the low and the high interest rate regimes are the following:  $\varepsilon(1) = \varepsilon(2) = 1.05$ ,  $\rho(1) = 0.995012$ ,  $\rho(2) = 0.991288$ ,  $\theta(1) = 0.2$  and  $\theta(2) = 0.21682$ . These choices imply a normalized average level of public consumption of  $g = 0.2049$ <sup>13</sup> Note that this value for  $g$  is close to the steady state public consumption of Experiments 1 and 2.

In Experiment 4 the policy parameter choices are the same as those for Experiment 3 for every component of government policy except for the average labor income tax rate under the high interest rate regime which are chosen to be  $\theta(2) = 0.203$ . This choices imply an average level of public consumption of  $g = 0.2028$ , again this value is close to the those in Experiments 1, 2 and 3.

#### Findings

Figures 1, 2, 3 and 4 show that permanent policy regime switches that occur after long periods of policy stability have significantly larger effects on aggregate output,

<sup>13</sup>The averages are taken over 51 independent 40 year samples.

consumption and asset holdings when the regimes are perceived as being permanent than when they are perceived as being transitory. Specifically, when policy regimes are persistent the differences between the asymptotic steady state<sup>14</sup> values of aggregate output, consumption and asset holdings under the low and the high interest rate regimes are, respectively 2.62, 3.44 and 4.55%, and when policy regimes are transitory these values are 0.09, 0.06 and 0.03%.

Figures 1, 2, 3 and 4 also show that when the policy regimes are perceived as being permanent, the effects of permanent policy switches take a few years to be fully realized. As can be seen from Figure 1, in the case of aggregate output, it takes almost 2 years to close 50% of the gap between the low and the high real interest regime asymptotic steady state values.

Finally, it is also interesting to note that in spite the discreteness of both the decision rules and the distribution of household-types, the responses of consumption and asset holdings are relatively smooth.

### 3. Concluding comments

We find that in our model economies, large quantities of nominal liquid assets are held by the households in equilibrium. The size of these holdings is approximately 40 per cent of the model economy annual output. Households hold these assets for the insurance substitution services that they provide. Another key finding is that the average real return on these assets depends upon monetary policy followed. Government policies which lower the real return on these assets drive a wedge between the intertemporal substitution rate of the household and the gross real return on these assets. In effect, such a policy change increases the tax rate on the insurance substitution services provided by these assets.

At the present stage of this research program we abstract from physical capital accumulation. But, given that monetary policy determines the tax rate on the insurance

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<sup>14</sup>By asymptotic steady state values we mean those to which the model economy aggregates would converge in the absence of regime changes.

substitution services provided by nominally denoted liquid assets and not the tax rate on the services of physical capital, we think that there is hope for an extension of this model in which there is capital accumulation and in which the rate of return on physical capital is significantly higher than liquid nominal assets, as in fact it is.

We emphasize that at the present stage of this research program it is still premature to use these results as a basis for policy discussions. We think, however, that these early findings are promising enough to suggest that applied monetary theory should not abstract from the precautionary motive for holding liquid assets. We also think that this line of inquiry warrants further development. One extension to this theory is to introduce banks that pool individual savings and effectively divide up the large denomination interest-bearing government debt. Another important extension is to allow for banks to intermediate between households with some households borrowing to finance the purchase of houses and to finance their small businesses. A final extension is to include capital accumulation also in the corporate business sector.

If this research program is to be successful, the aggregate behavior of the extended model must be consistent with the observations that lead to the neoclassical growth model, and with the data on aggregate stocks and average returns on the important classes of nominal assets held by households. We conjecture that this will probably require exploiting household heterogeneity along additional dimensions.

Figure 1: Output response (Low to High)

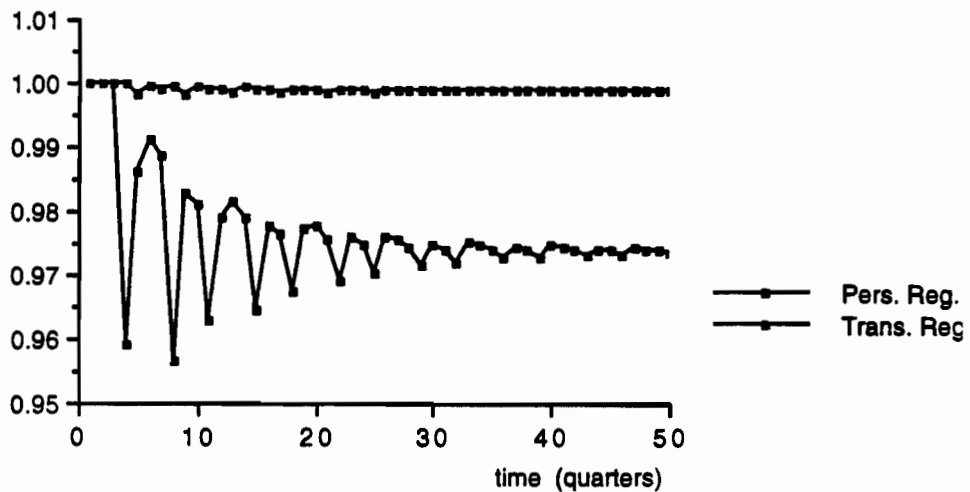
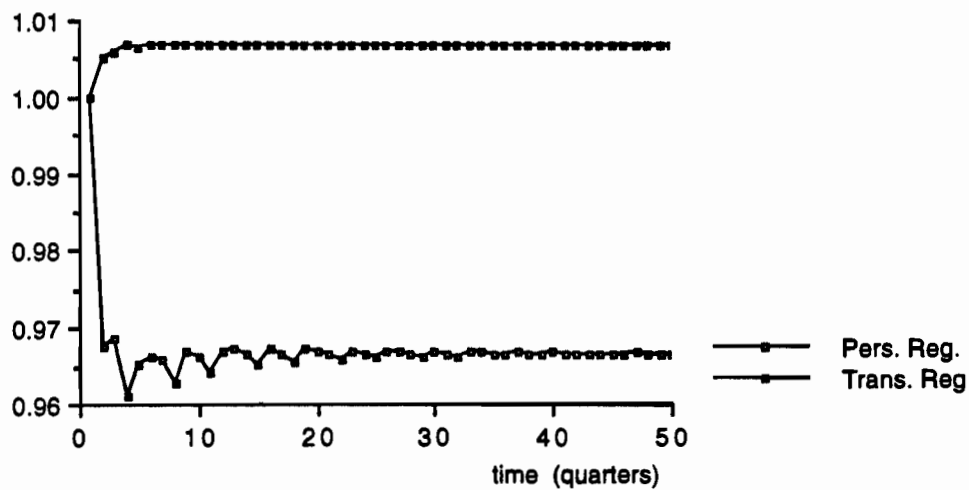


Figure 2: Private consumption response (Low to High)



Note that in all cases aggregates have been normalized at period 0 to be 1

Figure 3: Asset holdings response (Low to High)

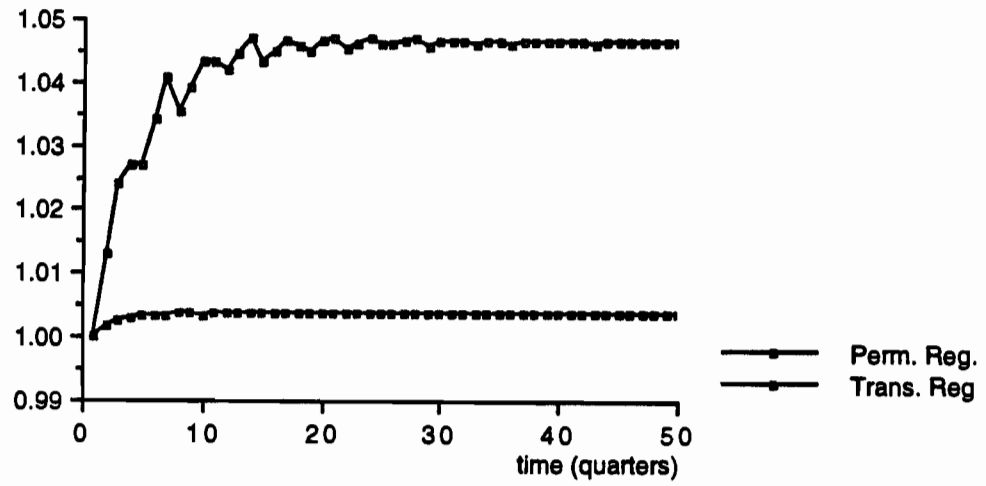
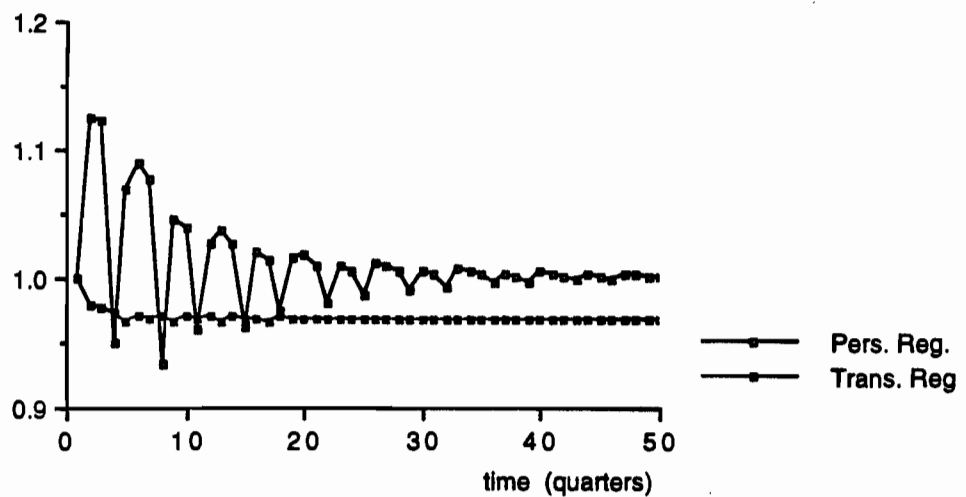


Figure 4: Public consumption response (Low to High)



Note that in all cases aggregates have been normalized at period 0 to be 1

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## Appendix 1

### a. Definitions of the model aggregates

For each simulation of the model economies we compute the following real aggregates:

#### 1. Output

$$y = \sum_{a,s} w(s, z) n(a, s, z) x(a, s) \quad (16)$$

#### 2. Employment<sup>15</sup>

$$h = \sum_{a,s} n(a, s, z) x(a, s) \quad (17)$$

#### 3. End-of-period real currency holdings

$$m = \sum_{a,s} m(a, s, z) x(a, s) \quad (18)$$

#### 4. End-of-period real T-bill holdings

$$b = \sum_{a,s} b(a, s, z) x(a, s) \quad (19)$$

#### 5. Beginning-of-period real asset holdings

$$a = \sum_{a,s} ax(a, s) \quad (20)$$

#### 6. Private consumption

$$c = \frac{a}{e(z)} + y(1 - \theta) - m - \gamma q(z)b \quad (21)$$

#### 7. Interest payments

$$int = q(z)\gamma b - \gamma b_{-1}/e \quad (22)$$

#### 8. Seignorage revenues

$$sgn = m - m_{-1}/e \quad (23)$$

### b. Definitions of the quarterly time series

We then used the model aggregates to construct quarterly time series for some of the basic macroeconomic variables. In so doing, we followed as closely as possible the procedures actually used for U.S. data. Flows are therefore quoted annually. Subscript  $i$  denotes the  $i$ -th subperiod of each quarter. Since the model period was chosen to be one-eighth of a year,  $i = 1, 2$ . I computed the following variables:

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<sup>15</sup>Since the measure of agents is 1, levels and rates are equal

1. Output

$$y = 4(y_1 + y_2) \quad (24)$$

2. Private consumption

$$c = 4(c_1 + c_2) \quad (25)$$

3. Public consumption

$$g = y - c \quad (26)$$

4. Hours

$$h = 4(h_1 + h_2)0.45 \quad (27)$$

5. Average labor compensation

$$w = y/h \quad (28)$$

6. Real currency holdings

$$m = (m_1 + m_2)/2 \quad (29)$$

7. Real T-bill holdings

$$b = (b_1 + b_2)/2 \quad (30)$$

8. Real end-of-period asset holdings

$$a' = m + b \quad (31)$$

9. Nominal interest rate

$$i = 4(-\log q_1 - \log q_2) \quad (32)$$

10. Inflation rate

$$\dot{p}/p = 4(\log p_{t+1,1} - \log p_{t,1}) \quad (33)$$

11. Real interest rate

$$r = i - \dot{p}/p \quad (34)$$

12. Tax revenues

$$\Theta = \theta y \quad (35)$$

13. Interest payments

$$int = 4(int_1 + int_2) \quad (36)$$

14. Government Deficit

$$def = g + int - \Theta = sgn \quad (37)$$

## Appendix 2

Let  $a' = f(a, s)$  be the end-of-period optimal asset holdings expressed as a function of beginning-of-period assets,  $a$ , and of the realization of the household-specific productivity shock,  $s$ . These functions for  $s = 1, 2$  for the economy described in Experiment 2 are plotted in Figures 5 and 6. The ergodicity of the equilibrium Markov chain follows immediately from the following two facts: *i*) that the transition probabilities,  $\pi(s' | s)$  are all positive, and *ii*) that  $a' = f(a, 2)$  lies uniformly below the 45 degree line. These two facts imply that there is a positive probability of reaching asset holdings  $a = 0$  in a finite number of periods, and, therefore, that state  $(a = 0, s = 1, 2)$  is recurrent and that the equilibrium Markov chain is ergodic.

The ergodic set for this economy is  $E = \{(a, s) : a < 3.59\}$ . This result can be seen from the following argument. The decision rule  $a' = f(a, 1)$  is the largest of the two and it crosses the 45 degree line from above at  $a = 3.59$ . Given fact *i*) there is a positive probability of reaching  $a = 3.59$  from any  $a < 3.59$  in a finite number of periods. Further, if  $a > 3.59$  then no point with  $a > 3.59$  can be reached with positive probability. Hence the set  $T = \{(a, s) : a > 3.59\}$  is transient and the set  $E = \{(a, s) : a < 3.59\}$  is ergodic. This argument is very similar to the one offered in the Appendix of İmrohoroğlu (1989).

Figure 5: Decision Rules for Exp 2 ( $s=1$ )

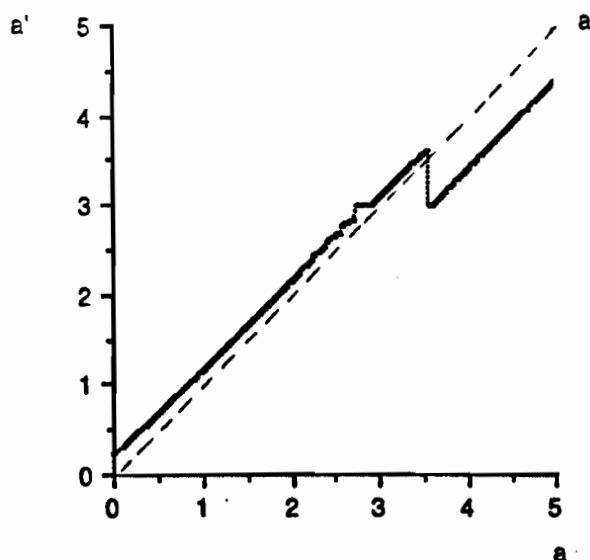


Figure 6: Decision Rules for Exp 2 ( $s=2$ )

