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THE EFFICIENCY OF FINANCIAL MARKETS WITH HIGH INFLATION

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Abstract

In a two period general equilibrium model with incomplete asset markets, it is shown that the contraction of nominal financial markets that occurs during high inflations can result from the variability of the future rate of inflation and from large bankruptcy costs. If the probability that inflation in the future will be high is sufficiently large, then, for a generic set of endowments, an increase in the variability of future prices reduces the utility possibilities set. In economies with only nominal assets more variable future prices lead to a Pareto fall in social welfare.

Key words:
Incomplete asset markets, inflation, welfare cost.

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1 Introduction

This paper is concerned with the welfare costs of inflation; it investigates how does monetary instability affect the way in which a society can allocate risk. Our research is motivated by the observation that in economies with high inflation nominal financial markets become very thin (Heyman and Leijonhufvud [19]). We examine the reasons underlying this phenomenon and analyze its welfare consequences. We find that the contraction of nominal financial markets observed during high inflations can be explained by the variability of the future rate of inflation and by large bankruptcy costs[1]. An important aspect of the contraction of nominal financial markets is that when future price levels are very variable, nominal securities cannot be used to transfer income into or out of high inflation states. As a result, in high inflation economies in which there is a small (but positive) probability that prices will be stabilized, increases in the variability of the future rate of inflation render nominal securities useless, and reduce the efficiency with which financial markets can allocate risk—i.e. they reduce the utility possibilities set.

In the economy we analyze asset markets are incomplete and money is used as a unit of account in nominal financial contracts. In such an economy, monetary authorities face a problem of security design[2] that arises because the value of money determines the real payoff of nominal securities. By choosing the price level monetary authorities determine the set of feasible reallocations of income across states of nature for any given array of real and nominal securities. We study how the economy adjusts to increases in the level of future prices in some *high inflation states* while prices are held fixed in *low inflation states*. That is, we analyze the consequences of increases in the *variability* of the future rate of inflation. We find that monetary policies characterized by (i) a large ratio between inflation in the high and the low inflation states and (ii) a large probability of high inflation are dominated polices.

The contraction of nominal financial markets induced by the variability of the future rate of inflation stems from restrictions on nominal portfolios imposed by a no-bankruptcy condition. In the low inflation states (for which the real

[1] Sudden and drastic changes in the rate of inflation, usually associated with attempts to stabilize the price level, are a common feature of high inflation economies. This is illustrated in the following table that shows summary statistics for the inflation process in Argentina.

Argentina: Feb 1977-Nov 1989

	p_t	$p_t - p_{t-1}$
Median	8.7%	0.2%
Std. Dev.	17.5%	11.6%
Min	2.0%	−85.1%
Max	170.5%	68.9%

p_t: monthly rate of inflation.

[2] In this paper we do not attempt to find an optimal monetary policy. We limit ourselves to the characterization of a class of dominated monetary policies.

payoff of nominal assets is high) the no bankruptcy constraint imposes bounds on the short positions that agents can take in nominal securities. In equilibrium, this also restricts long positions. As a result of these bounds on portfolios, in the high inflation states nominal portfolios will have very small payoffs. Thus, nominal assets become useless to transfer income to and from the high inflation states when future prices are extremely variable[3].

When the probability of a high inflation state is sufficiently large, the variability of the future price level has several additional effects. First, it reduces the value of trade in nominal assets at date 0. For, as the price level in the high inflation states increase and the probability of a high inflation state rises, the price of nominal securities falls while the size of nominal portfolios is restricted by the no bankruptcy constraints. Second, extremely variable rates of inflation reduce the set of feasible utility levels of a typical economy. A highly variable inflation prevents the use of nominal assets to redistribute income between high inflation states, curtailing the gains that can be attained in those states from trading nominal securities. Thus, when it is very likely that a high inflation state will occur extremely variable inflations reduce the efficiency of financial markets. If all the assets that are traded in financial markets are nominal, then an increase in the variability of the rate of inflation leads to a Pareto decline in social welfare. Of course, this will not occur in economies with Pareto optimal endowments in which there are no gains from trade.

Our main theorem shows that if the probability of high inflation is sufficiently high, then, for a generic set of endowments, an increase in the variability of the future rate of inflation reduces the utility possibilities set. However, this does not imply that high inflations make everyone worse-off. When there are real, as well as nominal assets in the economy, the shift in monetary policy may redistribute income in such a way that some agents benefit from the highly variable monetary policy. An example at the end of the paper shows that when these redistributive effects arise, they stem from changes in the prices of real securities. Another example highlights the role played by the probability distribution over the future rate of inflation. If the probability that inflation will be low is sufficiently high, a variable rate of inflation is not necessarily harmful from a social point of view. In fact, we provide an example where an increase in the variability of the rate of inflation leads to a Pareto improvement in social welfare. Even when the rate of inflation is very variable, nominal assets are still useful to reallocate income between low inflation states. Hence, if the probability of low inflation states is sufficiently high it is possible for increases in the variability of the rate of inflation to be socially beneficial.

This article contributes to the discussion of the welfare effects of inflation[4]. Other studies of the welfare costs of inflation examine, among other things, (i) the "shoe leather" costs associated with low levels of real money balances (Bai-

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[3]Notice that nominal assets can still be used to transfer income in the low inflation states.

[4]For a general discussion of the welfare costs of inflation see Driffill et. al. [10] and Modigliani and Fischer [13].

ley, [3]), (ii) the "Tobin effect" of inflation on capital accumulation (Tobin, [27]), (iii) the distortionary effects of the inflation tax (Phelps, [25] and Drazen, [9], (iv) the inefficiencies in the allocation of resources that arise as a result of imperfect information and the confusion of relative and aggregate prices (Lucas, [20], [21] and Cuckierman, [6]), (v) the menu costs of price adjustment (Sheshinski and Weiss, [26]), (vi) the inefficiencies introduced by variable relative prices in search models (Tommasi, [28]) and (vii) the inefficiencies introduced in financial intermediation by high inflation (Azariadis and Smith [2]). Our model adds to this large literature by focusing on the impact of monetary instability on the efficiency with which financial markets allocate risk. While most of the literature on the welfare effects of inflation focuses on the role of money as an asset, a medium of exchange and a unit of account in commodity markets, we address the issues arising from the role that money has as a unit of account in nominal financial contracts. In the economy we study, the only channel through which monetary policy can affect the economy is through the redistributive effects that it has when nominal securities are traded and asset markets are incomplete. If nominal securities are not traded or asset markets are complete, the allocation of resources is independent of the actions of monetary authorities.

The article also contributes to the literature on general equilibrium with incomplete financial markets (GEI). When asset markets are incomplete and some securities have payoffs specified in terms of units of money, the value of the unit of account matters because it leads to changes in the subspace of income transfers across states of nature achievable through trade in financial markets. This is the key insight of a series of papers that study the real effects of changes in the price level in economies with nominal assets and incomplete financial markets. Cass [5] showed that in this type of economies, for a generic choice of endowments, there is a continuum of equilibria that depend on the value of money. The dimensionality of the set of equilibria has been characterized by Balasko and Cass [4] and Geanakoplos and Mas-Colell [15]. Magill and Quinzii [22] have shown that when outside money is added to the model, and agents have a motive for holding it, equilibrium consumption allocations will be locally unique and will depend on the quantity of outside money. This article extends this literature by analyzing the welfare consequences of different monetary policies. It also highlights the role of no-bankruptcy constraints in models of incomplete markets that has been previously studied by Dubey et al. [11] and Zame [29].

The remainder of the paper is organized as follows. Section 2 describes monetary policy, the GEI model and lays down our assumptions. Section 3 studies the effect of the variability of the rate of inflation on the size of nominal financial markets, section 4 discusses the consequences of high expected inflation on economic efficiency and section 5 contains several examples.

2 The Model.

The purpose of the model presented bellow is to study how the actions of policy makers influence the behavior of an economy via money's role as a unit of account in financial contracts . For this reason we present a monetary economy in which this is the only role for money. We assume that there are no trading frictions and therefore money is not necessary for transaction purposes. Money is just an abstract unit of account used to denominate financial contracts. The value of money—the inverse of the price level—is modeled as an exogenous variable that is determined by monetary authorities.

Consider a two date economy with one perishable commodity and denote the states of the world by $s = 0, 1, \ldots, S$ and their respective probabilities by $\pi(s)$, with state 0 corresponding to date 0 and states $1, \ldots, S$ corresponding to date 1. At the beginning of date 0 agents know all the information characterizing the economy, but they do not know which of the S states of nature will be realized at date 1.

INSERT FIGURE 1

Definition 1 (Price Policy) *A price policy is a function*

$$p : S + 1 \longmapsto \Re^{S+1}_{++}$$

that assigns to each realization of the state of the world, s, a price level. Without loss of generality, we assume that $p(0) = p(1) = 1$.

Agents have perfect conditional foresight and know the price policy chosen by the central bank.

The states of nature at date 1 are indexed by a rule that orders them according to the rate of inflation: $s < s' \Rightarrow p(s) < p(s')$. We partition[5] the states of nature into L low inflation states indexed by $s = 1, \ldots, L$ and $S - L$ high inflation sates indexed by $s = L + 1, \ldots, S$. The probability of a low inflation state is $\pi(L) = \sum_{s=1}^{L} \pi(s)$ and the probability of a high inflation state is $\pi(H) = \sum_{s=L+1}^{S} \pi(s)$.

The variability of future inflation (prices) is characterized by two numbers: $\underline{\sigma} = \frac{p(L+1)}{p(L)}$ is a lower bound on the ratio between prices in the high and low inflation states; $\overline{\sigma} = \frac{p(S)}{p(1)}$ bounds the ratio of prices between high and low inflation states from above. A large value of $\underline{\sigma}$ (small value of $\overline{\sigma}$) implies a large (small) variance of future prices.

[5] We could easily incorporate some intermediate inflation states into the analysis. If there are I states with intermediate inflation then the high inflation states will be those with $L + I + 1 < s < S$.

Later in the paper we compare the properties of equilibria in which the parameters $\underline{\sigma}$ and $\overline{\sigma}$ take different values. We adopt the convention that when $\underline{\sigma}$ or $\overline{\sigma}$ vary, the prices in the low inflation states remain constant and the prices in the high inflation states change[6]. As $\underline{\sigma}$ grows the rate of inflation in the high inflation states becomes higher, thus inducing a more variable inflation. (Recall that $p(0) = 1$.) We say that a price policy is extremely variable when $\underline{\sigma} \rightarrow \infty$. The expected rate of inflation is $\sum_s \pi(s)p(s)$. An economy with a variable and high expected inflation is one where $\pi(H)$ and $\underline{\sigma}$ are large.

The asset structure of the economy is exogenously given and consists of a set of J financial contracts. This set contains a subset $\mathbf{J}_n = \{1, \ldots, J_n\}$ of nominal contracts and a subset $\mathbf{J}_r = \{J_n + 1, \ldots, J_n + J_r\}$ of real ones. $J_r + J_n = J$. Real contracts are promises to deliver units of the single good in the economy, while nominal contracts are promises to deliver units of account (dollars). The payoff matrices of the nominal and the real contracts are denoted by N and R, respectively. N is of dimension $S \times J_n$, while R is of dimension $S \times J_r$.

$$N = \begin{pmatrix} n_1(1) & \ldots & n_{J_n}(1) \\ \vdots & \ddots & \vdots \\ n_1(S) & \ldots & n_{J_n}(S) \end{pmatrix} \qquad R = \begin{pmatrix} r_{J_n+1}(1) & \ldots & r_J(1) \\ \vdots & \ddots & \vdots \\ r_{J_n+1}(S) & \ldots & r_J(S) \end{pmatrix}$$

The column vectors of N and R represent the state contingent payoff of each security and are denoted by n_j and r_j; while the rows represent the payoff of all the securities in state s and are denoted by $n(s)$ and $r(s)$.

Since the terms of the nominal contracts, the $n_j(s)$'s, are expressed in terms of units of account the matrices R and N are not directly comparable. It is necessary to divide the rows $n(s)$ of N by the price level $p(s)$ in order to obtain the real payoff of nominal securities. Let $p \in \Re^S_{++}$ denote the date 1 vector of the prices of the consumption good in terms of money for each state, and define $\Lambda^{-1}(p)$ as

$$\Lambda^{-1}(p) = \begin{pmatrix} \frac{1}{p(1)} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \frac{1}{p(S)} \end{pmatrix}$$

Real asset payoffs are then given by the matrix

$$A(p) = \left[\Lambda^{-1}(p)N, R\right].$$

Let $A_J(p)$ be the matrix formed by the first J rows of $A(p)$, $\Lambda_H(p)$ be the H dimensional diagonal matrix of price levels in the high inflation states, I_H be

[6] If we fix prices in the high inflation states and let prices in the low inflation states shrink then the all the results of the paper are valid, albeit with a different interpretation. The source of monetary variability will now be the possibility of an hyperdeflation.

an H dimensional identity matrix and p_1 be a price policy characterized by a finite $\bar{\sigma}$.

Assumption 1 **(i)** *Rank* $N = J_n$, *Rank* $R = J_r$.

(ii) $\begin{pmatrix} 0 & 0 \\ 0 & \Lambda_H^{-1}(p_1) \end{pmatrix} n_1 \notin \text{span}\left[\begin{pmatrix} 0 & 0 \\ 0 & I_H \end{pmatrix} R\right]$

(iii) $p \in \mathcal{P} = \{p \mid Rank\ A_J(p) = J\}$.

Assumption 1 rules out the existence of redundant assets[7]. Any asset that has payoffs that can be expressed as linear combinations of the payoffs of other assets is redundant and we will not consider it. Part (ii) of the assumption requires at least one nominal asset, say asset 1, to be non-redundant in the high inflation states for a policy with a finite $\bar{\sigma}$. It implies that n_1 has to have some non-zero elements in the high inflation states. As it will become clear in the proof of theorem 2, this part of the assumption is necessary to assure that when $p(s) \to \infty$ in all the high inflation states the loss of the nominal assets is costly. Part (iii) restricts price policies to prevent a drop of rank of $A_J(p)$. Observe that for any payoff matrices R and N the admissible class of price policies, $\mathcal{P}$, is an open and dense set of full Lebesgue measure in $\Re^S_{++}$— i.e. given an asset structure almost any price policy satisfies assumption 1.

Assumption 2 (Strongy Incomplete Markets) $J \leq L$.

This assumption is implicitly implied by assumption 1(iii) for if $L < J$ and $p(J) \to \infty$ then rank$[A_J(p)] < J$. The full rank of $A_J(p)$ is necessary to generate enough restrictions on portfolios in the proof of Lemma 1 bellow.

Asset prices are denoted by $q \in \Re^J$ and portfolios are $y^h \in \Re^J$. At date 0 each of the J securities is traded at a price q_j. Each component y^h_j of y^h represents the holdings of security j by individual h; when y^h_j is positive (negative) individual h is long (short) in security j. A portfolio of nominal assets is $y^h_n \in \Re^{J_n}$ and a portfolio of real assets is $y^h_r \in \Re^{J_r}$.

Individuals are denoted by $h = 0, \ldots, H$. An individual is characterized by her consumption set, preferences and endowments.

Assumption 3 **(i)** *The consumption set is* $\Re^{S+1}_+$ *with consumption bundles* $x^h \in \Re^{S+1}_+$.

(ii) *Every individual has a finite endowment vector* $\omega^h \in \Omega = \Re^{S+1}_{++}$.

(iii) *Preferences are described by the utility function* $u^h : \Re^{S+1}_+ \mapsto \Re$. u^h *can be written as* $u^h = (1, \pi)v^h$, *where* π *is a probability measure*, $\pi \in \Re^S_{++}$.

[7] Adding redundant assets to the economy does not alter equilibrium consumption allocations.

$\sum_s \pi(s) = 1$, *and* v^h *is a vonNeuman-Morgenstern utility index,* $v^h \in \Re_+^{S+1}$ *and has components* $v_s^h : \Re \rightarrow \Re$. *Each function* v_s^h *is assumed to be continuous, strictly monotonically increasing and differentiably strictly concave. Furthermore,* $\lim_{x^h(s)\rightarrow 0} \frac{\partial v_s^h}{\partial x^h(s)} (x^h(s)) = \infty$ *and we adopt the normalization* $u^h(\omega^h) = 0$.

Individuals maximize their utility in the budget set $\mathcal{B}^h(p, q)$.

$$\begin{aligned}\mathcal{B}^h(p,q) = \{ (x^h, y^h) \; : \; x^h(0) \; &- \; \omega^h(0) + qy^h = 0 \text{ and} \\ x_1^h \; &- \; \omega_1^h = A(p)y^h \; \}\end{aligned}$$

where x_1^h and ω_1^h are the S dimensional vectors of date 1 consumption and endowments. In each state, the numeraire is the consumption good in that state.

The definition of the budget set and of the consumption set imply that individuals always honor their commitments: they only choose financial and consumption plans that are consistent with the resources that they have in each state. The assumption on the boundary behavior of the utility function implies that consumers will never adopt a strategy where they cannot consume in some state. It captures the costs of bankruptcy.

The data for an economy E_i with price policy p_i can be summarized in the array

$$E_i = \left[\left(v^h, \omega^h\right)_{h=0}^{H}, R, N, \pi, p_i \right]$$

Asset prices do not allow for arbitrage when there are no trading strategies that allow consumers to obtain a positive income in some state without paying for it in some other state. The set of asset prices that does not allow for arbitrage is denoted by Q and it is defined as follows:

$$Q = \left\{ q \mid \nexists y^h \in \Re^J \text{ such that } \begin{pmatrix} -q \\ A(p) \end{pmatrix} y^h > 0 \right\}.$$

where $y > 0$ means that $y(s) \geq 0$ for all s and $y(s) > 0$ for at least some s. If $q \notin Q$ individuals will be able to attain an infinite amount of wealth in some state.

Definition 2 (Equilibrium) *For a given price policy* $\overline{p}_i$, *the consumption and portfolio allocations* $\left(x^{*h}, y^{*h}\right)_{h=0}^{H}$ *are an equilibrium for the economy* E_i *if, and only if, there is an asset price vector* $q^* \in Q$ *for which:*

(i) *markets clear*

$$\sum_h y^{*h}(q^*, \bar{p}_i, \omega^h) = 0$$

(ii) *every h maximizes u^h in the budget set $\mathcal{B}^h(\overline{p}_t, q^*)$*

For any given price policy the economy is in equilibrium when markets clear and individuals carry out their optimal programs. The existence of an equilibrium for this economy is shown in Geanakoplos and Polemarchakis [16].

3 The Effect of Inflation on the Market for Nominal Securities.

In this section we show that, when markets are incomplete, high and variable rates of inflation induce thin nominal financial markets.

Lemma 1 and its corollary show that extremely variable monetary policies prevent the use of nominal securities to transfer income to and from the high inflation states. The lemma establishes that the payoffs of nominal portfolios in the high inflation states have bounds that are inversely proportional to the lower bound on the variability of future inflation, $\underline{\sigma}$. Hence, as $\underline{\sigma} \to \infty$ the payoff of nominal portfolios in the high inflation states goes to zero.

Lemma 1 *Suppose that assumptions 1,2 and 3 hold, for every h $\exists y^h \mid x_1^h - \omega_1^h = A(p)y^h$ and $\sum_h y^h = 0$. Then, there exist finite numbers $\underline{\theta}$ and $\overline{\theta}$ (that are independent of $\underline{\sigma}$) for which*

$$\frac{\underline{\theta}}{\underline{\sigma}} < \frac{n_j(s)}{p(s)} y_j^h < \frac{\overline{\theta}}{\underline{\sigma}} \quad \textit{for all } h,\ j \in \mathbf{J}_n \textit{ and for } s = L + 1^8, \ldots, S.$$

The transfers of income to and from the high inflation states attainable by trading nominal assets have an upper and a lower bound that are inversely proportional to the lower bound on the variability of the rate of inflation.

Proof: $\exists y^h \mid x_1^h - \omega_1^h = A(p)y^h$ and $\sum_h y^h = 0 \Rightarrow \sum_h x_1^h = \sum_h \omega_1^h$. For ω^h finite and $x^h \in \Re_+^{S+1}$ this implies that $x_1^h \in$ compact set $\Rightarrow \tau^h = x_1^h - \omega_1^h \in$ compact set. Let τ_J^h be the vector formed by the first J rows of τ^h. Then, as $A_J(p)$ has full rank by assumption 1, $\tau^h = A(p)y^h \Rightarrow y^h = [A_J(p)]^{-1}\tau_J^h \Rightarrow y^h \in$ compact set. Assumption 2 ($J < L$) implies that $[A_J(p)]^{-1}$ is independent of $\underline{\sigma}$ because prices levels are fixed in the low inflation states. Hence, $\exists \underline{\theta}'$ and $\overline{\theta}'$ (independent of $\underline{\sigma}$) such that $\underline{\theta}' < n_j(s)y_j^h < \overline{\theta}'$ for all s, h and $j \in \mathbf{J}_n$. It follows that $\frac{\underline{\theta}}{\underline{\sigma}} < \frac{\underline{\theta}'}{p(L)\underline{\sigma}}\frac{p(L+1)}{p(s)} < \frac{n_j(s)y_j^h}{p(s)} < \frac{\overline{\theta}'}{p(L)\underline{\sigma}}\frac{p(L+1)}{p(s)} < \frac{\overline{\theta}}{\underline{\sigma}}$ for $s = L+1, \ldots, S$ and $j \in \mathbf{J}_n$; with $\underline{\theta} = \frac{\underline{\theta}'}{p(L)}$ and $\overline{\theta} = \frac{\overline{\theta}'}{p(L)}$.□

[8] If there are I intermediate inflation states, $L+1$ should be replaced by $L+I+1$.

Corollary 1 *Extremely variable monetary policies prevent the use of nominal securities to transfer income to and from the high inflation states.*

$$\frac{1}{\underline{\sigma}} \rightarrow 0 \Rightarrow \frac{n(s)}{p(s)} y_n^h \rightarrow 0 \qquad \text{for } s = L+1, \ldots, S$$

The intuition of this result is very simple. Assume that there are exogenous bounds on the short positions that agents can take in nominal assets. Then, the fact that $\frac{n_j(s)}{p(s)} y_j^h \rightarrow 0$ when $p(s) \rightarrow \infty$ follows trivially because the borrowing constraints will prevent y_j^h from going to infinity when the real payoff of nominal assets goes to zero. The bulk of the proof of lemma 1 is devoted to derive bounds that prevent nominal portfolios from going to infinity when the level of prices explodes in the high inflation states (and only in the high inflation states). Assumptions 1 and 2 guaranty that these bounds exist and are independent of prices in the high inflation states. The bounds on portfolios that we derive in the proof of the lemma stem from the restrictions imposed, in the low inflation states, by the requirement that agents have to pay their debts with their available resources. This can be illustrated with a simple example. Observe that the non-negativity of consumption (no bankruptcy) implies that $-\omega^h \leq \tau^h = x_1^h - \omega_1^h = A(p)y^h \leq \sum_h \omega^h$ and assume

$$[A_J(p)]^{-1} = \begin{bmatrix} \Lambda_{J_n}(p) & 0 \\ 0 & I_{J_r} \end{bmatrix}.$$

Nominal portfolios are then restricted by the value of the endowments in the low inflation states—i.e. $y_n^h \in \{ y_n^h : -p(j)\omega^h(j) \leq y_j^h \leq p(j) \sum_h \omega^h(j) \text{ for } j \in \mathbf{J}_n \}$. It follows that for this simple asset structure[9]

$$-\frac{p(L)}{p(s)} \left[\frac{n_j(s)}{p(L)} p(j) \omega^h(j) \right] \leq \frac{n_j(s) y_j^h}{p(s)} \leq \frac{p(L)}{p(s)} \left[\frac{n_j(s)}{p(L)} p(j) \sum_h \omega^h(j) \right]$$

for $j = 1, \ldots, J_n$ so $\underline{\theta} = \left[\frac{n_j(s)}{p(L)} p(j) \omega^h(j) \right]$ and $\overline{\theta} = \left[\frac{n_j(s)}{p(L)} p(j) \sum_h \omega^h(j) \right]$.

Theorem 1 and its corollary state that in an economy with strongly incomplete asset markets ($J < L$) where the rate of inflation is extremely variable ($\underline{\sigma} \rightarrow \infty$) and the probability of high inflation is large ($\pi(H) \rightarrow 1$), the value of the volume of trade in nominal securities shrinks to zero.

Theorem 1 *Suppose assumptions 1, 2 and 3 hold. Then, in equilibrium,*

$$\{ \underline{\sigma} \rightarrow \infty \text{ and } \pi(H) \rightarrow 1 \} \Rightarrow q_j y_j^h \rightarrow 0$$

for all h and for all $j \in \mathbf{J}_n$.

[9]Provided $n_j(s) > 0$. If $n_j(s) < 0$ the inequalities are reversed.

Proof: At a competitive equilibrium $q_j = Du^h\Lambda^{-1}(p)n_j$ for every h. Therefore, $q_j y_j^h = \Sigma_{s=1}^{L}\pi(s)v'(x(s))\frac{n_j(s)y_j^h}{p(s)} + \Sigma_{s=L+1}^{H}\pi(s)v'(x(s))\frac{n_j(s)y_j^h}{p(s)}$. The first sum vanishes when $\pi(H) \to 1$ for at an equilibrium v' and y^h are bounded. The second sum vanishes by Lemma 1.□

Corollary 2 *An extremely variable and highly expected inflation induces thin nominal financial markets. For all* $j \in \mathbf{J}_n$

$$\{\underline{\sigma} \to \infty \text{ and } \pi(H) \to 1\} \Rightarrow \sum_h q_j \left|y_j^h\right| \to 0$$

The idea behind the proof is very simple. The price of a nominal assets can be decomposed into the present value of its payoff in the low and in the high inflation states. The present value of the payoff of the portfolio in the low inflation states is small if these states occur with a negligible probability. For the high inflation states, we have just proved that the ex-post payoff of a nominal portfolio goes to zero as $\underline{\sigma}$ grows without bound.

4 Efficiency and High Expected Inflation.

We have established that, when asset markets are incomplete, a variable inflation reduces the ability of nominal assets to transfer income to and from high inflation states (Lemma 1). Thus, some potential gains from trade are forfeited. The next two theorems show that when there is *a high expected inflation* (when stabilization plans are possible but unlikely) this *reduces the efficiency with which financial markets allocate risk*.

We compare the economies E_1, E_2, E_∞ and E_r. E_r is an economy where there are only real assets—i.e. $y_n^h = 0$ for all h. E_1 is an economy with a finite $\overline{\sigma}_1$. E_2 and E_∞ are economies with monetary policies characterized by $\underline{\sigma}_1 < \underline{\sigma}_2 < \underline{\sigma}_\infty = \infty$.

Theorem 2 refers to an economy with real and nominal securities and studies the behavior of the sets of utility vectors (U_i) that can be reached by attainable allocations in the economy E_i. It compares the utility set in the benchmark economy E_1, where the variability of the future rate of inflation is finite ($\bar{\sigma}$ is finite), with the one that is attained when the variability of the rate of inflation is higher. If the probability of a high inflation state is sufficiently high then, for almost any distribution of endowments, different monetary policies induce the following ordering of utility sets: $U_r \subset U_\infty \subset U_2 \subset U_1$. The last three inclusions are a direct consequence of the bounds on nominal portfolios established in Lemma 1. The relation $U_r \subset U_\infty$ stems from the gains from trading nominal securities derived from the low inflation states.

In order for the theorem to hold it is important to have an economy with heterogeneous agents and a high probability of high inflation. Extremely variable monetary polices reduce social welfare because they annihilate the gains from trading nominal assets that are attained in the high inflation states. Therefore, if these states are not likely to occur, or if there are no potential gains from trade that are forfeited, the theorem does not apply.

Even though as $\underline{\sigma}_i \to \infty$ and $\pi(H) \to 1$ the utility possibilities set shrinks, it is not possible to Pareto rank the equilibria corresponding to different monetary policies. Changes in monetary policy can lead to changes in the prices of real assets that redistribute income across individuals. Theorem 3 shows that this cannot occur when there are no real assets.

Definition 3 (Attainable Allocation) *We say that an allocation x is attainable in the economy E_i if $x \in \mathcal{F}_i$:*

$$\mathcal{F}_i = \left\{ x \in \Re_+^{(H+1)(S+1)} : \sum_h (x^h - \omega^h) = 0 \text{ and } \exists y^h \in \Re^J \mid x_1^h - \omega_1^h = A(p_i)y^h \right\}$$

Definition 4 (Utility Set) *The utility set $U_i \subset \Re_+^{H+1}$ is the set of utility vectors $u = (u^0, \ldots, u^H)$ which can be reached by attainable allocations in the economy E_i: that is,*

$$U_i = \left\{ u \in \Re_+^{H+1} : \text{ for an attainable } x \text{ in } E_i, u^h = u^h(x^h) \text{ for all } h \right\}$$

We will abuse the notation and write $u(x)$ for $\left[\cdots, u^h(x^h), \cdots\right]$.

A generic set of endowments is an open and dense set of full Lebesgue measure in $\Re_{++}^{(H+1)(S+1)}$.

Theorem 2 *Suppose $2 \le H$, $\overline{\sigma}_1$ is finite and assumptions 1,2 and 3 hold. Then, there exists a generic set of endowments, Ω^*, and positive numbers δ and $\underline{\sigma}_2$ such that if $\omega \in \Omega^*$, $0 < \pi(L) < \delta$ and $\underline{\sigma}_1 < \underline{\sigma}_2 < \underline{\sigma}_\infty = \infty$:*

$$U_r \subset U_\infty \subset U_2 \subset U_1.$$

INSERT FIGURE 2

Proof: The proof proceeds in several steps. Let $\lambda \in \Re_+^{H+1}$ and define $W_i(\lambda) = \max \lambda u$ subject to $u \in U_i$.

Step 1: Suppose $2 \le H$, $\overline{\sigma}_1$ is finite and assumptions 1 and 3 hold. Then, there exists a generic set, Ω^*, such that if $\omega \in \Omega^*$ for any $\lambda > 0$ and for any $\pi(L) > 0$, (i) $W_r(\lambda) < W_1(\lambda)$ and (ii) $W_r(\lambda) < W_\infty(\lambda)$, where $W_\infty(\lambda) = \lim_{\underline{\sigma}_i \to \infty} \max \lambda u$ subject to $u \in U_i$.

Proof: Let $i = 1, \infty$. $\mathcal{F}_r \subset \mathcal{F}_i$ and, hence, $U_r \subset U_i$. So we have to show that under the assumptions of the theorem there is a generic set of endowments for which $\bar{u}_i = \arg\max \lambda u$ subject to $u \in U_i$ implies $\bar{u}_i \notin U_r$. Let $\bar{x}_i = \arg\max \lambda u(x)$ subject to $x \in \mathcal{F}_i$. By the separability of each u^h the problem $\max_x \lambda u(x)$ subject to $x \in \mathcal{F}_i$ can be transformed into two independent problems: (i) $\widehat{W} = \max_{x(0)} \lambda u_0(x(0))$ subject to $\sum_h x^h(0) - \omega^h(0) = 0$, and (ii) $\widetilde{W}_i = \max_y \lambda u_1(\omega + A(p_i)y)$ subject to $\sum_h y^h = 0$ and $w^h(s) + r(s)y_r^h + \frac{n(s)}{p(s)}y_n^h \geq 0$, where each component of u_1 is the expected utility of agent h at date 1. $W_i(\lambda) = \widehat{W}(\lambda) + \widetilde{W}_i(\lambda)$. We only need to analyze the second problem for $\widehat{W}$ is independent of monetary policy. The first order necessary and sufficient conditions for solving this problem are

$$\begin{gathered} \sum_h \bar{y}^h = 0 \\ \sum_s \lambda^h \frac{\partial u^h}{\partial x^h(s)} \left(\omega^h(s) + a(s)\bar{y}^h\right) a_j(s) = q_j \quad \text{for every } h, j \end{gathered} \tag{1}$$

In the appendix we show that if $2 \leq H$, $\bar{\sigma}_1$ is finite and assumptions 1 and 3 hold, then there exists a generic set of endowments, Ω^*, such that if $\omega \in \Omega^*$; the solution of (1) has the property that $\bar{y}_1^h \neq 0$ for every h. Assumption 1 then implies that for any $\pi(L) > 0$ $\bar{u}_i(\bar{x}(0), \omega_1 + R\bar{y}_r + N(p_i)\bar{y}_n) \notin U_r$. □

Step 2: $\lim_{\pi_i(L)\to 0} \lim_{\frac{1}{\underline{\sigma}_i}\to 0} W_i(\lambda) = \lim_{\pi_i(L)\to 0} W_r(\lambda)$, for any $\lambda > 0$.

Proof: Again, we write $W_i(\lambda)$ as $\widehat{W}(\lambda) + \widetilde{W}_i(\lambda)$ and concentrate on $\widetilde{W}_i(\lambda)$.

$$\lim_{\pi_i(L)\to 0} \widetilde{W}_i = \max_{y^0,\dots,y^H} \sum_h \lambda^h \sum_{s=L+1}^{S} \pi_i(s) v^h \left(w^h(s) + r(s)y_r^h + \frac{n(s)}{p_i(s)}y_n^h\right)$$

$$\text{subject to } \sum y^h = 0 \text{ and } w^h(s) + r(s)y_r^h + \frac{n(s)}{p_i(s)}y_n^h \geq 0$$

Corollary 1 implies that if $\underline{\sigma}_i \to \infty$ an allocation is feasible only if $\frac{n(s)}{p(s)}y_n^h \to 0$ for $s = L+1, \dots, H$. Hence,

$$\lim_{\pi_i(L)\to 0} \lim_{\frac{1}{\underline{\sigma}_i}\to 0} \widetilde{W}_i = \max_{y^0,\dots,y^H} \sum_h \lambda^h \sum_{s=L+1}^{S} \pi_i(s) v^h \left(w^h(s) + r(s)y_r^h\right)$$

$$\text{subject to } \sum y_r^h = 0 \text{ and } w^h(s) + r(s)y_r^h \geq 0.$$

This last expression is $\lim_{\pi_i(L)\to 0} \widetilde{W}_r(\lambda)$.□

Step 3: U_i can be written as $U_i = \left\{u \in \Re_+^{H+1} : \lambda u \leq W_i(\lambda) \text{ for all } \lambda \in \Re_+^{H+1}\right\}$

Proof: $\mathcal{F}_i$ is convex for it can be written as the intersection of the complete markets feasible set and the linear subspace spanned by the assets, two convex sets—$\mathcal{F}_i = \mathcal{F}_{GE} \cap \omega + \langle A \rangle$. Let $u, u' \in U_i$ and $0 \leq \alpha \leq 1$. Take attainable allocations x and x', with $u = u(x), u' = u(x')$. Then $x'' = \alpha x + (1-\alpha)x'$ is also attainable, that is, $u'' = u(x'') \in U_i$ and by the concavity of each u^h, $u'' > \alpha u + (1-\alpha)u' \geq 0$. Hence, $\alpha u + (1-\alpha)u' \in U_i$. A vector $\bar{u}$ is supported by $\lambda > 0$ if $\bar{u} = \arg\max \lambda u$ subject to $u \in U_i$. Knowing that U_i is convex the supporting hyperplane theorem implies that if $u \in \partial U_i$ then u is supported by a $\lambda > 0$. Then, every $u \in \partial U_i$ has an associated value $W_i(\lambda) = \max \lambda u$ in U_i and the claim follows since U_i is closed and convex.□

The first two steps imply that there is an $\varepsilon > 0$ such that for economies with $\omega \in \Omega^*$, $W_r = \lim_{\pi,(L)\to 0} W_\infty = \lim_{\pi,(L)\to 0} W_1 - \varepsilon$. Therefore, for every $\epsilon > 0$ there exists a positive number $\delta > 0$ such that if $\pi(L) < \delta$ then $|W_1 - W_\infty - \varepsilon| < \epsilon$. Then for a small enough ϵ, $\pi(L) < \delta$ implies $W_\infty < W_1$. Since W_i is continuous on $\mathcal{P}$, the intermediate value theorem implies that there exists a number $\underline{\sigma}_2$ satisfying $0 < \underline{\sigma}_2^{-1} < \underline{\sigma}_1^{-1}$ for which $W_\infty < W_2 < W_1$. So we know that $W_r < W_\infty < W_2 < W_1$ and from step 3 it follows that $U_r \subset U_\infty \subset U_2 \subset U_1$.
□

The previous theorem establishes the efficiency costs of a variable and high expected inflation in terms of the size of the utility sets. Theorem 3 sharpens this result by ranking monetary policies according to Pareto's welfare criterion. It asserts that in economies with only nominal assets, for a sufficiently high probability of high inflation, extremely variable monetary policies make everyone worse off . The difference between the two theorems lies in the fact that in economies with real and nominal assets changes in monetary policy lead to changes in the prices of real assets[10], redistributing wealth between those that buy and sell these assets. In the case where real assets are not available and the conditions of the theorem are met, when $\pi(H) \to 1$ and $\underline{\sigma} \to \infty$ nominal assets become useless, their value is zero and every agent's utility converges to the autarky point.

Theorem 3 *Suppose that $2 \leq H$, $\overline{\sigma}_1$ is finite and assumptions 1,2 and 3 hold.*

Then, for any $\omega \in \Omega^*$ (a generic set of endowments), there exists positive numbers δ and $\overline{\sigma}_2$ such that if $0 < \pi(L) < \delta$ and $\underline{\sigma}_1 < \underline{\sigma}_2 < \underline{\sigma}_\infty = \infty$:

$$u^h(\omega^h) < u^h_\infty < u^h_2 < u^h_1 \text{ for all } h.$$

Note: u^h_i denotes the utility of individual h at an equilibrium allocation in the economy E_i.

Proof: [sketch] The structure of the proof is the same as the one of theorem 2.

1. For economies with endowments belonging to the generic set Ω^*, equilibrium allocations in the economies E_1 and E_∞ (with $\pi(L) > 0$) have the following property: $u^h(\omega^h) < u^h_1$ and $u^h(\omega^h) < u^h_\infty$ for all h. As in step 1 in Theorem 2, this is a consequence of the fact that in a typical economy agents trade nominal asset 1.

[10] This does not occur if agents have quadratic vonNeumann-Morgenstern utilities. This type of preferences are ruled out by assumption 5.

2. $\lim_{\pi(L)\to 0}\lim_{\frac{1}{\underline{\sigma}}\to 0} u_i^h = \lim_{\pi(H)\to 1} u^h(\omega^h)$ for all h.

Equilibrium utility functions are

$$u_i^h = v^h(\omega^h(0) - q_n^* y_n^{*h}) + \sum_{s=1}^{S} \pi_i(s) v^h \left(\omega^h(s) + \frac{n(s)}{p_i(s)} y_n^{*h} \right).$$

Lemma 1 and theorem 1 imply that

$$\lim_{\pi(L)\to 0} \lim_{\frac{1}{\underline{\sigma}_i}\to 0} u_i^h = v^h(\omega^h(0)) + \lim_{\pi(H)\to 1} \sum_{s=L+1}^{S} \pi_i(s) v^h(w^h(s)).$$

3. 1. and 2. imply that there is a δ such that if $0 < \pi(L) < \delta$ then $u_\infty^h < u_1^h$.

4. The continuity of u_i^h on $\mathcal{P}$ and the intermediate value theorem imply that there exists a number $\underline{\sigma}_2$ satisfying $\underline{\sigma}_1 < \underline{\sigma}_2 < \infty$ for which $u_\infty^h < u_2^h < u_1^h$ for all h. If $\pi(L) > 0$, $u^h(\omega^h) < u_\infty^h$ from step 1.□

5 Examples

This section contains four examples that illustrate the lemmas and theorems established in the previous two sections. The first two examples consider an economy with a real and a nominal bond that pay one in every state. In example 1 an increase in the variability of inflation leads to a less efficient allocation of risk but does not induce a Pareto ranking of equilibria. Example 2 portrays an economy where an increase in the variability of the future rate of inflation induces a Pareto inferior equilibrium. The last two examples deal with an economy with only nominal assets. Example 3 illustrates theorem 3. The last example underscores the role that the probability distribution π has in the theorems. It depicts an economy where an increase in the variability of inflation induces a Pareto improvement because $\pi(H)$ is not large enough for the theorem to apply.

In all the examples the number of states is $S = 3$ and individuals are $h = 0, 1$. The utility function if of the C.E.S. type with a risk aversion parameter of 2.

$$u^h = 10 - \frac{1}{x^h(0)} - \sum_{s=1}^{3} \pi(s) \frac{1}{x^h(s)}.$$

Endowments are ω in examples 1, 3 and 4 and ω' in example 2, where

$$\omega = \begin{bmatrix} .5 & .5 \\ .1 & .9 \\ .5 & .5 \\ .9 & .1 \end{bmatrix} \qquad \omega' = \begin{bmatrix} .5 & .5 \\ .4 & .6 \\ .5 & .5 \\ .6 & .4 \end{bmatrix}.$$

The vector of prices and the financial structure of the economy are

$$p = \begin{bmatrix} 1 \\ 1 \\ 1.004 \cdot \sigma \\ 1.008 \cdot \sigma \end{bmatrix} \qquad A(p) = \begin{bmatrix} \frac{1}{p(1)} & 1 \\ \frac{1}{p(2)} & 1 \\ \frac{1}{p(3)} & 1 \end{bmatrix}$$

The vector of price levels in states $s = 0, 1, 2, 3$ is p. The numbers $\underline{\sigma}$ and $\overline{\sigma}$ defined in section 2 are $\underline{\sigma} = 1.004 \cdot \sigma$ and $\overline{\sigma} = 1.008 \cdot \sigma$. Bellow, we look at the equilibria corresponding to $\sigma = 1$ and $\sigma = 30$. The financial structure of the economy is depicted by the matrix $A(p)$. In the last two examples the only asset in the economy is the nominal bond, so the second column of $A(p)$ disappears.

The following table summarizes the welfare effects of increasing σ from 1 to 30 in the four cases considered bellow.

Table I

Example	Real Bond	π	Endowments	Welfare Effect
1	yes	(.20, .40, .40)	ω	No Pareto Ranking.
2	yes	(.20, .40, .40)	ω'	Pareto Decline.
3	no	(.02, .49, .49)	ω	Pareto Decline.
4	no	(.20, .40, .40)	ω	Pareto Improvement.

5.1 Real and Nominal Assets

Example 1 *No Pareto Ranking*

This example illustrates theorem 2. In the example, a shift in monetary policy from $\sigma = 1$ to $\sigma = 30$ causes the utilities possibilities set to shrink and reduces the efficiency with which financial markets allocate risk. The change in monetary policy also redistributes income in such a way that the equilibria under the two policies are not Pareto comparable. The equilibria are described in the following table[11] and they are represented by the points a, b and c in figure 2.

Table II

	q_n	q_r	y_n^0	y_r^0	x^0	u^0	u^1
$\sigma = 1$, ω	.995	1	100.9	-100.4	(.54, .54, .54, .54)	6.296	5.652
$\sigma = 30$, ω	.244	1.52	.652	$-.163$	(.59, .59, .36, .76)	6.32	4.8
$\sigma = 1$, $\tilde{\omega}$	.995	1	100.9	-100.4	(.55, .55, .55, .55)	6.348	5.58

[11] Equilibrium conditions imply that $y^1 = -y^0$ and $x^1 = 1 - x^0$.

The switch from $\sigma = 1$ to $\sigma = 30$ makes agent $h = 0$ better-off and agent $h = 1$ worse-off. The redistribution occurs because the monetary policy shift induces a decrease in the price of the nominal bond and an increase in the price of the real bond. Agent $h = 0$ that, in equilibrium, purchases nominal bonds and sells real ones gains, while agent $h = 1$ loses[12].

The inefficiency created by the variability of the rate of inflation can be illustrated by computing the equilibrium that arises if we redistribute date 0 income in the economy with $\sigma = 1$ and then let people trade. If the transfers are $d\omega^0(0) = -d\omega^1(0) = 0.015$ then $\tilde{\omega}^0 = (.515, .1, .5, .9)$ and $\tilde{\omega}^1 = 1 - \tilde{\omega}^0$. The new equilibrium is described in the last row of the table. Observe that the equilibrium with monetary policy $\sigma = 1$ and endowments $\tilde{\omega}$ Pareto dominates the equilibrium with monetary policy $\sigma = 30$ and the original endowments.

Another way of illustrating the inefficiency introduced by the variability of future price levels is by measuring the gains and losses of each individual in terms of date 0 consumption. This can be done by calculating the change in consumption in the economy with $\sigma = 1$ that will induce the change in utility generated by the shift in monetary policy. This gain or loss is captured by γ^h in the following expression

$$u^h_{\sigma=1} - u^h_{\sigma=30} = v^h\left(x^{*h}_{\sigma=1}(0) + \gamma^h\right) - v^h\left(x^{*h}_{\sigma=1}(0)\right).$$

In the example

$$\gamma^h = \frac{\left(u^h_{\sigma=1} - u^h_{\sigma=30}\right)}{1 - \left(u^h_{\sigma=1} - u^h_{\sigma=30}\right) x^{*h}_{\sigma=1}(0)}$$

and, hence, $\gamma^0 = 0.013$ and $\gamma^1 = -.2816$. Mr. $h = 0$ gains the equivalent of 1.3% of the aggregate endowment at date 0 and Ms. $h = 1$ loses the equivalent of 28%. The increase in monetary instability is inefficient in the sense that Ms $h = 1$ loss exceeds Mr. $h = 0$ gain. If Ms. $h = 1$ gives Mr. $h = 0$ 0.013 units of date 0 output when $\sigma = 1$ she would be better-off than when monetary variability increases to $\gamma = 30$, while Mr. $h = 0$ would be indifferent.

Example 2 *Pareto decline.*

In this example an increase in monetary instability makes both agents worse-off. Endowments are ω' and the change in asset prices is smaller than in the previous case[13].

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[12] An important element of this example is that agent 0 has strong incentives to shift income towards state 1, while agent 1 wants to shift income to state 3. The marginal utility at the endowment point (before multiplying by $\pi(s)$) in each of the states is $v'(0.1) = 100$, $v'(0.5) = 4$ and $v'(0.9) \approx 1.23$. Hence, agent 1's strong demand for the real assets that allows her to shift income to state 3.

[13] Endowments ω' are such that agent 1's need to transfer income into state 3 is much less severe and his demand for the real bond is weaker.

Table IV

	q_n	q_r	y_n^0	y_r^0	x^0	u^0	u^1
$\sigma = 1$	.995	1	25.2	−25.2	(.51, .51, .51, .51)	6.078	5.918
$\sigma = 30$	.227	1.02	.155	−.045	(.51, .51, .46, .56)	6.067	5.897

5.2 Only Nominal Assets

Example 3 *High Probability of High Inflation: Pareto Decline.*

This example illustrates very nicely lemma 1 and theorems 1 and 3. As σ increases from 1 to 30, the payoffs of nominal portfolios in the high inflation states become infinitesimal and the value of the volume of trade shrinks from 0.14 units of the consumption good to 0.004 units. The increase in monetary instability makes both agents worse-off.

Table IV

	π	q_n	y_n^0	x^0	u^0	u^1
$\sigma = 1$	(.02, .49, .49)	3.473	−.04	(.64, .06, .46, .86)	6.467	2.794
$\sigma = 30$	(.02, .49, .49)	.437	.009	(.496, .109, .5, .1)	6.277	2.098

When $\sigma = 1$ agent 0 borrows (in nominal terms) from agent 1 shifting income from date 0 to date 1. When σ increases to 30 no income can be shifted to states 2 and 3, which concentrate 98% of the probability. The gains from trade become so small that both agents' utility shrinks to the one corresponding to their autarky point.

Example 4 *Low Probability of High Inflation: Pareto Improvement.*

It is of interest to point out that if we change the vector of probabilities to $\pi = [.2, .4, .4]$ the welfare effect of an increase in σ is reversed. That is, when moderate inflation is more likely an extremely variable price level can make everyone better-off and theorem 3 does not apply.

Table V

	π	q_n	y_n^0	x^0	u^0	u^1
$\sigma = 1$	(.2, .4, .4)	7.515	−.06	(.548, .094, .494, .894)	4.781	3.015
$\sigma = 30$	(.2, .4, .4)	.633	.135	(.414, .235, .504, .904)	5.502	3.036

With $\sigma = 1$ the nominal bond is practically identical to a real bond that pays one unit of the consumption good in every state. In equilibrium agent 0 borrows .048 units of the consumption good from agent 1 at date 0 and repays her only .006 units; the real interest rate is approximately -87%![14] The reason

[14] The real interest rate with complete markets would be zero.

why the terms of the loan are so disadvantageous to agent 1 is that her valuation of consumption in state 3 is very high and it is very costly for agent 0 to give up consumption in state 1. When $\sigma = 30$ the nominal bond pays, approximately, one unit of the consumption good in state 1 and zero in states 2 and 3. In equilibrium now agent 0 is long in the nominal bond transferring income from date 0 to state 1. Agent 1 borrows in nominal terms and pays an ex-post real interest rate of, approximately, 58% in state 1 and -95% in states 2 and 3. In this equilibrium, both agents enjoy a higher level of utility than in the equilibrium with $\sigma = 1$. Most of these gains come from the income that agent 1 hands over to agent 0 in the low inflation state.

Appendix:

Proposition 1 *If $2 \leq H$ and assumptions 1,2 and 3 hold, then there exists a generic set of endowments, Ω^*, such that if $\omega \in \Omega^*$: $\bar{y}_1^h \neq 0$ for every h and $\bar{y}$ solves* (1),

$$\sum_h \bar{y}^h = 0$$
$$\sum_s \lambda^h \frac{\partial u^h}{\partial x^h(s)} \left(\omega^h(s) + a(s)\bar{y}^h\right) a_j(s) = q_j \quad \textit{for every } h, j.$$

Proof: We show that if we parameterize the economy by the agent's endowments $\omega \in \Omega = \Re^{(S+1)(H+1)}$, the set of endowments for which the equations are satisfied with $\bar{y}_1^h = 0$ for some h is exceptional.

Monotonicity of preferences implies that for q to be a solution to (1) it must belong to the set $Q = \left\{q : q = \alpha A(p) \text{ for some } \alpha \in \Re^S_{++}\right\}$. Let Ω_n be a sequence of relatively compact open sets in $\Re^{(H+1)(S+1)}$, with $\Omega_n \subset \Omega_{n+1}$, let $y_j^h : Q \times \Omega_n \times \mathcal{P} \times \Re^{H+1}_+ \to \Re$ be the solution to $\sum_s \lambda^h \frac{\partial u^h}{\partial x^h(s)} \left(\omega^h(s) + a(s)\bar{y}^h\right) a_j(s) = q_j$ corresponding to consumer h and security j and let $y : Q \times \Omega_n \times \mathcal{P} \times \Re^{H+1}_+ \to \Re^J$ be $y = \sum_h y^h$. Define $h : Q \times \Omega_n \times \mathcal{P} \times \Re^{H+1}_+ \to \Re^{J+1}$ as (y, y_1^h). We analyze the behavior of the family of functions h that are parameterized by fixed p and λ—i.e. $h(\cdot, p, \lambda) : Q \times \Omega_n \to \Re^{J+1}$. h is a smooth function on $Q \times \Omega_n$. We use the transversality theorem to show that 0 is a regular value of h for almost every $\omega \in \Omega_n$ and then apply the pre-image theorem to prove the proposition.

First we must show that 0 is a regular value of the restricted h, $h(\cdot, p, \lambda)$. That is, either $h^{-1}(0) = o$ or $D_{\overline{q},\overline{\omega}} h : T_{\overline{q},\overline{\omega}}(Q \times \Omega_n) \to \Re^{J+1}$ is surjective for all $(\overline{q}, \overline{\omega}) \in h^{-1}(0)$. We prove that for all $(\overline{q}, \overline{\omega}) \in h^{-1}(0)$ and for any $dh \in \Re^{J+1}$ there exists a vector $(dq, d\omega) \in Q \times \Omega_n$ such that

$$dh = D_{\overline{q},\overline{\omega}} h \begin{pmatrix} dq \\ d\omega \end{pmatrix}$$

Let $e^1 = (1, \ldots, 0), \ldots, e^{J+1} = (0, \ldots, 1)$ be the standard basis in $\Re^{J+1}$. It suffices to show that for each e^j; $j = 1, \ldots, J+1$ there exists a $(dq, d\omega) \in Q \times \Omega_n$ such that

$$e^j = D_{\overline{q},\overline{\omega}} \begin{pmatrix} dq \\ d\omega \end{pmatrix}$$

Each of the first J vectors e^j can be obtained by setting $dq = d\omega^1 = \ldots = d\omega^H = 0$ and $d\omega^0 = (0, -a_j(1), \ldots, -a_j(S))$. This perturbation of endowments allocates to agent 0 an additional unit of security j, while leaving q and the portfolio allocations for $h = 1, \ldots, H$ unchanged. y_1^h remains unchanged for $h \neq 0$. In order to obtain e^{J+1} we set $dq = d\omega^1 = \ldots = d\omega^{h-1} = d\omega^{h+1} = \ldots = d\omega^H = 0$ and $d\omega^h = -d\omega^0 = \left(0, -\frac{n_1(1)}{p(1)}, \ldots, -\frac{n_1(S)}{p(S)}\right)$. This perturbation allocates an additional unit of nominal security 1 to agent h, while agent 0 ends up with one unit less of the same security, leaving the first J equations unchanged.

Given that Q and Ω_n are open, h is smooth in $Q \times \Omega_n$ and 0 is a regular value of $h(\cdot, p, \lambda) : Q \times \Omega_n \longrightarrow \Re^{J+1}$, the transversality theorem implies that there exists a set of full measure Ω_n^* in Ω_n such that 0 is a regular value of $h(\cdot, \omega, p, \lambda)$ for all $\omega \in \Omega_n^*$.

In order to prove that the set Ω^* is open we must show that the set $h^{-1}(0)$ is included in a compact set. Observe that $h^{-1}(0) \subset y^{-1}(0)$, so it suffices to show that there exist compact sets K_ω, K_q such that $y^{-1}(0) \subset K_\omega \times K_q$. The existence of K_ω is assured since we can let K_ω be the closure of Ω_n. We now show that $K_q = \{q \mid y(q, \omega) = 0\}$ is (i) bounded and (ii) closed. (i) K_q is bounded because w.l.o.g. we can restrict q to belong to the normalized set $Q \cap S^J$, where $S^J = \{q \mid \sum_j q_j = 1\}$. (ii) K_q is closed, for all sequences $q^v \in K_q$, $q^v \longrightarrow \overline{q} \Rightarrow \overline{q} \in K_q$. We show that there is no sequence q^v converging to the boundary of Q ($\overline{q} \notin \partial Q$). Notice that $q^v \in y^{-1}(0)$ implies that $y(q^v, \omega)$ is well defined. If $q^v \longrightarrow \overline{q}$ then by continuity $y(\overline{q}, \omega)$ should be well defined as well. But if $\overline{q} \in \partial Q$, $y(\overline{q}, \omega)$ is not well defined. Therefore, there is no sequence $q^v \longrightarrow \overline{q}$ with $q^v \in y^{-1}(0)$ and $\overline{q} \in \partial Q$. K_q can be the closure of Q.

We conclude that for each n there exists an open set of full measure Ω_n^* in Ω_n such that 0 is a regular value of $h(\cdot, \omega, p)$ for all $\omega \in \Omega_n^*$. It follows from the pre-image theorem that the set $h_{\omega,p}^{-1}(0) = \{q \mid h(q, \omega, p) = 0\}$ is empty for all $\omega \in \Omega_n^*$. Now, observe that since $\Omega = \bigcup_{n=1}^{\infty} \Omega_n$ and the countable union of sets of measure zero in $\Re^{(H+1)(S+1)}$ has measure zero, $\Omega^* = \bigcup_{n=1}^{\infty} \Omega_n^*$ is an open and dense set of full measure in Ω. Repeating the argument for every h we obtain the result. □

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Figure 1: Representation of Uncertainty

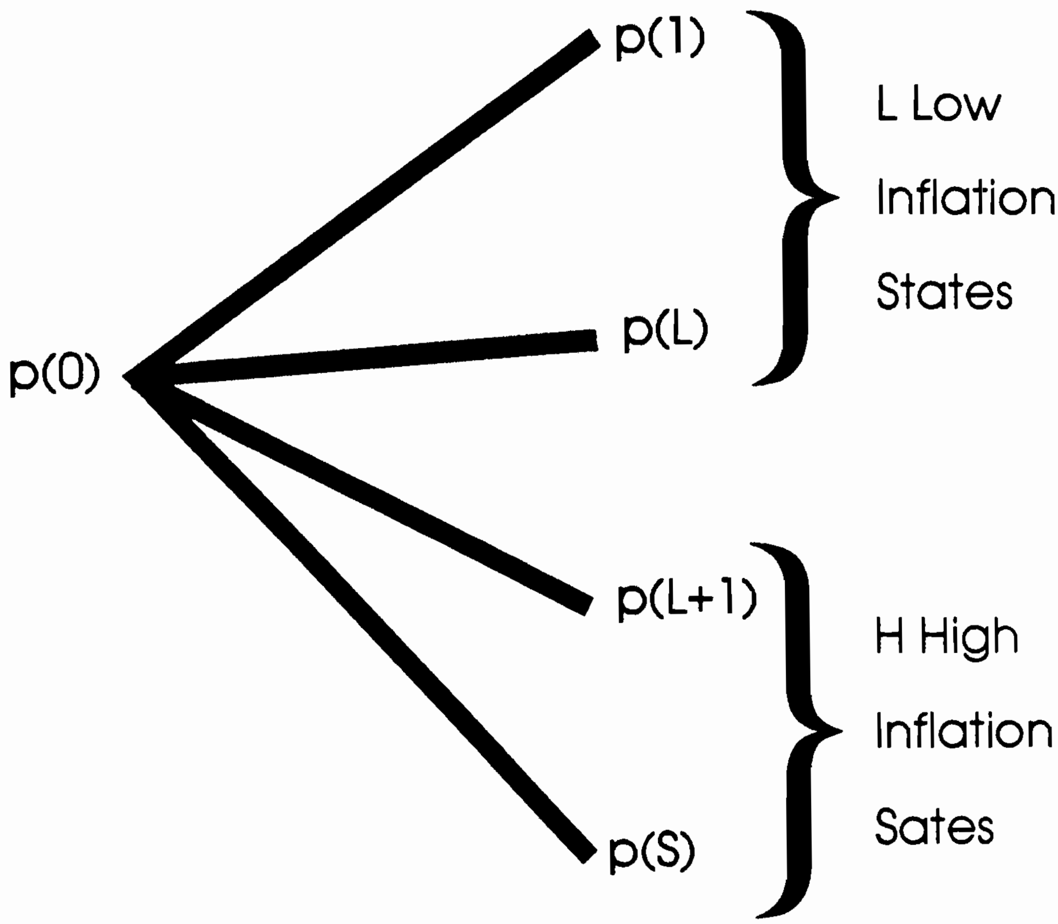

Figure 2: Efficiency and the Variability of the Rate of Inflation.